Pseudo-Parity-Time Symmetry in Optical Systems

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We introduce a novel concept of the pseudo-parity-time (pseudo-$\mathcal{PT}$) symmetry in periodically modulated optical systems with balanced gain and loss. We demonstrate that whether or not the original system is $\mathcal{PT}$ symmetric, we can manipulate the property of the $\mathcal{PT}$ symmetry by applying a periodic modulation in such a way that the effective system derived by the high-frequency Floquet method is $\mathcal{PT}$ symmetric. If the original system is non-$\mathcal{PT}$ symmetric, the $\mathcal{PT}$ symmetry in the effective system will lead to quasistationary propagation that can be associated with the pseudo-$\mathcal{PT}$ symmetry. Our results provide a promising approach for manipulating the $\mathcal{PT}$ symmetry of realistic systems.

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Parity-time ($\mathcal{PT}$) symmetry, the invariance under parity-time reflection, is an important concept in physics recently developed in application to optical systems. The parity reflection operator ($\hat{P}$: $\hat{x} \to -\hat{x}$, $\hat{p} \to -\hat{p}$) and the time reversal operator ($\hat{T}$: $\hat{x} \to \hat{x}$, $\hat{p} \to -\hat{p}$, $i \to -i$, $t \to -t$) are defined by their action on the position operator $\hat{x}$, the momentum operator $\hat{p}$, and the time t. In quantum mechanics, the requirement of Hermitian Hamiltonians guarantees the existence of real eigenvalues and probability conservation. However, as their Hamiltonian and $\mathcal{PT}$ operators share common eigenfunctions, a wide class of non-Hermitian $\mathcal{PT}$-symmetric Hamiltonians can still possess entirely real eigenvalue spectra [1–4]. Although the extension of quantum mechanics based upon non-Hermitian $\mathcal{PT}$-symmetric operators is still a subject to debate, optical systems with complex refractive indices [5–12] are widely used to test the $\mathcal{PT}$ symmetry in non-Hermitian systems, because of the equivalence between the Schrödinger equation and the optical wave equation [13]. In the last few years, the $\mathcal{PT}$ symmetry has been observed in several optical systems, such as optical couplers [14,15], microwave billiard [16], and large-scale temporal lattices [17].

Similar to the electron transport in periodic crystalline potentials and the quantum tunneling in periodically driven systems [18], the light propagation in waveguides can be effectively controlled by periodic modulations [19–23]. In an optical system, periodic modulation is associated with a periodic refractive index. Mathematically, an optical system with periodic complex refractive index is equivalent to a time-periodic non-Hermitian quantum system. Given the resonant frequency ω_0 for the system without modulation, the modulation frequency ω, and the modulation amplitude A, if $\omega_0 \ll \max[\omega, \sqrt{|A|\omega}]$, the modulated system can be mapped into an effectively unmodulated one with rescaled parameters [18,24]. Like the case of no modulation, the $\mathcal{PT}$ symmetry may appear in the effective system if the periodically modulated system may be described by a $\mathcal{PT}$-symmetric Hamiltonian [25]. Naturally, an important question arises: Can the $\mathcal{PT}$ symmetry appear in an effective system even if the periodically modulated system is non-$\mathcal{PT}$ symmetric? In other words, can we employ periodic modulations to manipulate the $\mathcal{PT}$ symmetry?

In this Letter, we study the light propagation in a periodically modulated optical coupler with balanced gain and loss and apply a biharmonic modulation along the propagation direction. The Hamiltonian for the modulated system is non-$\mathcal{PT}$ symmetric if the relative phase between the two applied harmonics is not 0 or π. With application of the high-frequency Floquet approach, the modulated system is effectively described by an effective averaged system, whose $\mathcal{PT}$ symmetry can be manipulated by tuning the modulation amplitude or frequency. More importantly, the $\mathcal{PT}$ symmetry can appear in the effective system corresponding to a non-$\mathcal{PT}$-symmetric and non-Hermitian Hamiltonian. Different from the $\mathcal{PT}$ symmetry from a $\mathcal{PT}$-symmetric Hamiltonian, which leads to stationary light propagation of bounded intensity oscillation, the $\mathcal{PT}$ symmetry from a non-$\mathcal{PT}$-symmetric Hamiltonian will lead to quasistationary light propagation of unbounded intensity oscillation. Therefore, we term the induced symmetry associated with modulated systems as the "pseudo-$\mathcal{PT}$ symmetry."

In optics, the electric field $E(x, z)$ of light obeys the wave equation

$$i\frac{\partial E(x,z)}{\partial z} = -\frac{1}{2k}\frac{\partial^2 E(x,z)}{\partial x^2} + V(x,z)E(x,z), \quad (1)$$

where $k = k_0 n_0$, $k_0 = 2\pi/\lambda$, and $V(x,z) = k_0[n_0 - n(x)]$ with the substrate index n_0, the free-space wavelength λ, and the complex refractive index distribution $n(x) = n_0 + n_R(x,z) + in_I(x)$, where n_R and n_I are real and imaginary parts of $n(x)$. Therefore, the effective potential reads as $V(x,z) = V_R(x,z) + iV_I(x) = -k_0[n_R(x,z) + in_I(x)]$. With the experimental techniques developed in recent years [6,10,12,14,15], one can make $V_I(-x) = -V_I(x)$ and $V_R(x,z) = V_0(x) + V_1(x,z)$ with the unmodulated part $V_0(x)$ being a symmetric double-well function and the modulation $V_1(x,z) = V'(x)F(z)$ described by an antisymmetric function $V'(-x) = -V'(x)$ and a biharmonic function $F(z)$; see Fig. 1.

With the use of the coupled-mode theory, the electric field for a two-channel coupler can be expressed as a two-mode ansatz with the localized waves $\{\psi_1(x), \psi_2(x)\}$ and the complex amplitudes $\{c_1(z), c_2(z)\}$ (Supplemental Material [26]). Thus, we have

$$i\frac{d}{dz}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} +\frac{i\gamma}{2} + \frac{S(z)}{2} & v \\ v & -\frac{i\gamma}{2} - \frac{S(z)}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad (2)$$

with the interchannel coupling strength v, the gain or loss strength γ, and the biharmonic modulation

$$S(z) = -A[\sin(\omega z) + f\sin(2\omega z + \phi)]. \quad (3)$$

Here, $\phi \in [0, 2\pi)$ denotes the relative phase between the two harmonics, ω is the modulation frequency, A is the modulation amplitude, and f is a dimensionless coefficient. Since the system is invariant under the transformation $c_2 \to -c_2$ and $v \to -v$, below we will only consider the case of $v > 0$. With definition of the parity operator as $\hat{P}$, which interchanges the two channels labeled by 1 and 2, and the time operator as $\hat{T}$: $i \to -i$, $z \to -z$, which reverses the propagation direction, the Hamiltonian $\hat{H}$ for the system (2) is $\mathcal{PT}$ symmetric if $\hat{P}\hat{T}\hat{H} = \hat{H}\hat{P}\hat{T}$. If $\phi = 0$ or π, $S(-z) = -S(z)$, $\hat{H}$ is $\mathcal{PT}$ symmetric.

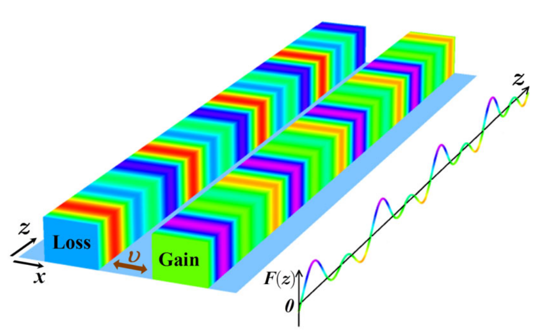

FIG. 1 (color online). Schematic diagram of a modulated two-channel optical coupler with balanced gain and loss. The periodic change of color along the z axis denotes the periodic modulation $F(z)$.

Otherwise, if $\phi \neq 0$ and π, $S(z_0 - z) \neq -S(z_0 + z)$ for arbitrary constant z_0, $\hat{H}$ becomes non-$\mathcal{PT}$ symmetric.

Under the condition of $v \ll \max[\omega, \sqrt{|A|\omega}]$, one can implement the high-frequency Floquet analysis. Introducing the transformation

$$c_1 = c_1' \exp\left\{-i\left[\frac{A}{2\omega}\cos(\omega z) + \frac{Af}{4\omega}\cos(2\omega z + \phi)\right]\right\}, \quad (4)$$

$$c_2 = c_2' \exp\left\{+i\left[\frac{A}{2\omega}\cos(\omega z) + \frac{Af}{4\omega}\cos(2\omega z + \phi)\right]\right\}, \quad (5)$$

and averaging the high-frequency terms, one can obtain an effectively unmodulated system

$$i\frac{d}{dz}\begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} +\frac{i\gamma}{2} & J \\ J^* & -\frac{i\gamma}{2} \end{pmatrix}\begin{pmatrix} c_1' \\ c_2' \end{pmatrix}, \quad (6)$$

with the rescaled coupling strength

$$J = v\sum_{m=-\infty}^{\infty}(i)^{-m}J_{-2m}\left(\frac{A}{\omega}\right)J_m\left(\frac{Af}{2\omega}\right)\exp(im\phi). \quad (7)$$

The modulus of J depends on the values of A/ω and ϕ. If A/ω is relatively small, the modulus $|J|$ is almost independent on the relative phase ϕ. When A/ω increases, the modulus $|J|$ becomes sensitively dependent on ϕ. In particular, the modulus $|J|$ equals zero at some specific values of A/ω (such as $A/\omega \simeq 2.4$ and 5.52) and $\phi = \pi/2$ or $3\pi/2$. In Fig. 2(a), choosing $f = 1/4$, we show the contour plot of $|J|$ as a function of A/ω and ϕ.

By diagonalizing the Hamiltonian for the effective system (6), we give the two eigenvalues as

$$\varepsilon = \pm|J|\sqrt{1 - [\gamma/(2J)]^2}. \quad (8)$$

Obviously, dependent on the values of $\gamma/(2|J|)$, the two eigenvalues can be real or complex. The two eigenvalues are real if $\gamma < 2|J|$, and they become complex if $\gamma > 2|J|$. Therefore, $\gamma_{\text{critical}} = 2|J|$ is the critical point for the phase transition between real and complex spectra in the effective system, which corresponds to the original system (2) under high-frequency modulations. The spontaneous $\mathcal{PT}$-symmetry-breaking transition takes place in the effective model (6) when the imaginary part of ε changes from zero to nonzero. Surprisingly, unlike our conventional understanding, we find that the quasienergies can be real even if the modulated system (2) is non-$\mathcal{PT}$ symmetric (i.e., $\phi \neq 0$ and π).

The parametric dependence of $|\text{Im}(\varepsilon)|$ is shown in Figs. 2(b)–2(e). In Figs. 2(b) and 2(c), we show $|\text{Im}(\varepsilon)|$ as a function of γ and ϕ for $f = 1/4$. For small A/ω, such as $A/\omega = 1$ in Fig. 2(b), $|\text{Im}(\varepsilon)|$ is almost independent on ϕ and the transition from a completely real quasienergy spectrum ($|\text{Im}(\varepsilon)| = 0$) to a complex spectrum ($|\text{Im}(\varepsilon)| \neq 0$) takes places when γ increases. Near a minimum of $|J|$, such as $A/\omega = 2.4$, $|\text{Im}(\varepsilon)|$ strongly depends on ϕ; see Fig. 2(c). In Figs. 2(d) and 2(e), we show $|\text{Im}(\varepsilon)|$ as a

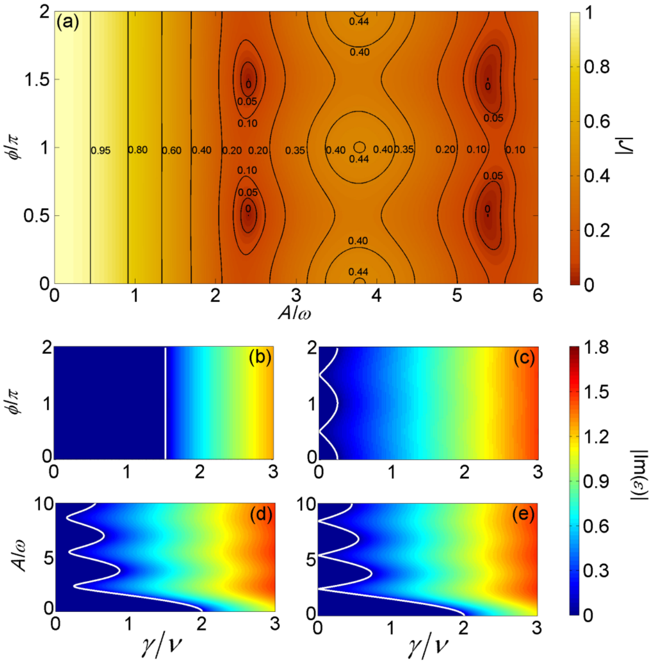

FIG. 2 (color online). Parametric dependence of the effective coupling $|J|$ and the imaginary parts of quasienergies $|\mathrm{Im}(\varepsilon)|$. Top [(a)]: $|J|$ versus A/ω and ϕ for $f = 1/4$. Middle row [(b) and (c)]: $|\mathrm{Im}(\varepsilon)|$ versus ϕ/π and γ for (b) $A/\omega = 1$ and (c) $A/\omega = 2.4$. Bottom row [(d) and (e)]: $|\mathrm{Im}(\varepsilon)|$ versus A/ω and γ for (d) $\phi = 0$ and (e) $\phi = \pi/2$. The other parameters for $|\mathrm{Im}(\varepsilon)|$ are chosen as $v = 1$, $\omega = 10$, and $f = 1/4$. The white curves are the boundary ($\gamma_{\mathrm{critical}} = 2|J|$) between $|\mathrm{Im}(\varepsilon)| = 0$ and $|\mathrm{Im}(\varepsilon)| \neq 0$.

function of γ and A/ω for (d) $\phi = 0$ and (e) $\phi = \pi/2$. Near the minima of $|J|$, such as $A/\omega \simeq 2.4, 5.52, \ldots$, $|\mathrm{Im}(\varepsilon)|$ shows significant difference between the two cases of $\phi = 0$ and $\phi = \pi/2$. In particular, at the minimum points, $|J|$ vanished to zero for $\phi = \pi/2$, and the corresponding critical value $\gamma_{\mathrm{critical}} = 2|J|$ is reduced to zero. Similar to a non-Hermitian system with no modulations, the spontaneous $\mathcal{PT}$-symmetry-breaking transition ($|\mathrm{Im}(\varepsilon)| = 0 \Rightarrow |\mathrm{Im}(\varepsilon)| \neq 0$) can be observed by tuning the gain or loss strength γ. More interestingly, for our modulated system (2) of fixed γ, it is possible to observe the spontaneous $\mathcal{PT}$-symmetry-breaking transition by tuning ϕ and A/ω; see Figs. 2(c)–2(e).

On the basis of the high-frequency Floquet analysis, it seems that whether or not the modulated system (2) obeys a $\mathcal{PT}$-symmetric Hamiltonian, a completely real quasienergy spectrum always appear if $\gamma < 2|J|$. This is obviously inconsistent with the previous theory [1–4], which tells us that only $\mathcal{PT}$-symmetric Hamiltonian systems can support completely real spectra. So, what really happens in the modulated non-Hermitian and non-$\mathcal{PT}$-symmetric Hamiltonian system?.

In general, according to the Floquet theorem, one can use a numerical method to calculate the Floquet states and their quasienergies for arbitrary modulation frequency and amplitude. Similar to the Bloch states, the Floquet states of the modulated system (2) satisfy $\{c_1(z), c_2(z)\} = e^{-i\varepsilon z}\{\tilde{c}_1(z), \tilde{c}_2(z)\}$. Here, the propagation constant ε is called the quasienergy, and the complex amplitudes $\tilde{c}_1(z)$ and $\tilde{c}_2(z)$ are periodic with the modulation period $T = 2\pi/\omega$.

To show the validity of the high-frequency Floquet analysis, we compare the numerical quasienergies obtained from the original model (2) and the analytical formula (8) obtained from the effective model (6). In the high-frequency regime, $v \ll \max[\omega, \sqrt{|A|\omega}]$, the analytical and numerical values for the quasienergies ε are in good agreement and only show a tiny difference dependent upon ϕ. As two examples, we show $\mathrm{Im}(\varepsilon)$ (the imaginary part of quasienergy) versus γ for $\phi = 0$ and $\phi = \pi/2$ in Figs. 3(a) and 3(b), respectively. It clearly shows that the analytical results (red lines) agree well with the numerical results (black lines). Below the critical point ($\gamma < \gamma_{\mathrm{critical}} = 2|J|$), for $\mathcal{PT}$-symmetric Hamiltonian systems ($\phi = 0$ or π), the numerical results confirm the entirely real quasienergy spectrum; see Fig. 3(c). However, for non-$\mathcal{PT}$-symmetric Hamiltonian systems ($\phi \neq 0$ and π), the numerical quasienergies ε still have small nonzero imaginary parts even if $\gamma < \gamma_{\mathrm{critical}} = 2|J|$; see Fig. 3(d). This means that, if the original system (2) obeys a non-$\mathcal{PT}$-symmetric Hamiltonian, the entirely real quasienergy spectrum for the effective model (6) does not correspond to a perfectly entirely real quasienergy spectrum for the original system (2). Therefore, such a $\mathcal{PT}$ symmetry in the effective model (6) corresponds to a kind of "pseudo-$\mathcal{PT}$ symmetry" in the original model (2). The appearance of pseudo-$\mathcal{PT}$ symmetry indicates that the deviation of the high-frequency Floquet analysis depends on both the modulation frequency ω and the Hamiltonian symmetry. Nevertheless, this deviation tends to be zero when $\omega \to \infty$ (Supplemental Material [26]).

Through numerical integration, we analyze the light propagations in the continuous system (1) and the coupled-mode system (2). The light propagation sensitively depends upon the quasienergies. Stationary light propagations of bounded intensity oscillations appear if all quasienergies are real. Nonstationary light propagations of unbounded intensity oscillations appear if at least one of the quasienergies is complex, in which quasistationary light propagations of slowly varying time-averaged intensities appear if the two quasienergies for the effective system (6) are real. In Fig. 4, for $v = 1$, $A = 10$, $f = 1/4$, $\omega = 10$, and $\gamma = 0.1$ (which is below the critical value $\gamma_{\mathrm{critical}}$), we show the intensity evolution of the coupled-mode system from $c_1(0) = 1$ and $c_2(0) = 0$; in which, the two intensities $I_j(z) = |c_j(z)|^2$, the total intensity $I_t(z) = I_1(z) + I_2(z)$ and the time-averaged total intensity $I_t^{\mathrm{av}}(z) = (1/T_s)\int_z^{z+T_s} I_t(\tilde{z})d\tilde{z}$ with $T_s = 2\pi/|\mathrm{Re}(\varepsilon_2) - \mathrm{Re}(\varepsilon_1)|$ and $\mathrm{Re}(\varepsilon_j)$ being the real part of

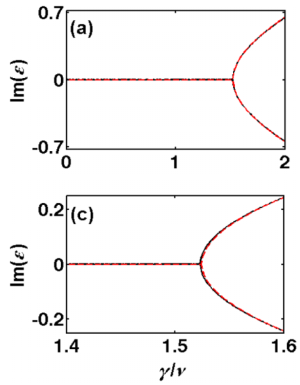

FIG. 3 (color online). Comparison between numerical and analytical results of $\mathrm{Im}(\varepsilon)$, the imaginary part of the quasienergy. Upper row [(a) and (b)]: $\mathrm{Im}(\varepsilon)$ versus γ for (a) $\phi = 0$ and (b) $\phi = \pi/2$. Solid lines are for numerical results obtained from the original model (2), and red dashed lines are analytical results given by the formula (8) for the effective model (6). Lower row [(c) and (d)]: the enlarged regions of (a) and (b) near the bifurcation point given by the analytical formula (8). The other parameters are $v = 1$, $A = 10$, $f = 1/4$, and $\omega = 10$.

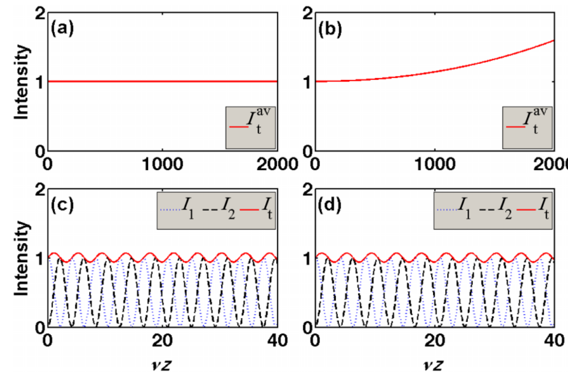

FIG. 4 (color online). Intensity evolution from the initial state of $c_1(0) = 1$ and $c_2(0) = 0$. Upper row: long-distance time-averaged intensity evolution for (a) $\phi = 0$ and (b) $\phi = \pi/2$. Lower row: short-distance intensity evolution for (c) $\phi = 0$ and (d) $\phi = \pi/2$. The other parameters are chosen as $v = 1$, $A = 10$, $f = 1/4$, $\omega = 10$, and $\gamma = 0.1$.

ε_j. In short-distance propagations, $I_{1,2}(z)$ and $I_t(z)$ oscillate periodically and it is hard to see the difference between the cases of $\phi = 0$ and $\pi/2$; see Figs. 4(c) and 4(d). However, significant difference appears in long-distance propagations. For a $\mathcal{PT}$-symmetric Hamiltonian system of $\phi = 0$, $I_t^{av}(z)$ remains unchanged; see Fig. 4(a). For a non-$\mathcal{PT}$-symmetric Hamiltonian system of $\phi = \pi/2$, $I_t^{av}(z)$ slowly increases; see Fig. 4(b). The quasistationary light propagation of slowly varying $I_t^{av}(z)$ is a direct signature of the pseudo-$\mathcal{PT}$ symmetry. Moreover, our numerical simulations of the continuous wave equation (1) perfectly confirm the pseudo-$\mathcal{PT}$ symmetry predicted by the corresponding coupled-mode system (Supplemental Material [26]).

Now, we discuss the experimental possibility of observing our theoretical predictions. Recently, several $\mathcal{PT}$-symmetric optical systems were experimentally realized [6,10,12,14,15]. Complex refractive index of gain or loss effects can be obtained from quantum-well lasers or photorefractive structures through two-wave mixing [27]. Periodic modulations can be introduced by out-of-phase harmonic modulations of the real refractive index [13,21,23] or periodic curvatures along the propagation direction [13,19,23,28]. For a short optical coupler under periodic modulations, spontaneous $\mathcal{PT}$-symmetry-breaking transitions can be observed, whether the system Hamiltonian is $\mathcal{PT}$ symmetric or not. In such a system, the light propagation will be periodic and stable if $\gamma < 2|J|$, and an instability will be observed if $\gamma > 2|J|$. The critical point $\gamma_{\mathrm{critical}} = 2|J|$ can be adjusted by controlling the modulation parameters A/ω, f, and ϕ in addition to controlling γ. However, for a long optical coupler under periodic modulations, the light propagation below the critical point ($\gamma < 2|J|$) depends on the Hamiltonian symmetry. If the Hamiltonian is $\mathcal{PT}$ symmetric, the light propagation is periodic and stable, in which the time-averaged total intensity remains unchanged. Otherwise, if the Hamiltonian is non-$\mathcal{PT}$ symmetric, the light propagation is quasistationary, in which case the time-averaged total intensity slowly grows.

In summary, we have studied the non-Hermitian Hamiltonian systems under periodic modulations and introduce the concept of pseudo-$\mathcal{PT}$ symmetry. If the modulated system obeys a $\mathcal{PT}$-symmetric Hamiltonian, there exists a truly spontaneous $\mathcal{PT}$-symmetry-breaking phase transition from a real quasienergy spectrum to a complex one. If the modulated system obeys a non-$\mathcal{PT}$-symmetric Hamiltonian, although a spontaneous $\mathcal{PT}$-symmetry-breaking phase transition in the effective system derived from the high-frequency Floquet analysis exists, there is no truly spontaneous $\mathcal{PT}$-symmetry-breaking phase transition in the original system. Corresponding to the real spectrum for the effective system, the original system has a quasireal spectrum of small imaginary parts, which leads to a quasistationary light propagation of slowly varying time-averaged total intensity. This is the pseudo-$\mathcal{PT}$ symmetry in the non-Hermitian system described by a non-$\mathcal{PT}$-symmetric Hamiltonian.

In addition to the discovery of the pseudo-$\mathcal{PT}$ symmetry, we believe that our work brings three key advances to related fields. First, although high-frequency Floquet analysis can capture most key features, some important information (such as the so-called pseudo-$\mathcal{PT}$ symmetry) may be lost. Second, periodic modulations provide a new route to the observation of the spontaneous $\mathcal{PT}$-symmetry-breaking transition. Third, as the interchannel coupling can be effectively switched off by controlling the modulation, and the corresponding intensity grows exponentially even for arbitrarily weak gain or loss, this exponential growth offers an efficient way to beam amplification in optical waveguides.

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