Essays on Bond Yields

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Declaration

The work in this thesis is my own except where otherwise stated.

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Vijay Austin Murik
For my parents
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Contents

Abstract viii

1 Introduction 1

2 Bond pricing with a surface of zero coupon yields 8
  2.1 Introduction ........................................... 8
  2.2 Zero coupon yield surface .............................. 11
  2.3 Risk premia in Australian bond yields .................. 14
  2.4 Conclusion ............................................ 23

3 Conditional tests of monotonicity in term premia 28
  3.1 Introduction ........................................... 28
  3.2 Determinants of term premia ........................... 31
  3.3 Monotonicity tests ................................... 33
  3.4 Monotonicity in U.S. term premia ..................... 36
  3.5 Conclusion ............................................ 43

4 Measuring monetary policy expectations 50
  4.1 Introduction ........................................... 50
  4.2 Measuring expectations ................................. 53
  4.3 The fixed income universe ............................. 57
  4.4 Expectations in Australian bond pricing ............... 58
  4.5 Conclusion ............................................ 72

5 Conclusion 74

References 77
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Descriptive statistics, surface fitting errors (per cent)</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Descriptive statistics, sample term premia (per cent)</td>
<td>38</td>
</tr>
<tr>
<td>3.2</td>
<td>Correlation matrix: excess returns and factors</td>
<td>39</td>
</tr>
<tr>
<td>3.3</td>
<td>Monotonicity test ( p )-values, by conditioning factor</td>
<td>40</td>
</tr>
<tr>
<td>3.4</td>
<td>Empirical power: Frequencies of monotonicity outcome in tests</td>
<td>42</td>
</tr>
<tr>
<td>4.1</td>
<td>Forecast error descriptive statistics</td>
<td>62</td>
</tr>
<tr>
<td>4.2</td>
<td>OLS estimates for equation (4.1)</td>
<td>64</td>
</tr>
<tr>
<td>4.3</td>
<td>GMMIV estimates for moment conditions (4.3)</td>
<td>68</td>
</tr>
<tr>
<td>4.4</td>
<td>Forecast accuracy, GMMIV model (4.3)</td>
<td>70</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Observed yield curves, 31 March 2011 ..................................... 15
2.2 Zero coupon surface, 31 March 2011 ...................................... 16
2.3 Decomposition, Treasury Corporation of Victoria yields ............. 18
2.4 Decomposition, Inter-American Development Bank yields .......... 19
2.5 Decomposition, European Investment Bank yields ..................... 20
2.6 Relative pricing, long semi-government bonds .......................... 22
2.7 Relative pricing, short supranational and agency bonds ............... 23

3.1 Sample mean term premia .................................................. 37
3.2 Sample mean conditional term premia ................................... 39

4.1 Fixed income market pricing .............................................. 59
4.2 Short end root mean squared forecast errors ........................... 61
Abstract

This doctoral dissertation comprises three essays which study the determinants of bond yields.

The dissertation is organised around the idea that bond yields can be partitioned into a risky component which prices for the risk of illiquidity and default; and a risk-free component which prices for investors’ time preferences, and expected monetary policy movements [Homer and Leibowitz, 2004]. The first essay considers the liquidity and credit premia in supranational, semi-government and agency bond yields; term premia in sovereign bond yields and their relation to the economy constitute the focus of the second essay; and the third essay is devoted to an inquiry into the nature of expectations of future monetary policy movements in bond yields.

The first essay presents a new method for consistent cross-sectional pricing of all traded bonds in the fixed income market. By applying thin plate regression splines ([Wood, 2003] to bootstrapped zero coupon bond yields [Hagan and West, 2006], the method decomposes traded yields into a risk-free component plus premia for credit and liquidity risks, where the decomposition is consistent with the market valuations and underlying cash flows of the bonds. We apply the framework to end of quarter yield data from 2008 to 2011 on Australian dollar denominated semi-government, supranational and agency bonds, and find that the surface provides an excellent fit to the underlying zero coupon yield curves. Further, the decomposition of selected yield time series and cross sections demonstrate how credit premia increased for Australian semi-government, supranational and agency bonds through the Global Financial Crisis, but were counterbalanced by liquidity discounts as investors sought safe haven securities.
The second essay designs conditional tests for the liquidity preference hypothesis, which predicts monotonicity in term premia. Drawing on the excess return forecasting literature (Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2009), the tests are conditioned on information from macroeconomic variables and the current yield curve. Specifically, a filter is constructed to use this conditioning information set in new versions of the Wolak test (Boudoukh et al., 1999a) and Monotonicity Relation test (Patton and Timmermann, 2010) for the liquidity preference hypothesis. Consistent with the literature, our tests conclude that raw, unconditional term premia in U.S. Treasury bills between 1965 and 2001 do not increase monotonically. However, we find that the tests indicate term premia in Treasury bills do increase monotonically when the sample term premia are conditioned on the excess return forecasting factors. This confirms the explanatory power of the excess return forecasting factors, and also suggests that conditioning information should be used in applying inequality constraints tests to determine whether the liquidity preference hypothesis holds empirically.

The third essay evaluates the accuracy of the fixed income market in pricing for future movements in monetary policy. By generalising the approach in Gürkaynak et al. (2007) and Goodhart and Lim (2011), we compare yields and forward rates implied by market pricing on various fixed income securities to averages of the cash rate over corresponding periods with an ordinary least squares regression model. Where the market pricing is subject to risk premia, instrumental variables are used to strip away the effects of the risk premia as if they were measurement errors. When we apply our framework to Australian fixed income pricing from 2004 to 2010, we find that, consistent with findings in the extant literature, the market is quite effective in forecasting cash rate movements over horizons of up to six months. Beyond that horizon, the presence of risk premia diminishes to a large extent the signal on expectations in market pricing, but our instrumental variables framework suggests nonetheless that there is important information in fixed income market pricing regarding expected cash rate movements over the one to three year horizon.
Chapter 1

Introduction

The bond market lies at the intersection between the real economy and financial markets. With regard to the economy, the risk-free Treasury yield curve responds to shocks in inflation and growth expectations. The benchmark provided by the yield curve underpins all other interest rates in the economy, and thus determines the supply and price of credit. There is a strong relationship between the business cycle and the yield curve. The supply of bonds is determined by fiscal policy, and monetary policy influences the short end of the yield curve. Offshore flows of funds to and from the domestic bond market constitute an important determinant of the capital account in the balance of payments.

Turning to financial markets, the risk-free yield curve sets the discount factors according to which all other financial assets are priced. The yield curve thus determines the time value of money. In a very broad sense the cashflows of almost any financial security (and by extension, any government, corporate or trust structure) can be treated as a bond or bond option. Furthermore, derivatives and securitisation technologies provide almost unlimited flexibility to market participants in structuring fixed income securities to meet their investment, financing and hedging requirements. It is therefore clear that the bond market plays a fundamental role in the valuation of all financial securities in other financial markets.

At the heart of the bond market is the pricing mechanism where investors
ascribe a value to the deterministic or risky sets of cashflows that are attached to fixed income securities. This dissertation explores empirical aspects of bond market pricing. The underlying philosophy of our research is that raw market pricing is informative: important information about investors’ risk sentiment and interest rate, inflation and growth expectations can therefore be extracted from observed market pricing with minimal interference from models.

Specifically, in this dissertation, we consider how the bond market prices for risk. The aim is to measure and characterise the credit and liquidity premia in Australian dollar denominated semi-government, supranational and agency bonds (Chapter 2); the term premia in U.S. Treasury bills (Chapter 3); and the expectations of future monetary policy movements in various Australian fixed income instruments and interest rate futures (Chapter 4). In each case, we find that valuable information can indeed be extracted from the market pricing implied by bond yields. The remainder of the present Chapter provides an overview of the research problem, methodology, findings and significance of each Chapter in the dissertation.

Chapter 2, ‘Bond pricing with a surface of zero coupon yields’, explores whether the fixed income market prices in a consistent manner Treasury bonds along with other bonds of different credit qualities and with different liquidity characteristics. A model is proposed for risk free zero coupon yields, credit premia and liquidity premia for all tenors across all securities in the entire fixed income market at a single point in time.

Our model builds on ideas from the fixed income literature, especially with respect to estimating zero coupon yields. Of course, most of this literature simply addresses the bootstrapping of Treasury zero coupon yields, so our model represents an important extension of the conventional methods. A surface of zero coupon yields capable of pricing the entire fixed income market would be useful in assessing relative value, and thus could assist investment and issuance decisions. While there are separate streams of the literature on credit premia (eg. Krishnan et al., 2010) and liquidity premia (eg. Bao et al., 2011) in bond yields, there is a paucity of scholarship on simultaneous estimation of risk free rates, credit spreads and liquidity spreads (Houweling et al., 2001; Jarrow et al., 2004; Cruz-Marcelo et al., 2011); and on applications
of multivariate smoothing splines to yield curve estimation (Krivobokova et al., 2006).

To implement the model, we use end-of-day market yields for bonds issued by a wide range of issuers as the key input for the model. Zero coupon yield curves are then estimated for each issuer’s bonds with a standard bootstrap and smoothing splines interpolation (starting with the risk-free Treasury zero curve) (Hagan and West, 2006). Next, we compute risk free asset swap margins for each bond. These are margins to the floating rate on an interest rate swap based on the Treasury zero curve that match exactly the cash flows of the bond (Manning, 2004). We fit an approximation to a multivariate smoothing spline (a thin plate regression spline) through the cloud of points to get a zero coupon surface (Wood, 2003). Given the surface and the underlying zero coupon bond yields, we can then assess how well the surface prices bonds and swaps. For each issuer, we also solve for the constant risk free asset swap margin that best prices its bonds off the zero coupon surface. This margin is the fundamental measure of the credit premia, and the zero coupon yield errors to the surface are the fundamental measures of liquidity premia.

We find that the zero coupon yield surface summarises effectively the information in traded yields from the semi-government, supranational and agency segment of the Australian fixed income market. Looking at the quarter-end fit of the surface from 2008Q1 to 2011Q1, it is clear that the disruption in pricing through the Global Financial Crisis affects the surface; but that the surface still captures much of the cross-sectional variation in bond yields for different issuers. Furthermore, the surface can be used to provide decompositions of traded bond yields into a risk-free component plus premia for credit and liquidity. We explore the decomposition in a cross-sectional (yield curve) sense and in a time series sense for selected bond lines of a prominent issuer. The decomposition shows how credit yield premia widened during the Crisis, but there were liquidity yield discounts for high quality semi-government, supranational and agency bonds.

Taking a step back, the zero coupon yield surface model has potential to change fundamentally our understanding of the fixed income market from a correlated set of two dimensional yield curves and spreads to an evolving three
dimensional surface that can be decomposed into term premia (the pure time value of money component), credit premia (associated with default risk) and idiosyncratic liquidity premia.

Chapter 3, ‘Conditional tests of monotonicity in term premia’, considers the expected shape of term premia in U.S. Treasury bills across tenors, drawing on the econometric methodology of multiple inequality constraints testing. Existing specifications of inequality constraints tests for monotonicity in term premia (Boudoukh et al., 1999a; Patton and Timmermann, 2010) suffer from low power because they utilise an insufficient set of conditioning information. Hence, we improve the tests by incorporating excess return forecasting factors from the forward curve (Cochrane and Piazzesi, 2005); and factors estimated via principal components analysis from a large panel of macro variables as conditioning information (Ludvigson and Ng, 2009).

An inequality constraints test for term premia conditioned on macro and forward curve factors is the logical next step in relation to at least three strands of the literature. First, the literature on excess return forecasting factors does not consider the conditional shape of premia across tenors (Ludvigson and Ng, 2009; Cochrane and Piazzesi, 2005; Duffee, 2011). Second, scholarship on inequality constraints testing does not accommodate the use of principal components as conditioning information (Boudoukh et al., 1999a; Patton and Timmermann, 2010). Third, the literature on arbitrage free term structure modelling does not address traditional theories about the term structure, instead adopting parametric forms for risk premia that do not necessarily reflect empirical term structure dynamics (eg. Joslin et al., 2010, 2011).

To conduct the conditional monotonicity tests, we start with a monthly dataset of zero coupon U.S. Treasury bill yields as inputs. We compute excess returns over a one year horizon with various long and short tenors. We collect the excess return forecasting factors (Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2009) from the respective authors’ websites and align these datasets with our excess returns data. Then, we filter the factors to be non-negative, multiply them by our data and perform the Monotonicity Relations test (Patton and Timmermann, 2010) and Wolak test (Boudoukh et al., 1999a) on the conditional sample data. Finally, we compare the $p$-values
of the conditional tests with those of the unconditional tests, and perform a simulation study to assess the empirical power of the two tests, with and without the use of the conditioning information.

Our tests suggest that the use of conditioning information changes the outcome of the tests from the unconditional case. Both the Monotonicity Relations Test and the Wolak Test suggest that U.S. Treasury bill term premia are non-monotonic using the unconditional sample. However, when we condition our tests on the positive elements of the Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009) factors in particular, the tests suggest that the conditional term premia are monotonically increasing. This is evidence that the excess return forecasting factors do have explanatory power in respect of the conditional shape of term premia. Further, our results imply that the conditional tests shed new light on how the monotonicity tests can be applied to term premium data, by better capturing the information that market participants are ostensibly considering when they determine U.S. Treasury bill pricing. The simulation experiments then show that the conditional tests are more powerful in detecting monotonicity than their unconditional equivalents.

More generally, our approach to testing for the conditional shape of term premia across tenors provides an effective method for empirically characterising term premia, and thereby describing the evolution of the yield curve over time in terms of departures from the expectations hypothesis. Our approach does not impose the á-priori structure of an arbitrage free term structure model, and yet it allows for statistically powerful tests of hypotheses about the ex ante shape of term premia across tenors.

Chapter 4, ‘Measuring monetary policy expectations’, assesses the accuracy of the Australian fixed income market in pricing for future movements in the monetary policy instrument. We develop an econometric framework for measuring monetary policy expectations from fixed income pricing that is well founded in asset pricing theory. The framework allows us to abstract away from risk premia in market pricing, and also facilitates the measurement of hitherto elusive long term interest rate expectations of horizons between one and three years.

There is a paucity of scholarly literature on measuring monetary policy
expectations from fixed income securities without using the prism of a term structure model. The literature on extracting policy expectations from asset prices is mainly limited to money markets and related futures contracts (e.g. [Kuttner, 2001], [Taylor, 2010]). While money markets and short term interest rate futures have a direct relation to the policy instrument, they often suffer from a lack of liquidity and are therefore subject to additional risk premia ([Piazzesi and Swanson, 2008]).

Our econometric framework provides a mechanism for comparing yields and implied forward rates associated with market pricing for various types of Australian fixed income securities with the ex-post average cash rate over the same period. The universe of fixed income securities under consideration in our study includes: overnight indexed swaps, interbank futures, bank bills, bank bill futures, and Treasury bond futures. Once we have calculated the yields from market prices on the one hand, and corresponding average cash rates on the other, we compute cash rate forecast errors for each type of fixed income security at each observation date. This leads to root mean squared forecast errors for each type of fixed income security. Also, in a simple attempt to abstract away from risk premia down to a linear approximation, we regress the cash rate returns onto the yields and forwards. Then, to provide further robustness around our results, and better control for the effects of risk premia, we use a generalised method of moments instrumental variables estimation framework. Under this framework, the yields and forwards of securities that are subject to term, credit and liquidity risks are instrumented with other yields and forwards that are not subject to those risks. In this manner, we improve on the simple regression framework that has been adopted by most other studies of market-based measures of monetary policy expectations ([Gürkaynak et al., 2007], [Goodhart and Lim, 2011]).

The principal findings of our study are as follows: Overnight indexed swaps perform best at forecasting the Australian cash rate over the nearest two quarters. Beyond that, accuracy drops off substantially over longer horizons. The OLS regressions show that some of the influence of risk premia can be incorporated into the intercept term for each security, thus improving the forecast efficacy of all of the Australian fixed income securities. However,
these regressions still leave much of the variation in the cash rate unexplained. This is because, beyond the horizon of six months, the presence of risk premia diminishes to a large extent the signal on expectations in market pricing. Nonetheless, our instrumental variables framework suggests there is valuable information embedded in pricing on three and ten year bond futures contracts regarding expected cash rate movements over the one to three year horizon.

Our framework is preferable to other methods such as dynamic term structure models, because the components of the framework are all observable. Furthermore, by constructing a composite measure that incorporates information from liquid long term securities and those with payoffs dependent directly on future policy movements with an instrumental variables framework, our framework extends the extant literature on assessing policy expectations in fixed income securities.
Chapter 2

Bond pricing with a surface of zero coupon yields

2.1 Introduction

Each trade in the fixed income market represents an individual market participant’s view on the time value of money, and the amount of compensation required by investors for taking on credit and liquidity risk. Nonetheless, bond market pricing is not determined in isolation from one trade to the next. To the extent that it is an efficient market, the fixed income market does not just price consistently between the different securities of the same issuer — it prices consistently between the securities of different issuers. At a single point in time, the market’s pricing for sovereign, semi-sovereign, bank and corporate bonds is jointly determined. The principal argument of this essay is that the fixed income market differentiates between security specific characteristics in a consistent manner, and applies the same approach to pricing for risk between different fixed income securities.

Hence, it should be possible to model a set of zero coupon yields that reflects consistently the information in all of the separately determined zero coupon yield curves associated with the traded securities of each issuer, and that can thus be used to price all bonds in the market. What does consistency mean in the context of synthesising information from different zero coupon
yield curves? From the econometrician’s perspective, this means that if the zero coupon yield curves of different issuers are placed side by side (in order of credit risk) in a three dimensional space with axes for tenor, credit risk and zero coupon yields, it should be possible to fit a smooth surface through the yield curves. Such a surface would then summarise all of the discount factors that are being used to price traded bonds in the market.

Our argument constitutes a fundamental challenge to the extant literature on fixed income market valuation. To date, authors have focused on single aspects of the fixed income market in isolation\(^1\). There are growing bodies of scholarship aiming to measure and explain credit spreads (Martell, 2008; Liu et al., 2009; Jacoby et al., 2009; Krishnan et al., 2010), and liquidity premia (Chordia et al., 2005; Chen et al., 2007; Goyenko et al., 2010; Bao et al., 2011) in bond yields, that complement, but do not address directly the underlying literature on zero coupon yield curve estimation from risk free Government bond yields (Fama and Bliss, 1987; Nelson and Siegel, 1987; Fisher et al., 1994; Anderson and Sleath, 2001; Hagan and West, 2006) and defaultable bond yields (Houweling et al., 2001; Jarrow et al., 2004; Jarrow, 2004; Jankowitsch and Pichler, 2004; Cruz-Marcelo et al., 2011). However, if risk free zero coupon yields, credit premia and liquidity premia are determined jointly across the fixed income market, then the extant literature is not taking into account the fundamental interactions between these quantities that influence market valuations.

Accordingly, we present a unified method that prices simultaneously all risk free and defaultable bonds in the market, and thereby leads to consistent measures of credit and liquidity premia. Under this method, a smooth surface is fit through a three dimensional cloud of zero coupon yields. The co-ordinates of these zero coupon yields within the cloud are determined by their tenor and their risk free asset swap margin, which is the margin to the floating rate for an interest rate swap based on the risk free zero coupon yield curve that matches exactly the cash flows of the underlying bond. The smooth surface is

\(^1\)This is no surprise, given that credit and liquidity premia cannot be directly observed from market pricing data (Boyle et al., 2009), which makes their dynamics through time quite difficult to characterise (Allen and Powell, 2009).
then modelled as a thin plate regression spline, which is an approximation to a multivariate smoothing spline. Constructed in this manner, such a “zero coupon yield surface” constitutes a model of all zero coupon yield curves (equivalently, discount factors) in the market. Under additional identification assumptions, the model can then be used to decompose traded bond yields into a risk free component plus premia for credit and liquidity. The zero coupon yield surface is an important extension of extant methods in fixed income analysis, where zero coupon yield curves are typically estimated only from Government bond yields and interest rate swap yields. Our technique generalises traditional bootstrap and interpolation methods (Hagan and West, 2006) to other fixed income securities, and facilitates the decomposition of traded bond yields into their basic constituents.

We find that the zero coupon yield surface provides an accurate characterisation of the information in traded yields from the semi-government, supranational and agency (SSA) segment of the Australian fixed income market. The root mean squared fitting errors of the surface over 2008Q1 to 2011Q1 hit their peak of 15 basis points at the height of the Global Financial Crisis in 2009Q2, but quickly recede thereafter. We show that despite the market turbulence, the surface still captures much of the cross-sectional variation in bond yields across different issuers. Furthermore, we use the surface to decompose traded bond yields into a risk-free component plus premia for credit and liquidity. We explore the decomposition in both a cross-sectional (yield curve) sense and in a time series sense for selected bond lines of three prominent issuers. The decomposition shows how credit premia spiked during the Crisis, but the impact on pricing was dampened by the presence of liquidity discounts for high quality SSA bonds. This is consistent with investor sentiment through the Crisis period, where SSA bonds were perceived by portfolio managers as a safe haven asset class.

The remainder of this essay is organised as follows. Section 2.2 describes the construction of the zero coupon yield surface. In Section 2.3 the resulting zero coupon yield surface is shown to price accurately the entire cross-section of SSA securities in the Australian fixed income market using end of quarter valuation yields. This Section also shows how the surface can be used to
decompose traded bond yields under additional identification assumptions. Finally, Section 2.4 concludes.

2.2 Zero coupon yield surface

2.2.1 Zero coupon yields

At the outset, we bootstrap zero coupon yields from traded bond yields for each issuer across the market, using linear interpolation on log discount factors. This is a standard technique in fixed income analysis, and is well documented in eg. Hagan and West (2006) and Fama and Bliss (1987) (details are provided in the Appendix). This gives us separate zero coupon yield curves for each issuer in the market.

2.2.2 Risk free asset swap margins

Once zero coupon yields have been computed for each of the issuers in the fixed income market, the next step is to calculate the measure of credit risk. This measure is used to project the zero coupon yields into a three dimensional space with axes for yield, tenor and credit risk. The relevant measure of credit risk to be used is the riskfree asset swap margin, which is an asset swap margin using a swap based on the Treasury zero coupon yield curve. These asset swap margins are a purely market-based measure of credit risk, which are superior to alternatives such as credit ratings, benchmark par yield spreads, exchange for physical spreads and zero coupon yield spreads. This is because asset swap margins take account of the cash flows of the underlying securities; and they can measure credit risk for any fixed income security given its market price.

An asset swap is a type of trade in the fixed income market where the investor purchases a bond and an interest rate swap, such that the investor’s fixed coupon receipts on the bond are converted into floating rate cashflow receipts (Fabozzi, 1991). Under the interest rate swap, the investor pays fixed coupons that match the cashflows that she receives from the bond, and receives floating rate coupons on the swap. The asset swap margin is then the additional margin over the market floating rate that the investor receives on the interest rate swap. See the Appendix for a formal definition of an asset swap margin.

2
With their flexibility in mind, we use riskfree asset swap margins to assess the credit risk of risky bonds relative to the risk free Government zero coupon yield curve. The riskfree asset swap margin is calculated with the discount factors taken from the riskfree zero coupon yield curve, and using the market prices of defaultable bonds across the cross-section of issuers in the fixed income market.

### 2.2.3 Surface interpolation method

Define \( r_{it} \) and \( \chi_{it} \) as the zero coupon yield and risk free asset swap margin associated with issuer \( i \) at tenor \( t \). Having obtained zero coupon yields and risk free asset swap margins, we project triplets of zero coupon yields, risk free asset swap margins and tenors into three dimensional space. Then we fit the smooth surface \( g \) through these triplets. The surface takes the form of a thin plate regression spline \(^3\) (Wood, 2003).

Specifically, we estimate the bivariate thin plate regression spline

\[
    r_{it} = g(t, \chi_{it}) + \epsilon_{it}\tag{2.1}
\]

through the triplets of zero coupon yields, tenors and asset swap margins \( \{r_{it}, t, \chi_{it}\} \), and the resulting zero coupon yield surface summarises the entire set of zero coupon yield curves for the fixed income market.

The errors \( \epsilon_{it} \) are crucial to the analysis of the zero surface. Clearly, small errors indicate a close fit, and therefore constitute evidence of consistency in fixed income market pricing from one issuer to the next.

### 2.2.4 Credit and liquidity yield premia

Beyond summarising the information in individual issuers’ zero coupon yield curves, the zero coupon yield surface fulfills another important function. It allows for traded yields to be decomposed into a risk free component along

\(^3\)The construction of the splines is covered in the Appendix, here it suffices to describe thin plate regression splines in heuristic terms as an approximation to multivariate smoothing splines that retain the automatic knot selection property of univariate smoothing splines.
with premia for credit risk and liquidity risk. This depends on additional assumptions.

Now, the risk free asset swap margins as calculated for each traded bond conflates credit and liquidity premia, and is not a pure credit spread. This is because it captures the difference between the price and the risk free price, which could involve either credit or liquidity premia. Given the zero coupon yield surface, we implement the decomposition of observed bond prices with

\[ p_{it} = p_{it}^{(\gamma)} - p_{it}^{(\chi_i)} - p_{it}^{(\lambda_i)}, \]

where \( p_{it} \) is the observed bond price, \( p_{it}^{(\gamma)} \) represents the net present value of the bond’s cashflows using discount factors from the riskfree zero coupon yield curve; \( p_{it}^{(\chi_i)} \) is a price discount (yield premium) for credit risk, and \( p_{it}^{(\lambda_i)} \) is a price discount (yield premium) for liquidity risk. To be explicit, this specification is predicated on the assumption that the risk-free component of bond yields is additive with the credit risk and liquidity risk price discounts, and that together all three components sum to the observed bond price.

Now, we estimate the pure credit price discount \( p_{it}^{(\chi_i)} \) by solving the problem

\[ p_{it}^{(\chi_i)} \equiv p_{it}^{(\gamma)} - p_{it}(g(t, \chi_i)), \quad \chi_i : \min_{\chi} \| p_i - p_i(g(t, \chi)) \|, \]

where \( p_i \) is a vector containing the observed prices \( p_{it} \) for all bonds associated with issuer \( i \). Also, \( p_i(g(t, \chi_i)) \) denotes the prices of the same bonds calculated with the estimated zero coupon surface using the discount factors corresponding to the points on the surface with tenor equal to \( t \) and constant riskfree asset swap margins equal to \( \chi_i \).

4Our intention here is not to assert that this is the only way or the correct way to decompose bond yields with the zero coupon surface, but to show that this is one possible approach to decomposition using the surface. Of course, this additive decomposition would be expected to be valid only in fixed income markets where there are few liquid bond lines per issuer, and where extrinsic considerations such as tax, operational and sovereign risk are not influential in pricing. This is reasonable for well developed fixed income markets such as Australia.

5Geometrically, \( g(t, \chi_i) \) can be represented as selecting the “slice” of the zero coupon surface perpendicular to the riskfree asset swap margin axis that best prices a given issuers’ bonds. Note that \( \chi_i \) is not the same as \( \chi_{it} \), as the latter incorporates both credit and
risk free equivalent price less its pure credit price, where the latter is priced off the zero coupon yield surface along a single risk free asset swap margin. This is consistent with the situation where the market prices equally for credit risk across tenors.

The liquidity price discount is then the idiosyncratic remainder between the risk free price less the observed price on the one hand and the pure credit price discount on the other, according to the decomposition (2.2), so that

\[ p^{(\lambda_{it})}_{it} = p^{(\gamma)}_{it} - p_{it} - p^{(\chi_{i})}_{it}. \] (2.4)

These credit and liquidity premia can also be expressed in yield to maturity terms by treating \( p^{(\cdot)}_{it} \) as the price in the bond pricing formula (see equation (2.7) in the Appendix) and then solving for the yield to maturity \( y_{it} \) given the underlying cash flows.

We have shown that observed bond yields can be decomposed into a risk free component along with premia for credit risk and liquidity risk in the manner described in this Section. The separation between credit and liquidity premia is achieved through the use of a constant risk free asset swap margin. The difference in par yield terms between the pure credit price and the risk-free price is the credit premium; and the difference in par yield terms between the market price and the pure credit price is the liquidity premium. Taking a step back, it is clear that our decomposition method imposes minimal structure on traded bond yield data, and yet provides an effective means through which to extract credit and liquidity premia.

### 2.3 Risk premia in Australian bond yields

In this Section, we start by motivating the need for our method in practice. Then we fit the zero coupon yield surface to Australian fixed income market end of quarter revaluation pricing (2008Q1 to 2011Q3), as sourced from Yieldbroker, and show an example of our decomposition method with Treasury Corporation of Victoria bond yields and the zero coupon surface. We have liquidity premia.
restricted our sample to sovereign risk free bonds, semi-government bonds, and supranational/agency bonds. The sample covers a total of 13 quarters, with 37 unique issuers from the semi-government, supranational and agency segment of the bond market, and 1,941 observations of end of quarter bond yields.

The fixed income market pricing implied by observed yield curves incorporates a lot of information, not all of which is obvious. Figure 2.1 shows Australian Government yields, as compared to Queensland Treasury Corporation (QTC) yields and Kreditanstalt fur Wiederaufbau (KfW) yields. This Figure illustrates market pricing for the bonds of each of the three issuers, and therefore contains sufficient information for one to analyse whether the market is pricing consistently for each issuer’s bonds. However, it is not obvious how consistency in pricing should be assessed just by looking at the observed yields (eg. is there an investment reason why QTC spreads are lower than KfW spreads by the amount observed?). Further, the presence of coupons distorts comparison between observed yields, and it is impossible to distinguish credit spreads from liquidity spreads in Figure 2.1.

Figure 2.1: Observed yield curves, 31 March 2011

Instead, the zero coupon yield surface is necessary. The zero coupon
surface makes it possible to assess consistency in the fixed income market and decompose bond yields. Fundamentally, the surface is the true primitive of the fixed income market, showing discount factors for all credit qualities at all tenors. An example of the zero coupon surface is set out in Figure 2.2. Intuitively, this graph shows that as far as the eyeball test is concerned, the surface fits zero coupon yields fairly closely in March 2011. The remainder of this Section is devoted to showing how surfaces like the one above fit Australian fixed income market pricing data through time, and how they can be used to decompose bond yields.

When the surface is fit to zero coupon yield, tenor and riskfree asset swap margin triplets for each of the thirteen observation dates in our sample, we find that it generally achieves an excellent fit to the zero coupon yields. Descriptive
statistics for errors $\epsilon_{it}$ are provided in percentage terms by Table 2.1. As can be seen, the mean errors are all close to zero, indicating the surface is unbiased. The quartiles show that the errors are relatively symmetric around zero, but the minima and maxima do indicate the presence of a few outliers on each sample date. Further, the root mean squared errors are all small, but they did increase to a peak of 15 basis points in 2009Q2 (ostensibly as a result of the turbulence in the Australian market associated with the Global Financial Crisis and the introduction of the Australian Government’s guarantee on State Government bond issuance, which operated to segment the market). Since that time though, the errors have decreased, indicating the return of coherent pricing. This Table provides strong evidence for consistency in Australian fixed income market pricing, and facilitates the decomposition of observed yields, which are of course predicated on a zero coupon surface that fits well.

Table 2.1: Descriptive statistics, surface fitting errors (per cent)

<table>
<thead>
<tr>
<th>Date</th>
<th>Min</th>
<th>First Quarter</th>
<th>Median</th>
<th>Mean</th>
<th>Third Quarter</th>
<th>Max</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-03-28</td>
<td>-0.0816</td>
<td>-0.0247</td>
<td>-0.0029</td>
<td>0.0000</td>
<td>0.0221</td>
<td>0.1032</td>
<td>0.0337</td>
</tr>
<tr>
<td>2008-06-30</td>
<td>-0.0803</td>
<td>-0.0109</td>
<td>0.0023</td>
<td>-0.0000</td>
<td>0.0129</td>
<td>0.0643</td>
<td>0.0222</td>
</tr>
<tr>
<td>2008-09-30</td>
<td>-0.1295</td>
<td>-0.0379</td>
<td>-0.0027</td>
<td>-0.0000</td>
<td>0.0222</td>
<td>0.3465</td>
<td>0.0655</td>
</tr>
<tr>
<td>2008-12-31</td>
<td>-0.1527</td>
<td>-0.0439</td>
<td>0.0051</td>
<td>-0.0000</td>
<td>0.0407</td>
<td>0.1915</td>
<td>0.0679</td>
</tr>
<tr>
<td>2009-03-31</td>
<td>-0.1607</td>
<td>-0.0392</td>
<td>-0.0056</td>
<td>0.0000</td>
<td>0.0319</td>
<td>0.2474</td>
<td>0.0641</td>
</tr>
<tr>
<td>2009-06-30</td>
<td>-0.5643</td>
<td>-0.0750</td>
<td>-0.0018</td>
<td>0.0000</td>
<td>0.0898</td>
<td>0.3480</td>
<td>0.1451</td>
</tr>
<tr>
<td>2009-09-30</td>
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<td>-0.0618</td>
<td>0.0074</td>
<td>0.0000</td>
<td>0.0570</td>
<td>0.1921</td>
<td>0.0972</td>
</tr>
<tr>
<td>2009-12-31</td>
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<td>-0.0277</td>
<td>0.0004</td>
<td>0.0000</td>
<td>0.0350</td>
<td>0.1196</td>
<td>0.0543</td>
</tr>
<tr>
<td>2010-03-31</td>
<td>-0.2113</td>
<td>-0.0474</td>
<td>0.0068</td>
<td>0.0000</td>
<td>0.0493</td>
<td>0.1640</td>
<td>0.0754</td>
</tr>
<tr>
<td>2010-06-30</td>
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<td>-0.0545</td>
<td>-0.0028</td>
<td>0.0000</td>
<td>0.0398</td>
<td>0.1823</td>
<td>0.0606</td>
</tr>
<tr>
<td>2010-09-30</td>
<td>-0.0530</td>
<td>-0.0151</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0136</td>
<td>0.0535</td>
<td>0.0216</td>
</tr>
<tr>
<td>2010-12-31</td>
<td>-0.1010</td>
<td>-0.0215</td>
<td>0.0035</td>
<td>0.0000</td>
<td>0.0211</td>
<td>0.0945</td>
<td>0.0321</td>
</tr>
<tr>
<td>2011-03-31</td>
<td>-0.0439</td>
<td>-0.0098</td>
<td>-0.0001</td>
<td>-0.0000</td>
<td>0.0099</td>
<td>0.0431</td>
<td>0.0164</td>
</tr>
</tbody>
</table>

To that end, we turn now to the decomposition. Effectively, we use the surface to provide a scalar to three element vector map from each observed yield to the risk free component, credit yield premia and liquidity yield premia associated or priced into that yield. Of course, this triples the result set from our sample size to 5,823 yield components. Thus, for the sake of brevity, we illustrate the decomposition for a cross section and time series of yields for a selected issuer, namely the Treasury Corporation of Victoria (TCV).

Figure 2.3 shows how par yields can be broken down, both on a time series...
and cross sectional (yield curve) sense. As a general, rule observed yields are higher than credit yields, which in turn are greater than risk-free yields – implying positive liquidity and credit premia (in yield terms). To interpret the graph, the red lines show observed yields, and the dashed green and blue lines show the risk-free and credit yields under the decomposition, and are the par yields that correspond to $p_{it}^γ$ and $p_{it}^χ$ respectively. The vertical green hatching shows the yield if the bond was issued by the government. The slanted blue and red hatching show the discount/premium associated with credit and liquidity risks respectively, under the zero coupon surface.

Figure 2.3: Decomposition, Treasury Corporation of Victoria yields

In the TCV yield curve from March 2009, one can see that liquidity yield premia are priced into the middle part of the curve, and the extremes of the curve at both ends include liquidity discounts; while riskfree and credit yields are broadly parallel across tenors. This is to be expected: short and long tenor bonds tend to be thinly traded relative to medium tenor bonds. The timeseries in the lower panel of Figure 2.3 shows the repricing of risk through the GFC for the October 2022 TCV bond. While the credit premia priced into
this bond according to our surface and identification assumptions appeared to spike in during the GFC period around 2009, these were counterbalanced by a compression of liquidity premia (to the point where there were liquidity yield discounts) arising ostensibly from investors’ flight to quality in reaction to the stressed market condition.

These graphs and the underlying decomposition illustrate the power and flexibility of the zero coupon surface in facilitating fixed income investment analysis. Further examples of the decomposition, for a representative supranational issuer (Inter-American Development Bank, IADB), and a representative agency issuer (European Investment Bank, EIB), are set out in Figures 2.4 and 2.5. In both cases, these decompositions tell a similar story to that for TCV which we have told here.

Figure 2.4: Decomposition, Inter-American Development Bank yields

Now the important question arises as to whether the objects that we have designated as credit and liquidity premia in our model, and represented as such in our empirical analysis of Australian bond yields, actually correspond to investors’ views on the credit and liquidity risks in individual bond lines. One
way to provide evidence in support of our claims is to regress our estimates of credit premia from the zero coupon surface onto conventional determinants of credit risk, such as credit ratings and measures of financial or fiscal position. The same could be done for our liquidity premia estimates in relation to bid-ask spreads and face value on issue. The results of these regressions could then be strengthened with reference to surveys of financial market dealers and investors of Australian semi-government, supranational and agency bonds.

Such an approach might be able to establish a definitive link between our estimates and the underlying credit and liquidity risks implied by market pricing. However, this approach ignores a fundamental philosophical point: to the extent that the market is efficient, investors will have already priced for all of this information in the outright level of yields or in spreads to benchmark government bonds. Thus, the question of whether our credit and liquidity premia estimates’ labels are justified is a circular one, in that credit and liquidity premia may simply be in essence what investors deem them to be. The important point is that our estimates are entirely consistent with
Instead, the preferable approach is therefore to explore how the zero coupon surface and the underlying market pricing is capable of accommodating a range of alternative views on credit and liquidity risk for particular bond lines and issuers’ securities. One example of how this might work is as follows: Suppose we wish to analyse a set of bonds of similar tenors, each issued by different entities that nonetheless have broadly the same credit quality. Thus, we might define our sample as all bonds of tenor between five and seven years issued by the largest State government issuers (New South Wales Treasury Corporation [NSWTC], Queensland Treasury Corporation [QTC], and TCV). Or we might consider three to five year bonds issued by the significant supranational and agency issuers (KfW, EIB and the Asian Development Bank [ADB]).

Once we have our sample, we can assume that the credit quality of these bonds is priced across the different lines according to the surface. Hence, we can use a constant slice of the surface that is perpendicular to the tenor axis and parallel to the credit margin axis (i.e. the opposite direction to the decomposition presented above) to price the bonds and explore relative credit and liquidity premia. In this setup, we set the tenor constant as the average tenor of the bonds under consideration, as we believe that term risk has an approximately equal effect across our sample of bonds. The setup also implies that the risk-free asset swap margin will vary over a small interval that captures the margins of the bonds being examined – this allows us to control for variation in issuer credit quality.

The results of our analysis are presented in Figures 2.6 and 2.7 which show line segments of the zero coupon surface (blue lines) from 31 March 2011 for the two samples of issuers mentioned above (long semis and short supras), along with labelled points showing zero coupon yields estimated for each issuer at equivalent tenors to those of their traded bonds. The residuals between the line and the points can then be thought of as liquidity premia measured on a relative basis within the sample of bonds, since we have controlled for credit

---

6 Of course, to avoid solving across segments of the surface and reconstructing the NPVs of traded bonds, we only consider zero coupon bonds of the same tenor as traded bonds here, but this will suffice, as the zero coupon bonds have the highest delta with respect to the underlying bonds, given their tenor.
and term premia through our use of the zero coupon surface.

Figure 2.6: Relative pricing, long semi-government bonds

These graphs illustrate our method for comparing the market pricing of similar bonds (by issuer credit quality and tenor). They show how the zero coupon surface can be used to evaluate the relative pricing of similar bonds. The first (second) graph is predicated on a line segment of the zero coupon surface with tenor set to 7.14 (3.58) years, corresponding to the average tenor of the bonds in the sample. The graphs show that liquidity premia are fairly idiosyncratic across issuers for long NSWTC, TCV and QTC bonds with average tenor around seven years, but that EIB, KFW and ADB bonds with tenors less than three and a half years tend to attract a liquidity yield discount (price premium). Taking a step back, the sensitivity analysis of our assumptions presented here suggests that the zero coupon surface is capable of accommodating a wide range of assumptions about the underlying risks that drive bond pricing.
2.4 Conclusion

We have shown that notwithstanding the effects of the Global Financial Crisis on the Australian fixed income market, the zero coupon surface summarises market pricing relatively well. This suggests that the market is pricing consistently between issuers. Further, the decomposition shows that it is possible to use the surface in more than one way to identify hitherto elusive credit and liquidity premia.

We are not claiming that our decomposition necessarily represents the underlying subjective premia that Australian bond investors charge for taking on credit and liquidity risks. Instead, our aim here is to show that the surface can be combined with identification assumptions to calculate credit and liquidity yield premia in a consistent manner. As we have shown, the surface is flexible to accommodate a wide range of assumptions. Indeed, this is in keeping with the spirit of traditional relative value analysis in fixed income markets, where the model is just a tool, an artifice for interpreting pricing in an objective manner. There is, of course, no substitute for judgment.
Appendix

Bootstrapping zero coupon yields  The approach to bootstrapping taken here is to blend zero coupon yields from deposit instruments with those from bond yields. As deposit instruments are already zero coupon bonds, the only calculation necessary for deposit rates is to convert them from simple interest to continuous compounding. In this essay, we use the cash rate and one, three and six month overnight indexed swap rates as the yields on deposit instruments.

The rest of the zero coupon yield curve is obtained by bootstrapping and interpolating between bond yields or swap rates. A bond of any credit quality pays the coupon $c_1, \ldots, c_n$ at evenly spaced tenors $t_1, \ldots, t_n$ and principal of $1$ at time $t_n$. The bond price $p_n$ is then the sum of discounted interest payments and principal, which can be written as

$$p_n = \sum_{j=1}^n c_j \delta_j + \delta_n,$$  

(2.5)

where $\delta_j \equiv e^{-r_j t_j}$ and $r_j$ are the discount factor and zero coupon yield for tenor $t_j$.

Assume $\delta_1, \ldots, \delta_{n-1}$ are known. Then, by solving (2.5) for $\delta_n$, the following bootstrap relation emerges [Hagan and West, 2006]:

$$\delta_n = \frac{p_n - \sum_{j=1}^{n-1} c_j \delta_j}{1 + c_n}$$

$$\Rightarrow r_n = -\frac{1}{t_n} \log \left( \frac{p_n - \sum_{j=1}^{n-1} c_j \delta_j}{1 + c_n} \right).$$  

(2.6)

The next step is to convert all quoted bond yields from their native compounding basis into continuously compounding equivalent rates $y_n$. The initial guesses of the zero coupon yields $r_n$ in the bootstrap relation are given by $y_n$. Then the dirty price of each bond is calculated with

$$p_n = \sum_{j=1}^n c_j e^{-y_n t_j} + e^{-y_n t_n},$$

(2.7)

where $y_n$ is the yield to maturity of bond with tenor $n$. These dirty prices are treated as the bond prices $p_n$ in the bootstrap relation (2.6). Finally, discount factors are interpolated at all coupon dates (due to the bootstrap framework, one can use any interpolation method, but we use a standard approach – linear interpolation on log
discount factors \citep{Hagan2006} and substituted into (2.6) to obtain new estimations of \( r_n \). This final step should be iterated, and subsequent estimates of \( r_n \) will converge rapidly onto the desired zero coupon yield curve \( r_t \forall t \).

As mentioned in the text, the process described here is repeated for all issuers in the market, to derive a complete set of zero curves for the market. While this does not pose any problems for riskfree Government bonds, further assumptions are required on the default and recovery processes for the framework to be successfully applied to yields on defaultable bonds \citep{Jarrow2004}.

**Calculating riskfree asset swap margins**  Formally, the riskfree asset swap margin \( \chi_n \) is the difference between the net present value of a bonds’ cashflows \( c_j \) (where the discount factors \( \delta_j \) used to calculate the net present value are taken from the risk-free zero coupon yield curve) and the traded bond price \( p_n \), expressed in basis point terms:

\[
\chi_n = \sum_{j=1}^{n} c_j \delta_j + \delta_n - p_n, \tag{2.8}
\]

where \( \alpha_j \) is the day count fraction applicable to period \( j \). It is also worth noting for those familiar with fixed income markets that risk-free asset swap margins are related to Z-spreads \citet{Fabozzi1991}.

**Constructing thin plate regression splines**  A thin plate regression spline is an approximation to a thin plate spline, which in turn is a multivariate smoothing spline. The outline of thin plate regression splines set out here follows closely the original exposition in \citep{Wood2003}. In the univariate case, suppose there is a model

\[
r_i = f(x_i) + \epsilon_i, \quad i = 1, \ldots, n, \tag{2.9}
\]

where \( r_i \) is the response variable, \( f \) is a smooth univariate function, \( x_i \) is a single covariate and \( \epsilon_i \) is a mean zero error term. Smoothing splines provide a method to estimate the smooth function \( f \) such that it minimises the error and roughness of the fit, so that

\[
\min_{f} \| r - f \| + \sigma \int f''(x)^2 dx, \tag{2.10}
\]

where \( r \) is a vector of \( r_i \)'s, \( \| \cdot \| \) is the Euclidean norm, \( f \) are the corresponding \( f(x_i) \) values, and \( \sigma \) is the parameter that controls the tradeoff between fit and smoothness.
Thin plate splines generalise smoothing splines to include any finite number of covariates, and allow for higher orders of differentiation \( m \) satisfying \( 2m > d \) in the roughness penalty [Wahba 1990; Gu 2002]. In this case, the model becomes

\[
 r_i = g(x_i) + \epsilon_i \quad i = 1, \ldots, n,
\]

where \( g : \mathbb{R}^d \rightarrow \mathbb{R} \) is an unknown multivariate smooth function to be estimated, \( x \) is a vector of length \( d \) from \( n \geq d \) observations and \( \epsilon_i \) is again a zero mean random error term. Thin plate splines estimate \( g \) by solving the problem

\[
 \min_g \| r - g \| + \sigma J_m(g)
\]

(2.11)

where \( r \) is the vector of zero coupon yields \( r_i \), \( g \equiv (g(x_1), \ldots, g(x_n))' \) is the smooth unknown multivariate function to be estimated, \( J_m(g) \) is a penalty measuring the roughness of \( g \), and \( \sigma \) controls the trade off between fit and smoothness. The roughness penalty is defined in the general case as

\[
 J_m(g) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \sum_{\nu_1 + \cdots + \nu_n = m} \frac{m! \nu_1! \cdots \nu_d!}{\nu_1! \cdots \nu_d!} \left( \frac{\partial^m g}{\partial x_1^{\nu_1} \cdots \partial x_d^{\nu_d}} \right)^2 dx_1 \cdots dx_d. \quad (2.12)
\]

and in the bivariate case \((d = 2, m = 2, g = g(x_1, x_2))\) this reduces to

\[
 J_2(g) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( g_{x_1x_1}^2 + 2g_{x_1x_2}^2 + g_{x_2x_2}^2 \right) dx_1 dx_2. \quad (2.13)
\]

It can be shown that the thin plate spline \( g \) which minimises (2.12) and (2.13) is characterised by the unknown parameter vectors \( \psi \) and \( \phi \) which are estimated by solving the problem

\[
 \min_{\phi, \psi} \| r - E\phi - T\psi \|^2 + \sigma \phi' E\phi,
\]

(2.14)

given the \( n \times n \) and \( n \times m \) weighting matrices \( E \) and \( T \) (subject to \( T'\phi = 0 \)) [Wahba 1990]. This multivariate problem is directly comparable with the univariate smoothing spline problem: the term within the Euclidean norm captures fitting errors, and the quadratic form is the roughness penalty.

The thin plate spline \( g \) can be shown to be an ideal smoother in the sense that it characterises smoothness, determines the optimal tradeoff between fit and smoothness, and finds the function that best meets this objective. On the other hand \( g \) is of high rank, to the extent that there are as many parameters as there
are data. This means that for \( d > 1 \) there are \( O(n^2) \) operations for each thin plate spline fit, and implies that estimation is significantly expensive in computational terms.

Thin plate regression splines were introduced by Simon Wood in a series of papers \( \text{Wood 2003, 2004, 2006, 2008; Marra and Wood 2011} \) to solve the problem with computational intractability in the estimation of thin plate splines. This is achieved by forming a rank \( k \) low rank approximation to the parameter space and restating the thin plate spline problem with this approximation. In other words, the basis of \( \phi \) is truncated to rank \( k \), and the \( E \) matrix as it appears in the fitting error and penalty term is adjusted in a consistent manner. Parameters are then estimated by minimising given \( k \) the worst possible changes in the fitted values and penalty induced by the approximation.\(^7\) The resulting thin plate regression spline can be shown to be an optimal approximation to the thin plate spline, that is computationally tractable because of the low rank parameter space.

\(^7\)In this essay, we set \( k = 30 \) when we use thin plate regression splines to fit the zero coupon surface. Having experimented with different values for \( k \), we find that this value achieves the best balance between fit and parsimony, as evidenced in Table 2.1.
Chapter 3

Conditional tests of monotonicity in term premia

3.1 Introduction

From the perspective of market participants, term premia are the essence of the Treasury yield curve. In the absence of term premia, investors would be indifferent between Treasury bonds of different maturities, as would the government in formulating its issuance strategy. Hence, to understand term premia is to understand the dynamics of the yield curve.

Specifically, competing theories of the yield curve can be reduced to three statements regarding term premia (Jarrow, 2010; Campbell et al., 1997). According to the expectations hypothesis, term premia are non-existent because expected returns are equal for all investment strategies in Treasury bonds. The liquidity preference hypothesis holds that term premia increase monotonically with tenor, due to investors demanding higher compensation for receiving their principal later. The preferred habitat hypothesis assumes that investors possess heterogeneous preferences across the yield curve, and that flows of funds resulting from supply and demand at different parts of the curve lead to general, non-monotonic relationships between term premia. Now, the

---

1 Term premia represent the difference in expected returns from Treasury securities of maturities greater than one period to market expectations of future monetary policy movements, usually proxied by the single period Treasury yield.
expectations hypothesis has been rejected consistently in empirical studies
(Campbell and Shiller, 1991; Bekaert and Hodrick, 2001; Sarno et al., 2007),
so the appropriate task for a conditional test of term structure theories is to
distinguish between monotonicity and non-monotonicity in term premia, and
thereby discriminate between the liquidity preference and preferred habitat
hypotheses.

To choose between the competing theories, one might estimate a no-
arbitrage dynamic term structure model in order to decompose Treasury
yields into expectations of future monetary policy and term premia over time,
and then examine the model-based term premia (Duffee, 2002; Finlay and
Chambers, 2008; Wright, 2011). However, this approach is problematic because
any statistical test of the term structure theories applied to model-based term
premia is by definition a joint test of the model and the data. This diminishes
the scope of the findings of such a test. Further, the empirical efficacy of
a dynamic term structure model depends crucially on the flexibility of the
functional form of the factor risk premia in the model. But these factor
risk premia predetermine the model-based term premia estimates, and thus
predetermine the outcome of a test for the term structure theories. Researchers
have formulated increasingly flexible specifications for the factor risk premia
(Cheridito et al., 2007; Joslin et al., 2010, 2011), but these suffer from a
significant loss of parsimony.

The alternative approach is to construct a test for the shape of expected
term premia across tenors based solely on observed Treasury yields, without
imposing the a-priori structure of a term structure model on the data. As
Boudoukh et al. (1999a) and Patton and Timmermann (2010) recognise when
they proposed their Wolak and Monotonicity Relations tests respectively,
multiple inequality constraints tests provide an ideal platform for testing
the shape of expected term premia across tenors. These tests allow the
econometrician to consider simultaneously the empirical relationships between
term premia across all tenors in an effort to establish whether monotonicity
holds, whilst imposing minimal distributional assumptions and structure on
the underlying bond return data. As we will see later in this essay, the Wolak
test is based on a comparison of sample mean term premia with term premia

29
that are estimated under the condition that they be non-negative; and the
Monotonicity Relations test uses the insight that expected term premia must
be monotonically increasing across tenors if the minimum expected term
premium (of any tenor) is positive.

When conducting inequality constraints tests, the crucial problem is the
choice of the information set to be used as conditioning information. The
empirical literature on excess return forecasting offers importance guidance
here. It has been established that forward curve factors that are linear
combinations of forward rates implied by the current yield curve can be used to
forecast term premia (Cochrane and Piazzesi 2005), which are in turn related
to macroeconomic factors extracted from a panel dataset of macroeconomic
variables (Ludvigson and Ng 2009). It is therefore appropriate to characterise
the conditioning information to be used in tests for monotonicity in term
premia on the forward curve and macroeconomic factors.

Accordingly, we construct conditional tests of monotonicity in term premia
using information in the current yield curve and macroeconomic variables. The
tests are conditional versions of the Wolak test (Boudoukh et al. 1999a) and
Monotonicity Relations test (Patton and Timmermann 2010) for monotonicity
in term premia. Given that the latter paper did not implement conditional
tests, and the earlier paper relied principally on an uninformative indicator
variable as the conditional information set, we improve the two extant tests
by using a comprehensive information set.

The results of our inequality constraints testing suggest that the use of
conditioning information changes the outcome of the tests from the uncondi-
tional case. Both the Monotonicity Relations and the Wolak tests suggest
that U.S. Treasury bill term premia are non-monotonic when unconditional
sample term premia are used. But when we condition our tests on the positive
elements of the Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009)

\[2\]In a recent paper, Duffee (2011) argues that an additional excess return forecasting
factor that can be derived in the context of a Gaussian dynamic term structure model.
This “hidden factor” contains information about expected returns, but is hidden from
the current yield curve, and appears to be unrelated to the Cochrane and Piazzesi (2005)
and Ludvigson and Ng (2009) factors. We do not analyse Duffee’s hidden factor in any detail in
this essay.
factors, the tests suggest that the conditional term premia are monotonically increasing. The same effect is apparent, although to a lesser magnitude, when we condition on both signs of the factors. This constitutes evidence that the excess return forecasting factors do have explanatory power in respect of the conditional shape of term premia. Further, our results indicate that the conditional tests shed new light on how the monotonicity tests can be applied to term premium data, by better capturing the information that appears to influence market participants’ investment decisions when they determine U.S. Treasury bill pricing.

This essay is organised as follows. Section 3.2 discusses the conditions necessary for monotonicity in term premia to hold, in order to frame an argument for why monotonicity tests should be conditional on information from the macroeconomy and the forward curve. Section 3.3 describes the conditioning information and describes the construction of the tests. Section 3.4 applies the tests to observed U.S. Treasury bill yields. Finally, Section 3.5 concludes.

### 3.2 Determinants of term premia

At the outset, it is important to develop intuition around the economic forces behind the liquidity preference hypothesis. To foreshadow the discussion in this Section, the asset pricing theory identifies covariances between marginal rates of substitution as the principal driver of term premia. The literature then indicates that the current yield curve and macroeconomic variables may be of use in testing for monotonicity in term premia.

Specifically, whenever covariances between the marginal rates of substitution are lower than zero and monotonically decreasing with tenor, it will be the case that term premia are monotonically increasing in tenor (see eg. Boudoukh et al. (1999b); the Appendix sets out full details). The intuition behind this argument is that the marginal rates of substitution reflect intertemporal consumption preferences, that in turn determine demand for bonds. A negative monotonically decreasing set of covariances indicates that agents value consumption in the next period higher than consumption in later periods.
They therefore demand short tenor bonds, which drives yields lower relative
to long tenor bonds, resulting in positive term premia. The opposite will hold
if the covariances are greater than zero and monotonically increasing.

Economic agents’ consumption and saving decisions depend on their prefer-
ences and on expected economic conditions (Varian 1999). While preferences
are not directly observable, the asset pricing theory suggests that there is a
direct link between zero coupon bond prices and average stochastic discount
factors (see the Appendix). Hence, there should be information in the cur-
rent yield curve about the covariances between marginal rates of substitution
(Cochrane and Piazzesi 2005). Another determinant of the covariances is
economic conditions, as it is clear that the marginal rate of substitution is
affected by expectations of the business cycle and real activity in the economy
(Ludvigson and Ng 2009, Hansen and Singleton 1982). Thus, it is clear that
any test for monotonicity in term premia should utilise the information in
the current yield curve and the state of the macroeconomy as conditioning
variables.

The forward curve factor proposed by Cochrane and Piazzesi (2005) (‘CP
factor’) is an ideal candidate for use as conditioning information in tests
for monotonicity in term premia, because it summarises the information in
the current yield curve about expected term premia. Cochrane and Piazzesi (2005)
and Kessler and Scherer (2009) experienced considerable success in
regressing Treasury excess returns of various tenors onto the CP factor, but
our focus will be on using the CP factor as conditioning information in the
conditional monotonicity tests. Ludvigson and Ng (2009) explored the extent
to which factors extracted from principal components analysis of a large panel
dataset of macroeconomic variables assists in explaining term premia alongside
the CP factor. The resulting economic factors (‘LN factors’) were found to
contain a significant amount of additional forecasting power to the CP factor
in capturing future variability in Treasury excess returns.

Apart from the CP and LN excess return forecasting factors, other ex-
planations have been put forward for the consumption and savings decisions
that drive the shape of term premia across tenors. A clear starting point is
supply and demand in the bond market. Possible measures of these forces
in the bond market include net buying pressure (Bollen and Whaley, 2004) and market flows (Vayanos and Vila, 2009), although these rely on tick level trade data, which is difficult to source in over the counter fixed income markets. Additional economic influences on covariances between marginal rates of substitution include learning (Sinha, 2010), subjective expectations (Xiong and Yan, 2010), habits (Wachter, 2006) and structural breaks in the short rate process (Bulkley and Giordani, 2011). However, because these factors are unobservable and have uneven effects across term premia of different tenors, it is arguable that the clearest empirical evidence that might be found in their favour is a rejection of monotonicity in term premia.

3.3 Monotonicity tests

3.3.1 Testing framework and inputs

To fix notation and provide a formal statement of the liquidity preference hypothesis, we start by defining $p_t^{(n)}$ as the log riskfree zero coupon bond price. Then the riskfree zero coupon yield is $y_t^{(n)} \equiv -p_t^{(n)}/n$. It follows that the log holding period return on an $n$ period bond is given by

$$r_t^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)}.$$ 

The excess return $r x_{t+1}^{(n)}$ is the holding period return less the spot yield, and corresponds to the trade where one borrows for a single period to finance an investment in a long bond, which is unwound at the end of the period. It can be written as

$$r x_{t+1}^{(n)} \equiv r_{t+1}^{(n-1)} - y_t^{(1)}.$$ 

Finally, the term premium $\delta_{t+1}^{(n)}$ is the difference between adjacent excess returns, so that

$$\delta_{t+1}^{(n)} \equiv r x_{t+1}^{(n)} - r x_{t+1}^{(n-1)}.$$ 

Now define $\bar{\Delta}(n) \equiv E[\delta_{t+1}^{(n)}]$. Then the liquidity preference hypothesis, which predicts that expected term premia are monotonically increasing, can written
as
\[ \Delta^{(i)} > 0, \quad \text{for } i = 2, \ldots, n. \]

A natural statement of the liquidity preference hypothesis in the language of statistical hypothesis testing is therefore

\[ H_0 : \text{Any element of } \Delta \leq 0 \quad H_A : \text{All elements of } \Delta > 0. \] (3.1)

where the parameter is defined with \( \Delta \equiv (\Delta^{(2)}, \ldots, \Delta^{(n)})' \). These definitions provide the basis for the monotonicity tests and facilitate the computation of sample term premia from zero coupon yields.

Apart from sample term premia, the other input for our conditional monotonicity tests comes from the conditioning information vector \( Z_t \), which is defined as

\[ Z_t \equiv \{ \hat{C}P_t, \hat{L}N_t \}. \]

In this expression, \( CP_t \) is the Cochrane and Piazzesi (2005) forward curve factor and \( LN_t \) are the Ludvigson and Ng (2009) macroeconomic factors (the Appendix sets out full details on how to estimate these factors). By construction, there are no restrictions on the sign of the elements of \( Z_t \). However, the inequalities in term premia to be tested will only be preserved if the elements of the original conditioning information vector \( Z_t \) are all positive. To this end, we follow Boudoukh et al. (1993) and redefine the conditioning information \( Z_t \) as

\[ Z_t^* \equiv \{ Z_t^+, Z_t^- \} \]

where the filters are \( Z_t^+ \equiv \max(0, Z_t) \) and \( Z_t^- \equiv \max(0, -Z_t) \) so that \( Z_t \) captures all possible states of the world.

The conditioning information can then be incorporated into the excess returns with the multiplication (Boudoukh et al., 1999a; Patton and Timmermann, 2010)

\[ r_{x_{t+1}}^{(n)} \equiv r_{x_{t+1}}^{(n)} \otimes Z_t^*. \] (3.2)

It is worth emphasising that this multiplication operation constitutes the principal contribution of our research to the extant literature. Patton and
Timmermann (2010) suggested that such an approach could be taken to conducting conditional monotonicity tests, but did not actually perform conditional tests. Boudoukh et al. (1999a) did use an approach akin to equation (3.2) to condition their tests on the slope of the yield curve, but they did not have the opportunity to incorporate the Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009) factors, because these factors had not yet been proposed in the literature.

Finally, let $\bar{\Delta} \equiv (\bar{\Delta}^{(1)}, \ldots, \bar{\Delta}^{(n)})'$ denote the vector of term premia across tenors by where $n$ is the longest tenor in the sample. Then conditional (unconditional) tests for monotonicity in term premia focus on the parameter $\bar{\Delta}^* (\bar{\Delta})$, whose constituent sample means are based on the conditional (unconditional) excess returns $r_{x_t^{(n)}} + 1 (r_{x_{t+1}^{(n)}})$.

### 3.3.2 Monotonicity Relations test

Patton and Timmermann (2010) design their “Monotonicity Relations” test around the fact that if the smallest difference in adjacent excess returns is positive, then all differences must be positive and monotonicity must hold.

$$
H_0 : \bar{\Delta} \leq 0 \quad H_A : \min_{i=2, \ldots, n} \bar{\Delta} > 0,
$$

(3.3)

where the minimum is taken on a piecewise basis across the conditional expected values of the parameter $\bar{\Delta}^*$.

The distribution of the test statistic, which is the smallest average adjacent difference in excess returns, is obtained with the stationary bootstrap of Politis and Romano (1994) (see Appendix).

### 3.3.3 Wolak test

The Wolak test is stated in different terms to the Monotonicity Relations test. Instead of treating monotonicity as an alternative, the Wolak test posits weak monotonicity under the null and sets an unrestricted alternative.

Romano and Wolf (2011) argue that this specification of the null hypothesis misses the important case where the term premium is unrestricted but non-monotonic. We do not attempt to address this potential problem with the Monotonicity Relations test specification in this paper.
so that
\[ H_0 : \bar{\Delta} \geq 0 \quad H_A : \bar{\Delta} \text{ unrestricted}. \] (3.4)

The intuition behind the Wolak test is that if the sample term premia are “close” in a statistical sense to artificial nonnegative term premia obtained from the same sample, then the liquidity preference hypothesis must hold (Boudoukh et al., 1999a). Again, full details are provided in the Appendix.

### 3.4 Monotonicity in U.S. term premia

In this Section, we apply the testing framework set out in the previous Section, and implement the conditional tests for monotonicity in term premia on U.S. Treasury bill excess returns conditional on the excess return forecasting factors. Following Patton and Timmermann (2010) and Boudoukh et al. (1999a), sample term premia are calculated with U.S. Treasury bill zero coupon yields (tenors from two to eleven months) from the CRSP Fama–Bliss bond files dataset. This dataset is most amenable to our study because there is a long sample of historical data available. The series for the forward curve factor and the macroeconomic factors are sourced from the respective authors’ websites. All zero coupon yield data and conditioning information factors are sampled monthly, from January 1965 to December 2001, for a total of 420 observations. We use this sample period, with these particular start and end points, in order to align our sample with Patton and Timmermann (2010) (and the sample of the seminal Cochrane and Piazzesi (2005) paper), so that as far as possible our results are directly comparable. The sample mean term premia, which form the basis for our tests, are depicted in Figure 3.1.

This Figure shows that sample mean term premia are not monotonically

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4Thanks to Andrew Patton (Duke) for MATLAB code to perform unconditional Wolak and Monotonicity Relation tests, which is available on his website. We have adapted this code to perform the conditional tests and compute empirical power.

5The zero coupon Treasury bond yield dataset available on the Federal Reserve Board website is an alternative source of data, and encompasses longer tenors than the Fama–Bliss dataset.

6Thanks to Monika Piazzesi (Stanford), Sydney Ludvigson (NYU) respectively for making the series, underlying data and associated MATLAB code available on their websites.
increasing in our sample. The sample mean term premia increase out to 6 months, but decrease between 7 and 8 months, and 9 and 10 months. This provides a strong hint that the tests will reject monotonicity for the sample, on an unconditional basis. A superficial empirical analysis of the liquidity preference hypothesis might conclude at this point. However, a closer look at the sample term premia data indicates a considerably more complicated story. There is a significant amount of volatility in the term premia series, as illustrated by the descriptive statistics for the sample term premia set out in Table 3.1. The Table demonstrates that sample term premia are highly dispersed, with large standard deviations and extreme minima and maxima.

This Table constitutes an important justification for bootstrap-based or data-driven approaches for testing monotonicity, as it is unlikely that a parsimonious term structure model with non-latent state variables could adequately fit these sample moments. Rather than making the restrictive assumptions about the data generating process that are implied by term structure models, the preferable approach is to let the dataset speak for itself, as it were. The monotonicity tests impose minimal structure on the sample term premia, and yet facilitate tests of the liquidity preference hypothesis that are entirely
Table 3.1: Descriptive statistics, sample term premia (per cent)

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Med</th>
<th>Mean</th>
<th>3Q</th>
<th>Max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 months</td>
<td>-0.3585</td>
<td>0.0009</td>
<td>0.0216</td>
<td>0.0274</td>
<td>0.0452</td>
<td>0.4393</td>
<td>0.0618</td>
</tr>
<tr>
<td>3 months</td>
<td>-0.3522</td>
<td>0.0068</td>
<td>0.0349</td>
<td>0.0507</td>
<td>0.0783</td>
<td>0.8217</td>
<td>0.1044</td>
</tr>
<tr>
<td>4 months</td>
<td>-0.5911</td>
<td>-0.0082</td>
<td>0.0361</td>
<td>0.0518</td>
<td>0.0913</td>
<td>1.1828</td>
<td>0.1470</td>
</tr>
<tr>
<td>5 months</td>
<td>-0.7851</td>
<td>-0.0188</td>
<td>0.0482</td>
<td>0.0668</td>
<td>0.1255</td>
<td>1.5667</td>
<td>0.1963</td>
</tr>
<tr>
<td>6 months</td>
<td>-0.9937</td>
<td>-0.0243</td>
<td>0.0560</td>
<td>0.0711</td>
<td>0.1479</td>
<td>1.8417</td>
<td>0.2361</td>
</tr>
<tr>
<td>7 months</td>
<td>-1.3578</td>
<td>-0.0436</td>
<td>0.0498</td>
<td>0.0661</td>
<td>0.1654</td>
<td>2.1284</td>
<td>0.2790</td>
</tr>
<tr>
<td>8 months</td>
<td>-1.4714</td>
<td>-0.0443</td>
<td>0.0708</td>
<td>0.0839</td>
<td>0.1995</td>
<td>2.4547</td>
<td>0.3238</td>
</tr>
<tr>
<td>9 months</td>
<td>-1.6390</td>
<td>-0.0467</td>
<td>0.0784</td>
<td>0.0905</td>
<td>0.2243</td>
<td>3.0701</td>
<td>0.3750</td>
</tr>
<tr>
<td>10 months</td>
<td>-2.3792</td>
<td>-0.0966</td>
<td>0.0665</td>
<td>0.0742</td>
<td>0.2174</td>
<td>3.2588</td>
<td>0.4253</td>
</tr>
<tr>
<td>11 months</td>
<td>-2.5410</td>
<td>-0.1102</td>
<td>0.0698</td>
<td>0.0802</td>
<td>0.2515</td>
<td>3.6797</td>
<td>0.4650</td>
</tr>
</tbody>
</table>

consistent with these sample moments.

It is also important to recall that the unconditional sample means could change if they reflected conditioning information. As suggested in Section 3.2, one potential determinant of term premia dynamics could be the excess return forecasting factors. These factors therefore provide an ideal source of conditioning information for the monotonicity tests. In fact, the factors are correlated with the term premia data, and this relationship (along with the weight of the literature) suggests that they could play a role in conditional tests of monotonicity. Table 3.2 sets out the correlation matrix.

Indeed, these correlations are borne out in the conditional sample means, which are the unconditional sample means multiplied by the excess return forecasting factors. In particular, Figure 3.2 shows how the conditional sample means tend to become monotonically increasing when multiplied by some of the factors. In interpreting this Figure, the unconditional sample means are the sample averages of the raw term premia on Treasury bills. The other series refer to the conditioning information that has been applied to (multiplied
Table 3.2: Correlation matrix: excess returns and factors

<table>
<thead>
<tr>
<th></th>
<th>CP</th>
<th>LN</th>
<th></th>
<th>CP</th>
<th>LN</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 months</td>
<td>-0.0197</td>
<td>0.2212</td>
<td>7 months</td>
<td>0.0901</td>
<td>0.2445</td>
</tr>
<tr>
<td>3 months</td>
<td>0.0117</td>
<td>0.2898</td>
<td>8 months</td>
<td>0.1009</td>
<td>0.2485</td>
</tr>
<tr>
<td>4 months</td>
<td>0.0610</td>
<td>0.2619</td>
<td>9 months</td>
<td>0.1017</td>
<td>0.2357</td>
</tr>
<tr>
<td>5 months</td>
<td>0.0890</td>
<td>0.2535</td>
<td>10 months</td>
<td>0.1336</td>
<td>0.2384</td>
</tr>
<tr>
<td>6 months</td>
<td>0.0960</td>
<td>0.2434</td>
<td>11 months</td>
<td>0.1192</td>
<td>0.2178</td>
</tr>
</tbody>
</table>

Figure 3.2: Sample mean conditional term premia

Of course, when one considers the sample average of a single signed factor, one can just append the plus and minus and one gets the sample average vector for the case where both signs of the factors are conditioned upon.
Turning now to the central results of our analysis, Table 3.3 sets out \( p \)-values for the unconditional and conditional monotonicity tests. The unconditional tests are the same as the ones conducted in [Patton and Timmermann (2010)](https://doi.org/10.1016/j.ijforecast.2009.06.003) (but for our slightly different sample end points). The conditional tests are defined by which factor is used, for instance the factor “CP+” uses the positive CP factors as conditioning information; and the factor “CP” comprises both the CP+ and CP− factors. As there are eight LN factors (that correspond to the first eight principal components of their macro panel dataset), we have only used the first LN factor. The Table provides compelling evidence that the acceptance or rejection of monotonicity by each test is influenced strongly by the use of conditioning information.

Table 3.3: Monotonicity test \( p \)-values, by conditioning factor

<table>
<thead>
<tr>
<th></th>
<th>None</th>
<th>CP</th>
<th>CP+</th>
<th>CP−</th>
<th>LN</th>
<th>LN+</th>
<th>LN−</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top less bottom</td>
<td>0.0532</td>
<td>-0.3100</td>
<td>0.1718</td>
<td>-0.0114</td>
<td>-0.0187</td>
<td>0.0925</td>
<td>-0.0032</td>
</tr>
<tr>
<td>( t )-test (( t ) stat)</td>
<td>2.4873</td>
<td>-0.5710</td>
<td>2.7502</td>
<td>-0.2239</td>
<td>1.7701</td>
<td>2.6824</td>
<td>-0.4287</td>
</tr>
<tr>
<td>( t )-test (( p )-val)</td>
<td>0.0064</td>
<td>0.7160</td>
<td>0.0030</td>
<td>0.5886</td>
<td>0.9616</td>
<td>0.0037</td>
<td>0.6659</td>
</tr>
<tr>
<td>MR (( p )-val)</td>
<td>0.9540</td>
<td>0.9130</td>
<td>0.1830</td>
<td>0.9310</td>
<td>0.9880</td>
<td>0.0040</td>
<td>0.9950</td>
</tr>
<tr>
<td>Wolak (( p )-val)</td>
<td>0.0465</td>
<td>0.0000</td>
<td>0.8228</td>
<td>0.0663</td>
<td>0.0001</td>
<td>0.8300</td>
<td>0.0000</td>
</tr>
<tr>
<td>Bonferonni (( p )-val)</td>
<td>0.0206</td>
<td>0.0450</td>
<td>1.0000</td>
<td>0.0665</td>
<td>0.0023</td>
<td>1.0000</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

When we consider the results of the conditional tests that use both signs (the full factors, CP and LN), both the Wolak test and the MR test \( p \)-values indicate that term premia do not increase monotonically across tenors. Interestingly, the strength of the unconditional tests’ outcomes is magnified in the conditional case. That is, the \( p \)-values indicate more or less significance for the outcome each test respectively when conditioning on CP or on LN relative to the unconditional case. This constitutes an initial indication that conditioning on the factors affects the outcome of the test.

To push our analysis further, we have also conditioned on the signed components of each factor separately. This enables us to study the impact of

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8For comparison, we also include test outcomes for \( t \)-tests of the difference between the
the signed components of the factors on the \( p \)-values in isolation, and should thus help us to understand the underlying drivers of the test results. We find that the positive and negative CP and LN factors give conflicting test results – non-monotonicity tends to be rejected (not rejected) for the conditional sample means by the MR test when conditioning on \( CP^{+} \) and \( LN^{+} \) (\( CP^{-} \) and \( LN^{-} \)). Consistent results hold for the Wolak test, but in that test the hypotheses are of course flipped (so care needs to be taken in interpreting the \( p \)-values).

Hence, our conditional inequality tests suggest that the signs of the factors play a role in determining the monotonicity of term premia. Put another way, our conditional tests show that the signs and magnitudes of the CP and LN factors each appear to be correlated with the states of the world in which the liquidity preference hypothesis holds.

Finally, we note that the power of the MR and Wolak tests in detecting monotonicity is very high. This can easily be confirmed by imposing monotonicity on the data and re-running the tests. Specifically, empirical power is obtained by imposing monotonicity on the data, resampling from the data according to the stationary bootstrap \cite{Politis1994} and calculating the test \( p \)-values from each bootstrap sample. Then the power is reported as the proportion of monotonicity outcomes detected by the tests over the bootstrap samples (where the \( p \)-value is less than [greater than] the nominal test size of 0.05 for the \( t \)-Test and MR Test [Wolak Test]) out of the total number of bootstrap iterations (200 in our case). To impose monotonicity, we have followed \cite{Patton2010} and added \{1, 2 \ldots, 10 basis points\} multiplied by the step sizes of 0, 1 and 2 to every observation of the 2 to 11 month sample term premia, respectively.

Table 3.4 compares the empirical power of the conditional and unconditional tests. In this Table, as expected, the monotonicity outcome frequencies for the \( t \)-test follow from the averages. Once we add the spreads, the differences between sample average term premia across tenors widen. Hence, the \( t \)-test almost always reports a significant difference (ie. monotonicity between the longest tenor mean term premium and the shortest tenor mean term premium and for Bonferroni bounds on the minimum \( t \)-test statistic across tenors. For details see \cite{Patton2010}.
Table 3.4: Empirical power: Frequencies of monotonicity outcome in tests

<table>
<thead>
<tr>
<th>Factor (Step size)</th>
<th>t-Test</th>
<th>MR Test</th>
<th>Wolak Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>None (0)</td>
<td>0.805</td>
<td>0.000</td>
<td>0.310</td>
</tr>
<tr>
<td>None (1)</td>
<td>1.000</td>
<td>0.165</td>
<td>0.900</td>
</tr>
<tr>
<td>None (2)</td>
<td>1.000</td>
<td>0.695</td>
<td>1.000</td>
</tr>
<tr>
<td>CP$^+$ (0)</td>
<td>0.980</td>
<td>0.150</td>
<td>0.980</td>
</tr>
<tr>
<td>CP$^+$ (1)</td>
<td>1.000</td>
<td>0.925</td>
<td>1.000</td>
</tr>
<tr>
<td>CP$^+$ (2)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>CP$^-$ (0)</td>
<td>0.085</td>
<td>0.000</td>
<td>0.485</td>
</tr>
<tr>
<td>CP$^-$ (1)</td>
<td>0.535</td>
<td>0.075</td>
<td>0.930</td>
</tr>
<tr>
<td>CP$^-$ (2)</td>
<td>0.920</td>
<td>0.310</td>
<td>0.995</td>
</tr>
<tr>
<td>LN$^+$ (0)</td>
<td>0.965</td>
<td>0.565</td>
<td>1.000</td>
</tr>
<tr>
<td>LN$^+$ (1)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>LN$^+$ (2)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>LN$^-$ (0)</td>
<td>0.015</td>
<td>0.000</td>
<td>0.030</td>
</tr>
<tr>
<td>LN$^-$ (1)</td>
<td>1.000</td>
<td>0.635</td>
<td>1.000</td>
</tr>
<tr>
<td>LN$^-$ (2)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

end point tenors) by the time we add a two basis point spread, no matter which factor we are conditioning on.\(^9\) Given how they reflect the sample average term premia, adjusted by the conditioning information and the spreads, the \(t\)-test monotonicity outcome frequencies provide a benchmark for our MR and Wolak test simulation experiments.

Now, we turn to the conditional experiments. As before, we can best understand the empirical power results by separating out by the signs of the conditioning factors. Considering the positive components of the CP and LN factors for a given step size, we find that the monotonicity outcome frequency is always higher for the MR and Wolak tests, when conditioning on the CP$^+$ and LN$^+$ factors, relative to the unconditional case where no

\(^9\)By the time a step size of two is used, 22 (11) basis points are added to every eleven (six) month term premium relative to the two month term premium, thus effectively ensuring monotonicity.
factors are conditioned upon. Again, this suggests that conditioning on the factors enables the inequality constraints tests to be better able to detect monotonicity in the sample term premia data. The simulation results for the tests that are conditioned on the negative factors are less conclusive. This is because, given the sample averages, we expect a finding of non-monotonicity where the step size is zero, and, as for the $t$-statistic simulations, the step sizes take over when larger step sizes are applied to the spreads.

The key implication of our analysis in this Section is that it is preferable to use a conditional test for monotonicity in term premia that reflects ex-ante expectations by incorporating excess return forecasting factors than to use an unconditional test on raw sample term premia. The signs of the factors determine the outcome of the tests, and we have shown that the Wolak and MR tests are more powerful when applied to conditional term premia data.

3.5 Conclusion

We have demonstrated that the sign and magnitude of the excess return forecasting factors are a key determinant of monotonicity in U.S. term premia. More broadly, our research provides an important confirmation of the utility of the excess returns forecasting literature. While it is well known that the CP and LN factors perform well in forecasting excess returns, we have used the methodology of conditional multiple inequality constraints testing to show that these factors also influence the shape of term premia across tenors. Finally, we have shown that the empirical power of inequality constraints tests for the liquidity preference hypothesis increases when the excess return forecasting factors are used as conditioning information.

Returning now to the aim of this essay, our results suggest that the CP and LN factors should be used as conditioning information when assessing the empirical validity of the liquidity preference hypothesis in U.S. Treasury bills. Our analysis also confirms the versatility of the MR and Wolak tests in accommodating different forms of conditioning information.
Appendix

Relating term premia to the pricing kernel  The exposition here is based on the theoretical framework set out in Boudoukh et al. (1999b). Let $X_t^{(n)}$ be the returns vector. Standard asset pricing theory states that in the absence of arbitrage, the conditional expectation of the returns vector weighted by the stochastic discount factor $M_t^{(n)}$ is the unit of account (Duffie 2001), so that

$$E_t[X_t^{(n)} M_t^{(n)}] = 1. \quad (3.5)$$

A riskfree bond pays the unit of account in all states of the world, so by the asset pricing formula (3.5), the bond price is equal to the conditional expectation of the stochastic discount factor of matching tenor,

$$P_t^{(n)} = \frac{1}{Y_t^{(n)}} = E_t[M_t^{(n)}], \quad (3.6)$$

where $Y_t^{(n)}$ is the gross yield to maturity. The holding period return is the return on an investment strategy to purchase a $n$ period bond at time $t$ and sell it at time $t+1$ (when it is a $n-1$ period bond),

$$R_t^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} = \frac{E_{t+1}[M_{t+1}^{(n-1)}]}{E_t[M_t^{(n)}]} \quad (3.7)$$

The expected present value of the profit or loss of this trading strategy is the expected price of the long bond at the beginning of the next period discounted back to the present less the current price of the long bond

$$E_t[P_t^{(1)} P_{t+1}^{(n-1)} - P_t^{(n)}]. \quad (3.8)$$

Using equation (3.6) and the definition of covariance, it can be shown that the expected profit or loss of the trading strategy equals the negative conditional covariance between the spot stochastic discount factor and the forward stochastic
discount factor

\[- \text{Cov}_t(M^{(1)}_t, M^{(n-1)}_{t+1}) = E_t[M^{(1)}_t]E_t[M^{(n-1)}_{t+1}] - E_t[M^{(1)}_t M^{(n-1)}_{t+1}] \]

\[= E_t[M^{(1)}_t]E_t[M^{(n-1)}_{t+1}] - E_t[M^{(n)}_t] \]

\[= E_t[P^{(1)}_t P^{(n-1)}_{t+1} - P^{(n)}_t]. \tag{3.9} \]

where the second line follows from the first due to the absence of arbitrage. Therefore, the expected profit or loss that characterises the holding period return on the excess returns trading strategy is driven by the covariances between the stochastic discount factors. By dividing this relation through by \(P^{(1)}_t P^{(n)}_t\) or equivalently multiplying by \(Y^{(1)}_t Y^{(n)}_t\) we can express it in terms of excess returns, which yields the relation

\[E_t[R^{(n)}_{t+1}] - Y^{(1)}_t = -\text{Cov}_t(M^{(1)}_t, M^{(n-1)}_{t+1})Y^{(1)}_t Y^{(n)}_t. \tag{3.10} \]

Let \(p^{(n)}_t\) be the log \(n\) period bond price (or discount factor) at time \(t\). Then define

\[y^{(n)}_t \equiv p^{(n)}_t / n, \]

\[r^{(n)}_{t+1} \equiv p^{(n-1)}_{t+1} - p^{(n)}_t, \]

\[r^{(n)}_{x,t+1} \equiv r^{(n-1)}_{t+1} - y^{(1)}_t, \]

where \(y^{(n)}_t\) is the \(n\) period yield at time \(t\), \(r^{(n)}_{t+1}\) is the log holding return from buying an \(n\) period bond at time \(t\) and selling it at time \(t + 1\) as an \(n - 1\) period bond, and excess returns over spot yields \(r^{(n)}_{x,t+1}\) are the log holding period returns on longer period bonds less log spot yields. Now, a further linear approximation may be made to get

\[E_t[r^{(n)}_{x,t+1}] \approx -\text{Cov}_t(M^{(1)}_t, M^{(n-1)}_{t+1}). \tag{3.11} \]

The theory implies that term premia are driven by covariances between stochastic discount factors of different tenors. \textbf{Lucas (1978)} showed that stochastic discount factors can be interpreted as marginal rates of substitution between present and future consumption. In particular, by equation (3.10) the liquidity preference hypothesis

\[E_t[R^{(2)}_{t+1}] < \ldots < E_t[R^{(n-1)}_{t+1}] < E_t[R^{(n)}_{t+1}] \tag{3.12} \]
will hold whenever

\[ \text{Cov}(M_{t}^{(1)}, M_{t+1}^{(n-1)}) < \text{Cov}(M_{t}^{(1)}, M_{t+1}^{(n-2)}) < \ldots < \text{Cov}(M_{t}^{(1)}, M_{t+1}^{(2)}) \] (3.13)

because \( Y_t^{(1)} Y_t^{(n)} \) is always positive and close to one.

**Estimating the forward curve factor** Cochrane and Piazzesi (2005) discovered a tent shaped pattern in the slope coefficients of regressions of excess returns onto linear combinations of the spot rate and forward rates

\[ rx_t^{(i)} = \beta_0^{(i)} + \beta_1^{(i)} y_t^{(1)} + \beta_2^{(i)} g_t^{(2)} + \ldots + \beta_n^{(i)} g_t^{(n)} + \epsilon_t^{(i)}, \quad i = 2, \ldots, n \] (3.14)

where \( g_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)} \) is the log forward rate. This led to the idea that a single linear combination of the spot rate and forward rates explains excess returns across tenors, which can be expressed via the specification

\[ rx_t^{(n)} = b_n CP_t + \epsilon_t^{(n)}, \quad CP_t \equiv \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 g_t^{(2)} + \ldots + \gamma_n g_t^{(n)}, \] (3.15)

Now, in this specification, \( b_n \) and \( \gamma(\cdot) \) are not separately identified, but estimation can proceed by restricting the average value of \( b_n \) to be 1, and regressing the average excess returns across tenors onto the linear forward rates,

\[ \bar{rx}_{t+1} = \hat{CP}_t + \epsilon_{t+1}, \quad \bar{rx}_{t+1} \equiv \frac{1}{n-1} \sum_{i=2}^{n} rx_t^{(i)}, \quad i = 2, \ldots, n. \] (3.16)

The forward curve factor is then defined as the fitted values \( \hat{CP}_t \) for this regression.

**Estimating macroeconomic factors** Suppose there is a panel of macroeconomic variables \( h_{it} \) with the approximate factor structure,

\[ h_{it} = \lambda_i f_t + e_{it}, \] (3.17)

where \( \lambda_i \) are factor loadings, \( f_t \) are macroeconomic factors, and \( e_{it} \) is the error process. The factors \( f_t \) are estimated with principal components analysis

\[ \min_{f_t} (h_t - \Lambda f_t)^2, \]
where \( h_t \) is a cross-sectional vector of macroeconomic variables and \( \Lambda \) is a vector of factor loadings.

In this context, Ludvigson and Ng (2009) devised a procedure for selecting the macroeconomic factors which possess optimal explanatory power in respect to term premia. Specifically, the optimal subset \( \hat{LN}_t \) of \( f_t \) that spans the space of \( f_t \) can be chosen by forming different subsets \( LN^{(s)}_t \) of \( f_t \) and evaluating the Bayesian Information Criterion of the regression models

\[
rx_{t+1}^{(n)} = \alpha CP_t + \beta^t \hat{LN}^{(s)}_t + \gamma_t.
\]

Despite the use of factors as explanatory variables in this regression, ordinary least squares can still be shown to lead to consistent coefficient estimates (Bai and Ng, 2006). The \( \hat{LN}^{(s)}_t \) with the lowest Bayesian Information Criterion is chosen as \( \hat{LN}_t \).

An economic interpretation of the factors \( \hat{LN}_t \) can be obtained by considering the marginal \( R^2 \) of regressions of factors onto each component of the macroeconomic variable panel. In this essay, for the sake of simplicity, we do not follow the procedure of Ludvigson and Ng (2009). Instead, we use the first principal component of \( f_t \) as \( \hat{LN}_t \).

**Implementing the monotonicity relations test** As above, let the sample equivalents of the term premia postmultiplied by the conditioning information be denoted \( \bar{\Delta}^* \). Patton and Timmermann (2010) mentioned the possibility of incorporating conditioning information in this way into their implementation of the Monotonicity Relations test, but did not pursue this idea any further. We implement the test with the forward curve factors and macroeconomic factors as conditioning information.

Applying the alternative hypothesis to the sample term premia\(^{10}\) leads to the test statistic

\[
J_T = \min_{i=2,\ldots,n} \bar{\Delta}^{(i)}.
\]

The Monotonicity Relations test avoids the approximations that follow from the large sample theory of the test statistic \( J_T \) by instead obtaining critical values for the test with a bootstrap (Patton and Timmermann, 2010). Let the returns data

\(^{10}\)Romano and Wolf (2011) critique the monotonicity relations test on the basis that non-monotonicity in the context of expected asset returns really means that adjacent differences in returns are unrestricted, not just less than or equal to zero.
be denoted
\[ \{ r_{t+1}^{(i)}, t = 1, \ldots, T - 1; i = 2, \ldots, n \} . \]

A stationary bootstrap (Politis and Romano, 1994) can then be implemented by
drawing a new sample of returns across all tenors with a randomly selected time
index from the original data
\[ \{ \tilde{r}_{\tau(t+1)}^{(i)}(b), \tau(1), \ldots, \tau(T - 1); i = 2, \ldots, n \} , \]
where \( b \) is an index for the bootstrap iteration number \( \{ b : 1, \ldots, N \} \) such that \( N \)
is large. This resampling process preserves the cross-sectional dependence in the
returns data. Time series dependence is accounted for by resampling in a block,
where the block length is a random variable drawn from a geometric distribution
with a random starting point. This gives the bootstrap distribution of \( \hat{\Delta}_{t+1} \).

The null (\( \bar{\Delta} = 0 \)) is imposed by subtracting \( \bar{\Delta}^{(i)} \) from the bootstrap sample
equivalents \( \bar{\Delta}^{(i)}(b) \). A count is then made of the number of times when a pattern in
the bootstrapped samples emerges that is at least as unfavourable relative to the
null as that observed in the real data. This leads to the bootstrapped test statistic
and \( p \)-value
\[ J_T(b) = \min_{i=2,\ldots,n} (\hat{\Delta}^i(b) - \bar{\Delta}^{(i)}), \quad b = 1, \ldots, B \] (3.19)
\[ \hat{p} = \frac{1}{B} \sum_{b=1}^{B} 1\{ J_T(b) > J_T \}. \] (3.20)
Under this test framework, the null hypothesis is rejected in favour of the liquidity
preference hypothesis when the \( p \)-value is less than 5%.

**Implementing the Wolak test** Drawing on the Wolak test, Boudoukh et al.
(1999a) formulate a conditional test of the liquidity preference hypothesis. They
postmultiply \( \bar{\Delta}^{(i)} \) by the conditioning information \( Z_t^* \) and apply the law of iterated
expectations to restate the null hypothesis as
\[ (\bar{\Delta} - \theta) \otimes Z_t^* \geq 0 , \] (3.21)
where the parameter \( \theta \equiv (\theta_1, \ldots, \theta_{n-1}) \in R^{(n-1)+} \) is positive under the null.

The parameter \( \theta \) is estimated as the sample means of the term premia in equation
conditional on $Z_t^*$, so that

$$\hat{\theta} = \bar{\Delta} \times Z_t^* \geq 0. \tag{3.22}$$

There is no restriction on the sign of these estimates, and they may be negative either because the null is false or because sampling error is present. Let the covariance matrix of the sample moment vector be denoted $\Omega$. This covariance matrix may have non-zero entries off the diagonal, and therefore account for cross-correlation, autocorrelation and heteroskedasticity in the series. In this context, Boudoukh et al. (1993) show that the vector $\hat{\theta}$ is asymptotically normal with $\hat{\theta} \overset{\text{a.s.}}{\sim} N(\theta, \Omega)$. To implement the Wolak one need not know $\Omega$. Instead, a consistent estimate of the sample covariance matrix $\hat{\Omega}$ will suffice. This estimate may be derived with the Newey–West procedure among others.

The sample mean $\hat{\theta}$ needs to be estimated under the restriction that it be nonnegative. Following Boudoukh et al. (1993) the restricted sample mean $\hat{\theta}^R$ can be written as the solution to the problem

$$\min_{\hat{\theta}^R} Q = (\hat{\theta}^R - \hat{\theta})'\Omega^{-1}(\hat{\theta}^R - \hat{\theta}), \tag{3.23}$$

subject to $\hat{\theta}^R \geq 0$. Finally, to test the liquidity preference hypothesis $\theta \geq 0$ with a multivariate one-sided Wald statistic, calculate

$$W \equiv T(\hat{\theta}^R - \hat{\theta})'\hat{\Omega}^{-1}(\hat{\theta}^R - \hat{\theta}). \tag{3.24}$$

This statistic is evaluated at an appropriate level of significance, using the asymptotic distribution

$$\sum_{k=0}^{N} P_{\theta}[\chi_k^2 \geq c] \times w(N, N - k, \hat{\Omega}/T), \tag{3.25}$$

where $c \in R^+$ is the critical region for a given size, $N$ is the number of inequality restrictions, and the weight $w(N, N - k, \hat{\Omega}/T)$ is the probability that $\hat{\theta}^R$ has exactly $N - k$ positive elements.
Chapter 4
Measuring monetary policy expectations

4.1 Introduction

Expectations of future movements in monetary policy play a crucial role in the policy setting process, and the policy transmission mechanism (Woodford, 2010). Of course, expectations are unobservable, and may differ from one economic agent to the next. Nonetheless, as many researchers and practitioners have recognised, financial markets provide a convenient medium through which to measure policy expectations, because market prices for many different financial instruments are sensitive to future movements in policy.

Within the universe of financial instruments, it is clear that fixed income securities bear the closest relation to policy settings, and therefore constitute the best type of financial instrument for measuring policy expectations. Intuitively, the entire fixed income market can be conceptualised as a collection of signals related to private sector expectations of future movements in monetary policy. The pricing of fixed income securities provides a real time, albeit noisy, market based measure of monetary policy expectations. The noise arises from the presence of risk premia that differ in magnitude according to the particular term, credit and liquidity risks that are associated with each traded fixed income security. Therefore, if one has a means to abstract away from these
risk premia, then one can directly measure monetary policy expectations in the traded yields of fixed income securities.

This essay presents an empirical framework for extracting the signal on monetary policy from fixed income market pricing. While it is widely known that the market prices fairly well for movements in the policy instrument over a horizon of up to six months, an application of our framework to Australian fixed income pricing shows that the policy expectations embedded in liquid securities of tenor up to three years still retain a fair degree of accuracy.

Our work addresses an important gap in the literature. While the extant literature acknowledges the existence of risk premia in market pricing for financial instruments, there is no purely empirical way to abstract away from those risk premia when evaluating the accuracy of market expectations with regard to future policy movements. Many authors have fit affine term structure models augmented with survey data on expectations (Kim and Wright, 2005; Kim and Orphanides, 2005; Finlay and Chambers, 2008; Lee Chun, 2011), but these models often assume restrictive functional forms for risk premia and stochastic processes for the short rate. Another stream of the literature looks at money market and interest rate futures pricing (Kuttner, 2001; Gürkaynak et al., 2007; Goodhart and Lim, 2011), but does not suggest a way to strip out risk premia in such pricing (Piazzesi and Swanson, 2008; Hamilton, 2009) without relying on exogenous sources such as survey data (Ichiue and Yuyama, 2009). The foreign exchange market (Engel and West, 2005; Fatum and Scholnick, 2006) and stock market (Bernanke and Kuttner, 2005) literature on policy expectations can be put to one side, as these markets only have an indirect or tangential relation to monetary policy. Some research has been conducted on the extraction of expectations from bond and swap markets (Nagano and Baba, 2008; Joyce et al., 2008; Söderlind and Svensson, 1997) and bond options (Vahamaa, 2005), which are highly sensitive to movements in the policy instrument. But this research has not proposed an adequate method for controlling for the term premia and the credit and liquidity premia that are often priced into the longer term fixed income securities used as inputs into their curve construction efforts. Finally, hybrid approaches have been proposed that combine elements of the Taylor rule with market pricing to
measure expectations (Smith and Taylor, 2009; Taylor, 2010; Hamilton et al., 2011). While these approaches reflect central banks’ price stability and full employment objectives, they do not fully reconcile the difference in frequency between macro-economic variables pertaining to the output gap and inflation expectations on the one hand, and fixed income market pricing on the other.

We construct an empirical framework for measuring the explanatory power of a wide range of fixed income securities, including liquid money market securities, short term interest rate futures and Treasury bond futures in relation to movements in monetary policy. Following Gürkaynak et al. (2007), yields and implied forward rates extracted from market pricing for each of the different fixed income securities under consideration are compared to the cash rate when grossed up or averaged over the appropriate horizon in an effort to see how accurately the market prices for policy movements. Specifically, the implied cash rate is regressed onto the yields and implied forward rates with ordinary least squares (OLS), where the cash rate is the dependent variable in the regression to reduce measurement error (Fama, 1975). When the yields and forwards are subject to additional risk premia, the rates are combined as instruments and regressors in an instrumental variables (IV) specification that strips out the effect of risk premia and thereby identifies the underlying signal on policy expectations in market pricing. The IV model is estimated with the generalised method of moments (GMM).

We apply this approach to examine market pricing in Australia for policy movements over horizons of up to three years, in contrast to the vast majority of the literature on market-based measures of policy expectations (with the exception of Goodhart and Lim (2011), who examine short to medium term policy expectations priced into the United Kingdom interest rate swap curve and government bond curve). While extant studies of Australian interest rates have focused on term premia (Walsh and Tan, 2008; Guido and Walsh, 2005), yield curve forecasting (Bilson et al., 2008; Murik, 2006), and the short rate process (Gray and Smith, 2008; Sanford and Martin, 2006; Tre pongkaruna and Gray, 2006; Chan, 2005; Gray, 2005; Tre pongkaruna and Gray, 2003), our work constitutes the first empirical examination of policy expectations in the Australian fixed income market. In particular, our approach complements
the literature on estimating and forecasting the policy rate according to the
short rate process by incorporating information from many different classes of
fixed income securities, whose tenors span the entire yield curve.

We find that overnight indexed swaps outperform other fixed income
securities at forecasting the Australian cash rate over the nearest two quarters,
with one month ahead root mean squared forecast errors inside the typical 25
basis point cash rate movement. Beyond that, accuracy drops off substantially
over longer horizons. The OLS regressions show that some of the influence
of risk premia can be incorporated into the intercept term for each security,
thus improving the forecast efficacy of all of the Australian fixed income
securities. However, these regressions still leave much of the variation in the
cash rate unexplained, especially over longer horizons of up to three years.
The GMM IV framework allows us to address this problem by instrumenting
the bond futures pricing with overnight indexed swap rates and implied yields
on interbank futures contracts. Using our GMM IV framework, we find that
the bond futures pricing contains policy expectations that forecast the average
of future movements in the Australian overnight cash rate over horizons of
one to three years from 2004 to 2010 to well within 75 basis points.

The remainder of the essay is organised as follows. Section 4.2 constructs
the measures of monetary policy expectations. Section 4.4 estimates the
measures with Australian data and Section 4.5 concludes.

4.2 Measuring expectations

The essence of our approach is as follows. Suppose that, in implementing our
model, we have perfect foresight of the cash rate. That is, in each period
of our sample, we know the cash rate for all other periods of the sample.
Then we can use OLS equations to compare the ex post average cash rate to
market implied yields and forward rates (‘market yields’) in order to gauge
the accuracy of traded fixed income securities in forecasting the cash rate.\footnote{The use of returns on financial securities to gauge private sector expectations of future movements in monetary policy can be justified with reference to standard asset pricing theory (Gürkaynak et al., 2007). See the Appendix for an exposition.}
Where market pricing is subject to noise from risk premia, we employ GMM IV estimators to assist in isolating the portion of market yields that pertain solely to expectations. The use of GMM IV estimators constitutes the principal contribution of our work to the literature. This Section describes the approach in detail.

Following the notation in Gürkaynak et al. (2007), let \( i_t \) be the overnight cash rate at time \( t \) and define the average cash rate as

\[
\bar{i}_{t,t+k} \equiv \prod_{j=t}^{t+k-1} (1 + i_j) - 1 \approx \frac{1}{k} \sum_{j=t}^{t+k-1} i_j
\]

Assuming that the daily cash rate through the entire sample is known, we can calculate \( \bar{i}_{t,t+k} \) for any \( t \) and \( k \) within the sample. Let \( r_{t,t+j,t+k}^s \) be the forward rate implied by time \( t \) pricing on fixed income security \( s \) for a loan beginning at time \( t + j \) and ending at time \( t + j + k \). This notation encompasses zero coupon yields, where \( j = 0 \). For money market securities, quotes are already given as zero coupon rates and forward rates. For short term interest rate futures and Treasury bond futures, quotes are converted to implied yields.

By selecting appropriate starting points \( t \) and horizons \( k \), we then compute returns on the monetary policy instrument \( \bar{i}_{t+j,t+k} \) such that they line up with the corresponding forward rates \( r_{t,t+j,t+k}^s \) implied by market pricing for all fixed income securities \( s \) at all times \( t \) in the sample. This leads to the central linear model to be used in this study (Gürkaynak et al., 2007; Goodhart and Lim, 2011), which compares forward rates implied by the market pricing of fixed income securities to returns on the cash rate over the corresponding period:

\[
\bar{i}_{t+j,t+k} = \alpha + \beta r_{t,t+j,t+k}^s + \epsilon_{t,t+j,t+k}^s. \tag{4.1}
\]

This equation corresponds to the asset pricing theory (see the Appendix for details), but the cash rate is on the left hand side (so the risk premium is the negative intercept where the slope coefficient is one). This is reasonable because to the market, \( r_{t,t+j,t+k}^s \) is known at time \( t \), whereas \( \bar{i}_{t+j,t+k} \) is not. Furthermore, Fama (1975) regressed inflation onto interest rates to mitigate measurement error in inflation. Similarly, we put the cash rate on the left.
hand side of the regression specification to mitigate the ex-post to ex-ante measurement error.

It can be shown that the error term $\epsilon_t$ in this specification incorporates the risk premium $\rho_{t,t+k}$ (see the Appendix), thereby allowing for variation over time in the risk premium without affecting the integrity of the model. The model (4.1) provides the central empirical framework for this paper. The idea is to use this model to assess the extent to which variability in the monetary policy instrument is explained by market pricing on various fixed income securities. Specifically, the goodness of fit of the model and the behaviour of its error term for different assets provide measures of the accuracy of financial market expectations regarding future movements in monetary policy.

The error term $\epsilon_{t+j+k}$ is fundamental to the specification of the central linear model. Depending on the particular characteristics of security $s$, this error term may contain time varying, security specific risk premia; which may cause the model to be mis-specified. Our solution for this problem is to use instrumental variables. Specifically, we recast model (4.1) as an instrumental variables regression, so as to derive a composite measure of fixed income expectations for future monetary policy. This measure combines the accuracy of the short term securities and the liquidity of the long term securities. Hence, market yields for securities $s_1$ that are subject to risk premia are instrumented by corresponding market yields over the same time horizon for securities $s_2$ that are not subject to the risk premia. The following instrumental variables specification emerges,

$$r_{t,t+j,t+k} = \delta + \gamma r_{t,t+j,t+k} + \zeta_t$$
$$\bar{r}_{t,t+j,t+k} = \alpha + \beta r_{t,t+j,t+k} + v_t,$$

where $r_{t,t+j,t+k} \equiv \delta + \gamma r_{t,t+j,t+k}$.

The validity of the instrumental variables specification is predicated on the quality of the market yields on the $s_2$ securities as instruments. To satisfy the qualities of a good instrument, the market yields on the $s_2$ securities must be orthogonal to the risk premia in the $s_1$ securities, and must be correlated with the portions of the market yields on the $s_1$ securities that reflect investors’
pricing for future movements in the cash rate. It is therefore appropriate to
count tests for weak instruments to assist the construction of the market
based measures of policy expectations. Specifically, a simple $F$-test for an OLS
regression of long yields onto short (instrument) yields will be used (Stock
et al., 2002).

Having performed tests for weak instruments, we proceed to estimate the
IV model (4.2) with the generalised method of moments (GMM). The GMM
moment conditions for market yields of horizon $k$ are then

$$g_{T,k}(\alpha, \beta) \equiv \frac{1}{T} \sum_{t=1}^{T} [i_{t,t+k} - \alpha - \beta r_{t,t+k}^{(s_1)}] \otimes r_{t,t+k}^{(s_2)}.$$  \hspace{1cm} (4.3)

The GMM instrumental variables approach is effective in this context because
the market yields on the short term securities are correlated with the market
yields on the overnight rate, but are uncorrelated with the extra risk premia
that affect long term securities. The result is a revision of the original OLS
regression model (4.1) that combines the best of both worlds, in that it
incorporates both the accuracy of short term securities with the liquidity of
long term securities. Also, GMM has better asymptotic properties than two
stage least squares, and there is an exact solution in the linear case, meaning
that the system is just-identified (Hayashi, 2000).

We conclude this Section by comparing our approach to the literature.
Gürkaynak et al. (2007) and Goodhart and Lim (2011) use model (4.1) to
consider which financial market security provides the most accurate monetary
policy expectations from a broad set of fixed income securities and derivatives.
They made adjustments to the model to account for the use of securities
referenced over forward interest rates, and to correct for cointegration between
the overnight rate and market yields for other fixed income securities. However,
they did not adapt the framework to deal with what are arguably the strongest
impediments to the use of the model in practice — the lack of liquidity on a
duration adjusted basis in short term securities that are directly related to the
overnight rate, and the contamination of market yields on more liquid longer
term securities by risks that are beyond the scope of the basic asset pricing
theory. Our empirical framework provides a means to address these issues.

Finally, we note that there is a clear portfolio management interpretation of the central linear model and its underlying economic intuition. Suppose a fixed income investor wants to trade at time $t$ over the time horizon $t + j$ to $t + k$, where $j$ may be equal to zero. Based on her preferences in respect of term, credit and liquidity risks, and the characteristics of security $s$, the investor decides to trade security $s$. Whilst other securities may act as a benchmark for $s$, the most fundamental counterfactual is the cash rate. In other words, the investor can substitute a strategy to invest in $r_{t,t+j,t+k}^s$ with a strategy based on $\bar{r}_{t+j,t+k}$. For this reason, our empirical framework actually reflects investment decisions that fixed income portfolio managers make in practice.

### 4.3 The fixed income universe

Having presented the modelling framework, we turn to a discussion the various Australian money market and fixed income securities that will be used to implement the instrumental variables regression. The overnight cash rate is the Australian monetary policy instrument, and will be used as $i_t$. Short term instruments $s_2$ which correspond to zero coupon bonds and futures contracts with tenor less than one year and always use simple compounding include:

**Overnight indexed swaps** An interest rate swap where the floating rate is the monthly average overnight cash rate. Illiquid but directly related to overnight cash rate.

**Bank bills** A short term borrowing instrument of a major financial institution. Liquid but subject to credit risk.

**Interbank futures** A futures contract written on the average monthly overnight cash rate. Illiquid but directly related to overnight cash rate.

**Bank bill futures** A futures contract written on the three month bank bill rate set. Highly liquid but subject to credit risk.
The principal long term instrument \( s_1 \) that we use in our study are Treasury bond futures contracts. These contracts summarise the pricing in the Australian Treasury yield curve, and are far more liquid than the underlying Treasury nominal bonds.\(^2\) Market participants actively use bond futures to hedge their interest rate exposures. For this reason, the bond futures contracts are a more reliable indicator of fixed income market pricing for term interest rate risk than the interest rate swap curve, which also include an element of counterparty risk.

These traded short and long term instruments are all sensitive in different ways to the overnight cash rate, and thus pricing for each instrument necessarily sheds light on the fixed income markets’ expectations regarding future movements in monetary policy. The judicious application of our GMM instrumental variables framework in combining instruments and regressors to control for risk premia should assist in extracting and evaluating the market’s expectations in each case.

### 4.4 Expectations in Australian bond pricing

In this Section, we apply the empirical framework for assessing the accuracy of expectations in fixed income pricing to the Australian market. Data on the pricing of Australian fixed income securities is sourced from Reuters. Rates for are collected daily from 1 January 2004 to 22 October 2010, a total of 1,735 observations. The cash rate series is sourced from the Reserve Bank of Australia.\(^3\)

At the outset, Figure 4.1 depicts selected rates from our dataset. It is clear

\(^2\)Our use of bond futures incorporates the futures to physical basis, and also prices for coupon payments rather than being a direct reflection of Treasury zero coupon yields. However, given the status of the bond futures as the primary mechanism for price discovery in the Australian fixed income market, we believe that the bond futures are preferable to the underlying Treasury bonds for use in our study. Note that the bond futures data (along with interbank and bank bill futures data) used in our study have been adjusted to smooth for roll trades and associated price volatility around the expiry date.

\(^3\)While it is always better to use longer sample periods in studies of this nature, we note that interbank futures first commenced trading on the Australian Securities Exchange in August 2003. Hence, to include this fundamentally important futures contract in our study, we commence our sample in January 2004.
that the short end rates tend to track the cash rate very closely, but also that
in 2008, the implied yields on 3Y and 10Y bond futures fell dramatically, well
before the cash rate was cut by the RBA. Similarly, the pricing on bond futures
seemed to anticipate the normalisation of the monetary policy instrument
from late 2009 onwards. Of course, these market movements in the bond
futures pricing may reflect risk sentiment or market reactions to Australian
economic fundamentals as opposed to interest rate expectations. Nonetheless,
our aim in this Section will be to apply our GMM IV empirical framework in
an analysis of the expectations embedded in the market pricing of the different
fixed income securities in our sample.

Figure 4.1: Fixed income market pricing

As is generally the case with yield panel data, the rates on fixed income
securities depicted in Figure 4.1 appear to be non-stationary and co-integrated.
These statistical properties in the data have the potential to affect the results
of our econometric analysis. However, in separate results not reproduced
here, we have confirmed the robustness of our modelling to the presence of
non-stationarity and co-integration in the yield. Further, we also confirmed

\[4\text{Specifically, Augmented Dickey–Fuller tests with a single lag show that the yields in our dataset are non-stationary in levels, but are stationary in first differences. Nonetheless, the}\]
that our analysis is robust to the presence of the structural breaks in the series, which were caused by the Global Financial Crisis\(^5\).

Armed with this set of time series, we next construct implied forwards from the cash rate series that correspond to the market pricing quotes for each class of fixed income securities under consideration. We illustrate the root mean squared errors for the short end securities. Figure 4.2 shows the root mean squared forecast errors of the forward rates of short end instruments of different tenors relative to the actual cash rate over the corresponding periods. Note that this Figure only includes instruments with a tenor of less than one year. It is clear that the overnight indexed swap rates contain the most accurate set of cash rate forecasts out to six months. However, there is a surprising degree of dispersion in accuracy between the different short end instruments. Specifically, bank bill futures outperform bank bills, and interbank futures appear to be less accurate than overnight indexed swaps.

Table 4.1 drills down into the descriptive statistics for the underlying forecast errors by instrument, that also underpin the RMSE graph. In the Table, “OIS” stands for overnight indexed swap, “BAB” denotes bank bills, “YIB” refers to monthly interbank futures (the Australian equivalent to futures on the federal funds rate), “YBA” stands for quarterly bank bill futures, and “YTT” and “YTC” denote three year and ten year Treasury bond futures, respectively. The suffixes “#m” and “c#” indicate the tenor in months and futures contract number, respectively. The Table shows how forecast errors tend to fan out by tenor, and also illustrates the manner in which risk premia combinations of yields that are used in our main results generally imply two co-integrating relations according to the Johansen maximum eigenvalue and trace tests. While these co-integrating relations may not be entirely consistent with our parameter estimates from the OLS and GMM IV models, they confirm the existence and strength of the empirical relationships between the cash rate and the fixed income market pricing variables under consideration in our study.

\(^5\)We used empirical fluctuation processes (Zeileis et al., 2002) to determine that the parameters of OLS relationships between the fixed income market yields and the cash rate change through time. One way to address these changes is to break down the sample into subsamples. Instead, we stochastically detrended our yield dataset, and re-run our models. Here, we do not find an improvement in the fit of the GMM IV model to the detrended data. Further, we consider that any benefits of stochastic detrending are outweighed by the clarity with which the original non-detrended model can be interpreted, and its direct connection to market pricing.
dilute the signal on expectations from one type of fixed income security to the next. For example, as one would expect, OIS and interbank futures tend to have the smallest mean forecast errors, whereas bank bills have slightly larger mean forecast errors, which reflect credit risk. Also, beyond the six month horizon, the accuracy of forecasts tend to wane substantially, suggesting that term premia and other risk premia play an increasingly important role in determining forward rates at longer tenors.

When we reach the tenors beyond one year, we need to put aside the short end instruments (OIS, BAB, YIB and the first four YBA contracts) because they ostensibly do not directly price for longer term expectations. However, the three year and ten year bond futures contracts, YTTc1 and YTCc1 respectively, clearly span the horizon of between one and three years which is of interest to us. We can posit that the implied yields on three year and ten year bond futures contracts contain information on expectations over horizons of one year, two years and three years (hence the 1Y, 2Y and 3Y suffixes to YTTc1 and YTCc1 in Table 4.1) and then compare the implied

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6We also note here that, to accommodate the long horizons, the dataset for the one, two and three year horizons end in October 2009, 2008 and 2007 respectively, to ensure that the
yields to the average cash rates over these horizons for each day in our sample.

The performance of the three and ten year futures in forecasting the average cash rate over these horizons is nonetheless poor, as suggested by the large interquartile ranges and forecast error standard deviations for these contracts in the Table.

Table 4.1: Forecast error descriptive statistics

<table>
<thead>
<tr>
<th>Series</th>
<th>Min</th>
<th>1Q</th>
<th>Med</th>
<th>Mean</th>
<th>3Q</th>
<th>Max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>OIS.1m</td>
<td>-0.2762</td>
<td>0.0115</td>
<td>0.0175</td>
<td>0.0411</td>
<td>0.0300</td>
<td>0.8759</td>
<td>0.0985</td>
</tr>
<tr>
<td>OIS.3m</td>
<td>-0.2720</td>
<td>0.0200</td>
<td>0.0500</td>
<td>0.1042</td>
<td>0.1100</td>
<td>1.0910</td>
<td>0.2088</td>
</tr>
<tr>
<td>OIS.6m</td>
<td>-0.2902</td>
<td>-0.0090</td>
<td>0.0825</td>
<td>0.1802</td>
<td>0.1851</td>
<td>2.5010</td>
<td>0.3921</td>
</tr>
<tr>
<td>BAB.1m</td>
<td>-0.0518</td>
<td>0.0950</td>
<td>0.1400</td>
<td>0.1856</td>
<td>0.2000</td>
<td>1.7070</td>
<td>0.1703</td>
</tr>
<tr>
<td>BAB.3m</td>
<td>-0.0734</td>
<td>0.1250</td>
<td>0.1800</td>
<td>0.2806</td>
<td>0.3188</td>
<td>1.9470</td>
<td>0.2907</td>
</tr>
<tr>
<td>BAB.6m</td>
<td>-0.1442</td>
<td>0.1315</td>
<td>0.2200</td>
<td>0.4018</td>
<td>0.3600</td>
<td>2.9120</td>
<td>0.5303</td>
</tr>
<tr>
<td>YIBc1</td>
<td>-0.2630</td>
<td>-0.0150</td>
<td>-0.0050</td>
<td>0.0112</td>
<td>0.0000</td>
<td>1.1210</td>
<td>0.1726</td>
</tr>
<tr>
<td>YIBc2</td>
<td>-0.4280</td>
<td>-0.0100</td>
<td>0.0000</td>
<td>0.0495</td>
<td>0.0298</td>
<td>1.9130</td>
<td>0.2985</td>
</tr>
<tr>
<td>YIBc3</td>
<td>-0.4248</td>
<td>-0.1072</td>
<td>0.0110</td>
<td>0.0919</td>
<td>0.0750</td>
<td>2.4130</td>
<td>0.4266</td>
</tr>
<tr>
<td>YIBc4</td>
<td>-0.4792</td>
<td>-0.1793</td>
<td>0.0150</td>
<td>0.1252</td>
<td>0.1010</td>
<td>3.1070</td>
<td>0.5654</td>
</tr>
<tr>
<td>YBAc1</td>
<td>-0.7596</td>
<td>0.1400</td>
<td>0.2000</td>
<td>0.2627</td>
<td>0.3300</td>
<td>1.6500</td>
<td>0.2639</td>
</tr>
<tr>
<td>YBAc2</td>
<td>-0.6475</td>
<td>0.1100</td>
<td>0.2023</td>
<td>0.3273</td>
<td>0.3700</td>
<td>3.2020</td>
<td>0.4981</td>
</tr>
<tr>
<td>YBAc3</td>
<td>-0.5277</td>
<td>-0.0200</td>
<td>0.1300</td>
<td>0.4496</td>
<td>0.4010</td>
<td>4.9020</td>
<td>0.9554</td>
</tr>
<tr>
<td>YBAc4</td>
<td>-0.6600</td>
<td>-0.1676</td>
<td>0.0600</td>
<td>0.5598</td>
<td>0.4539</td>
<td>5.6270</td>
<td>1.3364</td>
</tr>
<tr>
<td>YTTc1/1Y</td>
<td>-0.8700</td>
<td>-0.4141</td>
<td>-0.1905</td>
<td>0.0414</td>
<td>0.2900</td>
<td>2.0480</td>
<td>0.6429</td>
</tr>
<tr>
<td>YTTc1/2Y</td>
<td>-1.0370</td>
<td>-0.6442</td>
<td>-0.1970</td>
<td>0.1198</td>
<td>0.6436</td>
<td>2.7550</td>
<td>0.9623</td>
</tr>
<tr>
<td>YTTc1/3Y</td>
<td>-1.3550</td>
<td>-0.7980</td>
<td>-0.2620</td>
<td>-0.0990</td>
<td>0.3687</td>
<td>1.8890</td>
<td>0.7996</td>
</tr>
<tr>
<td>YTCc1/1Y</td>
<td>-1.1620</td>
<td>-0.5393</td>
<td>-0.1171</td>
<td>0.1668</td>
<td>0.8308</td>
<td>2.2300</td>
<td>0.8567</td>
</tr>
<tr>
<td>YTCc1/2Y</td>
<td>-1.1670</td>
<td>-0.6466</td>
<td>-0.1478</td>
<td>0.1054</td>
<td>0.5642</td>
<td>2.4820</td>
<td>0.9183</td>
</tr>
<tr>
<td>YTCc1/3Y</td>
<td>-1.2580</td>
<td>-0.6689</td>
<td>-0.1361</td>
<td>-0.1077</td>
<td>0.3297</td>
<td>1.4390</td>
<td>0.6717</td>
</tr>
</tbody>
</table>

cash rate averages were not being cut off beyond the end of our dataset in October 2010.

Another way to consider longer term expectations is to examine pricing on longer term (5th to 12th) bank bill futures contracts. However, these contracts tend to be illiquid, and contain both term premia and credit premia along with futures-to-forward basis risk. We believe that these risk premia are higher in magnitude than those applicable to Treasury bond futures, so we stick with the latter contracts in our study.
It is clear that the key limitation of the forecast errors discussed above is that they contain risk premia. In other words, the fixed income market’s pricing for these instruments does not purely reflect expectations – the market yields also prices for various risks such as term risks, credit risks and liquidity risks. A simple way to address this problem is to assume that market yields price for risk down to a linear projection of the average cash rates – this is the regression equation (4.1). The OLS estimates set out in Table 4.2 demonstrate that it is indeed possible to abstract away from some of the risk premia for shorter dated securities. We are not suggesting here that risk premia are constant through time, instead the idea is that they tend to average to a constant at the short tenors, consistent with the asset pricing theory (set out in the Appendix). The high \( R^2 \) numbers for the shorter dated securities bear testament to this, and imply that risk premia are being absorbed into the regression intercept terms across instruments at shorter tenors.

More importantly, given the focus of our research on extracting expectations from longer dated securities, the OLS estimates suggest that yields on the longer dated three year and ten year bond futures contracts explain substantial amounts of the variability in the cash rate. For instance, the \( R^2 \) for the three year bond futures contract over a one year horizon is 64.6% and that for ten year contract over a three year horizon is 64.4%. Hence, the pricing in bond futures contracts does seem to incorporate a signal on expectations, albeit a weaker one than the short end securities. Nonetheless, the appropriate inference is not to reject long dated securities as market measures of expectations, but to recognise that risk premia are distorting those signals in a manner that cannot be corrected by linear models alone.

However, there may be lot more information on expectations priced into longer dated securities than is reflected in the OLS results. If this is the case, the challenge is therefore to model the additional risk premia that would otherwise be part of the OLS error term (forecast errors). This may improve the overall fit of the regression, and therefore facilitate identification of the underlying signal on expectations in implied yields on bond futures contracts. As explained above, our preferred mechanism for achieving this is instrumental variables. This is because the risk premia can be treated as measurement errors,
and using an instrumental variables framework, we can specify combinations of regressors and instruments that allow us to strip out the effects of various types of risk premia.

Of course, the validity of this approach depends on the quality of our instruments. We have selected rates on the one, three and six month OIS securities and the implied yields on second, third and fourth interbank futures contracts as instruments in our analysis. We use the implied yields on the three and ten year Treasury bond futures contracts as regressors.\footnote{We performed \textit{F}-tests on each of the pairs of instruments and regressors in order to}

\begin{table}
\centering
\caption{OLS estimates for equation (4.1)}
\begin{tabular}{lcccc}
\hline
Series & $\alpha$ & se($\alpha$) & $\beta$ & se($\beta$) & $R^2$ \\
\hline
OIS.1m & 0.0565 & 0.0154 & 0.9825 & 0.0027 & 0.9892 \\
OIS.3m & 0.4290 & 0.0245 & 0.9106 & 0.0043 & 0.9690 \\
OIS.6m & 0.8446 & 0.0394 & 0.8293 & 0.0069 & 0.9100 \\
BAB.1m & 0.0639 & 0.0216 & 0.9567 & 0.0037 & 0.9789 \\
BAB.3m & 0.2226 & 0.0334 & 0.9145 & 0.0056 & 0.9478 \\
BAB.6m & 0.6826 & 0.0549 & 0.8204 & 0.0092 & 0.8469 \\
YIBc1 & -0.0952 & 0.0250 & 1.0124 & 0.0044 & 0.9733 \\
YIBc2 & 0.1339 & 0.0371 & 0.9681 & 0.0066 & 0.9378 \\
YIBc3 & 0.5570 & 0.0472 & 0.8915 & 0.0083 & 0.8879 \\
YIBc4 & 0.8866 & 0.0584 & 0.8307 & 0.0103 & 0.8176 \\
YBAc1 & 0.4832 & 0.0279 & 0.8751 & 0.0047 & 0.9595 \\
YBAc2 & 0.7562 & 0.0499 & 0.8193 & 0.0084 & 0.8672 \\
YBAc3 & 1.3313 & 0.1014 & 0.7040 & 0.0170 & 0.5413 \\
YBAc4 & 2.5937 & 0.1456 & 0.4769 & 0.0244 & 0.2094 \\
YTTc1/1Y & -0.1416 & 0.1079 & 1.0184 & 0.0196 & 0.6461 \\
YTTc1/2Y & 6.1874 & 0.2152 & -0.1034 & 0.0375 & 0.0062 \\
YTTc1/3Y & 10.2088 & 0.0981 & -0.7953 & 0.0174 & 0.6814 \\
YTCc1/1Y & -2.0842 & 0.2406 & 1.3442 & 0.0430 & 0.3973 \\
YTCc1/2Y & 8.9949 & 0.3125 & -0.5960 & 0.0547 & 0.0880 \\
YTCc1/3Y & 11.9909 & 0.1491 & -1.1135 & 0.0265 & 0.6435 \\
\hline
\end{tabular}
\end{table}
we use the average cash rate over horizons of one, two and three years as the dependent variables in our instrumental variables framework.

Results for the GMM IV estimation of model (4.2) with moment conditions (4.3) for the various combinations of regressors and instruments are set out in Tables 4.3 and 4.4. Starting with the coefficients and root mean squared forecast errors (RMSE) set out in Table 4.3 it is clear that the GMM IV model performs almost equally as well as the OLS model in forecasting the average cash rate with implied yields on three and ten year bond futures contracts instrumented by pricing on OIS and interbank futures. This is to be expected, given that the objective function for the OLS regressions is directly defined in terms of minimising the RMSE. Interestingly, with the exception of the two year forecast horizon for three year bond futures, the forecast accuracy for both the three year and ten year futures increases as we push out the forecast horizon further. As suggested above, this result could be driven by the flight to quality during the Global Financial Crisis. But it is also arguable that medium term policy expectations were being adjusted during this period, and that this adjustment was reflected in the implied yields on three year and ten year bond futures over the three year horizon.

Table 4.3 also indicates that the GMM IV coefficient estimates are often quite different from the corresponding OLS coefficient estimates, and the standard errors of the coefficients are always much higher in the GMM IV case. This does not necessarily translate into a clearer interpretation of the coefficients, though. If we had managed to control for all risk premia in the bond futures pricing with our GMM IV framework, and the fixed income market’s ex-ante expectations for the cash rate are unbiased, then we should see the slope coefficients converge towards one, and the intercept coefficients fall away to zero. In the result, the direction and magnitude of the coefficient changes from the GMM IV model relative to the OLS model are far from clear. For example, it is encouraging that the GMM IV intercept (slope) coefficients for the three year futures over the three year horizon for all six test for the presence of weak instruments. In results not reproduced here, our tests rejected at the 1% level the hypotheses that the slope coefficients were zero for all of the pairs under considerations. This constitutes evidence that our selected instruments are not weak instruments in the context of our model.

65
instruments are slightly closer to zero (one) than their OLS equivalents. But this result is reversed for the ten year futures contract with the six instrument combinations.

Nonetheless, despite their similar forecasting performance, it is arguable that the GMM IV models provide a better description of the underlying data. This is because OLS model can only control for risk premia via the intercept term, whereas the GMM IV model adjusts for risk premia with both the intercept term and the use of yields on short term securities as instruments. In other words, while the RMSE may be slightly lower for the OLS model than for the GMM IV model across the sample, the GMM IV forecasts could be expected to be superior at any particular point in time within the sample due to the greater capacity of this model to control for risk premia.

Table 4.4 presents results on the forecast accuracy results of the GMM IV models, in order to present a deeper analysis of its performance as a market based measure of policy expectations. In this Table, it is clear that the models are all unbiased, regardless of the regressor, instrument and horizon. The mean absolute errors (MAE), mean percentage errors (MPE) and mean absolute percentage errors (MAPE) are all consistent with the RMSE results for each model presented already, and reproduced in this Table. Finally, the test statistic and \( p \)-value for the two-sided Diebold and Mariano (2002) test for forecast equivalency are set out in the last two columns of the Table. In almost all cases\(^9\) the test rejects the null hypothesis of forecast equivalency between the forecast errors of the GMM IV and OLS models. This shows that our empirical framework complements the existing OLS models commonly used in the literature by such authors as Gürkaynak et al. (2007) and Goodhart and Lim (2011).

In circumstances like these, where there is a degree of uncertainty about model specification, it is appropriate to take averages across the results of the different GMM IV model specifications that we have applied to our dataset. Specifically, the simple average of the GMM IV RMSE results in Tables 4.3 and 4.4 across the OIS and interbank futures instruments over the one to three year horizon.

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\(^9\)Except for the problematic two year horizon, which is affected by the end of the dataset for that horizon in October 2008, at the height of the Global Financial Crisis.
horizon is 55 basis points for the three year futures contract, and is 63 basis points for the ten year futures contract. These RMSE numbers indicate that there is a fair degree of accuracy in the policy expectations embedded in bond futures pricing during our sample period. Indeed, one would be hard-pressed to find a professional forecaster (or to construct an econometric model without reference to financial market pricing) able to forecast the average cash rate over a one to three year horizon to this degree of accuracy.

Hence, the IV model helps uncover the signal on expectations in market pricing for the longer dated fixed income securities, and does constitute a substantial improvement on the OLS models, in a way that is driven by economic arguments. In this respect our work constitutes a natural extension of Gürgen et al. (2007) and Goodhart and Lim (2011), by translating their OLS models for extracting expectations from fixed income securities into an instrumental variables framework, thereby improving the extant models’ capacity to control for risk premia.
Table 4.3: GMMIV estimates for moment conditions (4.3)

<table>
<thead>
<tr>
<th>Regressor / Instrument / Horizon</th>
<th>$\alpha$</th>
<th>$se(\alpha)$</th>
<th>$\beta$</th>
<th>$se(\beta)$</th>
<th>RMSE$^{\text{GMM}}$ (%)</th>
<th>RMSE$^{\text{OLS}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTTc1 / OIS.1m / 1Y</td>
<td>-0.49</td>
<td>0.76</td>
<td>1.08</td>
<td>0.15</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>YTTc1 / OIS.3m / 1Y</td>
<td>-0.54</td>
<td>0.62</td>
<td>1.09</td>
<td>0.12</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>YTTc1 / OIS.6m / 1Y</td>
<td>-0.50</td>
<td>0.54</td>
<td>1.08</td>
<td>0.11</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>YTTc1 / YIBc2 / 1Y</td>
<td>-0.50</td>
<td>0.66</td>
<td>1.08</td>
<td>0.13</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>YTTc1 / YIBc3 / 1Y</td>
<td>-0.55</td>
<td>0.58</td>
<td>1.09</td>
<td>0.12</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>YTTc1 / YIBc4 / 1Y</td>
<td>-0.58</td>
<td>0.54</td>
<td>1.10</td>
<td>0.11</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>YTTc1 / OIS.1m / 2Y</td>
<td>9.22</td>
<td>1.42</td>
<td>-0.63</td>
<td>0.26</td>
<td>0.79</td>
<td>0.74</td>
</tr>
<tr>
<td>YTTc1 / OIS.3m / 2Y</td>
<td>7.92</td>
<td>0.65</td>
<td>-0.41</td>
<td>0.12</td>
<td>0.76</td>
<td>0.74</td>
</tr>
<tr>
<td>YTTc1 / OIS.6m / 2Y</td>
<td>7.50</td>
<td>0.26</td>
<td>-0.33</td>
<td>0.05</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>YTTc1 / YIBc2 / 2Y</td>
<td>8.47</td>
<td>0.94</td>
<td>-0.50</td>
<td>0.17</td>
<td>0.77</td>
<td>0.74</td>
</tr>
<tr>
<td>YTTc1 / YIBc3 / 2Y</td>
<td>7.62</td>
<td>0.57</td>
<td>-0.35</td>
<td>0.11</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>YTTc1 / YIBc4 / 2Y</td>
<td>7.13</td>
<td>0.14</td>
<td>-0.27</td>
<td>0.03</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>YTTc1 / OIS.1m / 3Y</td>
<td>9.91</td>
<td>1.13</td>
<td>-0.74</td>
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<td>0.23</td>
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Table 4.4: Forecast accuracy, GMMIV model (4.3)

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4.5 Conclusion

In summary, our results in respect of expectations in short dated securities were consistent with the literature. The short end securities contain different, complementary information on monetary policy expectations over horizons of up to two quarters ahead. Furthermore, in our OLS analysis, we confirmed that longer dated securities are subject to risk premia, which complicates the estimation of longer horizon measures of policy expectations from these securities. To address this problem, we constructed measures of policy expectations in Australian Treasury three year and ten year bond futures pricing over the one to three year horizon with our GMM IV framework. We found that the bond futures pricing contains policy expectations that are accurate to well inside 75 basis points (or three standard movements in the cash rate of 25 basis points) when forecasting the average of future movements in the cash rate over horizons of one to three years.

Hence, we have demonstrated with our simple empirical framework that pricing on three year and ten year bond futures contracts incorporates important information about monetary policy expectations at horizons of between one and three years — information that could be highly beneficial for policy makers and investors.
Appendix

The fundamental theorem of asset pricing states that in equilibrium, the conditional expectation of the gross return on an asset \((1 + r_{t,t+k})\) multiplied by the stochastic discount factor \(M_{t,t+k}\) is the unit of account (Duffie, 2001), so that

\[
E_t[(1 + r_{t,t+k})M_{t,t+k}] = 1. \tag{4.4}
\]

It follows from the definition of covariance that

\[
E_t[1 + r_{t,t+k}] = \frac{1 - Cov_t[1 + r_{t,t+k}, M_{t,t+k}]}{E_t[M_{t,t+k}]]. \tag{4.5}
\]

Consider the \(k\) period return on security \(s\), \(r^{(s)}_{t,t+k}\) and the return from a \(k\) period rolling return on a series of investments in the overnight interbank lending rate (monetary policy instrument), denoted by

\[
\dot{i}_{t,t+k} \equiv \prod_{j=t}^{t+k-1} (1 + i_j) - 1,
\]

where \(i_t\) is the overnight interbank lending rate at time \(t\). Writing out equation (4.5) for the two assets and differencing results in

\[
E_t[1 + r^{(s)}_{t,t+k}] = E_t[1 + \dot{i}_{t,t+k}] + \rho^{(s)}_{t,t+k}, \tag{4.6}
\]

where the risk premium on security \(s\) relative to the overnight interbank lending rate is defined as

\[
\rho^{(s)}_{t,t+k} = \frac{Cov_t[\dot{i}_{t,t+k}, M_{t,t+k}] - Cov_t[1 + r_{t,t+k}, M_{t,t+k}]}{E_t[M_{t,t+k}]}.
\]
Chapter 5

Conclusion

Throughout this dissertation, we have argued that bond market pricing reveals important economic and financial information. Thus, we explored credit and liquidity premia in Chapter 2 and showed how they could be derived in a consistent manner from traded bond yields using the zero coupon surface. Our empirical analysis illustrated the evolution of credit and liquidity premia in the Australian semi-government, supranational and agency bond market through the Global Financial Crisis. Chapter 3 demonstrated that there is a distinct relation between the conditional shape of term premia across tenors and factors summarising the forward curve and the macroeconomy. Our application of the inequality constraints tests to U.S. Treasury bill returns in conjunction with the excess return forecasting factors suggested that market pricing for term risk in the short end of the Treasury yield curve is best analysed jointly with the excess return forecasting factors that characterise the state of the economy and the forward curve. Finally, in Chapter 4 we found that there is different, but complementary information about market expectations of future monetary policy movements in Australian fixed income securities. Moreover, our instrumental variables framework showed that reasonably accurate forecasts of monetary policy beyond the next two quarters can be derived from longer term securities.

Taking a step back, the discriminating reader may notice a certain empirical philosophy behind our work. Specifically, in each Chapter, we have tried to
be as agnostic as possible about the bond market pricing data. Our aim has always been to let bond market pricing speak for itself, as it were. The modelling frameworks that we have developed and advocated throughout this dissertation impose minimal structure on the underlying bond yield data, and do not interfere with market pricing in examining the various components of bond yields.

The reason for this stance is not a naïve belief in market efficiency. Instead it is a sense of conservatism on the part of the econometrician in seeking to interpret the markets. Each trade in the fixed income market absorbs and reflects a plethora of economic and financial information. Most of the time, extensive analysis is performed and intricate judgment is exercised before trades are executed. Clearly, the decision making processes of market participants are complex – far more complex than could ever be incorporated fully into a model. Nonetheless, we believe firmly that our approaches constitute the next best alternatives to individual case studies of the actual trading, investment, hedging and issuance strategies employed by market participants that converge in the bond market to drive pricing. We find this empirical stance to be far more insightful and persuasive than imposing unnecessary structure on the bond market pricing data with elaborate models.

Hence we envisage that, beyond extending the extant scholarly literature on fixed income markets and empirical finance, our research will be highly useful in practice. Portfolio managers and dealers could use the zero coupon surface as a powerful relative value tool to inform their trades in non-benchmark segments of the bond market, such as financial and corporate bonds. Sovereign debt managers and corporate treasurers could apply the conditional monotonicity tests to term premia in their funding markets of choice and translate the results into issuance strategies that minimise ongoing accrual debt servicing costs subject to acceptable levels of refinancing risks. Central banks could assess the efficacy of the crucial monetary policy transmission mechanism into the bond market with the instrumental variables framework; and also use it to gauge market expectations for future policy movements.

It is clear that our work provides a starting point for a more comprehensive empirical examination of bond yields. Many extensions spring to mind as
we reflect on our methodologies and findings. The zero coupon surface could be fit across the entire bond market (not just Australian dollar denominated semi-government, supranational and agency bonds), and updated in real-time with live market pricing rather on a quarterly basis. This would lead to a much more complete picture of market pricing as it evolves. The conditional tests of the liquidity preference hypothesis could be applied to returns on longer term Treasury zero coupon bonds. Of course, this has the drawback of conflating expectations with term premia, but it would be of far more relevance to bond issuers to gauge the conditional shape of term premia across tenors of one to ten years rather than simply up to one year. The best comparators for the instrumental variable measures of monetary policy expectations from fixed income securities are actually surveys of forecasts by market economists. Endogeneity issues aside, it would be very interesting to evaluate these forecasts against our measures (but unfortunately there currently is a lack of historical data on Australian economists’ medium term monetary policy forecasts).

Following on from these leads, much work still needs to be done to extend the research undertaken in this dissertation. But our empirical frameworks are highly extensible, as they were designed to be easily adaptable to changing market conditions and new fixed income technologies and structures. Indeed, our research is testament to the forward-looking dynamics which manifest the functional essence (and aesthetic quality) of the bond market: fixed income pricing, expectations and risk premia are perpetually re-evaluated and updated by market participants as they absorb the continuous flow of economic and financial market information.
References


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