Three-Dimensional Entanglement: Knots, Knits and Nets

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Preface

I declare that the research presented in this thesis is my original work. Parts of this research however, are done in collaboration with others. Firstly, some of the research in Ch. 2, titled "Free Tilings of the Hyperbolic Plane", was undertaken in collaboration with Dr Vanessa Robins, Mr Stuart Ramsden and Prof. Stephen T. Hyde. Specifically, § 2.1 which covers the extension of Delaney-Dress tiling theory to free tilings was done in collaboration and the remainder of the Chapter is my original work. Secondly, the development of the ideas presented in Ch. 5, "Dilatancy of Woven Filament Arrays", was undertaken in collaboration with Hyde. I also declare that the length of this thesis is under 100,000 words. Further, I have rendered all images in this thesis, except where otherwise referenced.

Myfanwy E. Evans

Acknowledgements

"We must not forget that when radium was discovered no one knew that it would prove useful in hospitals. The work was one of pure science. And this is a proof that scientific work must not be considered from the point of view of the direct usefulness of it. It must be done for itself, for the beauty of science, and then there is always the chance that a scientific discovery may become, like the radium, a benefit for humanity."

– Marie Curie (1867 - 1934), Lecture at Vassar College, 1921.

To my supervisors and mentors, Stephen Hyde and Vanessa Robins, I certainly owe you both a beer. Your contributions to my work so far and to my future work is unprecedented. You have taught me the "beauty of science" that our old friend Marie talks about above, through countless days of discussion, feedback and occasional disagreements, and that will stay with me forever. I hope that you both enjoy reading this final version! Significant contributions to this thesis have also come from both Toen Castle and Stuart Ramsden: thanks to comrade Toen for a wonderful collaborative environment, and to Stu for his help and encouragement with 3D animation. I hope you both enjoy this 'colouring-in book' thesis!

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The Applied Mathematics department has always been a delightful environment in which to work, thanks to you all for help, seminars, questions and feedback, many afternoons at the pub and great excursions to Kioloa. Further, a big thanks to all of my friends and foes... I know you will never read this thesis, but maybe you will make it far enough to get to this page. And to my family... this thesis is for you all, I would not be here without you (both literally and metaphorically)!

Abstract

Three-dimensional entanglement, including knots, periodic arrays of woven filaments (weavings) and periodic arrays of interpenetrating networks (nets), forms an integral part of the analysis of structure within the natural sciences. This thesis constructs a catalogue of 3-periodic entanglements via a scaffold of Triply-Periodic Minimal Surfaces (TPMS). The two-dimensional Hyperbolic plane can be wrapped over a TPMS in much the same way as the two-dimensional Euclidean plane can be wrapped over a cylinder. Thus vertices and edges of free tilings of the Hyperbolic plane, which are tilings by tiles of infinite size, can be wrapped over a TPMS to represent vertices and edges of an array in three-dimensional Euclidean space. In doing this, we harness the simplicity of a two-dimensional surface as compared with 3D space to build our catalogue.

We numerically tighten these entangled flexible knits and nets to an ideal conformation that minimises the ratio of edge (or filament) length to diameter. To enable the tightening of periodic entanglements which may contain vertices, we extend the Shrink-On-No-Overlaps algorithm, a simple and fast algorithm for tightening finite knots and links. The ideal geometry of 3-periodic weavings found through the tightening process exposes an interesting physical property: *Dilatancy*. The cooperative straightening of the filaments with a fixed diameter induces an expansion of the material accompanied with an increase in the free volume of the material. Further, we predict a dilatant rod packing as the structure of the keratin matrix in the corneocytes of mammalian skin, where the dilatant property of the matrix allows the skin to maintain structural integrity while experiencing a large expansion during the uptake of water.

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