USE OF THESES

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STATISTICAL DISTRIBUTIONS
AND THEIR APPLICATION

by
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PREFACE

I

This thesis was written during my two-year term, between July 1954 and July 1956, as a Research Scholar of the Australian National University. This thesis is divided into two parts, the first consisting of chapters 1, 2, 3 on curve-fitting and the evaluation of certain integrals, and the second of chapters 4 and 5 on the testing of various unconnected hypotheses. The work in the first chapter is published in almost the same form in the Australian Journal of Physics (Vol. 8, Number 2, 1955); that in the second chapter is accepted for publication in the March (1956) issue of this journal. The work in chapter 3 is to be published in the Proceedings of the Cambridge Philosophical Society for 1956, while chapters 4 and 5 are also being submitted for publication.

II

The problems discussed in this thesis were suggested by Professor P.A.P. Moran, but the work was done by me partly under his supervision and partly under Dr G.S. Watson's; I am thankful to both Professor Moran and Dr Watson...
for their guidance. I would also like to take this opportunity of thanking the Australian National University for their financial support while carrying out this research.

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S.C. Das
SUMMARY

SOME PROBLEMS IN PROBABILITY DISTRIBUTIONS

Part I
SOME PROBLEMS OF CURVE FITTING AND
NUMERICAL INTEGRATION

Chapters 1 and 2 are concerned with a problem of curve fitting which arose in testing the hypothesis proposed by Bowen (1953) concerning daily rainfall data.

Chapter 1. The method of maximum likelihood has been used to fit a truncated type III (Gamma) distribution to daily rainfall data for Sydney over the period 1859-1952. An approximate test of the hypothesis that there is a singularity at the origin is suggested. This test is based on a comparison of the expected frequency in the truncated part, when the observed frequency in this part is taken into account in the fit, with the expected frequency when these observations are neglected. For Sydney data the test shows that there is no evidence in the rainfall data for a singularity at the origin.

Chapter 2. Meteorologists usually consider a log-normal curve to be appropriate for graduating rainfall data. Accordingly in this chapter we discuss the fitting of a log-
normal distribution to the above daily rainfall data for Sydney. The method used in fitting is also that of maximum likelihood. As judged by the \( \chi^2 \) test, the fit does not compare well with that previously obtained by the use of a type III distribution.

Chapter 3 discusses the numerical evaluation of a certain class of integrals, which is connected with some of the work done in the first two chapters. In calculating the expected frequencies as given in the column headed \( f_E \) of the table 2 in the second chapter, use was made of the univariate normal probability integrals. These are well known, but the similar integrals in the multivariate cases are difficult to evaluate; we devise an elementary method of evaluating normal probability integrals for the bivariate, and trivariate cases. A short discussion of the general multivariate case is also given, and this method is then applied to the univariate case as an alternative to the known methods.
Part II

PROBLEMS IN TESTING OF HYPOTHESES

In this part of the thesis, various unconnected tests of hypotheses, for points on a lattice, and for pressure data are examined and analysed in some detail. In chapter 4, it is shown how Pitman's criterion (Noether 1955) can be used to discriminate between different tests of randomness of points on a lattice.

Finally in chapter 5, we give a statistical analysis of the pressure data along the east coast of Australia, to test a hypothesis which arose out of Deacon's (1953) work, that there was a shift in the mean high pressure belt. It is shown that the belt is slowly moving southward.