

Galactic archaeology: mapping and dating stellar populations with asteroseismology of red-giant stars

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ABSTRACT

Our understanding of how the Galaxy was formed and evolves is severely hampered by the lack of precise constraints on basic stellar properties such as distances, masses and ages. Here, we show that solar-like pulsating red giants represent a well-populated class of accurate distance indicators, spanning a large age range, which can be used to map and date the Galactic disc in the regions probed by observations made by the *CoRoT*¹ and *Kepler* space telescopes. When combined with photometric constraints, the pulsation spectra of such evolved stars not only reveal their radii, and hence distances, but also provide well-constrained estimates of their masses, which are reliable proxies for the ages of the stars. As a first application, we consider red giants observed by *CoRoT* in two different parts of the Milky Way, and determine precise distances for ~ 2000 stars spread across nearly 15 000 pc of the Galactic disc, exploring regions which are a long way from the solar neighbourhood. We find significant differences in the mass distributions of these two samples which, by comparison with predictions of synthetic models of the Milky Way, we interpret as mainly due to the vertical gradient in the distribution of stellar masses (hence ages) in the disc. In the future, the availability of spectroscopic constraints for this sample of stars will not only improve the age determination, but also provide crucial constraints on age–velocity and age–metallicity relations at different Galactocentric radii and heights from the plane.

Key words: asteroseismology – stars: distances – Galaxy: disc.

1 INTRODUCTION

Accurate and precise distances to stars are a fundamental cornerstone of astronomy. Such data play a key role in shaping theories of the history and fate of our Galaxy. Another fundamental cornerstone

is provided by stellar ages. Ages can be estimated only for a limited sample of individual field stars, and then only by using either model-dependent techniques or empirical methods that still need careful calibration (Soderblom 2010). These two cornerstones are essential to Galactic Archaeology, the study of the chemical compositions and dynamic motions of stars of different ages, within which is coded information on the origin and subsequent evolution of the Milky Way (Freeman & Bland-Hawthorn 2002; Turon et al. 2008). There is a modern consensus among astronomers that a variety of processes play a role in shaping our Galaxy, for example gas accretion and in situ star formation, mergers and dynamical

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¹ The *CoRoT* space mission, launched on 2006 December 27, has been developed and is operated by CNES, with the contribution of Austria, Belgium, Brazil, ESA (RSSD and Science Programme), Germany and Spain.

processes in the Galaxy (such as diffusion and migration of stars). Galactic Archaeology research has now reached a point where so-called chemo-dynamical models are needed to quantify the relative importance of each of these processes, over the course of more than 10 Gyr of evolution of the Milky Way. Stringent observational constraints are in turn desperately needed to test these model predictions.

Particularly important observational constraints are the age–velocity and age–metallicity relations at different positions in the Galaxy (see e.g. Freeman & Bland-Hawthorn 2002; Minchev, Chiappini & Martig 2012). These relations are uncertain, or limited to neighbouring stars [e.g. from the Geneva-Copenhagen Survey (Holmberg, Nordström & Andersen 2009; Casagrande et al. 2011), which is confined to distances less than about 100 pc from us]. Future planned surveys hope to fill this gap by combining precise proper motions from the ESA-*GAIA* (Perryman et al. 2001) astrometric mission with massive spectroscopic follow-up from ground-based telescopes, to extend the region for which distances and chemical compositions are known out to as far as 10 000 pc. However, this will not happen before the beginning of the next decade, and it will still need to confront the large and systematic uncertainties affecting the age determinations of single stars.

Red giants are cool highly luminous stars, which, by virtue of covering a wide domain in mass, age, chemical composition and evolutionary state, are in principle an important source of data for testing chemo-dynamical models. Previously, age estimation of red giants has had to rely on constraints on their surface properties only. A severe limitation of such an approach is that during the red-giant phase stars of significantly different age and distance end up sharing very similar observed surface properties making it extremely hard to discriminate evolutionary states of field giants belonging to the Galactic-disc population. That situation has now changed thanks to asteroseismology, with the detection of solar-like pulsations in thousands of red giants (De Ridder et al. 2009; Bedding et al. 2010; Hekker et al. 2011) observed by the *CoRoT* (Baglin & Fridlund 2006) and *Kepler* (Gilliland et al. 2010) space telescopes. The pulsation frequencies may be used to place tight constraints on the fundamental stellar properties, including radius, mass and evolutionary state (Stello et al. 2008; Kallinger et al. 2010; Montalbán et al. 2010; Mosser et al. 2010; Bedding et al. 2011), the properties of helium ionization regions (Miglio et al. 2010) and internal rotation (see e.g. Beck et al. 2012). Here, we show that it is possible to use pulsating red giants as accurate distance indicators and tracers of stellar population ages.

2 METHOD

Radii and masses of solar-like oscillating stars can be estimated from the average seismic parameters that characterize their oscillation spectra: the so-called average large frequency separation ($\Delta\nu$), and the frequency corresponding to the maximum observed oscillation power (ν_{\max}).

The large frequency spacing is predicted by theory to scale as the square root of the mean density of the star (see e.g. Vandakurov 1967; Tassoul 1980):

$$\Delta\nu \simeq \sqrt{\frac{M/M_{\odot}}{(R/R_{\odot})^3}} \Delta\nu_{\odot}, \quad (1)$$

where $\Delta\nu_{\odot} = 135 \mu\text{Hz}$. The frequency of maximum power is expected to be proportional to the acoustic cutoff frequency (Brown

et al. 1991; Kjeldsen & Bedding 1995; Mosser et al. 2010; Belkacem et al. 2011), and therefore

$$\nu_{\max} \simeq \frac{M/M_{\odot}}{(R/R_{\odot})^2 \sqrt{T_{\text{eff}}/T_{\text{eff},\odot}}} \nu_{\max,\odot}, \quad (2)$$

where $\nu_{\max,\odot} = 3100 \mu\text{Hz}$ and $T_{\text{eff},\odot} = 5777 \text{ K}$.

When no information on distance/luminosity is available, which is the case for the vast majority of field stars observed by *CoRoT* and *Kepler*, equations (1) and (2) may be solved to derive M and R (see e.g. Stello et al. 2008; Kallinger et al. 2010; Mosser et al. 2010):

$$\frac{M}{M_{\odot}} \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}} \right)^3 \left(\frac{\Delta\nu}{\Delta\nu_{\odot}} \right)^{-4} \left(\frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{3/2} \quad (3)$$

$$\frac{R}{R_{\odot}} \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}} \right) \left(\frac{\Delta\nu}{\Delta\nu_{\odot}} \right)^{-2} \left(\frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{1/2}. \quad (4)$$

We determined stellar radii and masses by combining the available seismic parameters ν_{\max} and $\Delta\nu$ with effective temperatures T_{eff} . The latter were determined using 2MASS (Skrutskie et al. 2006) J and K_s photometry and the colour– T_{eff} calibrations by Alonso, Arribas & Martínez-Roger (1999), which depend only weakly on metallicity. 2MASS colours were transformed into the CIT photometric system used by Alonso et al. (1999) using the relations available in Alonso, Arribas & Martínez-Roger (1998) and Carpenter (2001).

We then computed luminosities L using the Stefan–Boltzmann law $L = 4\pi R^2 \sigma T_{\text{eff}}^4$, where σ is the Stefan–Boltzmann constant. The distance modulus of each star was determined as $K'_{s0} - K_{s0}$, where K_{s0} is the de-reddened apparent 2MASS K_s magnitude and K'_{s0} is the absolute K_s magnitude. The latter was obtained by combining L and the bolometric corrections from Girardi et al. (2005), which are based on the Castelli & Kurucz (2004) ATLAS9 model atmospheres, for the range of T_{eff} and $\log g$ under consideration.

We took into account the effect of interstellar extinction on the magnitude and colour of each star using the 3D model of Galactic extinction in the V band (A_V) by Drimmel, Cabrera-Lavers & López-Corroira (2003). The extinctions in the J and K_s bandpasses were determined following Fiorucci & Munari (2003) assuming the spectral energy distribution of a K1 giant. Since the extinction is distance dependent, we iterated the procedure until the derived distance does not vary by more than 1 per cent. We chose to consider magnitudes in the near-IR to reduce the effect of interstellar reddening in both the determination of T_{eff} and apparent de-reddened magnitudes.

By analogy with the well-known period–luminosity relation used to estimate distances of classical pulsators (see e.g. Feast & Walker 1987; Madore & Freedman 1991; Bono et al. 2010, for a review), we can explicitly write a relation between the distance and the pulsation properties of solar-like oscillators. In contrast to the case of single-mode, radial pulsators we can estimate stellar radii without making any assumptions² on mass–metallicity–luminosity relations. Hence we find

$$\log d = 1 + 2.5 \log \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} + \log \frac{\nu_{\max}}{\nu_{\max,\odot}} - 2 \log \frac{\Delta\nu}{\Delta\nu_{\odot}} + 0.2 (m_{\text{bol}} - M_{\text{bol},\odot}),$$

where d is expressed in parsec, m_{bol} is the apparent bolometric magnitude of the star and $M_{\text{bol},\odot}$ the absolute solar bolometric

² Excluding the weak dependence of bolometric correction on metallicity.

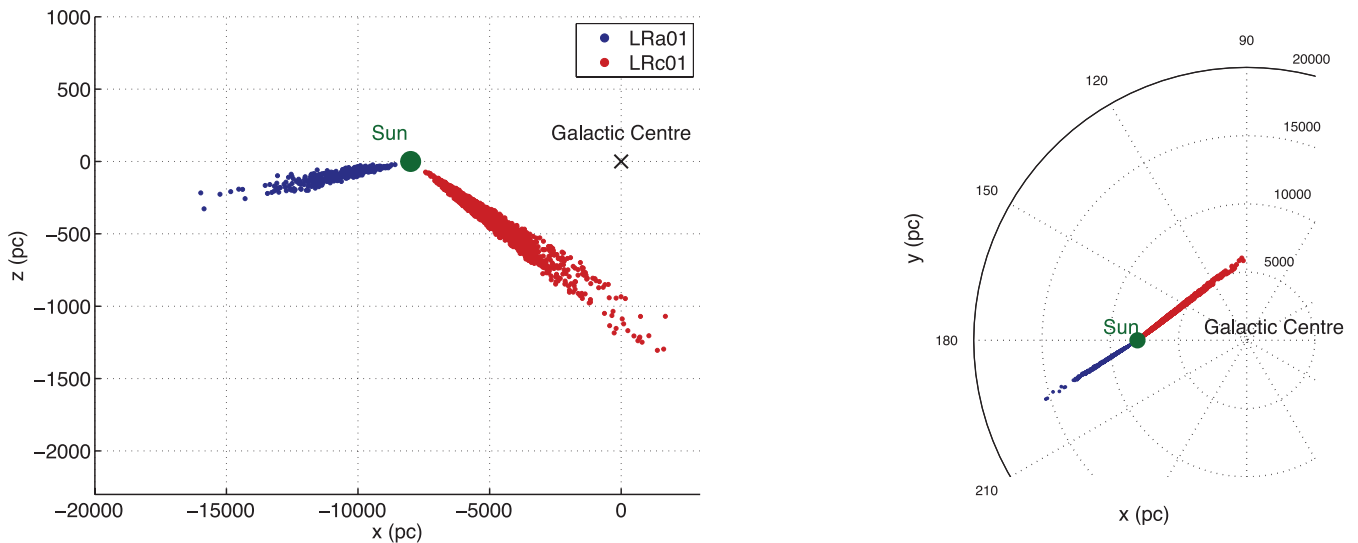


Figure 1. Positions of red giants in the two observed fields (*CoRoT* designations LRc01 and LRA01) projected on the Galactic plane (right-hand panel) and on a plane perpendicular to the disc (left-hand panel).

magnitude. Although this expression is model independent, it builds upon equations (1) and (2) which are based on simplifying assumptions that need to be independently verified and supported with empirical tests.

Tests against independent estimates of mass and radius suggest that R and M derived using average asteroseismic parameters are accurate to within 5 and 10 per cent, respectively (see Miglio et al. 2012 for a review on the tests performed so far, and see Huber et al. 2012, Miglio 2012 and Silva Aguirre et al. 2012 for tests on nearby stars).

3 DATA AND UNCERTAINTIES

The *CoRoT* photometric time series used in this work were obtained in the so-called exofield during the first long *CoRoT* runs: LRc01 ($l = 37^\circ$, $b = -7^\circ$) and LRA01 ($l = 217^\circ$, $b = -2^\circ$). These long runs lasted approximately 140 d, providing us with a frequency resolution of about $0.08 \mu\text{Hz}$. A first analysis of these data was made by De Ridder et al. (2009), Hekker et al. (2009) and Kallinger et al. (2010). Solar-like oscillations were detected in 435 and 1626 giants belonging to LRA01 and LRc01, respectively. To derive M , R and distance we use the values (and uncertainties) of ν_{max} and $\Delta\nu$ as determined by Mosser et al. (2010). The typical uncertainty on ν_{max} and $\Delta\nu$ for the 150-d long *CoRoT* observations is of the order of 2.4 and 0.6 per cent, respectively.

The uncertainties on the stellar properties are estimated using Monte Carlo simulations adopting the following constraints.

(i) Apparent magnitudes: uncertainties on J and K_s were taken from the 2MASS photometry available in the EXODAT (Deleuil et al. 2009) catalogue (the median uncertainty is 0.02 mag).

(ii) Extinction/reddening: a random error in A_V of 0.3 mag was considered both in determining T_{eff} photometrically and in de-reddening K_s apparent magnitudes.

(iii) T_{eff} : for each star we considered two sources of uncertainty. The first (100 K) was due to uncertainties on the colour- T_{eff} calibration itself (Alonso et al. 1999). The second one was due to uncertainties on reddening. This resulted in a median combined uncertainty on T_{eff} of ~ 190 K (calibration+reddening).

When determining the distance, reddening affects not only the de-reddened apparent bolometric magnitude, but also the determination of L (mostly via T_{eff}). A higher A_V increases the estimated T_{eff} , hence L , but it also increases the apparent de-reddened luminosity l . Since $d \propto (L/l)^{1/2}$, the overall effect of reddening on the distance itself is partly reduced. By performing 500 realizations of the data assuming the uncertainties described above to be Gaussian, we find a median intrinsic uncertainty of 5 per cent in distance, 3.6 per cent in radius and 10 per cent in mass. To explore the effect of possible systematic offsets in the T_{eff} scale (see e.g. Casagrande et al. 2010), we increased/decreased T_{eff} by 100 K, and found that the distance estimate was affected by 2.5 per cent (the radius by 1 per cent and the mass by 3 per cent). This leads to an overall uncertainty on the distance of 5 per cent (random) + 5 per cent (systematic due to radius determination via scaling relations) + 2.5 per cent (systematic due to the T_{eff} scale).

The positions of the giants in the LRc01 and LRA01 are illustrated in Fig. 1.

4 DIFFERENTIAL POPULATION STUDY

Before comparing populations of giants observed in the two fields, we checked whether different biases were introduced in the target selection in LRA01 and LRc01. We retrieved from EXODAT (Deleuil et al. 2009) photometric information on all the stars in the field, as well as the targets observed. Targets within each field of view were selected largely on the basis of colour-magnitude criteria. Within the observed targets, solar-like oscillations were searched for in stars belonging to a limited colour-magnitude domain: $0.6 < J - K_s < 1$ and $K_s < 12$ (see Mosser et al. 2010). Restricting to this domain, we find no significant difference in the target selection bias applied to LRc01 compared to LRA01 (see also Miglio et al. 2012).

We then compare the distributions of radius and mass of stars in LRA01 and LRc01. Since scaling relations were not tested at high luminosities we excluded from the sample the few stars with $L > 200 L_\odot$, and stars with no 2MASS photometry available. The fraction of targets excluded represents < 3 per cent of the whole sample. The distributions of R and M are reported in Fig. 2.

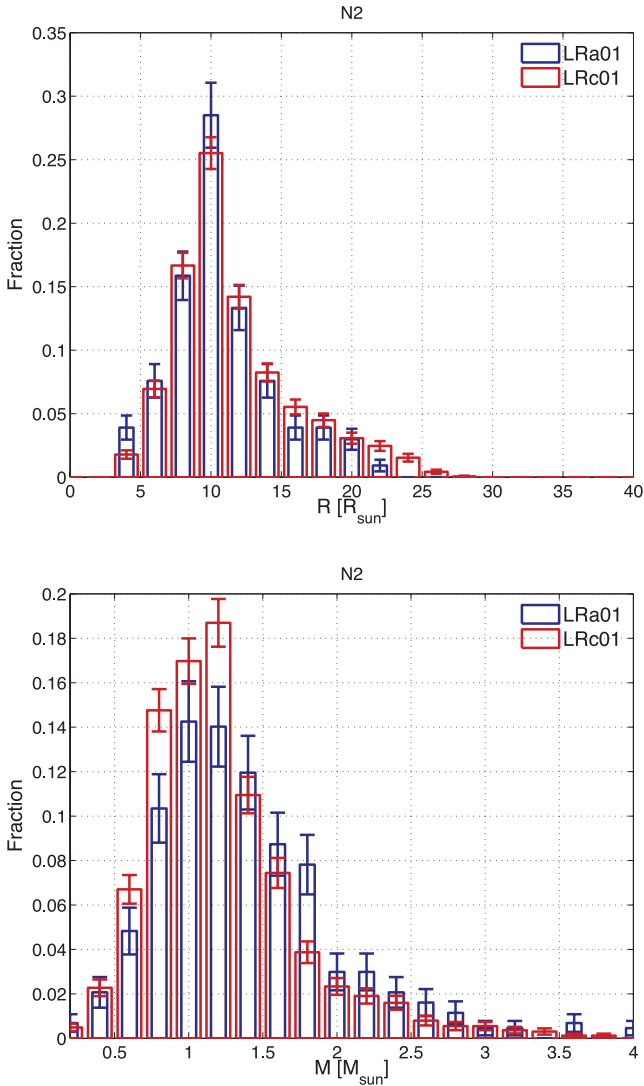


Figure 2. Radius and mass distributions of stars in LRA01 (blue) and LRC01 (red) obtained using the method described in Section 2.

We applied the Kolmogorov–Smirnov (K–S) test to the distributions of mass and radius to quantify differences between the two populations. When comparing radii, the null hypothesis (LRA01 and LRC01 samples are drawn from the same parent distribution) cannot be rejected, or the difference between the two populations is marginally significant at best. On the other hand, we find the difference between the mass distribution of the two populations to be highly significant (K–S probability higher than 99.9 per cent).

To test the impact on the K–S test of statistical uncertainties on mass and radius, we perturbed the observed radii and masses adding random offsets drawn from a Gaussian distribution having a standard deviation equal to the estimated uncertainty on the mass/radius. We generated 1000 realizations and performed a K–S test on each realization. The distribution of results from the 1000 K–S tests confirms that while comparing radius the null hypothesis cannot be rejected, the difference in the mass distribution is significant (in 95 per cent of the realizations the K–S probabilities are higher than 99 per cent).

4.1 Comparison with synthetic stellar populations

For a meaningful comparison between observed and synthetic populations, we applied to the synthetic population the same selection criteria based on colour and magnitude adopted in the target selection. Moreover, as discussed in detail in Mosser et al. (2010), since no significant bias in the distribution of the targets is present in the ν_{\max} range between 6 μHz and 80 μHz , we only considered observed and simulated stars with ν_{\max} in this frequency range. We considered synthetic populations computed with two codes: the Besançon model of the Milky Way (Robin et al. 2003), and with TRILEGAL (Girardi et al. 2005).

Simulations with both TRILEGAL (see Fig. 3) and the Besançon model show that, although a similar distribution of radius is expected in LRA01 and LRC01, the age (hence mass) distribution of the two populations of giants is expected to differ. Stars in LRC01 are expected to be older, on average, than those in LRA01. Additional tests presented in Miglio et al. (2012) show that, in the simulations, the different mass distribution expected in the two fields is indeed related to the assumed age-dependent vertical scale height of the thin disc. Our aim here is not to find the synthetic population that best matches the data, but to show that the difference we see in the observed distribution is in qualitative agreement with the simulations which include an increase of the disc scale height with age.

The estimated masses of our sample provide important constraints on the stellar ages. Once a star has evolved to the red-giant phase, its age is determined to good approximation by the time spent in the core-hydrogen burning phase, and this is predominantly a function of mass. The *CoRoT* giants cover a mass range from ~ 0.9 to $\sim 3 M_{\odot}$, which in turn implies an age range spanning ~ 0.3 to ~ 12 Gyr, i.e. the entire Galactic history. To estimate the age of giants in the two observed populations we use PARAM, a Bayesian stellar parameter estimation method described in detail in da Silva et al. (2006). For this work, we adapted PARAM³ to include as additional observational constraints ν_{\max} , $\Delta\nu$ and, when available, the evolutionary status of the star (core-helium or hydrogen-shell burning phase), as determined by the period spacing of gravity-dominated modes (see Mosser et al. 2011). Having knowledge of the evolutionary status is particularly useful in the age determination of stars with an estimated radius typical of RC giants. Since the mass of core-He burning stars is likely to be affected by mass loss, the age–mass relation in red-clump stars may be different from that of red giant branch stars (see e.g. fig. 1 in Miglio 2012).

We applied this parameter estimation method to the sample of giants observed in LRC01 and LRA01 and obtained distributions of age, mass and radius. The estimated uncertainty on the age is of the order of 30–40 per cent given that no information on the metallicity is available; hence, a broad prior on [Fe/H] is assumed. The results obtained with a model-based approach support the interpretation that the differences between the observed population in LRA01 and LRC01 are due to a different mass, hence age, distribution.

5 CONCLUSIONS

In this paper, we have shown that solar-like oscillating giants are key tracers of stellar populations in the Milky Way. When combined with photometric constraints, the pulsation spectra of solar-like oscillating giant stars not only reveal their radii, and hence distances,

³ This extended version of PARAM will be made available, via an interactive web form, at the URL <http://stev.oapd.inaf.it/param>.

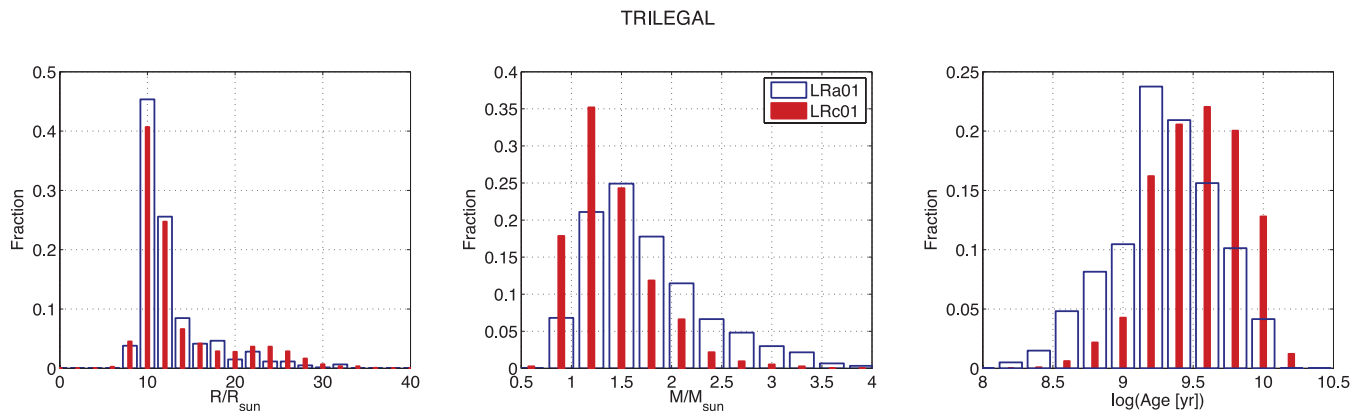


Figure 3. TRILEGAL simulations. Distributions of radius (left-hand panel), mass (middle panel), age (right-hand panel) of giants in synthetic populations computed with TRILEGAL for stars in the field LRA01 (blue) and LRC01 (red). Stars in the synthetic population were selected in magnitude ($K_s < 12$), colour ($0.6 < J - K_s < 1$) and ν_{max} ($6 < \nu_{\text{max}} < 80 \mu\text{Hz}$) to account for target selection effects.

but also provide well-constrained estimates of their masses, which are reliable proxies for the ages of the stars.

We considered red giants observed by the *CoRoT* space telescope in two regions located at different positions on the sky, with *CoRoT* designations LRC01 ($l = 37^\circ$, $b = -7^\circ$) and LRA01 ($l = 217^\circ$, $b = -2^\circ$). We found a significant difference in the mass distributions of the populations of the stars in the two fields, with stars in LRC01 having a lower average mass than those in LRA01. Differences in radius are in contrast marginal at best. To interpret these findings we have compared the observed distributions with the distributions given by synthetic population simulations of red giants for the same fields of the Galaxy covered by the observations. The synthetic populations show a difference in the mass distributions that agrees qualitatively with that found in the observed populations.

On the basis of these comparisons, we interpret the differences in the mass distributions as being due mainly to the different average heights of the observed fields below the galactic plane. Since it is believed that dynamical processes in the disc increase the velocity dispersion of stars with time, it follows that older stellar populations reach greater heights above and below the plane. What we observe with *CoRoT*, i.e. that the field higher below the plane has a larger fraction of old (i.e. low mass) stars, thus seems to support this theoretical expectation. Other data may hint at a similar dependence, as for instance an increase of the velocity dispersion of stars with increasing age (Holmberg et al. 2009), or the correlation between scale height and [O/Fe] (Bovy et al. 2012), but they are only available for stars in the solar vicinity. Future analyses of pulsating red giants sampling the Milky Way at different radii and heights from the plane, when complemented with their chemical abundances, will set even tighter constraints and enable to fully disentangle both the radial and vertical structure of the Galactic disc.

The findings presented here provide an example of the detailed picture of Galactic populations inferred from pulsating red giants. We can map regions that are more than a factor of 10 further away in distance compared to what has previously been possible, and we can explore (with a homogeneous sample) a wide age interval sampling look-back times as long as the age of the Galaxy. This method, when applied to the various regions explored by *CoRoT* and *Kepler*, and complemented by chemo-dynamical constraints from spectroscopic analyses, will provide the gold standard for current and future surveys of the Milky Way.

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