A STUDY OF

PHOTOPROTON REACTIONS IN HEAVY NUCLEI

by

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PREFACE

This thesis is concerned with experimental studies of the emission of photoprotons from heavy nuclei, performed with the bremsstrahlung radiation from the Canberra 33 MeV electron synchrotron.

Some of the investigations described here have been performed in collaboration with other workers. The measurements of the photoproton cross sections for Hg$^{201}$ and W$^{186}$ and the photoneutron cross section for W$^{186}$ were made in collaboration with Dr. J.H. Carver. The measurement of the photoproton cross section from tantalum was performed jointly with Dr. J.H. Carver and Dr. W. Turchinetz. Dr. Turchinetz showed that the Hf$^{180m}$ and the Ta$^{178}$ activities could be separated, and Dr. J.H. Carver and I made the measurements of the yield curve and analysed the data. The remaining experiments were carried out independently.

I am greatly indebted to Dr. J.H. Carver, who has supervised most of the investigations, and has given me much assistance and helpful advice in the course of this work.

I wish also to acknowledge the help of Dr. D.C. Peaslee who supervised the calculations presented in chapter 5 and Appendices III and IV.
I am also indebted to Mr. A. Bull and Mr. J. Gower for their assistance in running and maintaining the electron synchrotron.

I wish to thank the Australian National University for the award of a scholarship, during the tenure of which these studies were carried out.

Some of the work described in this Thesis has been reported in the following publications:

i) Photoprotons from Cs and I
   Nuclear Phys. 19, 453 (1960)

ii) Photoprotons from Tantalum (with Dr. J.H. Carver and Dr. W. Turchinetz)
    Australian Journal of Physics 13, 617 (1960)

iii) Three Quantum Transitions in Heavy Nuclei (with Dr. J.H. Carver and Dr. D.C. Peaslee)
    (to be submitted)

No part of this Thesis has been submitted for a degree at any other University.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td></td>
<td>(i)</td>
</tr>
<tr>
<td>Chapter 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>The Giant Resonance Region</td>
<td>2</td>
</tr>
<tr>
<td>1.2.1</td>
<td>The Collective Model</td>
<td>5</td>
</tr>
<tr>
<td>1.2.2</td>
<td>The Independent Particle Models</td>
<td>6</td>
</tr>
<tr>
<td>1.2.3</td>
<td>Relation between the Collective Models and the I.P.M.</td>
<td>8</td>
</tr>
<tr>
<td>1.3</td>
<td>Above the Giant Resonance</td>
<td>9</td>
</tr>
<tr>
<td>1.4</td>
<td>The Inverse Reaction</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>Conclusions</td>
<td>12</td>
</tr>
<tr>
<td>Chapter 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>14</td>
</tr>
<tr>
<td>2.2</td>
<td>Experimental Details</td>
<td>17</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Method</td>
<td>17</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Counter</td>
<td>18</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Energy Calibration</td>
<td>19</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Bremsstrahlung Beam</td>
<td>19</td>
</tr>
<tr>
<td>2.2.5</td>
<td>Crystal Size</td>
<td>20</td>
</tr>
</tbody>
</table>
# Measures to reduce electron background

2.2.6 Measures to reduce electron background 21

# Spurious Pulses

2.2.7 Spurious Pulses 22

# Correction for the escape of protons from the crystal

2.3 Correction for the escape of protons from the crystal 25

## Introduction

2.3.1 Introduction 25

## Calculations

2.3.2 Calculations 26

## Angular Distribution Effects on the Loss Correction

2.3.3 Angular Distribution Effects on the Loss Correction 28

## Other Sources of Error in Correcting for Escaping Protons

2.3.4 Other Sources of Error in Correcting for Escaping Protons 29

# Beam Monitoring and Absolute Yield Measurements

2.4 Beam Monitoring and Absolute Yield Measurements 31

# Results and Analysis

2.5 Results and Analysis 32

## Photoproton Spectra

2.5.1 Photoproton Spectra 32

## (γ,p) Cross Section

2.5.2 (γ,p) Cross Section 35

## Conclusions

2.5.3 Conclusions 36

# Chapter 3

The Measurement of the (γ,p) Cross Sections for Ta and Hg

## Introduction

3.1 Introduction 38

## Scintillation Spectrometer

3.2 Scintillation Spectrometer 40

## Ta(γ,p)

3.3 Ta(γ,p) 40

### Method

3.3.1 Method 40

### Absolute Yield

3.3.2 Absolute Yield 43

### Results

3.3.3 Results 43

### Discussion

3.3.4 Discussion 44
Chapter 4
The Measurement of Photonuclear Reactions in Tungsten

4.1 Introduction

4.2 Decay Schemes
4.2.1 W^{185m}
4.2.2 W^{185}
4.2.3 Ta^{185}
4.2.4 Ta^{183}
4.2.5 Ta^{182m}
4.2.6 W^{179m}
4.2.7 W^{187}
4.2.8 Conclusions

4.3 W^{186}(Y,n)W^{185}
4.3.1 Method
4.3.2 Absolute Yield Measurements
4.3.3 Results

4.4 W^{186}(Y,p)Ta^{185}
4.4.1 Method
4.4.2 Results
4.5 $^{184}(Y,p)^{183}$ Ta

4.5.1 Method

4.5.2 Results

4.6 Discussion

Chapter 5

Three Quantum Transitions

5.1 Introduction

5.2 The Independent Particle Model

5.3 $E_1$ Peak Energies

5.4 The Energy of the Overtones

5.5 Overlap Integrals

5.5.1 The Square Well Potential

5.5.2 The Modified Harmonic Oscillator

5.5.3 Conclusions

5.6 A comparison of the Experimental Results with the Theoretical Results

5.6.1 The Peak Energies

5.6.2 The Harmonic Cross Sections

5.6.3 Other Experimental Evidence Concerning Three Quantum Transitions

5.7 Conclusions

Appendix I
Appendix II
Appendix III
Appendix IV
References
1.1 Introduction

In 1934 Chadwick and Goldhaber (Ch35) disintegrated the deuteron using the 2.6 MeV gamma rays from ThC". This was followed by the disintegration of Be by Szilard and Chalmers (Sz34). The early work was limited to using gamma rays from naturally occurring radioactive sources and it was not until high energy electron accelerators were developed that it became possible to investigate the systematics of nuclear photodisintegrations. The problem of accurately determining photonuclear cross sections is complicated by the use of bremsstrahlung radiation which makes the data much more difficult to interpret than the data obtained using monochromatic gamma rays. However, recent improvements in the energy stability of electron accelerators, now reported as low as 5 keV (Go54), together with better knowledge of the bremsstrahlung spectrum, have made it possible to measure photonuclear cross sections with greater accuracy. Also the problem of producing a monochromatic variable energy γ-ray is being investigated (Mi59) and the results are encouraging. A recent measurement (Se60) produced cross sections which were very similar to other results obtained by using the conventional bremsstrahlung radiation. With the advent of suitable monochromatic gamma rays it will be possible to make a detailed investigation of
the region above the giant resonance, where at present any resonance structure is masked by the yield from the giant resonance.

The theoretical approach to nuclear photodisintegration has been analogous to that of the atomic photoeffect and it has been possible to explain the main features (of the nuclear photoeffect) by using non-relativistic wave functions. Although it has been realised that meson fields must play a part in the explanations, Siegert (Si37) has shown that they can be ignored in the giant resonance. At high energies it is no longer possible to use non-relativistic wave functions and here a different approach must be made to the problem. So far the theoretical predictions have been limited to the gross features, such as the position of the giant resonance, its width, magnitude and, in the case of deformed nuclei, the splitting of the giant resonance.

The remainder of this chapter gives a brief outline of the present state of photodisintegration theory and how it fits the experimental facts. The discussion deals mainly with the medium and heavy nuclei and with photon energies below the meson threshold.

1.2 The Giant Resonance Region

The main feature of the absorption of gamma rays by nuclei is the existence of a giant resonance, the position of which is about 20 MeV for light nuclei decreasing to 14 MeV for the heavy nuclei. Calculations by Bethe and Levinger (Le50) showed that the integrated
cross section for the giant resonance nearly exhausts the electric dipole sum rule. Their calculation gave the well known result:

$$\int_{\text{dip}} \sigma (\text{abs}) = \frac{\pi e^2 A}{M c} N Z (1 + 0.8x)$$

where $x$ is the fraction of exchange force. It was the good agreement between the experimental integrated cross sections and the sum rule, that gave rise to the generally accepted hypothesis that the giant resonance is predominantly dipole in character.

Although it has been shown that electric dipole absorption accounts for most of the absorption cross section the possibility of other modes of absorption cannot be ignored. The most important of these are the magnetic dipole and electric quadrupole. Similar sum rules to the E1 sum rule given above have been calculated for these other types of transitions (Le50, Kh57). The calculations which show that both the E2 and M1 transitions are small compared with the E1 transitions are, however, model dependent and the accuracy is therefore less than the E1 sum rule which is model independent if exchange forces are ignored. Experimentally the E2 transitions can be detected by measuring angular distributions of photonucleons. A mixture of E1 and E2 transitions leads to an interference term in the angular distribution and instead of measuring an $(a + b \sin^2 \theta)$ distribution from a pure E1 transition the angular distribution will be $(a + b \sin^2 \theta (1 + p \cos \theta))$, where $p$ is a measure of the relative strengths of the E1 and E2 transitions. The results of Barber et al. (Ba60) show an asymmetry in the emission of high energy photoprotons.
indicating the presence of E2 transitions. One does not expect to find E2 transitions contributing to photoneutrons because the effective charge is small $\left(\frac{2}{A^2}\right)$. The detection of M1 transitions is more difficult because the angular distribution of the emitted nucleons is similar to that from an E1 absorption.

It has been found that the width of the giant resonance, which is normally 6 or 7 MeV, is narrower for nuclei with magic numbers of protons or neutrons. Okamoto (Ok58) and Danos (Da58) showed that the width was associated with the amount of deformation in the nucleus, the wider resonances occurring in deformed nuclei and the narrow in the spherical nuclei.

Most of the knowledge of the absorption cross sections has been obtained from the study of the partial cross sections (e.g. $(\gamma,n)$, $(\gamma,p)$, $(\gamma,2n)$, $(\gamma,\gamma')$), which make up the absorption cross section. These cross sections are usually measured by detecting the emitted nucleons or by measuring a residual activity. It is possible to make direct measurements of the total absorption cross section (Ko55, Du59, Mi59); but these are more difficult than measurements of the partial cross sections because the nuclear cross section is only about 1% of the atomic absorption cross section.

Various models have been proposed to explain the giant resonance phenomena and these can be divided into two main types; the collective models and the shell or independent particle models (I.P.M.).
1.2.1 The Collective Model

The first model proposed to explain the giant resonance was suggested by Goldhaber and Teller (Go48), who postulated an ordered dipole vibration of the nucleus. It was assumed that the photons excited a motion in which the bulk of the protons move relative to the neutrons. Goldhaber and Teller showed that if the neutrons and protons were considered as two incompressable but interpenetrating fluids, the frequency (i.e. energy) of the vibration was proportional to $A^{-\frac{1}{3}}$. This was in reasonable agreement with the experimental results. This simple classical model could not however predict the width of the giant resonance and the observed widths were attributed to the transfer of energy from the simple dipole oscillation to other collective motions.

An important feature of this model was that it predicted that the giant resonance for deformed nuclei should actually be two resonances. The energies of the two resonances should be inversely proportional to the major and minor axes. Although these two resonances are smeared over several MeV it should be possible to separate them for very deformed nuclei. This prediction has now been substantiated experimentally and the giant resonance has been split for several nuclei (Fu58, Pa59).

There are other types of collective models which have been examined (St50, Da52, Je48) and whose basic assumptions are the same as the model just considered, although some details differ slightly.
These models, which successfully account for the absorption of photons, are not sufficiently detailed to predict nucleon emission. It is generally assumed that the ordered vibration induced by the photon absorption is broken up by inter-nucleon collisions, leading to the formation and subsequent decay of a compound nucleus. However Hirzel and Waffler (Hi47) showed that the emission of photoprotons from heavy nuclei was not consistent with the formation of a compound nucleus. Unfortunately there does not appear to be any mechanism in the collective models to allow for a "direct" emission of nucleons.*

1.2.2 The Independent Particle Models

Because of the failure of the collective models to explain the emission of photoprotons in heavy nuclei, Courant (Co51) and Wilkinson (Wi56) suggested models which led to the direct emission of nucleons, without passing through a compound nucleus stage. In these models the photon is absorbed by raising a nucleon from its ground state to an excited state by means of an E1 transition. The excited nucleon may then be emitted directly or absorbed by the nucleus to form a compound nucleus. Although the probability of a proton being emitted directly may only be several percent, it is sufficient to account for the measured proton yields.

Wilkinson has gone further than Courant and has used the I.P.M. to give a complete description of the nuclear photoeffect. The

* But see 1.2.3 where it is shown that there may be a link between the collective model and the I.P.M.
Wilkinson model pictures the giant resonance as the clustering of a few enhanced transitions, rather than a greater density of states at the peak. He shows that the sum of all the E1 transitions effectively exhausts the dipole sum rule, which was shown to be a requirement of any model (see 1.2). Wilkinson (Wi58) has also investigated the effect of deformed nuclei and has shown that his model predicts large widths of the giant resonance for deformed nuclei, with eventual splitting of the giant resonance for very deformed nuclei.

Although the Wilkinson model can satisfactorily explain most of the observed features of the giant resonance as well as enhancing proton emission, there is one feature of the model which is unsatisfactory. This objection is the use of an effective mass \( m^* / m = 1/2 \) to explain the position of the giant resonance. Calculations based on the correct nucleon mass would put the position of the giant resonance at approximately half the experimentally measured energy. Although there are theoretical reasons (We57) to support the use of an effective mass equal to a half, it is not clear that these are valid*. Also recent \( (d,p) \) experiments (Co60) would indicate that the single particle transitions, which are responsible for the giant resonance, have energies consistent with the use of the true nucleon mass.

* The theoretical calculations used to obtain \( m^* / m = 1/2 \) are based on an infinite mass and it is assumed that the calculations are valid for finite mass. However it is not obvious that surface effects do not play an important part (La58).
Attempts to overcome this problem of the level spacings for E1 transitions and the position of the giant resonance have recently been made by Brown and Bolsterli (Br59) and Carver and Peaslee (Ca60). They have considered the possibility of particle-hole interactions and Carver and Peaslee show that the position of the giant resonance can be expressed in the form:

$$E_m \simeq (40A^{-\frac{1}{3}}+7.5) \text{ MeV}$$

The first term equals the single-particle spacing for ordinary forces and an effective mass $$\frac{m^*}{m} \simeq 1$$. The second term, which is peculiar to the E1 mode of excitation, is the approximately constant (A-independent) shift produced by exchange forces. This model is discussed further in chapter 5, where the implications of the model are considered.

1.2.3 Relation between the Collective Models and the I.P.M.

The fact that both the I.P.M. and the collective models successfully predict the main features of the giant resonance, suggests that there should be some link between them. Brink (Br57) has in fact found that in the special case of a simple harmonic oscillator potential, the two types of models are identical*. He showed that the raising of a nucleon by an E1 transition was identical with displacing the centre of mass of all the protons relative to the centre of mass of the neutrons.

* To compare the Carver-Peaslee calculations it is necessary to remove the exchange potential term, because the collective models are classical models without exchange potential effects.
This exact equivalence disappears if the harmonic oscillator potential is altered; but it is not expected that the use of a more realistic potential would completely destroy the similarities. It should be possible to predict the direct emission of nucleons from the collective model but it is not clear how this could or should be done. The fact that the I.P.M. have been more successful than the collective models stems from the fact that they are much more detailed models and therefore should be able to predict the fine details as well as the gross structure.

1.3 Above the Giant Resonance

In the region just above the giant resonance it is still possible to explain the large measured cross sections in terms of single particle transitions, similar to those which contributed to the giant resonance. However for energies well above the giant resonance it has been found (Le51) that the absorption was more satisfactorily explained in terms of a quasi-deuteron model. This model predicts that the absorption of the photon will be followed by the emission of a proton and a neutron in opposite directions and this has been borne out experimentally (Od56). The model does not exclude the absorption of one or both of these nucleons in the nucleus leading to a compound nucleus formation.

In recent years there have been some detailed investigations (Fe57, Au59) made of the region 20-30 MeV, using fast neutron detectors to search for higher resonances. The results of these
investigations are conflicting, which is not surprising, as it is difficult to make accurate cross section measurements above the giant resonance using bremsstrahlung radiation. Ferrero et al. found evidence of a second resonance at about 24 MeV in four out of twenty-one nuclei, while Aull et al. did not find any higher resonances even in those nuclei in which Ferrero found them. Even if there were a second resonance in the region of 24 MeV, as suggested by the Carver-Peaslee calculation (see 5.4), it is doubtful whether it would be resolvable except perhaps in a spherical nucleus. The complete solution of the problems associated with the absorption of gamma rays above the giant resonance is likely to have to wait until measurements are made using monochromatic gamma rays. However, it will be shown here how certain aspects of the absorption process in this region can be studied by measuring photo-protons from middle and heavy weight nuclei.

1.4 The Inverse Reaction

A large amount of information about photodisintegration processes can be obtained by studying the inverse reaction \((p, \gamma)\). To do this, use is made of the reciprocity theory, which states that for any reaction

\[
a + A \leftrightarrow b + B + Q
\]

\[
(2I_a + 1)(2I_A + 1)p_a^2 \sigma(A \leftrightarrow B) = (2I_b + 1)(2I_B + 1)p_b^2 \sigma(B \leftrightarrow A)
\]

where \(I\) is the angular momentum and \(p_a\) and \(p_b\) are the momenta of the particles (or \(\gamma\)-rays) \(a\) and \(b\). The use of this relation is
limited to studying reactions where A and B are in the ground state which, in the case of \((\gamma, p)\) reactions, form only a small fraction of the reaction cross section. However, the inverse reaction \((p, \gamma)\) can still provide details of the absorption cross section which cannot be obtained from the \((\gamma, p)\) reaction.

One of the main uses of the inverse process \((p, \gamma)\) is to investigate the fine structure in the giant resonance. This can be performed more satisfactorily than by using the \((\gamma, p), (\gamma, n)\) reaction because of the higher resolution obtainable on the ingoing proton. It also provides information on the radiation widths \(\Gamma_{\gamma}\) which cannot be accurately determined from photodisintegration measurements but are generally assumed to be small compared with particle widths \(\Gamma_n, \Gamma_p\).

The comparison of the \((p, \gamma)\) and \((\gamma, p)\) reactions in light nuclei, where it is possible to see some structure in the \((\gamma, p)\) results, shows very good agreement both for the energy of the resonances and the magnitude of the cross section \((\text{Ge}59, \text{Co}59, \text{Gr}59)\). The measurements of the radiation widths for the resonances in the giant resonance region are about 10 kev which are in qualitative agreement with the shell model predictions of Elliott and Flowers (El57).

It can be seen from these preliminary \((p, \gamma)\) measurements that in certain light nuclei, fine structure should be observable in the corresponding \((\gamma, p)\) reaction and that the possibility of using the \((p, \gamma)\) measurements to supplement the \((\gamma, p)\) measurements should not be ignored. So far the excitation of nuclei in the giant resonance region by \((p, \gamma)\) reactions has been limited to the light nuclei.
1.5 Conclusions

It would now appear possible to give a complete explanation of the photonuclear effect for photon energies up to 40 MeV by using an I.P.M. At present this model is not satisfactory for energies higher than 40 MeV and it is necessary to invoke the quasi-deuteron model.* This I.P.M. need not be identical with the Wilkinson model. In fact it has already been shown where the latter conflicts with the experimental results (see 1.2.2). However, in view of the success of the Wilkinson model, a model based essentially on the same lines, with slight modifications to overcome the present objections, would appear to be the most suitable. The Carver-Peaslee calculation (Ca60) was an attempt to overcome some of these objections and was successful in explaining the position of the giant resonance without invoking an effective mass less than 1.

The object of the work described in this thesis was to test the I.P.M.s as described by Wilkinson and Carver et al. Since Carver and Peaslee in their paper (Ca60) only discussed the position of the giant resonance it was necessary to investigate the implication of their model to permit other conclusions to be drawn from it. The extension of the model to predict photoproton emission is discussed

* The quasi-deuteron model is an I.P.M. but is not a natural extension of the I.P.M.s used to explain the giant resonance.
in Chapter 5, which also shows the agreement between it and the experimental results. Various (γ,p) cross sections were measured using the Canberra 33 MeV electron synchrotron (Chapters 2, 3 and 4) and are discussed in the light of both models. Also measurements were made of photoproton spectra from Cs and I (Chapter 2) with the view of testing the various proposed direct photoproton emission spectra.
CHAPTER 2

THE MEASUREMENT OF PHOTOPROTONS FROM CESIUM AND IODINE
USING A THIN CsI SCINTILLATING CRYSTAL

2.1 Introduction

In studies of the yields of photoprotons from heavy nuclei, it has been established that the number of protons that are emitted are substantially more than would be expected from the evaporation of a proton from a compound nucleus \( \text{Hi}47 \). It has also been shown that the energy distribution of these protons is inconsistent with a compound nucleus theory.

It is now generally accepted that the majority of the protons from heavy and middle weight nuclei and also the high energy protons from the light nuclei are due to a direct interaction. The few efforts that have been made to calculate the shape of the photoproton spectra resulting from a direct reaction are based on the resonance-direct model proposed by Courant \( \text{Co51} \). Satisfactory calculations using the independent particle models cannot be made until there are more reliable results regarding energy levels in the nucleus. At present there is uncertainty about the position of the protons relative to the top of the nuclear potential well. Also it has now been proposed that the entire photon energy is not used in raising the nucleon from its ground state \( \text{Ca60} \), the residual energy being distributed amongst the remaining nucleons. This leads to an uncertainty in the energy of the emitted proton. However,
several attempts have been made to calculate the shape of the photoproton spectrum (Da56, To53) using simplifying assumptions and the results give reasonable agreement with experimental results on heavy nuclei, but are in error for the region \( Z = 50-60 \) (see 2.5.1).

The present experiment was undertaken because there were no satisfactory results on the \((\gamma,p)\) reaction in the region \( Z = 50-60 \) of the periodic table. Calculations by Wilkinson (Wi56) indicate that the yield of photoprotons from these nuclei would be mainly due to direct processes, the compound nucleus only accounting for a maximum of several percent of the yield. The measurements should determine whether the direct proton cross sections exhibit a resonance similar to that observed in the giant resonance. The independent particle models (Ca60, Wi56) predict that it would, although modified by the coulomb barrier, and this is supported by the measurement of photopros from Ag (Lo59).

Another purpose of the experiment was to find out if the shape of the proton spectra changed markedly with changes in the maximum energy of the bremsstrahlung*. Any changes would be attributed to changes in the relative importance of the \( E1 \) transitions contributing to the \((\gamma,p)\) cross section.

* Hereafter referred to as the bremsstrahlung energy.
The purpose of choosing Cs and I was twofold. The photo-protons were expected to have a low energy cut off at about 4 MeV and therefore there was a possibility of keeping the electron background clear of the proton spectrum. Also it was necessary to choose a scintillating crystal in which the constituents gave similar proton yields. Cs and I are extremely well suited in this aspect. They only differ in Z value by 2 (see table 2.1), they are both odd-even nuclei, have the same binding energies for protons and for neutrons and have the same filled proton and neutron shells.

Table 2.1

<table>
<thead>
<tr>
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<th>I</th>
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<tr>
<td>Z</td>
<td>55</td>
<td>53</td>
</tr>
<tr>
<td>A</td>
<td>133</td>
<td>127</td>
</tr>
<tr>
<td>Coulomb barrier (MeV)</td>
<td>10.7</td>
<td>10.6</td>
</tr>
<tr>
<td>($\gamma$,p) threshold (MeV)</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>($\gamma$,n) threshold (MeV)</td>
<td>9.0</td>
<td>9.3</td>
</tr>
<tr>
<td>($\gamma$,np) threshold (MeV)</td>
<td>15.4</td>
<td>15.3</td>
</tr>
<tr>
<td>($\gamma$,\alpha) threshold (MeV)</td>
<td>2.3</td>
<td>2.0</td>
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Threshold energies obtained from Wapstra's Table of Atomic Masses (Wa58)
Since the systematics of photoproton emission (We54, Ha51) do not predict large variations between neighbouring nuclei, it is assumed that the photoproton emission is identical for both Cs and I. This does not matter greatly as it is only the general trends with changing Z that are of interest.

2.2 Experimental Details

2.2.1 Method

The measurement of the photoprotons was based on the principle of using a scintillating crystal as the target as well as the detector. This idea was first developed by Mann, Ophel and Wright (Ma57) and has since been used extensively for observing ($\gamma$,p) reactions in various scintillating crystals (Bo59) using 17.6 MeV $\gamma$-rays. The only difference between these experiments and the one described here was the use of a bremsstrahlung output of an electron synchrotron as a $\gamma$-ray source instead of the monochromatic $\gamma$-rays. In principle the method was very simple. A scintillating crystal, in this case CsI(Tl), was placed in the bremsstrahlung beam from the synchrotron. The $\gamma$-rays produced protons in the crystal from the reactions Cs($\gamma$,p) and I($\gamma$,p). These photoprotons were detected in the crystal, the light pulse was converted to an electrical pulse, which was suitably amplified and displayed on a kicksorter.

* The variations that do occur (e.g. Mo$^{92}$ and Mo$^{100}$ (Bu53)) are due to variations in the binding energies which for Cs and I are seen to be the same (see table 2.1).
These proton spectra were then corrected for protons escaping from the crystal.

2.2.2 Counter

The scintillation detector, which was also the target, consisted of a 0.75" x 0.75" x 0.20" CsI(Tl) crystal mounted on a ¼" thick perspex disk with a clear araldite casting resin. The perspex was clamped to an E.M.I. type 6097 photomultiplier and there was a layer of silicone oil between the surfaces to ensure a good optical contact. The crystal and photomultiplier were enclosed in a light tight aluminium can which had a thin aluminium window (3 mgm/cm²) for beam entry. The pulses from the photomultiplier were fed through a cathode follower into a Higginbotham non-overload amplifier and analysed by a Hutchison-Scarrot kicksorter.

The performance of the counter was assessed by measuring the alpha particle spectrum from a radio thorium source. The energy resolution of the two alpha groups (8.8 and 6.1 MeV) was 6% (see fig. 2.1). Some of this energy spread may be attributed to non-uniformity in the photocathode and some to the scattering of the alpha particles which travel through 2 mm. of air to reach the crystal. The resolution for the protons was about the same, and more than adequate for the experiment.
Figure 2.1

Alpha-particle spectrum from a radio-thorium source
2.2.3 Energy Calibration

The crystal was calibrated by measuring the proton peaks from the reaction Cs(γ,p) using 17.6 MeV gamma rays. The proton energies were 7.2, 9.3 and 11.4 MeV (Wr59) and were obtained by comparison with the proton groups from the reaction B^{10}(d,p)B^{11}. The Cs proton peaks were compared with the radiothorium alphas, and the two alpha groups were then used for calibration checks during the experiment. For the particular crystal that was used, it was found that the 8.8 MeV alpha had the same light output as a 6.1 MeV proton.

2.2.4 Bremsstrahlung Beam

The beam from the synchrotron was hardened by passing it through 50" of graphite before entering the counter (see fig. 2.2). The purpose of the graphite was twofold. Firstly it reduced the number of low energy photons relative to the high energy photons. Fifty inches of graphite reduces the number of 1 MeV photons by $10^4$, while the same amount of graphite reduces 30 MeV photons by only a factor of 8. This reduction in the relative number of low energy photons was beneficial, as it was these photons which, without producing any photoprotons, were the main contributors to the production of electrons. Graphite has the advantage over most other materials that its total absorption coefficient has a minimum at an energy of $\sim$50 MeV (Wa59).
Figure 2.2

Experimental lay-out
Secondly it was found that without the graphite there was more than adequate beam and the effect of the graphite was to reduce the total beam to a suitable level. An excessive beam caused the electronic counting equipment to saturate, resulting in the spectra being distorted and also increased the problem of electron pile up in the crystal (see 2.2.6).

2.2.5 Crystal Size

The choice of the crystal dimensions was governed by three factors: (i) the electron edge; (ii) the correction factor for protons escaping from the crystal without being stopped; and (iii) the problem of making a uniform crystal. Photoelectrons which are produced predominantly in the forward direction (Ev55) will escape from a thin crystal with very little loss of energy (resulting pulse small). Therefore the thinner the crystal, the fewer the large pulses from electrons. On the other hand, the thinner the crystal the more uncertain becomes the correction for "escaping" protons due to non-uniformity in the crystal thickness (which varied by less than .001" over the entire crystal surface), and due to the uncertainty in the range of protons in CsI (see 2.3.4). It was therefore decided to strike a balance and use a crystal 0.020" thick. The effect of having the other two dimensions large compared with the thickness was investigated and found to have no measurable effect on the number of electron pulses*. The advantage

*This is contrary to what Wright (Wr59) found using 17.6 MeV γ-rays. However, his γ-ray source was not collimated, thus allowing γ-rays
of having these dimensions large compared with the thickness was to make the number of protons which escaped out of the crystal sides negligible. Therefore $\frac{3}{4}$ was chosen for the other two dimensions.

2.2.6 Measures to reduce electron background

The main difficulty in this experiment, in common with other techniques used to detect photoprotons, was to discriminate against electrons which have photon interaction cross sections four orders of magnitude greater than the photoproton cross section. This problem is aggravated further when bremsstrahlung is the source of gamma rays because:

i) there is a large flux of gamma rays which are below the photoproton threshold and can produce electrons

ii) electron accelerators are pulsed machines and as a result the entire beam arrives in a 50$\mu$s period and the resulting low energy electrons produced in the crystal can "pile up" to produce large pulses.

To help overcome the first difficulty, the beam was hardened with 50" of graphite (see 2.2.4). Another major factor in decreasing the electron background was to use a very thin crystal, which reduced the number of large electron pulses to enter the crystal at an angle. This would result in the crystal edges playing a more important part in determining the electron background.
relative to the number of proton pulses (see 2.2.5). Also the pulses were reduced to 1µs long, which was the lower limit for acceptance by the kicksorter. This considerably reduced the amount of electron "pile up."

The problem of the synchrotron being a pulsed machine was overcome to a certain extent by turning the radio frequency off slowly. This increased the total duration of the beam from 50µs to 200µs but even so a large proportion of the beam arrived within 100µs.

To test whether multiple electron pulses were being observed, several runs at different intensities were made. Since the number of multiple events increases exponentially with intensity, while single events increase linearly, the observed spectra at different intensities would differ in shape. Even at the maximum beam used, which was 3R per min. at the machine side of the graphite, no difference could be observed and it was concluded that the number of multiple events was negligible.

As a result of using all the methods described above, the pulses due to electrons were less than 4 MeV (see 2.5.1).

2.2.7 Spurious Pulses

There were two main types of spurious pulses: the natural background pulses, which were always present, and those associated with the beam. The natural background was reduced by a factor
of 100 by gating the kicksorter, so that it accepted pulses only when there was a beam from the synchrotron. This background mainly consisted of alpha contamination of the counter from the radio thorium source which was used to check the gain of the counter. The count rate of these alphas was kept below 1 per minute (kicksorter ungated) by allowing them to decay (half life of 10 hours).

The spurious pulses associated with the beam were partly due to protons produced in the aluminium window and in the perspex backing of the crystal and partly to charged particles formed in the crystal by nuclear reactions other than \( \text{Cs}(\gamma,p) \) and \( \text{I}(\gamma,p) \). The yields of protons from the aluminium and the perspex were calculated from the measured cross sections and proton distributions (Ho53; Li57; Ha51) and found to be less than 1% and 3% respectively. The low yield from the perspex was due to the high percentage of \( ^{12}\text{C} \) which has a high \( \gamma(p) \) threshold of 16 MeV. This meant that most of the \( ^{12}\text{C} \) protons would have energies less than 7 MeV, and these would have to enter the crystal with an energy greater than 4 MeV to be included in the Cs and I proton yields.

The only other major sources of extraneous pulses were nuclear reactions in the crystal leading to the emission of a charged particle. The following reactions were considered the
most important: \((n,p)\), \((\gamma,\alpha)\), and \((\gamma,np+d)\) in Cs and I. The \((n,p)\) reaction was measured by irradiating the crystal with a standard Ra-Be neutron source and counting the number of pulses observed in the crystal. To normalise the neutron flux of the Ra-Be source to that in the \(\gamma\)-ray beam, a block of aluminium was irradiated in the \(\gamma\)-ray beam and also by the neutron source. The relative neutron strengths were obtained from measuring the 9.7 min. residual activity from \(\text{Mg}^{27}\) which was produced by the reaction \(\text{Al}^{27}(n,p)\text{Mg}^{27}\). The result of these measurements was that the number of protons greater than 4 MeV produced in the crystal due to an \((n,p)\) reaction was (0±1)\% of the total photoproton yield.

Some \((\gamma,\alpha)\) cross sections in the region for \(Z > 40\) have been measured by Erdos et al. (Er57) who found the integrated \((\gamma,\alpha)\) cross sections to 32 MeV to be approximately 0.3 MeV-mb and that the value was independent of \(A\). This reaction was therefore considered to be a negligible source of spurious pulses.

The number of photodeuterons, and photoprotons in coincidence with neutrons that were produced in the crystal was difficult to estimate as there was little experimental data available in this region of the atomic table. Using the results of Hofman and Stoll (Ho58) it was estimated that below 25 MeV the cross section would
be negligible but in the region 27–32 MeV 30% of the measured photoproton cross section may have been due to photodeuterons and to photoproton-neutron pairs.

2.3 Correction for the escape of protons from the crystal.

2.3.1 Introduction

One of the problems associated with using thin crystals as detectors of charged particles, is to calculate the effect of particles which escape from the crystal. Usually the crystal is made large enough so that either all the particles are stopped, or, the number escaping is negligible. When the detector is also the source of the particles, some must escape and it is often impossible to reduce the number of such particles to an insignificant amount. This is the case in the present experiment, where photoprotons are produced uniformly throughout a thin crystal, and more than 50% of these protons escape. A correction can be made provided the range-energy relation of the protons in the crystal and their angular distribution are known.

The range-energy relationship for protons in CsI can be calculated theoretically and has also been measured experimentally (Wo60). For the purpose of correcting the measured proton spectra the range-energy relationship was obtained by interpolating Aron's range-energy tables (Ar51). The possible error that would be introduced by inaccuracies of the tables is discussed in 2.3.4.
The problem of determining the angular distribution is difficult, as there have been no measurements made of the distribution for the photoprotons from Cs or I. This is discussed further in 2.3.3.

Calculations for the loss correction (see 2.3.2) have been made assuming firstly, an isotropic distribution and secondly, a $\sin^2 \theta$ distribution of the emitted photoprotons.

2.3.2 Calculations

The correction for escaping protons was calculated for two different proton angular distributions, isotropic and $\sin^2 \theta$. To simplify the calculations it was assumed that the crystal was infinite in two of its dimensions. For the crystal used in the experiment (0.75" x 0.75" x 0.020") the error introduced by this assumption was found by calculating the number of protons which escape from the edges of the crystal. This was done in a similar manner to that described below for determining the number escaping from the front and back surfaces. For 20 MeV protons and an isotropic distribution, 2% will escape out of the edges and the percentage will decrease with decreasing proton energy.

The equations determining the number $N(R)$ of particles of range $R$ that are stopped completely in the crystal are:

\[
N(R) = \begin{cases} 
N_0(R) \frac{(1-R/4t)}{t/R}; & R \leq 2t \\
N_0(R) \frac{(t/R)}{R \gt 2t} & \text{isotropic distribution} 
\end{cases}
\]  

(Gr 48)
\[ N(R) = N_0(R) \left( 1 - \frac{3R}{16t} \right); \quad R \leq 2t \]  
\[ = N_0(R) \left[ \frac{3t}{2R} - \left( \frac{t}{R} \right)^3 \right]; \quad R > 2t \]  
\[ \sin^2 \theta \]

where 2t is the thickness of the crystal and \( N_0(R) \) is the total number of protons. Therefore the number (N') of protons that have a maximum possible range R in the crystal and only travel a distance r in the crystal is given by the relation:

\[ N'(r,R) = -\frac{dN(r)}{dr} \]

Using this relation together with the previous equations we get:

\[ N'(r,R) = N_0(R) \left( \frac{1}{4t} \right) dr; \quad r < R, r < 2t \]  
\[ = N_0(R) \left( \frac{t}{r^2} \right) dr; \quad 2t < r < R \]  
\[ \text{isotropic distribution} \]

\[ N'(r,R) = N_0(R) \left( \frac{3}{16t} \right) dr; \quad r < R, r < 2t \]  
\[ = N_0(R) \left( \frac{3t}{2r^2} - \frac{3t^3}{r^4} \right) dr; \quad 2t < r < R \]  
\[ \sin^2 \theta \]

Equations (1), (2), (4) and (5) together with the range-energy relation for protons in the crystal, are sufficient to determine the true proton spectrum from an observed spectrum and vice versa, where the angular distribution of the protons can be written in the form:

\[ N(\theta,E) = A(E) + B(E) \sin^2 \theta \]

Figure 2.3 shows a proton spectrum before and after correcting for proton losses for the case B = 0. Spectrum U is the observed (uncorrected) spectrum and C is the corrected (true) spectrum.
Figure 2.3

The effect of protons escaping from a thin crystal.

Spectrum C is the true proton spectrum of a source of monoenergetic protons produced uniformly throughout the crystal. Spectrum U is the observed spectrum calculated theoretically. The pulses appearing below the peak energy are due to protons only losing a fraction of their energy in the crystal.
FIG. 2.3

NO. of PROTONS (arbitrary units)

PROTON ENERGY (MeV)

U

C

(x\frac{1}{4})
2.3.3 Angular Distribution Effects on the Loss Correction

Since there was no data available on the angular distribution of photoprotons from Cs or I, it was necessary to (a) assume an angular distribution based on data from nearby nuclei and (b) investigate the possible error that could be introduced by the assumed distribution.

The only angular distributions that have been measured in the vicinity of Z = 54 are indium (Ba60; To53) and cerium (To53). These results show that no one angular distribution applies to all proton energies and that the angular distribution is also a function of the bremsstrahlung energy. There are some points however which are common to all the measured angular distributions. They all show a considerable isotropic component and the non isotropic component has a maximum in the region of $90^\circ$ with respect to the beam, but it may be displaced either in a forward or backward direction.

The experimental results agree with theoretical predictions (Co51); the displacement of the maximum from $90^\circ$ being attributed to an interference term from electric quadrupole absorption (see 1.2).

In view of the difficulty in determining a reliable angular distribution it was decided to correct the measured spectra using a pure isotropic distribution. Although this was incorrect and
perhaps a distribution with a \( \sin^2 \theta \) component would be more accurate, it will be shown that the difference in the final result for an isotropic and a \( \sin^2 \theta \) distribution is small. To calculate this difference, a measured spectrum, obtained from a crystal 0.020" thick, was corrected using both distributions, and the two resultant spectra are shown in fig. 2.4. The measured spectrum was smoothed before being corrected. It can be seen that the two spectra show the greatest difference at high energies and there is a small shift in the position of the peak. The total number of protons with energies larger than 4 MeV differed by only 4%, the number being larger for the isotropic distribution.

These measurements show that any error introduced by using the wrong angular distribution was less than 4% and due to the similarity between spectra for different bremsstrahlung energies (fig. 2.6) this error will be independent of the bremsstrahlung energy.

2.3.4 Other Sources of Error in Correcting for Escaping Protons

There are two other main sources of error in making the escape corrections; the crystal thickness and the range-energy relation. The 0.020" crystal used for the experiment was found to have a variation of 0.001" in thickness. The effect of this variation was measured by correcting a spectrum assuming the
Figure 2.4

The effect of assuming different angular distributions of the photoprotons to correct for protons escaping from the crystal. The two spectra have been obtained from a measured spectrum by correcting for proton losses assuming an isotropic (A) and a $\sin^2\theta$ (B) distribution.
crystal thickness was 0.020" and correcting the same spectrum assuming a thickness of 0.021". The two corrected spectra were almost identical in shape and the total yields of protons differed by 5%. The actual error is far less than 5% because the mean crystal thickness obtained by weighing the crystal, was used to correct the measured spectra.

The error introduced by using an incorrect range-energy relationship, was estimated by correcting a spectrum using two range-energy relationships, one of which was taken from Aron's tables (Ar51), and the other assumed a proton range in CsI which was 10% greater than that given by Aron's tables. A comparison of these spectra showed a very small change in the proton spectrum shape and a 3% difference in yield.

From these measurements the following conclusions can be drawn: Errors in the shape of the photoproton spectra will be almost entirely due to errors in the assumed angular distribution of the protons and these are expected to be small. Errors in the yield will be due mainly to error in the angular distribution (4%), variation in the crystal thickness (2%) and uncertainty in the range-energy relation (3%). All the errors in the yield will be independent of the bremsstrahlung energy and as a result will only affect the total cross section and not the shape of the cross section.
2.4 Beam Monitoring and Absolute Yield Measurements

The total bremsstrahlung dose was monitored by the activity induced in Ta foils placed in front of the graphite. The induced activity was measured by detecting the 55 keV k-capture γ-ray from the 8.15 hr. Ta$^{180m}$ using a scintillation spectrometer. Each foil was left for at least one hour after irradiation to allow the 10 min. activity produced by the Ta$^{181}(γ,3n)Ta^{178}$ reaction to decay away (ca58).

Because of the large separation of the monitoring foils and the CsI crystal, necessitated by the small beam intensity at the crystal, it was necessary to correct the proton yields for the shift in the γ-ray beam resulting from changes in the bremsstrahlung energy. One Ta disk was placed in the position of the monitor foil and another against the CsI crystal. The graphite was removed and the ratio of the induced activity in the two foils was measured as a function of the bremsstrahlung energy. The changes in the ratio were a direct measure of the beam shift.

The absolute yield of protons was determined relative to the known Cu$^{63}(γ,n)$ cross section. The CsI crystal was replaced by a Cu foil of the same dimensions and irradiated for ten minutes at 30 MeV, with the same monitoring procedure. By counting the annihilation quanta from the resulting 9.7 min. positron activity in Cu$^{62}$ in a NaI scintillation spectrometer,
it was possible to calculate the neutron yield per mole of Cu per unit Ta monitor count. From this the proton yield per mole of CsI was calculated relative to the Cu neutron yield. The absolute yield of photoprotons was calculated from Berman and Brown's Cu$^{63}(\gamma,n)$ results (Be54). A correction was made for the effect of the graphite on the bremsstrahlung shape. To obtain the absolute yield of Cu$^{62}$, it was necessary to know the efficiency of the NaI scintillation spectrometer for measuring the 511 kev annihilation quanta. This was measured using a simple coincidence method (Be58).

2.5 Results and Analysis

2.5.1 Photoproton Spectra

Photoproton spectra were obtained at 1 MeV intervals of the bremsstrahlung energy from 14 to 32 MeV. Fig. 2.5 shows some of the proton spectra before the corrections were made for protons escaping from the crystal. Each spectrum was the sum of at least 10 one hour runs and they all show evidence of a "valley" in the region of 4 MeV. The sharply rising low energy part of each spectrum was due to electrons and partially stopped protons. To see the extent of this electron edge, several runs were made at low bremsstrahlung energies (12-15 MeV), where the proton emission was negligible (see fig. 2.5). By extrapolating the results of these low energy runs, it was possible to make a good estimate
Figure 2.5

Photoproton spectra from CsI obtained at different bremsstrahlung energies. The spectra have not been corrected for the effect of protons escaping from the crystal. The ordinate is the number of protons per $\frac{1}{4}$ MeV interval. $E_m$ is the maximum bremsstrahlung energy.
$E = 32 \text{ MEV.}$

$E = 29 \text{ MEV.}$

$E = 25 \text{ MEV.}$

$E = 22 \text{ MEV.}$

$E = 20 \text{ MEV.}$

$E = 15 \text{ MEV.}$

**FIG. 25**
of the shape of the electron edge at higher bremsstrahlung energies. It was estimated that the number of pulses greater than 4 MeV, which were caused by electrons, was less than 2% for a bremsstrahlung energy of 30 MeV and less than 4% at 17 MeV.

The proton spectra, corrected on the assumption of an isotropic angular distribution, are shown in fig. 2.6. These show that very few photoprotons with energies less than 4 MeV, are emitted from Cs and I nuclei. The peak position of the proton spectra is at $7 - 7\frac{1}{2}$ MeV and this increases slightly with increasing bremsstrahlung energy.

To calculate the yield of photoprotons following the formation of a compound nucleus the following equations were used:-(B152).

$$
\sigma_{\gamma,p}(E) = \int_{0}^{E-\Delta_p} I(e) de = \int_{0}^{E-\Delta_p} Ke \sigma_{c,p}(e)W(E-e-\Delta_p) de
$$

(6)

$$
\sigma_{\gamma,n}(E) = \int_{0}^{E-\Delta_n} Ke \sigma_{c,n}(e)W(E-e-\Delta_n) de
$$

(7)

where

- $\sigma(E)$ = cross section for a gamma ray of energy $E$
- $\Delta_p, \Delta_n$ = thresholds for the $\gamma,p$ and $\gamma,n$ reactions respectively
- $e$ = energy emitted nucleon
- $W$ = level density of the residual nucleus
Figure 2.6

Photoproton spectra from CsI after correcting for the protons escaping from the crystal. The spectra have all been normalised to the same circulating electron current in the synchrotron. $E_m$ is the maximum bremsstrahlung energy.
$E = 32\text{MEV}$.

$E = 31\text{MEV}$.

$E = 30\text{MEV}$.

$E = 29\text{MEV}$.

$E = 28\text{MEV}$.

$E = 27\text{MEV}$.

$E = 26\text{MEV}$.

$E = 25\text{MEV}$.

$E = 24\text{MEV}$.

$E = 23\text{MEV}$.

$E = 22\text{MEV}$.

$E = 21\text{MEV}$.

PROTON ENERGY (MEV)

FIG. 2.6
where
\[ \sigma_{c,p}, \sigma_{c,n} = \text{capture cross section for protons and neutrons respectively in the residual nucleus}, \]
\[ K = \text{constant}. \]

Therefore the \((\gamma,p)\) cross section is obtained from the measured \((\gamma,n)\) cross section from the following equation:
\[ \sigma(\gamma,p)(E) = \left( \frac{\sigma(\gamma,p)(E)}{\sigma(\gamma,n)(E)} \right) \sigma(\gamma,n)(E) \] (8)

where the ratio is determined from equations (6) and (7) and \(\sigma(\gamma,n)(E)\) is the measured experimental \((\gamma,n)\) cross section.

The level density was assumed to be of the form:
\[ W(E) = \frac{1}{E^2} e^{2\sqrt{aE}} \] (9)

where \(a = 14 \text{ MeV}^{-1} \) (La61) and the capture cross sections were taken from Blatt and Weisskopf's tables (B152).

The calculation showed that the \((\gamma,p)\) cross section was approximately 0.3% of the \((\gamma,n)\) cross section for \(E > 20 \text{ MeV}\) and a crude absolute value for the integrated \((\gamma,p)\) cross section of 1 MeV·mb was obtained by extrapolating Nathan and Halpern's \((\gamma,n)\) cross section for \(^{127}\text{I}\) (Na54). This value can be compared with 136 MeV·mb for the total \((\gamma,p)\) cross section (see 2.5.2) and it is obvious that evaporation protons play an insignificant part.

*This calculation is based on the assumption that the \((\gamma,n)\) cross section is due to the formation of a compound nucleus. This, however, will not be true and according to the I.P.M. (Wi56) a large fraction of the neutrons will be emitted directly. This will greatly reduce the estimated yield of evaporated protons.*
The calculation of photoproton spectra from the direct interaction is extremely complex and it is doubtful if the theory of direct emission is sufficiently well understood to permit a reliable calculation. Various attempts have been made (To53; Da56; Ba60), but in every case the problem has been oversimplified. The fact that the results appear to fit the experimental data for some heavy nuclei is fortuitous and in the region of $Z = 50$ there is no agreement at all*

A detailed calculation such as is required by the theory cannot be reliably undertaken until there is more knowledge of the $E1$ transition energies and the position of the proton energy levels, especially those above the coulomb barrier.

2.5.2 $(\gamma,p)$ Cross Section

The $(\gamma,p)$ cross section for Cs and I (fig. 2.7) was obtained from the measured yield curve using the method described in Appendix I. Allowances were made for the change in the bremsstrahlung shape caused by the absorption in the graphite. The peak in the cross section was found at 25 MeV, the maximum cross section was 18 mb and the integrated cross section to 32 MeV was 136±30 MeV-mb. The estimation of the quoted error was based on the following errors: crystal thickness 2%, range-energy

*Attempts have been made (Ba60) to explain the spectra for nuclei in the region of $Z = 50$, by a combination of direct and compound nucleus protons. The results indicate that the necessary compound nucleus contribution is 5-10 times that predicted by the compound nucleus theory.
Figure 2.7

The yield curve and derived cross section for the CsI(γ,p) reaction.
CROSS-SECTION (mb.)

GAMMA RAY ENERGY (mev.)

FIG. 2.7
relation 3%, angular distribution 4%, protons from the perspex 4%, Berman and Brown's Cu$^{63}$($\gamma$,n) result 6% and the error in measuring the absolute yield 20%. All the other sources of error were less than 1%.

The striking feature of the experimental cross section was that it showed a maximum 10 MeV above the position of the peak of the ($\gamma$,n) cross section, which is at 15.2 MeV (Na54; Mo53). The position of the maximum of the ($\gamma$,p) cross section can be explained in terms of the coulomb barrier and of the high energy tail of the giant resonance. However there is an increasing amount of evidence, both theoretical and experimental, to support the suggestion that there is a second absorption resonance for photons in the region of 23-28 MeV for heavy nuclei. The main experimental evidence is the ($\gamma$,p) cross section measurements described here and in chapters 3 and 4. The manner in which they show evidence of the second resonance is shown in 3.4.4. In chapter 5 a theoretical calculation (Ca60), originally intended to explain the position of the giant resonance, has been extended to show that there may be a second smaller resonance at about 23 MeV for heavy nuclei. This new resonance can satisfactorily explain the observed ($\gamma$,p) cross sections for heavy nuclei.

2.5.3 Conclusions

The magnitude of the cross section agrees with the theoretical predictions of the new model and the comparison is plotted in
fig. 5.3 along with other experimental results.

The Wilkinson model can also be used satisfactorily to explain the \((\gamma,p)\) cross sections in the heavy nuclei. However, whichever model is used it must predict a large absorption cross section above the giant resonance, and on this aspect the Carver-Peaslee model appears more probable. A more detailed discussion of the two models is given in chapter 5.

An explanation of the shape of photoproton spectra seems to be the only main feature of the photonuclear process that cannot be satisfactorily provided by the I.P.M. It may be that with a more detailed knowledge a satisfactory explanation will be found which is in agreement with the present I.P.M. theories.
3.1 Introduction

The study of photomuluclear reactions resulting in the emission of a proton has mainly been concentrated on the light nuclei where the $(\gamma,p)$ cross sections are the same order of magnitude as the $(\gamma,n)$ cross sections. The experimental data in the middle and heavy nuclei is limited to the measurement of the photoproton yields for 23 MeV bremsstrahlung for a comprehensive list of nuclei (We54) and an occasional energy and angular distribution of the photoproteons for isolated bremsstrahlung energies (Ha51; Bu53; To55; Ho53). Also Barber and Vanhuyse (Ba60) have recently measured the proton spectra, angular distributions and cross sections for four nuclei ranging from Nb through to Au. These results of Barber et al. are the only detailed $(\gamma,p)$ cross sections that have been measured for the middle and heavy nuclei $(Z > 50)$.

The lack of data in this region can be attributed to the small $(\gamma,p)$ cross sections and the difficulty in separating the protons from the accompanying electrons (see 2.2.6).

However the photoproton measurements in this region of the periodic table are important, for it is only these measurements which permit an investigation of direct emission processes,
without the complication of competing evaporation processes (see 2.5.1). The experiments described below were carried out for two reasons: firstly, in conjunction with the results of chapters 2 and 4, to fill in the gap in the knowledge of this region of the periodic table*, and secondly to test the validity of the various independent particle models.

The cross section measurements were made using a scintillation spectrometer to count the residual activity induced in samples irradiated in the bremsstrahlung beam. This limited the choice of nuclei to those which led to the production of a radioactive nucleus after proton emission.

The use of residual activities for the measurement of cross sections has several advantages over the more conventional methods of detecting the emitted nucleons. They are especially advantageous in \((\gamma,p)\) measurements because the problem of differentiating between photoprotons and the large fluxes of electrons (see 2.2.6) is eliminated, although in some cases the required residual activity is masked by the more prolific \((\gamma,n)\) activities. Also if the gamma ray beam is monitored by measuring an induced activity, the possibility of the beam monitor and the sample being irradiated by different parts of the beam can be eliminated.

*At the time these measurements were made it was not known that Barber et al. (Ba60) were performing a similar experiment.
3.2 Scintillation Spectrometer

The scintillation spectrometer, which was used to measure the residual activities, consisted of a well shielded NaI(Tl) crystal, \( \frac{1}{2} \)" in diameter by 2" long, mounted on a Dumont type photomultiplier. Connected to the base of the photomultiplier was a cathode follower, the output of which was fed into a Higginbotham non-overload amplifier. The pulse spectrum from the amplifier was displayed on a 100 channel Hutchison-Scarrot kicksorter.

The crystal shielding consisted of 6" of iron, 3" of Pb and 12" of concrete. The inside measurements of the shielding were 10" x 10" x 18" and the crystal was situated at the centre to minimise back scattering. Some of the early Ta results were obtained in a "castle" consisting only of 3" lead with an inner sheath of liquid Hg to absorb the lead X-rays. The backgrounds for both castles were very similar.

The spectrometer was calibrated with standard gamma ray sources and was linear up to 2.6 MeV, which was the highest energy gamma-ray source available. The energy resolution of the NaI(Tl) crystal and the associated electronics was 9\% for the 511 kev \( \gamma \).

3.3 \( \text{Ta}(\gamma,p) \)

3.3.1 Method

The photoproton emission from Ta was studied by measuring
Figure 3.1

Decay scheme for Hf\(^{180}\) and Ta\(^{176}\)
the yield of the 5.5 hour isomer in Hf$^{180}$ produced in the 
$^{181}\text{Ta}(\gamma,p)Hf^{180m}$ reaction. It was not possible to measure 
the total proton yield, using residual activities, because 
the isomeric level in Hf$^{180}$ was not populated in 100% of the 
disintegrations and the ground state of Hf$^{180}$ was not radio-
active. It was estimated (see 3.3.4) that the measured yield 
was approximately 15% of the total.

The Ta targets consisted of large Ta blocks weighing 65 gms. 
These were wrapped in cadmium to stop thermal neutrons and 
irradiated in the bremsstrahlung beam for 1 hour. The residual 
activity was then measured in the scintillation spectrometer.

The main difficulty in measuring the residual activity lay 
in separating the 5.5 hour Hf$^{180m}$ activity from the more abundant 
2.1 hour Ta$^{178}$ activity, which was produced in the $^{181}\text{Ta}(\gamma,3n)\text{Ta^{178}}$ 
reaction. In the energy region 200 to 500 kev Hf$^{180}$ has $\gamma$-ray 
lines at 216, 332, 443 and 501 kev, and Ta$^{178}$ has lines at 214, 
326, 332 and 427 kev (St58). The decay scheme for these nuclei 
is shown in fig. 3.1. At lower energies the $\gamma$-ray spectrum is 
-dominated by the 8.15 hour Ta$^{180m}$ activity which is produced in 
the $^{181}\text{Ta}(\gamma,n)\text{Ta^{180m}}$ reaction. The difference between the two 
gamma ray spectra is illustrated in fig. 3.2, which shows two 
of the observed spectra. Spectrum B was taken 20 hours after 
spectrum A and corresponds mainly to Hf$^{180}$. The energy shift 
in the high energy peak was due to the decay of the 427 kev Ta$^{178}$
Figure 3.2

Gamma ray spectra of the 5.5 hr Hf^{180} and 2.1 hr Ta^{178} activities. Spectrum B was measured 20 hr after Spectrum A and corresponds mainly to Hf^{180}.
line leaving the 443 kev Hf$^{180}$ line. The decay of the $\gamma$-rays, an example of which is shown in fig. 3.3, was analysed by a method of least squares and showed that there were two activities of approximately 2.1 hours and 5.5 hours corresponding to the Hf$^{180}$ and Ta$^{178}$ activities respectively. The analysis included a correction for the bremsstrahlung produced by the 0.7 MeV $\beta$-particles from the 8.15 hour Ta$^{180}$ activity. This correction was made by irradiating a Ta target at 15 MeV, where there was no yield of Ta$^{178}$ or Hf$^{180}$, and measuring the bremsstrahlung intensity relative to the intensity of the 55 kev Ta$^{180}$ peak. Since the relative intensity was independent of the irradiation energy, the 55 kev line intensity was used to determine the bremsstrahlung intensity for higher irradiation energies.

The beam was monitored by measuring the 55 kev X-ray from the 8.15 hour Ta$^{180m}$ activity which was induced in the Ta targets by the Ta$^{181}(\gamma,n)$Ta$^{180m}$ reaction (see 2.4).

Because of the comparatively long dead time of the Hutchison-Scarrot kicksorter (850$\mu$s) and the large yield of Ta$^{180m}$ it was necessary to count the Ta$^{180}$ X-ray activity with the Ta targets removed 2" from the NaI(Tl) crystal, which reduced the dead time correction to less than 10%. The $\gamma$-rays from the Ta$^{178}$ and Hf$^{180}$ activities were counted with the targets directly on top of the crystal and the Ta X-ray line biased off the kicksorter.
Figure 3.3

Decay curve for the 5.5 hr Hf$^{180}$ and 2.1 hr Ta$^{178}$ gamma rays.
FIG. 3.3
3.3.2 Absolute Yield

The absolute yield of the $^{181}\text{Ta}(\gamma,p)^{180m}\text{Hf}$ reaction leading to the isomeric level was measured by comparing the $^{180m}\text{Hf}$ activity with the 9.7 min. activity in $^{62}\text{Cu}$ which was obtained from the $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$ reaction. This was done by irradiating identically shaped Cu and Ta disks at 30 MeV and comparing the intensity of the annihilation quanta from the Cu with the 332 kev line in $^{180}\text{Hf}$. Corrections were made for isotopic abundance, decay schemes (St58), self absorption in the disks and the NaI crystal efficiencies in the spectrometer (Mo58). From these measurements the number of $^{181}\text{Ta}(\gamma,p)^{180m}\text{Hf}$ disintegrations per unit $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$ disintegration was calculated. The absolute yield of $^{180m}\text{Hf}$ was obtained by using these measurements in conjunction with Berman and Brown's $^{63}\text{Cu}(\gamma,n)$ cross section data (Be54).

3.3.3 Results

The measured yield function and the derived cross section for the $^{181}\text{Ta}(\gamma,p)^{180m}\text{Hf}$ reaction are shown in fig. 3.4. The points on the yield curve were obtained from a least squares analysis of the 5.5 hour $^{180}\text{Hf}$ and 2.1 hour $^{178}\text{Ta}$ activities. This analysis was done independently for the three main gamma rays (see fig. 3.2) and also for the sum of these gamma rays. The four yield curves were in agreement except that the curve
Figure 3.4

The yield curve and the derived cross section for the Ta$^{181}(\gamma,p)$Hf$^{180m}$ reaction.
FIG. 3.4

CROSS SECTION (mb) vs GAMMA-RAY ENERGY (MeV)
obtained from the analysis of the 216 kev line showed a much larger scatter of the points. This was attributed to the larger correction for the 8.15 hour bremsstrahlung activity in this region.

The threshold for the Ta$^{181}(\gamma,p)$ reaction was 6.7 MeV (Wa58) which made the threshold of the Ta$^{181}(\gamma,p)$Hf$^{180m}$ 7.8 MeV (St58). However, because of the 12.4 MeV coulomb barrier there was no appreciable yield below a $\gamma$-ray energy of 16 MeV. The Ta($\gamma,p$) reaction leading to the isomeric level in Hf$^{180}$ has a peak cross section of 1.0 mb at a $\gamma$-ray energy of 28 MeV and an integrated cross section to 32 MeV of 10±2 MeV-mb. Approximately equal contributions to the error came from the analysis of the decay curves and from the comparison of the Hf$^{180}$ yield with the Cu$^{62}$ yield. There is a smaller error, 6%, in Berman and Brown's (Be54) estimate of the integrated Cu$^{63}(\gamma,n)$ cross section on which the present determination was based.

3.3.4 Discussion

The interesting feature of this cross section was that the peak occurred at a very high gamma ray energy, higher than in either mercury or tungsten (see figs. 3.8, 4.10 and 4.11), both of which are heavier nuclei. It is difficult to explain the difference in shapes between the Ta$^{181}(\gamma,p)$ and W($\gamma,p$) cross sections as they are neighbouring elements and therefore the same I.P.M. transitions should be involved.

A logical explanation of the anomalous position of the cross
section is that it is in some way connected with the deformation of the nucleus and the use of an isomeric transition. Wilkinson (Wi58) has indicated that for deformed nuclei there is a connection between the absorption process and the spin of the residual nucleus. This can be extended to show that the isomeric level may be populated preferentially from certain regions of the absorption cross section. This explanation is supported by the measurements of the Ta$^{181}(\gamma,n)$ reactions. Results from using the 8.15 hour isomeric transition (Ca59) are substantially different from those obtained by detecting the emitted neutrons (Sp58; Pa59; Fu58). The isomeric measurements appear to have preferentially selected the higher energy peak of the two shown in the neutron results. The difference between the methods is too large to be attributed to experimental errors.

It is interesting to compare the results of 3.3.3 with those obtained by Barber and Vanhuyse (Ba60), who detected the photoprotons from Ta, using a proton spectrometer. Their results showed a considerably higher cross section in the region 19-24 MeV.

*Wilkinson shows that different types of transitions populate the two resonances forming the giant resonance in deformed nuclei. This can lead to a difference in the spins of the residual nucleus, depending on which absorption resonance they originated from and hence to a preferential population of the isomeric level. This can be extended to cover the $(\gamma,p)$ cross section of Ta if the cross section is interpreted as being due to a second higher resonance (see chapter 5).
but this can be attributed to the fact that they measured the total \((\gamma,p)\) cross section whereas here only the cross section leading to the isomeric state of Hf\(^{180}\) was measured. The difference in the results supports the hypothesis suggested above, that the cross section as measured by an isomeric transition may not be truly representative of the total cross section.

It was also of interest to evaluate the fraction of disintegrations which proceed through the isomeric level. In view of the above discussion this fraction is expected to vary with the \(\gamma\)-ray energy. Using the results of Toms and Stephens (To55) who measured the yield of photoprotons from a 23 MeV bremsstrahlung irradiation it was found that the isomeric level was populated in 10\% of the disintegrations. The results of Barber et al., at 30 MeV would suggest a figure of 20\%. This large variation was not unexpected in view of the difference between the shape of the cross sections for the reaction leading to the isomeric level and the total \((\gamma,p)\) cross section.

3.4 \(\text{Hg}^{201}(\gamma,p)\)

3.4.1 Method

The cross section for the emission of photoprotons from mercury was studied by measuring the 48 min. activity in \(\text{Au}^{200}\) produced by the \(\text{Hg}^{201}(\gamma,p)\text{Au}^{200}\) reaction. Although Hg has six stable isotopes in varying abundance, of which five have \((\gamma,p)\)
reactions resulting in radioactive nuclei, only one, Hg$^{201}$, was measurable. The other four were discarded because their residual activities were masked by the competing Hg($\gamma$,n) activities.

The Hg targets consisted of a small glass container 1" in diameter by $\frac{1}{2}$" deep filled with 80 gms of natural mercury containing 13% of Hg$^{201}$ (St58). To monitor the beam, one inch diameter Ta disks were placed on the front and back surfaces of the glass container, which was then wrapped in cadmium. The purpose of the cadmium was to prevent the slow neutrons from the synchrotron from producing neutron capture reactions in the mercury.

These targets were then irradiated for 30 min. in the bremsstrahlung beam which was monitored by measuring the $^{180m}$ Ta activity in a similar manner to that described in 2.4. After the irradiation the mercury was extracted from the container and placed in a glass beaker with an internal diameter of $1\frac{1}{2}$". The beaker was placed on top of the NaI(Tl) crystal in the scintillation spectrometer where the residual activity was counted. The mercury was placed in the beaker to reduce its thickness while counting, thereby reducing the amount of absorption in the Hg. This reduction in the thickness of mercury increased the counting rate in the spectrometer by 20%. The Hg was weighed before irradiation and after counting,
Figure 3.6

Decay curve for the 1.24 MeV $\gamma$-ray from the decay of Au$^{200}$

Figure 3.5

Decay scheme for the 48 min Au$^{200}$ activity.
to guard against any loss while being transferred from the sample holder to the beaker.

$^{200}$Au emits two gamma rays of 368 keV and 1.23 MeV (St58). A proposed decay scheme (see fig. 3.5) shows the two gamma rays are in coincidence and are emitted in 24% of the disintegrations.

Fig. 3.6 shows a typical spectrum taken four minutes after the end of the irradiation. The high energy $\gamma$-ray at 1.23 MeV was found to decay with a 48 min. half life and was attributed to the decay of $^{200}$Au. A typical decay curve is shown in fig. 3.7. The 365 keV line decayed with a half life of 43 min. and this was attributed to the $^{200}$Hg($\gamma,n$)$^{199m}$Hg reaction. This reaction completely masked the 48 min. 368 keV line from $^{200}$Au. The lower energy lines were also found to result from Hg($\gamma,n$) reactions.

The energy of the 1.23 MeV line was checked against the 1.17 and 1.33 MeV lines from $^{60}$Co as there was conflicting evidence on its exact energy (St58).

3.4.2 Absolute Yield Measurement

The absolute yield was measured by comparing the 48 min. $^{200}$Au activity with the 9.7 min. $^{62}$Cu activity from the reaction $^{63}$Cu($\gamma,n$)$^{62}$Cu. The Cu activity was measured in the same geometry as the Au activity. This was done by filling the Hg target holder with a homogeneous mixture of 1 gm of Cu and 15 gms of HgI. This mixture was irradiated for 2 min. at 30 MeV and then counted in the glass beaker in exactly the same way as the mercury. The beam was monitored with Ta disks similar to those used to monitor

---

* The decay scheme for $^{200}$Au is uncertain and the value of 24% for the number of 1.23 MeV $\gamma$-rays has been taken from the results of Roy et al. as quoted by Strominger et al. (St58).
Figure 3.7

The gamma ray spectrum following the irradiation of natural Hg. The 1.24 MeV $\gamma$-ray is from the decay of Au$^{200}$ and the 0.365 MeV $\gamma$-ray is due to decay of Hg$^{199m}$ formed by the reaction Hg$^{200}(\gamma, n)$Hg$^{199m}$. 
FIG. 3.7

COUNTS/CHANNEL

365 keV.

1.24 MeV.

(x50)

CHANNEL NUMBER

400 800 1200 (keV.)
the mercury irradiations. The decay of the Cu$^{62}$ activity was followed carefully for 5 half lives to ensure that there were no conflicting activities from the HgI. From this measurement which was repeated several times, together with the relative Au$^{200}$ and Ta$^{180}$ activities, the ratio of the Au$^{200}$ to Cu$^{62}$ yields was obtained. Since the two yields were measured in the same geometry the only corrections that had to be made were for isotopic abundance, decay schemes, gamma ray absorption in the targets and the relative crystal efficiencies for the $\gamma^+$ and 1.23 MeV gamma rays. The absolute yield was then determined using Berman and Brown's cross section data for Cu$^{63}(\gamma,n)$Cu$^{62}$ (Be54).

3.4.3. Results

The measured yield curve obtained at 1 MeV intervals is shown with the derived cross section in fig. 3.8. The points on the yield curve were obtained from a least squares analysis of the 48 min. Au$^{200}$ activity. Each point represents the mean value of at least two separate irradiations. The cross section was derived using the method described in Appendix I.

The cross section shows a very wide resonance with a peak cross section of 4.1 mb at a gamma ray energy of 26 MeV. The integrated cross section from threshold to 32 MeV was 40±10 MeV-mb. The Hg$^{201}(\gamma,p)$ threshold was 6.7 MeV (Wa58) and the coulomb barrier was 13 MeV ($r_0 = 1.5 \times 10^{-13}$ cm).
Figure 3.8

The yield curve and the derived cross section for the $\text{Hg}^{201}(\gamma,p)\text{Au}^{200}$ reaction.
FIG. 3.8

GAMMA-RAY ENERGY (MeV)

CROSS SECTION (mb.)  YIELD (arbitrary units)

10  15  20  25  30  35

FIG. 3.8
3.4.4 Discussion

It would appear from the results, in conjunction with the Cs and I (γ,p) measurements (see 2.5.2) and the W(γ,p) results (see 4.4.2 and 4.5.2) that the emission of photoprotons for nuclei with Z > 50 follows the absorption of photons into a high energy resonance. If the absorption cross section was only a giant resonance with a high energy tail where the cross section was slowly decreasing the (γ,p) cross section would peak at higher and higher energies as Z increased. On the other hand if we interpret the cross section as being due to a higher resonance then there are two factors contributing to the position of the peak in the (γ,p) cross section: Firstly the position of the absorption cross section for this higher resonance, which will have an $A^{-\frac{1}{3}}$ dependence (see 5.4), similar to the giant resonance; and secondly, the coulomb barrier, which will increase with Z. These two factors will to some extent cancel each other out and as a result the peak in the (γ,p) cross section should not vary much with A.

It follows from the above arguments that the position of the (γ,p) cross section in the middle and heavy nuclei should test the validity of the various I.P.M. The experimental results mentioned above would indicate a preference to the Carver-Peaslee calculations (see chapter 5) which suggest a resonance in the region.

*It is assumed that absorption involving proton transitions will have the same general shape as the absorption involving neutron transitions. However there may be a slight difference in the peak positions and a small difference in the absolute magnitude.
22-30 MeV with the integrated absorption cross section up to 30% of the giant resonance. The Wilkinson model does not agree so favourably. The resonance which the Carver-Peaslee calculations place at 25 MeV would be 10-20 MeV higher in the Wilkinson model (see 5.2) and it is unlikely, although not impossible, that the 1-quantum transitions* which form the giant resonance will cluster in two widely separated regions. The whole question of the position of the $\gamma/p$ cross section and higher resonances in the absorption cross section is further discussed in chapter 5.

A comparison of the integrated cross section for the Hg$^{201}(\gamma,p)$ reaction with the prediction of the Carver-Peaslee calculation is given in fig. 5.3. This shows that the experimental results are consistent with the theoretical calculation. The whole problem of explaining the $\gamma/p$ cross sections for middle and heavy weight nuclei is further discussed in 5.6.2.

A comparison of the Hg($\gamma,p$) results with the Au($\gamma,p$) cross section measured by Barber et al. (Ba60) shows reasonable agreement in view of the large errors associated with both measurements. They found the integrated cross section to 40 MeV was $75\pm20$ MeV-mb as compared with 40 MeV-mb for the integrated cross section to 32 MeV measured above.

*The notation of 1, 2, 3 etc., quantum transitions refers to the E1 transitions which would correspond to 1, 2, 3 quantum jumps in the simple harmonic oscillation potential (see 5.1).
4.1 Introduction

It has been established that the yields of photoprotons from the light and middle weight nuclei may vary substantially in different isotopes (Bu53; Ca59a). These variations can be satisfactorily explained, provided that a substantial fraction of the protons are emitted from a compound nucleus. Therefore, it is of interest to study the relative proton yields for two isotopes of a heavy element, in which the compound nucleus plays an insignificant part in the emission of photoprotons (Wi56). If a heavy element with two isotopes, having the same ($\gamma$,p) threshold but different ($\gamma$,n) thresholds, is studied, then it would be expected from a simple independent particle model that the photoproton yields would be similar. However, if emission of the protons involves the formation of a compound nucleus, then the difference in the neutron energies will affect the relative probabilities of proton emission.

A study of the photodisintegration processes in tungsten permits a measurement of the photoproton yield from different isotopes. This can be done, using natural tungsten, by measuring the activities from the residual nuclei. Since the photodisintegration of tungsten had not been measured previously, it
was decided also to measure a $(\gamma,n)$ cross section in one of the isotopes. The $(\gamma,n)$ reaction in $W^{186}$ was the only suitable one as the others either gave stable residual nuclei or the residual nuclei could also be formed by a $(\gamma,2n)$ or $(\gamma,3n)$ reaction in another isotope of W.

4.2 Decay Schemes

Before attempting the measurement of yield curves and cross sections it was essential to understand the complex $\gamma$-ray spectra that were obtained after irradiating natural tungsten. Since all the activities produced have been previously studied, mainly by neutron capture reactions, the gamma ray spectrum associated with each of the individual isotopes was fairly well known. The $\gamma$-ray spectra observed following irradiation with a bremsstrahlung beam, were studied to see whether they were consistent with those obtained from other reactions and by using enriched isotopes.

Table 4.1 shows the tungsten isotopes together with their abundances and $(\gamma,n)$ and $(\gamma,p)$ threshold energies. Table 4.2 lists the important photonuclear reactions and the half life of the residual nucleus.
Figure 4.1

The gamma-ray spectrum obtained from a series of 2 min irradiations of natural tungsten and counting for 2½ min., 1 min after the irradiations. The gamma rays are mainly due to the decay of W$^{185m}$. 
Table 4.1

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Abundance</th>
<th>((\gamma,n)) Threshold (MeV)</th>
<th>((\gamma,p)) Threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W^{180})</td>
<td>0.135</td>
<td>(6.9)</td>
<td>(5.6)</td>
</tr>
<tr>
<td>(W^{182})</td>
<td>26.4</td>
<td>7.9</td>
<td>7.0</td>
</tr>
<tr>
<td>(W^{183})</td>
<td>14.4</td>
<td>6.7</td>
<td>7.7</td>
</tr>
<tr>
<td>(W^{184})</td>
<td>30.6</td>
<td>7.4</td>
<td>7.7</td>
</tr>
<tr>
<td>(W^{186})</td>
<td>28.4</td>
<td>6.4</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Threshold values obtained from Wapstra (Wa58); numbers in brackets from Metropolis et al. (Me50).

Measurements of \(\gamma\)-ray spectra were made from a series of irradiations at 30 MeV. The decay of the gamma ray spectra obtained after these irradiations, was followed until the main activities had decayed almost completely.

The initial irradiations were for 2 min. and the resultant gamma ray spectra showed two \(\gamma\)-rays at 125 and 175 kev respectively and the 55 kev W X-ray. (See fig. 4.1). The decay of these lines (see fig. 4.2) showed that they were mainly due to a short lived activity (~2 min.). The 55 and 175 kev lines also contained longer lived activities but these were not strong enough to measure the half lives.
Figure 4.2

The decay of the gamma rays observed after a two min irradiation (see fig. 4.1).

Curve A is the decay of the X-ray, Curve B the 175 kev line and C the 125 kev line.
FIG. 4.2
The next series of runs were 30 min. irradiations and the γ-ray spectra from these indicated more γ-rays and several half lives. A series of typical spectra recorded during the decay of the γ-rays is given in figure 4.3 and the decay of the main γ-rays in figure 4.4. Figure 4.3 clearly shows there are 4 main γ-rays at 125, 175, 220 and 240 kev and an X-ray line. Above 300 kev there was evidence of further γ-rays but these were too low in intensity to be resolvable. The decay curves were analysed by a method of least squares assuming that all the γ-rays had
Some typical gamma ray spectra observed after a 30 min irradiation of natural tungsten.

Spectrum A is a two minute count at $t = 1$ ($t = 0$ is at the end of the irradiation), B is a 15 min count at $t = 50$, C is a 25 min count at $t = 120$ and D is a 25 min count at $t = 300$. 
FIG. 4.3

Channel Number

Counts/Channel

A

B

C

D

\(x/4\)

\(x/8\)
Figure 4.4

The decay of the main gamma rays observed in figure 4.3

A - X-ray
B - 130 kev (x2)
C - 240 kev
D - 175 kev
E - 220 kev

The analysis of the decay curves for the components of the different half lives is given in table 4.3
components with the following half lives: 1.85 min., 5 min., 16.5 min., 49 min., 522 min. and greater than 1 day. The results of this analysis are given in table 4.3, which shows the various half life components of each gamma-ray.

Table 4.3

<table>
<thead>
<tr>
<th>Y-ray energy</th>
<th>Half-life (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.85</td>
</tr>
<tr>
<td>X-ray</td>
<td>9000</td>
</tr>
<tr>
<td>135</td>
<td>2500</td>
</tr>
<tr>
<td>175</td>
<td>3000</td>
</tr>
<tr>
<td>220</td>
<td>0</td>
</tr>
<tr>
<td>245</td>
<td>0</td>
</tr>
</tbody>
</table>

The numbers in the body of the table are the count rates per minute of the various half life components for each Y-ray at the end of the 30 min. irradiation.

A final series of irradiations lasting 12 hours was made to disentangle the long lived components. After waiting several days for the short lived activities to decay away there remained three distinct Y-rays at 245, 480 and 680 kev (see fig. 4.5). Below 240 kev there were the X-ray and a large number of smaller peaks. The 160 kev peak was the sum of several unresolved gamma rays with energies between 130 and 165 kev, and there were small peaks at 100 kev and 200 kev. The decay of the three high energy
Figure 4.5

Gamma ray spectra observed 3 days after a 12 hour irradiation of natural tungsten. The low energy $\gamma$-rays are mainly due to the Ta$^{183}$ activity and the two high energy $\gamma$-rays are from W$^{187}$. 
lines is shown in fig. 4.6. The 480 and 680 kev lines have a 24 hour half life and the 245 kev line has 24 hour and 5 day components.

From these measurements the following conclusions were reached regarding the various observed activities:

4.2.1 $^{185m}W$

This activity has been measured several times (Po55; Du50; Mo60) and the reported half lives range from 1.65-1.85 min. Gamma rays have been reported at 75, 100, 125 and 175 kev and an X-ray line at 55 kev (Mo60). The two high energy lines and the X-ray were seen (see fig. 4.1) but the intensity of the other two $\gamma$-rays was too weak. The intensity of the $\gamma$-rays was consistent with the reported values. A measurement of the half life was made from a series of 2 min. irradiations and gave $1.85 \pm 0.05$ min. In making the half life measurements a special dead-time correction factor was used to allow for the decay of the sample while counting (see Appendix I). A proposed decay scheme (Nd59), given in fig. 4.7, shows the 1.85 min. isomeric level feeding the 310 kev state in $^{185}W$. The energy of the isomeric $\gamma$-ray is unknown.

4.2.2 $^{185}W$

$^{185}W$ decays directly to the ground state of Re$^{185}$ with a 74 day half life by emitting a 0.43 MeV $\beta$-particle in over 99.9% of the disintegrations (Wi57). A 125 kev $\gamma$-ray associated
The decay of the 245 (A), 482 (B) and 686 (C) keV γ-rays.
Figure 4.7

Decay scheme of Ta$^{185}_{\text{a}}$ and W$^{185m}_{\text{m}}$ taken from the Nuclear Data Sheets.
FIG. 4.7
with a weak 0.307 MeV β-particle (0.02%), which feeds the first excited state in Re$^{185}$, has been reported (Co49). The beta-particles from an irradiated tungsten sample were counted in a thin end window Geiger counter and the activity followed for 90 days. After the short lived activities had decayed away, the half life of the β-activity was found to be 75±5 days (see fig. 4.8) which was in agreement with the reported value. No absorption measurements were made to determine the β-particle energy because of the low intensity.

4.2.3 Ta$^{185}$

The decay scheme of Ta$^{185}$ (see fig. 4.7) appears to be well understood. It has a 49 min. half life and apart from the K X-ray there are five γ-rays at 75, 100, 135, 175 and 245 kev. The proposed decay scheme is based on the work of Morinaga et al. (Mo60). Table 4.3 shows γ-rays with a 49 min. half life at 135, 175 and 245 kev and with intensities consistent with the previous measurements. There is a small peak at 100 kev (see fig. 4.3) which is probably due to the decay of Ta$^{185}$. The small 49 min. component in the 220 kev line is due to the Compton edge of the 245 kev line. A γ-γ coincidence measurement shows that the 135 and 245 kev γ-rays are in coincidence with the 175γ-ray.

4.2.4 Ta$^{183}$

From the series of 12 hour irradiations it was found that
Figure 4.8

Decay curve for the 0.43 MeV beta-particles from the decay of W$^{185}$. 
there were three \( \gamma \)-rays at 100, 165 and 245 kev, which had a half life of 5.1 days corresponding to the radioactive decay of Ta\(^{183}\). Because of the complex nature of the decay of Ta\(^{183}\) (Du53), the observed \( \gamma \)-rays were the sum of several unresolved \( \gamma \)-rays. The peak at 245 kev was, in fact, the sum of two \( \gamma \)-rays at 244 and 246 kev respectively, while the broad low energy lines were even more complex. However, the complexity of the 5 day \( \gamma \)-ray spectrum should not affect the measurement of the yield of Ta\(^{183}\).

A least squares analysis of the decay curves of the 245 kev line gave a value of 5.1\( \pm \)0.2 days for the half life (see fig. 4.5).

4.2.5 Ta\(^{182m}\)

Recently a comprehensive study was made by Sunyar and Axel (Su60) of the decay of the 16 min. isomer of Ta\(^{182}\) formed by the reaction Ta\(^{181}\)(n,\( \gamma \))Ta\(^{182}\). They found five \( \gamma \)-rays at 147, 172, 184, 319 and 356 kev of which the 184 kev one was the strongest. The analysis of the decay curves given in table 4.3 shows that there was no evidence of this activity. It can therefore be concluded that the isomeric level is only populated in a small percentage of the photodisintegrations which have Ta\(^{182}\) as the residual nucleus. This is not surprising since the spin of the isomeric level is 8+, the ground state 3- and the ground state of W\(^{183}\)\(^{1/2-}\) (Su60, Ma50).
4.2.6 $\text{W}^{179m}$

Very little is known about the isomeric level in $\text{W}^{179}$. It has been reported (Ro56) to decay to the ground state of $\text{W}^{179}$ by emitting a 220 kev $\gamma$-ray with a half life of 5 min. $\text{W}^{179}$ is a difficult element to study as it can only be produced by the $(\gamma,n)$ reaction in $\text{W}^{180}$ which is a 0.1% isotope, or a $(\gamma,3n)$ reaction in $\text{W}^{182}$.

The analysis of the 220 kev line (see table 4.3, fig. 4.4) clearly indicates that there is a short lived component. The possibility of it being a 1.85 min. $\gamma$-ray from $\text{W}^{185m}$ can be eliminated as it was not obvious in the short irradiations and was not seen by Morinaga et al. (Mo60). The decay curve shows that the half life is definitely less than 10 min. An accurate determination of the half life was not possible because of the complex nature of the decay of the 220 kev line but the best estimate would put it between 5 and 7 min.

4.2.7 $\text{W}^{187}$

The two $\gamma$-rays at 480 and 685 kev found in the 12 hour irradiations could not be attributed to any known photodisintegration reaction in tungsten. They were found to decay with a 24 hour half life (see fig. 4.6) and were therefore associated with the decay of $\text{W}^{187}$ which is formed by neutron capture in $\text{W}^{186}$. There are also other low energy $\gamma$-rays emitted in the decay of $\text{W}^{187}$ (Du57) but apart from one at 135 kev they would have too
low an intensity to be seen. In the region of 135 kev the
\( \gamma \)-ray spectra were complicated by the \( \gamma \)-rays from Ta\(^{183} \). However the line shape of the broad 165 kev line altered
with time, and this was consistent with the low energy
portion of the line decaying at a faster rate.

This activity proved annoying in the measurements of
the Ta\(^{183} \) activity. At low excitation energies it was
necessary to wait six to seven days before it was possible
to count the five day Ta\(^{183} \) activity.

4.2.8 Conclusions

The complex structure of the \( \gamma \)-ray spectra obtained
from irradiating natural tungsten targets with a bremsstrahlung
beam can be satisfactorily explained from the knowledge of the
individual residual activities. The results support the evi-
dence that W\(^{179m} \) decays to the ground state by emitting a
0.22 MeV \( \gamma \)-ray with a half life of \( \sim 6 \) min.

As a result it was decided to measure the following photo-
nuclear reaction cross sections:

i) \( W^{186}(\gamma,n)W^{185m}\rightarrow W^{185} \)

ii) \( W^{186}(\gamma,p)W^{185} \)

iii) \( W^{184}(\gamma,p)W^{183} \)

4.3 \( W^{186}(\gamma,n)W^{185} \)

4.3.1 Method

This reaction was studied by measuring the reaction
$W^{186}(Y,n)W^{185m}$, leading to the 1.85 min. isomeric level in $W^{185}$. From this the shape of the $W^{186}(Y,n)$ cross section was obtained.

The absolute yield was found by counting the 74 day $W^{185}$ β-particles.

The tungsten targets consisted of four tungsten foils 0.75" x 0.75" x 0.04" and a Ta monitor foil (0.75" x 0.75" x 0.01") was placed on either side to monitor the bremsstrahlung beam. This was enclosed in cadmium foil to reduce the probability of neutron capture reactions and then irradiated for two minutes.

After irradiation the tungsten foils were spread over the top of the 1\frac{1}{2}" diameter by 1" long NaI(Tl) crystal in the spectrometer and the decay of the 1.85 min. activity followed for twenty minutes. To facilitate the recording of data, only the X-ray line was recorded on the kicksorter. The intensity of the 175 kev line was measured using a single channel analyser, the output of which was recorded on a scaler.

## 4.3.2 Absolute Yield Measurements

The absolute yield of the reaction $W^{186}(Y,n)W^{185m}$ was measured by comparing the yield of the 175 kev γ-ray, from the decay of $W^{185m}$, with the yield of the positron annihilation quanta from the 9.7 min. Cu$^{62}$ activity, which was induced in Cu foils by the reaction Cu$^{63}(Y,n)Cu^{62}$. For this measurement W foils 0.003" thick and Cu foils 0.001" thick were irradiated together at 30MeV for 2 min. and the resulting activities counted in the NaI spectrometer.

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*The shape of the cross section measured from the decay of an isomeric level is not necessarily the same as the shape of the total cross section (see
(see 3.2). Before the Cu foils were counted in the spectrometer, they were placed between 0.035'' of Cu to ensure that the Cu$^{62}$ positrons annihilated near the Cu foils. These results were used in conjunction with the Cu$^{63}$(Y,n)Cu$^{62}$ results obtained by Berman and Brown (Be54) to calculate the absolute yield. For the purpose of the calculation the number of 175 kev γ-rays per 100 disintegrations of W$^{185m}$ was taken to be 44 (Mo60).

The measurement of the absolute yield of the reaction W$^{186}$(γ,n)W$^{185}$ was obtained by comparing the yield of the 0.43 MeV β-particle, from the decay of W$^{185}$, with the yield of Cu$^{62}$, and using Berman and Brown's results (see above). To measure the yield of the 0.43 MeV β-particles, a 0.003'' tungsten disk and two 0.001'' Cu foils were irradiated at 30 MeV. The W activity was counted between two thin end window Geiger counters and the Cu$^{62}$ γ+ counted in the scintillation spectrometer.

To compare the efficiency of the Geiger counters with that of the spectrometer, a radioactive source of Co$^{58}$ was prepared from the reaction Co$^{59}$(γ,n)Co$^{58}$. Co$^{58}$ decays emitting a 0.48 MeV positron. Therefore, by counting the Co$^{58}$ in both the Geiger counters and the spectrometer a direct measurement of the relative efficiencies for the two counters for 0.48 MeV positrons was

3.3.4) but it has been assumed here that the two shapes are identical
obtained. By making a small correction for the energy difference between the 0.43 MeV beta particles and the 0.48 MeV positrons, it was possible to relate the count rate of the tungsten in the Geiger counters to the count rate of the copper in the spectrometer. It was also necessary to make a correction for the thickness of the W and Co disks. The correction was made using the results of Baker and Katz (Ba53). The absolute yield of W^{185} was also measured by counting the Cu^{62} positrons in the Geiger counters, using the results of Baker and Katz to correct for the difference in the energy between the Cu^{62} positron and the W^{185} beta particle. The results from the two methods differed by 10%.

4.3.3 Results

The yield curve for the 1.85 min. activity which was obtained at 1 MeV intervals of the bremsstrahlung energy, and the derived cross section are shown in fig. 4.9. Each yield point was the average of at least three irradiations for which the X-ray line and 175 kev γ-ray were analysed separately. Also some data from the counting of β-particles are shown. These have been arbitrarily normalised and indicate that the yield curve in the region 22-32 MeV was the same for the 1.85 min. and 74 day W^{185} activities. Below 22 MeV the intensity of the β-particles was too low to make accurate measurements.

The W^{186}(γ,n) reaction leading to the 1.85 min. isomer in W^{185} has a peak cross section of 5.5 mb at a γ-ray energy of 14 MeV.
The yield curve and the derived cross section for the $^{186}_{\text{W}}(\gamma,n)^{185}_{\text{W}}$ reaction. The points marked X are obtained from the counting of the beta particles from the decay of the ground state of $^{185}_{\text{W}}$. These points have been normalised so that the 24 MeV point lies on the yield curve.
and an integrated cross section to 32 MeV of 40 MeV-mb. Assuming the same cross section shape for the total W^{185}(\gamma,n) reaction, the integrated cross section obtained from the \(\beta\)-particle counting was 3500 MeV-mb. This makes the relative population of the isomeric level 1.1%.

4.4 \(W^{186}(\gamma,p)Ta^{185}\)

4.4.1 Method

The yield of the reaction \(W^{186}(\gamma,p)Ta^{185}\) was obtained from measuring the 50 min. activity in Ta^{185}. The tungsten targets and monitoring foils were the same as those used in the \(W^{185m}\) measurements and the same counting method was used (see 4.3.1). The targets were irradiated for 30 min. and the activity was followed for 8 hours.

The intensity of the 50 min. Ta^{185} activity was measured by counting the 175 kev \(\gamma\)-ray, which was the most abundant \(\gamma\)-ray apart from the K X-ray (see fig. 4.3 and table 4.3). The X-ray, which was almost twice as abundant as the 175 kev \(\gamma\)-ray had the disadvantage of having large amounts of long lived activities, which gradually increased through the repeated use of the tungsten targets.

The absolute yield of Ta^{185} was found in a similar manner to the 2 min. \(W^{185}\) absolute yield (see 4.3.2). The 175 kev line was used and it was assumed that there were 44, 175 kev \(\gamma\)-rays per 100 Ta^{185} disintegrations (Mo60).
4.4.2 Results

The measured yield curve and the derived cross section are shown in figure 4.10. The points on the yield curve were obtained by a least squares analysis of the decay curves of the 175 keV $\gamma$-ray. Each point represents the mean of at least two irradiations. The cross section for the $^{186}\text{W}(\gamma,p)^{185}\text{Ta}$ reaction showed a broad resonance 7 MeV wide (full width at half height), peaking at 22.5 MeV. The peak cross section was 6 mb and the integrated cross section to 32 MeV was $55\pm11$ MeV-mb.

4.5 $^{184}\text{W}(\gamma,p)^{183}\text{Ta}$

4.5.1 Method

$^{183}\text{Ta}$ decays to $^{183}\text{W}$ with a 5.1 day half life and this activity was used to measure the yield of the reaction $^{184}\text{W}(\gamma,p)^{183}\text{Ta}$. The experimental set up was the same as that used in measuring the photodisintegration of $^{186}\text{W}$ (see 4.3.1). The targets were irradiated for a total of 12 hours. The irradiation period consisted of two six hour irradiations and new Ta monitoring foils were used for the second half of the irradiation. The activity in the W targets was then followed for at least twenty days.

The 245 keV $\gamma$-ray was used to measure the $^{183}\text{Ta}$ activity because it had no confusing activities after the 24 hour $^{187}\text{W}$ activity had decayed away and it was the most abundant $^{183}\text{Ta}$ gamma ray. It was necessary to wait at least five days before the 5 day activity could be counted because of the $^{187}\text{W}$ activity.
Figure 4.10

The yield curve and the derived cross section for the $W^{186}(Y,p)Ta^{185}$ reaction.

Figure 4.11

The yield curve and the derived cross section for the $W^{184}(Y,p)Ta^{183}$ reaction.
FIG. 4.10

Gamma ray energy ($\gamma$)

FIG. 4.11

Cross section (mb)

Yield (arbitrary units)

Gamma-ray energy (MeV)
The absolute yield of Ta$^{183}$ was calculated on the assumption that the 245 $\gamma$-ray contained the 244 and 246 kev and part of the 210 and 290 kev transitions. The relative $\gamma$-ray yields were taken from the decay scheme proposed by Dumond et al. (Du53) and the number of 246 $\gamma$-rays per 100 Ta$^{183}$ decays assumed to be 26.4 (Nd59).

4.5.2 Results

The W$^{184}(Y,p)$ results were very similar to those of W$^{186}(Y,p)$. The cross section and the yield curve for the reaction are shown in figure 4.11. The cross section has a peak value of 7 mb at 22.5 MeV, an integrated value to 32 MeV of $65\pm12 \text{ MeV-mb}$ and a width of 7 MeV. The main contributions to the error came from the analysis of the decay curves (5%), the determination of the absolute yield relative to the Cu$^{62}$ yield (18%) and a small error (6%) in Berman and Brown's (Be54) estimate of the integrated Cu$^{63}(\gamma,n)$ cross section on which the present calculations are made. These same errors also apply to the W$^{186}(Y,p)$ measurement described in 4.4. The relative error between the W$^{186}(Y,p)$ and W$^{185}(Y,p)$ cross sections is smaller (7%).

4.6 Discussion

A comparison of the results for W$^{186}$ and W$^{184}$ indicates that there is no significant difference between the $(Y,p)$ cross sections for the two nuclei. The small differences in the shapes could be entirely due to experimental errors.
in the measured yield curves. These results are in agreement with the accepted theory that the photoprotons from heavy nuclei are due to direct interactions and not to compound nucleus formation. This is also supported by the large W$^{186}(\gamma,n)$ cross section. The compound nucleus theory would predict that the ($\gamma,n$) cross section would fall rapidly to zero just above the ($\gamma,2n$) threshold, which for W$^{186}$ is 14 MeV. The fact that there is still a large cross section well above this threshold energy indicates that some nucleons are emitted by a direct interaction.

The fact that the peak in the ($\gamma,p$) cross section is at a lower energy than in Cs and I (see 2.5.2) is consistent with the idea that the absorption of photons in this region is due to 3-quantum transitions*. The idea that 3-quantum transitions play an important part in explaining the region above the giant resonance is discussed in chapter 5, where the comparison of measured integrated ($\gamma,p$) cross sections for heavy nuclei is also made.

*For the definition of 3-quantum transitions see 5.1
5.1 Introduction

In considering the nuclear E1 giant resonance little attention has been paid to the question of higher resonances than the first. Actually higher E1 resonances, of exactly the same character as the first but of smaller intensity, should exist on both the classical (Da51) and shell model calculations (Wi56). In the classical model of vibrating fluids, the ratio of the first overtone to the fundamental frequency is:

\[ \frac{W_1}{W_0} = 2.86 \]

and the corresponding ratio of peak heights (Da51) is:

\[ \frac{\sigma_1}{\sigma_0} \approx 2\% \]

In the shell model picture E1 excitation may in principle promote nucleons to the next, 3rd next, 5th next ... shells in accord with parity conservation; such transitions will be called "one-quantum, three-quantum, five-quantum ... jumps."

For the ideal harmonic oscillator (I.H.O.) the associated frequencies are in the ratio:

\[ W_1 : W_3 : W_5 : \cdots = 1 : 3 : 5 : \cdots , \]

while the corresponding matrix elements are in the ratio 1 : 0 : 0 ... Of course nuclei are not ideal harmonic oscillators, but explicit
calculations for a square well (Wi56) suggest that
\[ \oint \sigma_3 \, dw / \oint \sigma_1 \, dw \approx 3\%. \] These estimates of high frequency and low cross section have apparently discouraged interpretation of any experimental data in terms of, say, 3-quantum jumps.

Although the overlap integrals for 3-quantum transitions may be small relative to those of the 1-quantum, the actual transition strengths summed over all transitions are greatly enhanced for 3-quantum transitions. This is due to two effects: (i) The transition strength is proportional to the transition energy and using a simple classical model, the 3-quantum transitions are increased by a factor of three relative to the 1-quantum transitions. (ii) Due to the Pauli principle there are more possible 3-quantum than 1-quantum transitions. This increases the summed 3-quantum transition strength by a factor of two for light nuclei and nearly three for heavy nuclei.

The object here is to suggest that the earlier estimates of frequency and cross section should be substantially modified and that 3-quantum jumps provide an interpretation of the data presented in chapters 2, 3 and 4.

5.2 The Independent Particle Model

The nuclear photoeffect has been successfully explained using a model in which the photon energy is absorbed by a single nucleon which is raised to a higher energy level by an E1
transition. This model only considers shell model states and the transition strengths of all the E1 transitions (including 3-quantum, 5-quantum ... ) can be evaluated by the following equation:

\[ \sigma_{\ell \rightarrow \ell \pm 1} = (E_f - E_i) F_{j, \ell} T D_\ell R^2 \]  

(1)

where

\[ E_f - E_i = \text{the transition energy}, \]
\[ F_{j, \ell} = \text{spin orbit coupling enhancement factor}, \]
\[ T = \begin{cases} N/A \text{ for proton transition} \\ Z/A \text{ for neutron transition} \end{cases} \text{ effective charge}, \]
\[ D_\ell R^2 = \text{square of the radial overlap integral for dipole transitions, and} \]
\[ D_\ell R^2 = \int_0^{\infty} \psi_i r^3 \psi_f dr \]

where \( \psi_i \) and \( \psi_f \) are the initial and final state wave functions.

The integrated absorption cross section is obtained by summing over all possible E1 transitions:

\[ \int_0^{\infty} \sigma \, dE = \sum \sigma_{\ell \rightarrow \ell \pm 1} \]  

(2)

To evaluate \( \frac{1}{2} D_\ell R \), Wilkinson (Wi56) assumed that the nuclear potential was an infinite square well, in which the radial part of the wave function is a spherical Bessel function. Although this potential is unrealistic it has the advantage of simplicity, in that the excited state wave function is known and therefore
the energy of the transition is fixed. The effect of using a finite square well was investigated by Rand (Ra57), who found that the 1-quantum transitions are approximately the same as those obtained from the infinite square well and that the energy levels are a little more compact. The problem of 3-quantum transition strengths was not investigated for the finite well, but was assumed to be similar to the infinite well square/potential.

Wilkinson tabulated the values of $D_1R^2$ and these show that the strength of a 3-quantum overlap integral is about 0.4% of the corresponding 1-quantum transition. This means that the integrated cross section for 3-quantum transitions can account for about 3% of the total absorption cross section. The energy of these transitions will be approximately three times the giant resonance energy i.e. 40-60 MeV. On these results the 3-quantum transitions would be unimportant, especially in the region just above the giant resonance (20-30 MeV).

Wilkinson also discussed the problem of nucleon emission. After the nucleus has absorbed the photon and become excited to a higher level, which may or may not be in the continuum, then it can either be emitted "directly" or be reabsorbed in the nucleus to form a compound nucleus. The relative probability
of direct emission is given by:

\[ C = \frac{2kP}{2W} \frac{n^2}{MR} \tag{3} \]

where \( k \) is the nucleon wave number, \( \frac{n^2}{MR} \) the single particle reduced width, \( P \) the penetrability and \( W \) the imaginary part of the cloudy crystal ball potential.

### 5.3 E1 Peak Energies

Since the giant resonance is explained by electric dipole transitions, it follows logically that the energy of the E1 transitions should be the same as the energy of the giant resonance. However if the level positions are simply computed using \( R = 1.2A^{-\frac{1}{3}} \times 10^{-13} \) cm and adjusting the well depth to give the correct binding energy for the last nucleon it is found that the E1 transitions are too small by a factor of two to account for the giant resonance energy. In order to overcome this difficulty, Wilkinson (Wi56) suggested introducing a velocity dependent potential. Brueckner (Br54) has shown that the saturation of nuclear forces can be satisfactorily explained by the introduction of such a potential and that for infinite matter, this is equivalent to using normal forces with an effective mass \( m^* \), such that \( m^*/m \approx \frac{1}{2} \). If this effective mass \( m^*/m = \frac{1}{2} \), is assumed to be valid for finite matter, then the E1 transition energies are doubled and they agree with the giant resonance energy except for light nuclei.
However, some recent (d,p) measurements by Cohen et al. (Co60) have made the validity of the assumption that $m^*/m = \frac{1}{2}$ for finite nuclei doubtful. Their measurements, which are the first to reliably determine the energy spacings of E1 transitions, show the level spacing to be much smaller than is predicted by the Wilkinson model. The transition energies are consistent with those predicted using a value of $m^*/m = 1$ instead of the $\frac{1}{2}$ required to explain the giant resonance. This does not necessarily exclude the use of velocity dependent potentials, but suggests that the extension of Brueckner's theory for infinite matter to finite matter increases the value of $m^*/m$. If the results of Cohen et al. are accepted, then the Wilkinson model must be either discarded or modified to explain the position of the giant resonance using $m^*/m = 1$. The success of the model in all respects other than the use of an effective mass $<1$, suggests that a simple modification should be possible.

A possible explanation of the discrepancy between the giant resonance energy and the single particle levels has been given by Brown et al. (Br59) in terms of particle-hole interactions. Carver and Peaslee (Ca60) used sum rule calculations to show how the giant resonance position can be explained without invoking an effective nucleon mass less than one.

Carver and Peaslee have shown how the discrepancy between the separation of the single particle levels and the position
of the giant resonance can be explained by considering exchange forces. This is done by calculating the value of the harmonic mean energy $E_h$ defined by

$$E_h = I/I_h = \int \sigma \, dE / \int (\sigma / E) \, dE \tag{4}$$

The value of $E_h$ is then related to the energy of the giant resonance $E_g$, which experimental data suggests is nearly equal to $E_h$.

$$E_g \approx E_h >> |E_g - E_h| \tag{5}$$

The discussion is in terms of $E_h$ in preference to $E_g$ because $E_g$ depends strongly on the nuclear model invoked, while $E_h$ can be expressed in terms of sum rules and is considerably less model dependent. The sum rule expression for $I$ can be written in the form $I = I_o + I_x$, where $I_x$ is related to the nuclear exchange forces and depends on the form of the electric dipole operator. The corresponding $E_h$ is given by:

$$E_h = I_o/I_h + I_x/I_h = E_D + E_x \tag{6}$$

For the purpose of calculating the value of $E_h$, the wave function of the ground state was taken to be the ideal harmonic oscillator.

The results of the calculations show that $E_g$ can be expressed in the form:

$$E_g = (40 \pm 8)A^{1/3} + \Delta \quad \text{MeV} \tag{7}$$
where is to the first order independent of A. The first term of equation (7) is shown to correspond to the $E_D$ term in equation (6) and the $\Delta$ corresponds to $E_x$. The value of $\Delta$ is determined from experimental data and is $7.5 \pm 1.5$ MeV. Equation (7) then becomes

$$E_g = (40 \pm 8)A^{-\frac{1}{3}} + (7.5 \pm 1.5) \text{ MeV}$$

(8)

The interpretation of equation (8) is that the first term gives the single particle spacings for ordinary forces, and the second term, which is peculiar to the E1 mode of excitation, is the constant (A-independent) shift produced by exchange forces. Since this term is due to interactions between the excited nucleon and the remaining nucleons, the energy will not in general be available to the excited nucleon, but will be distributed amongst all the nucleons. This does not exclude the possibility that the energy $\Delta$ is concentrated in the excited nucleons in a portion of the absorptions. The result of equation (8) shows how the difference between the single particle transition energies and the position of the giant resonance can be satisfactorily explained.

The estimate of $\Delta$ should be interpreted as an average value. The actual value varies from nucleus to nucleus and may vary for different nucleons in the one nucleus. The calculations are not sufficiently detailed to obtain reliable estimates of the variation in $\Delta$. 
The value of $E_g$ obtained by Carver and Peaslee in equation (8) can be compared with the values obtained by Levinger (Le55). He found, using the harmonic oscillator potential, that

$$E_n = 46(1+0.8x)A^{-\frac{1}{3}} \text{ MeV}$$  \(9\)

where $x$ is the amount of exchange force. The result has been changed to allow for the different assumed nuclear radii.*

The first term, which is the harmonic energy given by ordinary forces, agrees with the value obtained by Carver and Peaslee. The exchange force term has a different $A$ dependence. The experimental results favour the expression obtained by Carver and Peaslee (equation (8)) in preference to equation (9).

In view of the success of the new calculations (by Carver and Peaslee) in evaluating the giant resonance position, it is of interest to investigate the implications of the results to see whether other photodisintegration results support this interpretation of the giant resonance. The results of this investigation are given in the following sections.

5.4 The Energy of the Overtones

The interpretation of the giant resonance as being due to 1-quantum transitions means that the energy of these

*The value of $E_n$ is calculated as a function of $R$, the radius of the nucleus, which is proportional to $A^{-\frac{1}{3}}$. Carver et al. assumed $A^{-\frac{1}{3}} = 1.1R$, while Levinger used $A^{-\frac{1}{3}} = 1.2R$. 
transitions \((W_1)\) can be written in the form

\[ W_1 = E_g \]

which from equation (8) gives

\[ W_1 = 40A^{3/2} + 7.5 \text{ MeV} \]  \(\text{(10)}\)

The energy of the 3-quantum transitions \((W_3)\) is not so easily estimated. Using similar arguments to those used to determine \(E_g\), the expression for \(W_3\) will be of the form

\[ W_3 = 40aA^{3} + \delta \] \(\text{(11)}\)

where the term \(\delta\) corresponds to the \(\Delta\) in equation (7). The classical model of vibrating fluids (Da51) would indicate a value of \(a=2.86\), while the ideal harmonic oscillator gives \(a=3\). The value of \(a\) will be taken as

\[ 40a = 120 \] \(\text{(12)}\)

The evaluation of \(\delta\) is difficult because it cannot be measured experimentally like \(\Delta\). For a 3-quantum transition the mean momentum difference between states is about three times that for a 1-quantum transition; a smooth dependence of the exchange potential on the momentum transfer would suggest a reduction in \(\Delta\) by a factor of two or three (Ca60). It is therefore proposed to take a value of \(\delta = 3 \pm 2 \text{ MeV}\) as a first estimate.

If it is accepted that the exchange term decreases as the transition energy increases, then for higher than 3-quantum transitions the exchange term can be ignored as it will always
be less than the error in the $A^{-\frac{3}{2}}$ term. The energy of the 5th and 7th quantum transitions can be written as $200A^{-\frac{3}{2}}$ and $280A^{-\frac{3}{2}}$ respectively.

5.5 Overlap Integrals

We consider the values of the overlap integral

$$D^2_{R} = \int_{0}^{\infty} \psi_f^* r^2 \psi_i \, dr$$  \hspace{1cm} (13)

from which the E1 transition strengths can be calculated using equation (1). The values of $D^2_{R}$ have been calculated for two different nuclear potentials: (a) the square well potential, and (b) a modified harmonic oscillator potential. These particular potentials were easy to handle and have a certain amount of flexibility that is not available in the infinite square or ideal harmonic oscillator potentials.

5.5.1 The Square Well Potential

This nuclear potential is basically the same as the infinite square well with one important modification. The wave function will not be restricted to zero values at the nuclear boundary as required by the infinite square well. We shall ignore the contribution to the overlap integral of the wave function outside the nuclear surface. Although the general formula for $D^2_{R}$ is derived, only one specific case is studied in detail, and this is when the ground state wave functions are in fact zero at the surface. In this case the potential
can be represented by two infinite square wells: one for the
ground state and one for the excited states where the radii of
the square wells differ.

The wave functions for the above potential can be written
in the form:

\[ \psi = A_l j_l (ar) T^m_l (\theta \phi) \quad r < R \quad (14) \]

where \( j_l (ar) \) is the spherical Bessel function of order \( l \),
\( T^m_l \) is the normalised spherical harmonic of order \( (l, m) \), \( a \)
is a factor with the dimensions of wave number and is determined
by the boundary conditions, and \( A \) is a normalising factor
such that

\[ \int \psi^2 \, d\tau = 1 \quad (15) \]

The value of \( D^R_l \) is given by:

\[ D^R_l = \int_0^\infty \psi_f r \psi_i \, d \quad (16) \]

where \( \psi_i \) and \( \psi_f \) are the initial and final state wave functions.

By integrating over \( \theta \) and \( \phi \) and defining \( k_l(x) \) as \( x j_l(x) \)

\[ D^R_l = \int_0^R \int_0^\pi k_l (ar) r k_l (ar) \, dr \, d\theta \, d\phi \quad (17) \]

\[ = I_l / N_l N_{l'} \quad (18) \]

where \( l' = l + 1 \)
The evaluation of equation (18) is given in Appendix III and the final result is:

\[ D_{l}^{R} = \left[ \frac{2y^2}{x^2 - y^2} k_{\ell}(x) k_{\ell-1}(y) - \frac{2xy}{x^2 - y^2} k_{\ell-1}(x) k_{\ell}(y) - \right. \\
\left. \quad y k_{\ell}(x) k_{\ell}(y) + x k_{\ell-1}(x) k_{\ell+1}(y) \right] \left( \frac{2R}{x^2 - y^2} \right)^{1/2} \\
\left[ k_{\ell}^2(x) - k_{\ell-1}(x) k_{\ell+1}(x) \right]^{1/2} \left[ k_{\ell+1}(y) - k_{\ell}(y) k_{\ell+2}(y) \right]^{1/2} \] (19)

where

\begin{align*}
x &= aR \\
y &\quad = bR \quad \text{for a } l \to l+1 \text{ transition}
\end{align*}

and

\begin{align*}
x &= bR \\
y &\quad = aR \quad \text{for a } l \to l-1 \text{ transition}.
\end{align*}

To attach any physical meaning to the evaluation of equation (19), boundary conditions must be applied to fix the values of \( x \) and \( y \). An investigation using different values of \( x \) and \( y \), showed that the only factor that plays a significant part in the value of \( D_{l}^{R} \), is the phase difference between the final and initial wave functions at the surface. Therefore the phase of either wave function at the surface can be arbitrarily chosen and for convenience the initial wave
The function is made zero at the boundary. To simplify the discussion only $\ell \to \ell + 1$ transitions will be considered, the modification for $\ell \to \ell - 1$ transitions being obvious from equation (19). The above boundary condition can then be expressed by the relation

$$k(x) = 0$$

(20)

Applying this to equation (19)

$$\frac{1}{2} \frac{2xR}{x^2 - y^2} \left[ \frac{-2x}{x^2 - y^2} k_\ell(y) + k_{\ell+1}(y) \right] \sqrt{\frac{k_{\ell+1}^2(y) - k_\ell(y) k_{\ell+2}(y)}{2}}$$

(21)

The boundary condition given by equation (20) only defines the value of the initial state and the conventional notation (Fe55) for the value of the $n$ quantum number will be used. That is the wave function for a state $(n, \ell)$ will have $n-1$ nodes between the centre and the surface of the nucleus. Although at this stage the value of $y$ will not be fixed, it is necessary to limit the range over which it can vary. The phase of the excited wave function will be permitted to vary $\pm \pi/2$ from the zero phase, which is the phase that makes the wave function zero at the boundary. This fixes the values of $y$ for a state $n$, to those between the $n^{th}$ and $n^{th} + 1$ zero of $\frac{d}{dy} k_{\ell+1}(y)$.

To evaluate $\frac{1}{2} \frac{2xR}{x^2 - y^2}$ well for an infinite square well it is necessary to make both the wave functions zero at the surface of the nucleus.
This condition is given by the relations: \( k_\ell(x) = 0 \),
\( k_{\ell+1}(y) = 0 \). Putting these in equation (19)

\[
\frac{\alpha}{D_{\ell}^2 R} = \frac{4xyR}{(x^2 - y^2)^2}
\]  

(22)

where \( x \) and \( y \) are the zeros of \( k_\ell(x) \) and \( k_{\ell+1}(y) \) respectively.

Figure 5.1 shows the values of \( D_{\ell} \) given by equation (21)
for specific values of \( x \) and a range of \( y \) values. Each plot
gives the range of values of \( D_{\ell} \) for 1-quantum, 3-quantum, and
5-quantum transitions corresponding to a given initial state.
The values of \( D_{\ell} \) at the points marked A correspond to the values
obtained using an infinite square well potential.

There are two facts which are immediately obvious. Firstly,
the values of \( D_{\ell} \) for the infinite square well are near maximum
for the 1-quantum transitions and near zero for the 3-quantum
transitions. Secondly, by suitably choosing the phase of the
excited wave function (i.e. value of \( y \)), the ratio of 3-quantum
to 1-quantum transition strengths can vary from 0 to \( \pm 1 \). These
results indicate that the value of \( D_{\ell} \) will strongly depend on
the phase of the wave function at the boundary. Therefore, the
choice of a particular model which will explicitly evaluate \( D_{\ell} \)
(i.e. harmonic oscillator, infinite square well), may give values
of \( D_{\ell} \) which are misleading.

Since the relative strengths of the 1-quantum and 3-quantum
transitions are dependent on the choice of \( y \), for which there is
no reliable physical basis for guidance, the value of \( D_y \) is averaged over all possible values of \( y \). The results of the calculations for a few typical transitions are shown in table 5.1 together with those obtained using an infinite square well potential. It can be seen from these results that the use of the above potential to evaluate \( D_y \) compared with the infinite square well potential, increases 3-quantum transitions by a factor of 10-20 and reduces the 1-quantum transitions by 10-30%.

In these calculations we have made no specific mention of the energy of the transition. In the original wave function the energy of the level and hence the energy of the transition, is given by the arbitrary parameter. The effect of averaging over \( y \) in equation (21) makes the determination of the transition energy meaningless and it is therefore not intended to use the results to determine the transition energies. Because of this uncertainty in the transition energy, only harmonic cross sections will be discussed. These have the advantage that they are proportional to the square of the overlap integrals and are independent of the transition energy. Also the integrated harmonic cross section can be measured more accurately than the integrated cross section.
Values of $D_j$. The values of $D_j$ for transitions with the ground states (1,0), (3,0) and (2,1) are given by equation (21). The values of $D_j$ for the transitions with the ground state (1,1) are obtained from equation (19) by interchanging $X$ and $Y$ and then putting $k_{f+1}(x) = 0$. The points marked A give the values of $D_j$ for an infinite square well.
Table 5.1

VALUES OF $D_{t}$

<table>
<thead>
<tr>
<th>Transition</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s $\rightarrow$ 1p</td>
<td>0.21</td>
<td>0.28</td>
</tr>
<tr>
<td>$\rightarrow$ 2p</td>
<td>0.02</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rightarrow$ 3p</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>1p $\rightarrow$ 1d</td>
<td>0.29</td>
<td>0.38</td>
</tr>
<tr>
<td>$\rightarrow$ 2d</td>
<td>0.019</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rightarrow$ 3d</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>1h $\rightarrow$ 1i</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td>$\rightarrow$ 2i</td>
<td>0.035</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rightarrow$ 3i</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>2p $\rightarrow$ 2d</td>
<td>0.19</td>
<td>0.28</td>
</tr>
<tr>
<td>$\rightarrow$ 3d</td>
<td>0.022</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rightarrow$ 4d</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>3s $\rightarrow$ 3p</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>$\rightarrow$ 4p</td>
<td>0.02</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rightarrow$ 5p</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>1p $\rightarrow$ 2s</td>
<td>0.68</td>
<td>0.10</td>
</tr>
<tr>
<td>$\rightarrow$ 3s</td>
<td>0.008</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\rightarrow$ 4s</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

Column A is obtained from equation (21) by averaging over all possible values of $y$.

Column B is the value for an infinite square well obtained from equation (22).
5.5.2 The Modified Harmonic Oscillator

The ideal harmonic oscillator is modified by assuming that the ground state nucleons and the excited state nucleons move in slightly different harmonic oscillator potentials.

The ground state wave function can be written in the form*

\[ \psi_0 = N_0 r^l e^{-r^2/2a^2} \]  

where \( N_0 \) is determined by

\[ \int_0^\infty \psi_0^2 r^{2l} dr = 1 \]  

The excited state wave function for a 1-quantum, 3-quantum, 5-quantum ... transition can be written in the form:

\[ \psi_1 = N_1 r^{l+1} e^{-r^2/2b^2} \]  

\[ \psi_3 = N_3 r^{l+1} (1 + cr^2/b^2) e^{-r^2/2b^2} \]  

\[ \psi_5 = N_5 r^{l+1} (1 + dr^2/b^2 + fr^4/b^4) e^{-r^2/2b^2} \]

Since \( \psi_1, \psi_3, \) and \( \psi_5 \) are wave functions in the same harmonic potential,

\[ \int_0^\infty \psi_3 \psi_1 r^2 dr = 0 \]  

\[ \int_0^\infty \psi_5 \psi_1 r^2 dr = 0 \]  

*Only transitions of the type \( l = N \rightarrow l+1, N+1 \) of the ideal harmonic oscillator are considered. However, it is these transitions which make the major contribution to the cross section.
\[ \int_0^\infty \psi_3 \psi_5 r^2 \, dr = 0 \quad (29b) \]

\( N_1, N_3 \) and \( N_5 \) are determined by normalising equations similar to equation (24) and \( c, d \) and \( f \) are determined from equations (28) and (29).

The values of \( \frac{\hbar}{E} R \) are given by:

\[ H_1^1 = \int_0^\infty \psi_1 \psi_0 r^3 \, dr \quad \text{1-quantum} \quad (30) \]

\[ H_3^3 = \int_0^\infty \psi_3 \psi_0 r^3 \, dr \quad \text{3-quantum} \quad (31) \]

\[ H_5^5 = \int_0^\infty \psi_5 \psi_0 r^3 \, dr \quad \text{5-quantum} \quad (32) \]

The solution of equations (30), (31) and (32) are given in Appendix IV, the results of which are:

\[ \int_0^\infty \psi_1 \psi_0 r^3 \, dr = a \sqrt{\ell+3/2} \left[ \frac{2ab}{a^2+b^2} \right]^{\ell+5/2} \quad (33) \]

\[ \int_0^\infty \psi_3 \psi_0 r^3 \, dr = a \sqrt{\ell+3/2} \sqrt{\ell+5/2} \left( \frac{2ab}{a^2+b^2} \right)^{\ell+5/2} \left( \frac{b^2-a^2}{b^2+a^2} \right) \quad (34) \]

\[ \int_0^\infty \psi_5 \psi_0 r^3 \, dr = \frac{1}{2} a \sqrt{\ell+3/2} \sqrt{\ell+5/2} \sqrt{\ell+7/2} \left( \frac{2ab}{a^2+b^2} \right)^{\ell+5/2} \left( \frac{b^2-a^2}{b^2+a^2} \right)^2 \quad (35) \]
Therefore the ratio of the squares of 1-quantum to 3-quantum overlap integrals \( \frac{H_3}{H_1} \) will be

\[
\frac{H_3}{H_1} = \left[ \int_0^\infty \psi_3 \psi_0 r^3 \, dr \right]^2 = (l+5/2) \left( \frac{b^2-a^2}{b^2+a^2} \right)^2
\]

(36)

\[
\frac{H_5}{H_1} = \frac{1}{2}(l+5/2)(l+7/2) \left( \frac{b^2-a^2}{b^2+a^2} \right)^4
\]

(37)

and the ratio \( \frac{H_{2n+1}}{H_1} \) can be obtained by inspection:

\[
\frac{H_{2n+1}}{H_1} = \frac{1}{n!} (l+5/2)(l+7/2) \ldots (l+\frac{2n+3}{2}) \left( \frac{b^2-a^2}{b^2+a^2} \right)^{2n}
\]

(38)

It can be seen from equations (34) and (35) that by putting \( b = a \), the results are the same as those obtained from an ideal harmonic oscillator, i.e. \( 0 = H_3 = H_5 = H_7 \ldots \) Since there have been no restrictions placed on \( a \) and \( b \), the value of \( \frac{H_3}{H_1} \) can take values between 0 and 2\( l+5 \). However the values of \( a \) and \( b \) determine the energy level spacing in the nucleus, and it is reasonable to expect that the excited levels have a similar spacing to the bound states. This would restrict the values of \( b \) to be near \( a \), which limits the values of \( \frac{H_3}{H_1} \) to smaller values than the 2\( l+5 \) given above.
5.5.3 Conclusions

The main conclusion that can be drawn from these results is that the strengths of the 3-quantum transitions may vary over a large range of values, and are not necessarily limited to the small values suggested by the infinite square well or harmonic oscillator potentials. The range of values can be reduced by imposing more stringent boundary conditions, but it is doubtful whether the structure of the nucleus is sufficiently well understood to justify a limitation of this kind.

Since the two potentials give a different $l$ dependence for the relative 3-quantum to 1-quantum overlap integrals, it was decided to take an average value of 7% for $D_l R^2$ for all transitions. It was also decided to use the infinite square well values less 20% for the 1-quantum overlap integrals. The 20% reduction is to allow for the transfer of cross section from the 1-quantum to the 3, 5 ...-quantum transitions. The value of 7% is the average value for the square well potential and corresponds to an average value of $b/a = 1.05$ in the harmonic oscillator potential.

It should be emphasised that these calculations are not intended to be a reliable calculation of transition strengths. They have been presented simply to indicate that it is possible to increase the contribution of 3-quantum transitions to at
least 30% of the E1 absorption cross section. It will be shown in 5.6 how the experimental results suggest that the 3-quantum transitions account for 25% of the total absorption cross section.

5.6 A Comparison of the Experimental Results with the Theoretical Results

It is intended to compare the two best known parameters of the \((\gamma,p)\) cross sections with the predictions of the calculations. The two parameters that will be considered are the peak energies and the harmonic cross sections. Only the middle and heavy nuclei in which it is expected that 3-quantum transitions will dominate the \((\gamma,p)\) cross sections, will be considered.

5.6.1 The Peak Energies

The peak energies are plotted in fig. 5.2. The result for Ta was not included, because it is doubtful whether the cross section shape measured from the decay of the isomeric level, is the same as for the total cross section (see 3.3.4). The two curves are the theoretical positions of the 1-quantum and 3-quantum transitions, the dotted line being an estimate of the effect of the coulomb barrier. It would appear from these results that the \((\gamma,p)\) cross section in the heavy nuclei may be attributed to 3-quantum transitions and in the light nuclei to 1-quantum transitions. For the middle weight nuclei with \(Z\) between 40 and 50, it appears that both 1-quantum and
Energy of the peak in the \((\gamma, p)\) cross section. The curves are the theoretical values of the 1-quantum \((E_1)\) and 3-quantum \((E_3)\) transition energies given by equations (8) and (11) respectively. The dotted line is a measure of the coulomb barrier effect. The Mo result was taken from the work of Ferrero et al. (Fe57), the Ni and Ti results were obtained by T.R. Sherwood, (private communication), and the other results were obtained by the author.
FIG. 5.2
3-quantum transitions may contribute to the \((\gamma,p)\) cross section. It is not expected that 1-quantum protons will be emitted from the heavy nuclei, because the interpretation of the \(\Delta\) term in the transition energy (see equations (7) and (8)) is, that it is not available for the emission of direct nucleons (see 5.3). This means that in heavy nuclei 1-quantum transitions will in general leave the excited nucleon in a bound state and therefore nucleon emission can only occur through a compound nucleus formation, where the entire transition energy is available.

In the light nuclei where proton emission can be explained by the formation of a compound nucleus, it would be expected that the 1-quantum transitions would be the dominant ones and the peak in the \((\gamma,p)\) cross section would be near the giant resonance energy.

It is interesting to note that the region \(Z = 40-50\), where both types of transitions contribute to the \((\gamma,p)\) cross section, is also the region where direct photoproton emission starts to become important.

Although the position of the \((\gamma,p)\) cross section in heavy nuclei suggests that there is a second resonance, there is not sufficient data to determine the value of \(\delta\) (the exchange force term).

5.6.2 The Harmonic Cross Sections

Figure 5.3 shows all the available integrated \((\gamma,p)\) harmonic
The integrated harmonic (Y,p) cross section expressed as a fraction of the harmonic absorption cross section integrated over the giant resonance. The line corresponds to a ratio of 5% for the square of the 3-quantum overlap integral to 1-quantum overlap integral. The results for Nb, In, Ta, Au have been taken from the results of Barber et al. (Ba60) and the Mo results from Ferrero et al. (Fe57).
cross sections (expressed as a fraction of the absorption harmonic cross section integrated over the giant resonance i.e. 1-quantum transitions) for nuclei with \( A \) greater than 90. The solid curve shown is the theoretical prediction obtained from the calculations in 5.4 and 5.5. Since these sections only discuss the problem of photon absorption, a simple shell model will be used to calculate the probability of proton emission.

In this model all levels within a closed shell are given the same energy, the energy separation of the shells is 7 MeV and the binding energy of the last shell is 7 MeV. The probability of an excited nucleon being directly emitted is given by equation (3) and \( W \) is taken to be 3 MeV. This value was obtained from the results of Lane and Wandel (La55)*.

The slope of the line, which is independent of the ratio of 3-quantum to 1-quantum overlap integrals, is calculated by summing over all possible transitions. The slope is determined by the number of possible E1 transitions and by the change in the coulomb barrier with the change in \( A \), both of which are relatively well known.

The magnitude of the line is determined by the ratio of the 3-quantum to 1-quantum overlap integrals. Since this factor

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* This is larger than the value of 1.5 MeV used by Wilkinson (Wi56).
cannot be accurately determined, the line in figure 5.3 was
drawn to fit the experimental points (keeping the correct slope)
and the ratio of the overlap integrals evaluated from it. The
actual line corresponds to an average ratio of 5% for the
3-quantum to 1-quantum overlap integrals squared.

Using this value of 5%, the contribution from 3-quantum
transitions to the total harmonic absorption cross section is
approximately 15%. Since the use of cross sections is more
general than the use of harmonic cross sections, the contribu-
tion of the 3-quantum transitions to the total absorption
cross section has been calculated from the result for the harmonic
cross section given above. This result indicates that 3-quantum
transitions contribute 25% of the total absorption cross section.
Although an oversimplified model is used in calculating the
results, it is found that a large fraction of the cross section
is due to protons being emitted from above or near the top of
the coulomb barrier, where the probability of direct emission
is insensitive to the position of the energy levels. The fact
that the model predicts a large fraction of the protons to be
emitted with energies greater than the coulomb barrier, is in
rough agreement with the experimental results (see fig. 2.6).
5.6.3 Other Experimental Evidence Concerning 3-Quantum Transitions

(a) The Total Absorption Cross Sections. - The total harmonic cross sections \( H_0 \) for three nuclei \( \text{Pr}^{141}, \text{Ni}^{58}, \text{Ta}^{181} \) were obtained by adding the \( (\gamma,n), (\gamma,2n), (\gamma,p) \) and \( (\gamma,np) \) harmonic cross sections together. Each resulting cross section was then divided into two parts: the giant resonance fraction, which was attributed to the 1-quantum transitions \( H_1 \), and the tail, which was taken to be due to 3-quantum transitions \( H_3 \). The ratios \( H_3/H_1 \) for the three nuclei were \( \text{Ta}^{181} (18\%), \text{Pr}^{141} (29\%) \) and \( \text{Ni}^{58} (25\%) \). These results would indicate a rather larger fraction of the cross section being due to 3-quantum transitions than is found from the \( (\gamma,p) \) cross sections. This difference may be due to the arbitrary manner in which the absorption cross section is divided into 1- and 3-quantum transitions. The experimental data was taken from the best available results (Ca59b, Sp58, Fu58, Pa59).

(b) The Fast Neutron Measurements. - Measurements of fast neutron emission (Au59; Fe57) indicate with some uncertainty that in the heavy nuclei, they account for 10% of the total neutron yield. If these neutrons are attributed entirely to direct emission from 3-quantum transitions, corrected for those neutrons forming a compound nucleus (40%), and multiplied by two to allow for the proton absorption, the total yield from 3-quantum
absorption is approximately 30% of the total absorption cross section. This result, although rather vague, is in general agreement with the estimates determined above.

5.7 Conclusions

There can be little doubt that 3-quantum transitions exist, but the problem is to determine how much effect they play in explaining photodisintegration results. The experimental evidence suggests that it may be of the order of 30% of the total absorption cross section. However, this evidence relies heavily on the interpretation of the exchange force term in the equation for the giant resonance energy. This interpretation, that the energy of the exchange term will not be available to promote single nucleons, is perhaps a logical interpretation, but it is a little too early to say it is the only interpretation.

If the absorption of photons above the giant resonance is to be predominantly due to 3-quantum transitions, it is necessary to increase the 3-quantum transition strengths by an order of magnitude from those calculated using the infinite square well potential. The calculations of transition strengths show that the results depend on the specific model chosen and even using one model, large variations in transition strengths may easily be obtained. However in the two specific models
considered it is seen that the 3-quantum transition strengths could be reasonably increased by an order of magnitude over the values obtained by an infinite square well potential.

It can be concluded from the results given in this chapter, that the interpretation of the absorption cross section in the region immediately above the giant resonance as being due to 3-quantum transitions, does not conflict with the experimental results. There is very little evidence that this interpretation gives better agreement to the photonuclear results, than say the model proposed by Wilkinson.

The primary objects of this chapter were to suggest that 3-quantum transitions may play an important role in explaining photonuclear results and to show that preliminary calculations do not produce any major conflicts between the theory and the experimental results.
APPENDIX I

ANALYSIS OF YIELD CURVES

OBTAINED FROM BREMSSTRAHLUNG RADIATION

It is evident that the yield $A(k_o)$ measured as a function of the maximum electron energy $k_o$, and the cross section $\sigma(k)$ of a photonuclear reaction are related by a Volterra equation of the form

$$A(k_o) = \int_T^{k_o} \sigma(k) P(k,k_o) \, dk$$

(1)

where $T$ is the threshold of the reaction and $P(k,k_o)$ is the gamma ray intensity.

If the R.H.S. of (1) is partially integrated

$$A(k_o) = P(k,k_o) \int_T^{k_o} \sigma(k) \, dk - \int_T^{k_o} \left[ P'(k_1,k_o) \int_T^{k_1} \sigma(k) \, dk \right] \, dk_1$$

(2)

where

$$P'(k,k_o) = \frac{d P(k,k_o)}{dk}$$

If the integral over $dk_1$ is replaced by a summation, successive values of the integral over $dk$ may be found from the corresponding values of $A(k_o)$ by induction; since all terms except the last in the summation are composed of known quantities. Using the Euler-McLaurin formula to evaluate the integral and denoting

$$\int_T^{k} \sigma(k) \, dk_1, \text{ by } S(k),$$
then

\[ S(k_0) = A(k_0) + \frac{13}{12} \hbar P'(k_0-h, k_0) S(k_0-h) + \]
\[ \frac{\hbar}{P(k_0, k_0)} P(k_0-h, k_0) \ldots P(k_0, k_0) + \frac{5}{12} \hbar P'(k_0, k_0) \] (3)

For the solution of (3) the formula for \( P(k, k_0) \) given by Schiff

\[ P(k, k_0) = \frac{1}{k} \left\{ \left[ 1 + \left( \frac{1-Z}{2} \right)^2 \right] \log a - \left( \frac{2-Z}{2} \right)^2 \right\} \]

was used, where

\[ Z = k/(k_0 + \mu) \]

\[ \mu \quad \text{electron mass} \]

\[ \frac{1}{a^2} = \frac{1}{a_1^2} + \frac{1}{a_2^2} \]

\[ a_1 = 2(k_0 + \mu)(1-Z)/\mu Z \]

\[ a_2 = 111/Z^{3/2} \]

\[ Z \quad \text{atomic number of bremsstrahlung radiator} \]

The author is indebted to D.W. Lang who programmed the solution of (3) on the electronic computer (Siliac) at the University of Sydney.
APPENDIX II

DEAD TIME CORRECTION FOR A DECAYING SOURCE

The problem of obtaining the dead time correction for a random (non-decaying) count-rate is trivial. If the observed count-rate is \( n \) and the true number of counts is \( N \), then the observed fraction of the true count which is recorded is

\[
\frac{n}{N} = 1 - \alpha n,
\]

where \( \alpha \) is the dead time of the recording instrument.

For a decaying source, \( N = N_0 \ e^{-\lambda t} \)

Then the number of counts in an interval \( T \) is

\[
K = \int_0^T N_0 \ e^{-\lambda t} \ dt
\]

\[
= \frac{N_0}{\lambda} \ (1 - e^{-\lambda T})
\]

The number of observed counts in this interval \( T \) is

\[
C = \int_0^T \frac{N_0 e^{-\lambda t}}{1 + \alpha N_0 e^{-\lambda t}} \ dt
\]

\[
= \frac{1}{a \lambda} \log \frac{1 + \alpha N_0}{1 + \alpha N_0 e^{-\lambda T}}
\]

\[
\therefore \ e^{\lambda a C} = \frac{(1 + \alpha N_0)}{(1 + \alpha N_0 e^{-\lambda T})}
\]

\[
\therefore \ N_0 = \frac{(e^{\lambda a C} - 1)}{(\alpha - e^{-\lambda T} e^{\lambda a C})}
\]

\[
\therefore \ K = \frac{(1 - e^{-\lambda T})(e^{\lambda a C} - 1)}{\lambda a (1 - e^{-\lambda T} e^{\lambda a C})}
\]
The dead time correction is given by

\[ \frac{K}{C} = \frac{e^{\lambda aC} - 1}{\lambda aC} \left[ 1 - \frac{e^{-\lambda T}}{1-e^{-\lambda T}} (e^{\lambda aC} - 1) \right]^{-1} \]

For a value of \( \lambda T = 0.937 \) and small values of \( \lambda aC \) the dead time correction approximates to:

\[ \frac{K}{C} = 1 + 1.144x + 0.655x^2 \quad (x = \lambda aC) \]
APPENDIX III

Evaluation of
\[ \int_0^R x k_{\ell+1}(\beta r) k_\ell(\alpha r) \, dr \]
\[ \int_0^R k_{\ell+1}^2(\beta r) \, dr \int_0^R k_\ell^2(\alpha r) \, dr \]

Let
\[ I_\ell = \int_0^R x k_{\ell+1}(\beta r) k_\ell(\alpha r) \, dr \quad (1) \]

Using the following standard recurrence formulae
\[ k_{n-1}(r) = \left( \frac{n}{x} + D \right) k_n(r) \quad (2) \]
\[ k_{n+1}(r) = \left( \frac{n+1}{x} - D \right) k_n(r) \quad (3) \]

where \( D = \frac{d}{dr} \) we obtain
\[ \left( D^2 \frac{n(n+1)}{x^2} + 1 \right) k_n(r) = 0 \quad (4) \]

From (3)
\[ k_{\ell+1}(\beta r) = \frac{1}{r} \left( \frac{\ell+1}{\beta} - \frac{d}{d\beta} \right) k_\ell(\beta r) \quad (5) \]

Therefore
\[ I_\ell = \left( \frac{\ell+1}{\beta} - \frac{d}{d\beta} \right) H_\ell \quad (6) \]

where
\[ H_\ell = \int_0^R k_\ell(\alpha r) k_\ell(\beta r) \, dr \quad (7) \]

which from (4)
\[ = \int_0^R k_\ell(\alpha r) \left[ \frac{\ell(\ell+1)}{x^2} \beta^2 - \frac{1}{\beta^2} \right] k_\ell(\beta r) \, dr \quad (8) \]

and
\[ = \int_0^R k_\ell(\beta r) \left[ \frac{\ell(\ell+1)}{\alpha^2} \beta^2 - \frac{1}{\alpha^2} \right] k_\ell(\alpha r) \, dr \quad (9) \]
From (8) and (9)
\[(\alpha^2 - \beta^2) H_{\ell} = \int_0^R \left[ k_{\ell}(\alpha r) D^2 k_{\ell}(\beta r) - k_{\ell}(\beta r) D^2 k_{\ell}(\alpha r) \right] dr\]
\[= \int_0^R D \left[ k_{\ell}(\alpha r) D k_{\ell}(\beta r) - k_{\ell}(\beta r) D k_{\ell}(\alpha r) \right] dr\]
\[= k_{\ell}(\alpha R) D k_{\ell}(\beta R) - k_{\ell}(\beta R) D k_{\ell}(\alpha R) \quad (10)\]

Using equation (5), equation (10) becomes
\[(\alpha^2 - \beta^2) H_{\ell} = \beta k_{\ell}(\alpha R) k_{\ell-1}(\beta R) - \alpha k_{\ell}(\beta R) k_{\ell-1}(\alpha R) \quad (11)\]

Therefore
\[I_{\ell} = \left( k_{\ell+1}(\beta) - \frac{d}{d\beta} \right) \left[ \beta k_{\ell}(\alpha R) k_{\ell-1}(\beta R) - \alpha k_{\ell}(\beta R) k_{\ell-1}(\alpha R) \right] \left( \frac{1}{\alpha^2 - \beta^2} \right) \quad (12)\]

From equations (2), (3) and (12),
\[I_{\ell} = - \frac{1}{\alpha^2 - \beta^2} \left[ \frac{2 \beta}{\alpha^2 - \beta^2} k_{\ell}(\alpha R) k_{\ell-1}(\beta R) - \frac{2 \alpha \beta}{\alpha^2 - \beta^2} k_{\ell-1}(\alpha R) k_{\ell}(\beta R) - \beta R k_{\ell}(\alpha R) k_{\ell}(\beta R) + \alpha R k_{\ell-1}(\alpha R) k_{\ell+1}(\beta R) \right] \quad (13)\]
putting \( x = \alpha R \) and \( y = \beta R \)

\[
I_\ell = \frac{-R^2}{x^2-y^2} \left[ \frac{2y^2}{x^2-y^2} k_\ell(x) k_{\ell-1}(y) - \frac{2xy}{x^2-y^2} k_{\ell-1}(x) k_\ell(y) - \right.
\]

\[
y k_\ell(x) k_\ell(y) + x k_{\ell-1}(x) k_{\ell+1}(y) \left. \right]\]  \hspace{1cm} (14)

similarly

\[
\int_0^R k^2_\ell(\alpha r) dr = \frac{1}{2} R \left[ k^2_\ell(x) - k_{\ell-1}(x) k_{\ell+1}(x) \right] \]  \hspace{1cm} (15)

and

\[
\int_0^R k^2_{\ell+1}(\beta r) dr = \frac{1}{2} R \left[ k^2_{\ell+1}(y) - k_\ell(y) k_{\ell+2}(y) \right] \]  \hspace{1cm} (16)

\[
\therefore D^{\frac{3}{2}}_\ell = \left[ \frac{2y^2}{x^2-y^2} k_\ell(x) k_{\ell-1}(y) - \frac{2xy}{x^2-y^2} k_{\ell-1}(x) k_\ell(y) - \right.
\]

\[
y k_\ell(x) k_\ell(y) + x k_{\ell-1}(x) k_{\ell+1}(y) \left. \right]\left( - \frac{2}{x^2-y^2} \right) \]

\[
\left[ k^2_\ell(x) - k_{\ell-1}(x) k_{\ell+1}(x) \right]^{\frac{1}{2}} \left[ k^2_{\ell+1}(y) - k_\ell(y) k_{\ell+2}(y) \right]^{\frac{1}{2}} \]  \hspace{1cm} (17)

**Boundary conditions**

i) \( k_\ell(x) = 0 \)

Therefore

\[
k_{\ell-1}(x) = -k_{\ell+1}(x) \]  \hspace{1cm} (18)

\[
D^{\frac{3}{2}}_\ell = \left[ \frac{-2x}{x^2-y^2} k_\ell(y) + k_{\ell+1}(y) \right] \left[ k^2_{\ell+1}(y) - k_\ell(y) k_{\ell+2}(y) \right]^{\frac{1}{2}} \]  \hspace{1cm} (19)
Boundary conditions (cont.)

ii) \( k_{\ell}(x) = 0, k_{\ell+1}(y) = 0 \)

\[
\frac{D^3}{k} = \frac{-4xy}{(x^2-y^2)^2}
\]

(20)

iii) \( k_{\ell+1}(y) = 0 \)

\[
\frac{D^3}{k} = \frac{-2y}{x^2-y^2} \left[ \frac{2x}{x^2-y^2} k_{\ell+1}(x) - k_{\ell}(x) \right] \left[ k_{\ell}(x)^2 - \frac{k_{\ell-1}(x) k_{\ell+1}(x)}{2} \right]^{\frac{1}{2}}
\]

(21)
i) **Normalising Integrals**

\[ 1 = \int_0^\infty \psi_0^2 r^2 \, dr \]

\[ = \int_0^\infty N_0^2 r^{2\ell} e^{-r^2/a^2} r^2 \, dr \]

\[ = N_0^2 a^{2\ell+3} \left( \frac{\sqrt{\pi}}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \cdots \cdot \frac{2\ell+1}{2} \right) \]

by partial integration

\[ \therefore N_0 = a^{-(\ell+3/2)} N_\ell \]

where \( N_\ell = \left( \frac{\sqrt{\pi}}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \cdots \cdot \frac{2\ell+1}{2} \right)^{-1/2} \)

\[ \psi_0 = N_\ell a^{-(\ell+3/2)} r^\ell e^{-r^2/2a} \]

similarly

\[ \psi_1 = N_{\ell+1} b^{-(\ell+5/2)} r^{\ell+1} e^{-r^2/2b} \]

\[ \psi_3 = N_{\ell+1} b^{-(\ell+5/2)} r^{\ell+1} \left( 1 + \frac{\alpha r^2}{b^2} \right) e^{-r^2/2b^2} \left( \frac{2}{2\ell+5} \right)^{1/2} \]

\[ \psi_5 = N_{\ell+1} b^{-(\ell+5/2)} r^{\ell+1} \left( 1 + \frac{\alpha r^2}{b^2} + \frac{\beta r^4}{b^4} \right) \]

\[ \quad e^{-r^2/2b^2} \left( \frac{2}{2\ell+5} \cdot \frac{2}{2\ell+7} \right)^{1/2} \]

ii) **Determination of \( c, d \) and \( f \).**

From the orthogonal properties of the excited state wave functions

\[ 0 = \int_0^\infty \psi_1 \psi_3 r^2 \, dr \]
\[ 0 = N_1N_0 \int_0^{\infty} r^{l+1} e^{-r^2/2b^2} \left( 1 + \frac{cr^2}{b^2} \right) e^{-r^2/2b^2} r^2 dr \]

\[ = \frac{a}{b^2} \int_0^{\infty} r^{2l+6} e^{-r^2/b^2} dr + \int_0^{\infty} r^{2l+4} e^{-r^2/b^2} dr \]

\[ = \frac{a}{b^2} N_{l+2}^2 b^{2l+7} + N_{l+1}^2 b^{2l+5} \]

\[ = N_{l+1}^2 b^{2l+5} \left[ \frac{(2l+5)}{2} + 1 \right] \]

Therefore

\[ c = -\frac{2}{2l+5} \]

Similarly

\[ d = -\frac{4}{2l+5} \]

\[ f = \frac{4}{(2l+5)(2l+7)} \]

iii) Overlap Integrals \( H_{\frac{1}{2}} \)

\[ H_{\frac{1}{2}} = \int_0^{\infty} \psi_1 \psi_0 r^3 dr \]

\[ = \int_0^{\infty} N_1N_0 r^{l+1} e^{-r^2/2b^2} r e^{-r^2/2a^2} r^3 dr \]

\[ = N_1N_0 \int_0^{\infty} r^{2l+4} e^{-r^2/A^2} dr \]

where

\[ \frac{2}{A^2} = \frac{1}{a^2} + \frac{1}{b^2} \]
\[ H_{1}^{1/2} = \frac{N_{l+1} N_{l}}{N_{l+1}^{2}} a^{-(l+3/2)} b^{-(l+5/2)} \frac{1}{N_{l+1}^{2/2}} A^{2l+5/l+1} \]

\[ = \frac{a}{N_{l+1}^{2}} \left( \frac{ab}{a^{2}} \right)^{-(l+5/2)} \]

\[ = a \sqrt{2l+3} \left( \frac{2ab}{a^{2}+b^{2}} \right)^{l+5/2} \]

Similarly

\[ H_{3}^{1/2} = a \sqrt{l+3/2} \left( \frac{2ab}{a^{2}+b^{2}} \right)^{l+5/2} \sqrt{l+5/2} \left( \frac{b^{2}-a^{2}}{b^{2}+a^{2}} \right) \]

and

\[ H_{5}^{1/2} = a \sqrt{l+3/2} \left( \frac{2ab}{a^{2}+b^{2}} \right)^{l+5/2} \frac{1}{2} \sqrt{(l+5/2)(l+7/2)} \left( \frac{b^{2}-a^{2}}{b^{2}+a^{2}} \right)^{2} \]

Therefore

\[ H_{3}/H_{1} = (l+5/2) \left( \frac{b^{2}-a^{2}}{b^{2}+a^{2}} \right)^{2} \]

\[ H_{5}/H_{1} = \frac{1}{2} (l+5/2)(l+7/2) \left( \frac{b^{2}-a^{2}}{b^{2}+a^{2}} \right)^{4} \]

\[ H_{2n+1}/H_{1} = \frac{1}{n!} (l+5/2)(l+7/2) \ldots (l+\frac{2n+3}{2}) \left( \frac{b^{2}-a^{2}}{b^{2}+a^{2}} \right)^{2n} \]
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