

Realization of Reflectionless Potentials in Photonic Lattices

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We realize experimentally a true reflectionless potential, which facilitates nonresonant unity transmission for all incident waves and at the same time supports localized modes. We utilize arrays of evanescently coupled optical waveguides, where a particular modulation of the transverse waveguide separations provides a physical realization of reflectionless Ablowitz-Ladik soliton potentials.

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It is a generic physical phenomenon that incident wave packets tend to be scattered by inhomogeneities, leading to partial reflection. In contrast, perfect transmission through inhomogeneities usually arises only at specific resonances. In the optical context, antireflection coatings are commonly based on the quarter-wavelength condition [1]. Although such transmission resonances can be broadened in multilayered structures, reflections still occur if the angle of incidence is varied. In contrast, it was predicted that certain smooth modulation profiles of the optical refractive index (as opposed to steplike profiles in multilayered coatings) yield nonresonant perfect unitary transmission for waves with broadband frequency spectra; such modulations are therefore called “reflectionless potentials” (RPs) [2]. Importantly, RPs can therefore be constructed without the use of optically resonant materials or negative refractive index metamaterials, and their profiles can be constructed systematically using a special mathematical procedure [3].

Since their prediction, RPs were explored in various physical contexts beyond optics. Indeed almost 80 years ago, in the early days of quantum mechanics, it was shown that the reflection of electrons can vanish for particular potentials [4], whose profiles can be mapped to the same class of functions as for optical waves. It was also shown that profiles of soliton solutions of integrable models generally constitute RPs [5]. Despite the mathematical advances and general physical analogies which were studied in detail in many theoretical works [6–8], to this date no experimental implementation of a true RP has been achieved. In optics, this is particularly due to difficulties in generating a smooth refractive index transition in order to reproduce the desired potential profile, which has been achieved so far only when using one-dimensional solitons in slab waveguides [9]. Hence, multilayered coatings remain as a common approach.

Recently, it was shown that the concept of RPs can be generalized for systems with a periodic modulation of their parameters, including coupled-resonator structures in photonic crystals and arrays of coupled waveguides [10].

If the structure is ideally periodic—which means in the case of coupled waveguide arrays that all the waveguides are identical and the separation between all pairs of neighboring waveguides is constant—then it supports propagation of waves without reflections (except at the structure boundaries). However, in such ideal structures the optical beams exhibit broadening due to the effect of discrete diffraction (see, e.g., [11]). If periodicity is broken by introducing inhomogeneities, i.e., detuned waveguides or altered separation between particular waveguides [12,13], then under appropriate conditions these defects can support localized modes which do not diffract. On the other hand, such defects tend to partially reflect incident waves. The concept of RPs allows us to overcome this drawback, realizing defects which support localized modes, but do not reflect any incident propagating waves.

In this Letter, we report the experimental demonstration of a true RP. We utilize an appropriately modulated array of evanescently coupled waveguides to realize the RP and directly visualize the wave propagation with the waveguide fluorescence technique [14]. The reflectionless modulation is based on a special transformation of soliton profiles of the discrete Ablowitz-Ladik (AL) model [15]. Accordingly, our results also constitute the first physical realization of a discrete AL soliton potential.

In order to explain our approach, we first discuss the mathematical basis for the realization of RPs in an array of optical waveguides. As was predicted in Ref. [10], such potentials can be created by modulating the coupling between the waveguides within a specific region, which can be practically realized by varying the transverse separation as illustrated in Fig. 1(a). Assuming identical waveguides, the beam propagation in such a structure can be modeled by a set of equations for the amplitudes of the guided modes (E_n) in the individual waveguides [10,11]:

$$i\frac{\partial E_n}{\partial z} + C_{n,n-1}E_{n-1} + C_{n,n+1}E_{n+1} = 0. \quad (1)$$

Here n is the waveguide number, z is the propagation distance along the waveguides, and $C_{n,m}$ are the coupling

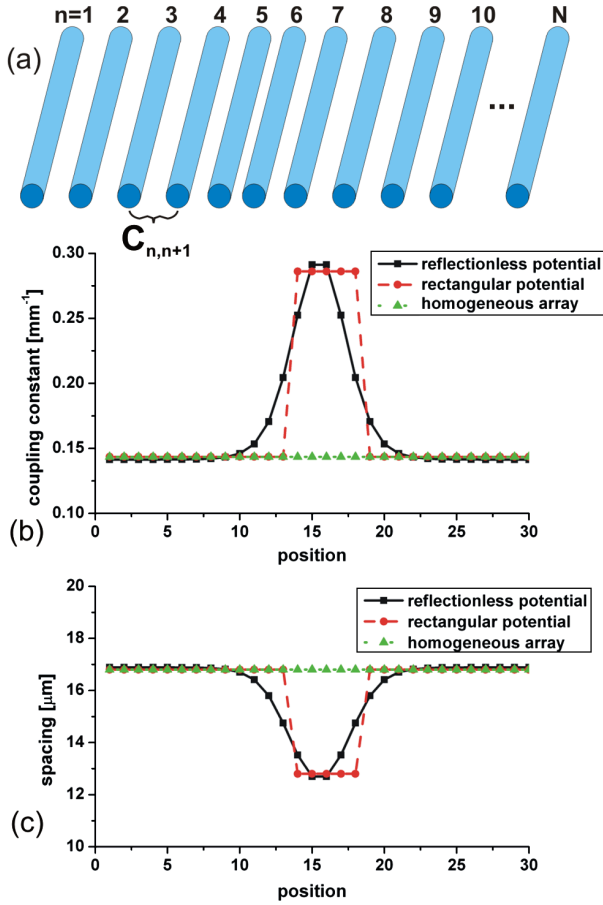


FIG. 1 (color online). (a) Setting of the waveguide array. (b) The coupling constants used in the experiments. (c) The respective waveguide spacing to achieve the desired intersite coupling.

coefficients between the modes of waveguides n and m . We note that in conservative dielectric structures, the coupling coefficients are symmetric, $C_{n,n-1} = C_{n-1,n}$. We consider localized modulations of the coupling coefficients $C_{n,n-1}$, whereas the structure approaches periodicity far away on either side of the lattice defect, i.e., $C_{n,n-1} \rightarrow C$ for $n \rightarrow \pm\infty$. Although we examine the *linear* regime of beam propagation, we use solutions of the *nonlinear* AL equation [15] to design reflectionless modulations [10]. The AL equation has the form

$$\frac{d\psi_n}{dz} + (\psi_{n-1} + \psi_{n+1})(C + |\psi_n|^2) = 0. \quad (2)$$

We note that in the AL model, the nonlinearity induces a modulation of the coupling coefficients as $C_{n,n\pm 1} = C + |\psi_n|^2$. It is evident that any inhomogeneous intensity distribution causes a coupling modulation. A stationary (z -independent) coupling modulation can be associated with a soliton solution, whose intensity profile remains fixed along the propagation direction. One family of such solitons is given by

$$\psi_n = \sqrt{C} \sinh\{\mu\} \operatorname{sech}\{\mu(n - n_0)\} \exp\{i\beta z\}, \quad (3)$$

where μ is the soliton parameter defining its width and amplitude, n_0 is the soliton position, and $\beta = 2C \cosh\{\mu\}$ is the propagation constant of the soliton. It is most important for our analysis that the AL equation is an integrable model [15], and accordingly its soliton solutions possess reflectionless characteristics. Therefore, modulations of the coupling coefficients in Eq. (2), caused by the soliton Eq. (3), fall into the class of RPs. Unfortunately, the AL model has a crucial downside. Because of the non-Hermitian Hamiltonian caused by the asymmetric coupling coefficients (in general, $C_{n,n-1} = C + |\psi_n|^2 \neq C_{n-1,n} = C + |\psi_{n-1}|^2$), it does not conserve the total power traveling in the array $P = \sum_n |\psi_n|^2 \neq \text{const}$. Hence, such modulation cannot be implemented in real physical waveguide lattices. However, it was shown that by applying a special transformation [16], it is possible to symmetrize the coupling coefficients such that the reflectionless characteristics are preserved:

$$C_{n,n+1} = C_{n+1,n} = \sqrt{(C + |\psi_n|^2)(C + |\psi_{n+1}|^2)}. \quad (4)$$

This modulation of the coupling coefficients can be implemented in Eq. (1) and therefore realized in practice, and will form a RP when ψ_n is taken as an AL soliton solution according to Eq. (3).

Our experimental study of RPs is based on arrays of coupled optical waveguides as shown schematically in Fig. 1(a), which were created in fused silica samples using the femtosecond laser direct-writing technology [17]. The specific fabrication parameters are provided in Ref. [18]. The waveguide arrays consist of 31 waveguides, with a length of 100 mm. We note that our system exhibits only a single band of Floquet-Bloch modes, since all higher bands are embedded in the continuum of radiation modes, and it is therefore excellently modeled by a single-band tight-binding system. For comparison, we fabricated three waveguide array structures with different modulations of the coupling coefficients presented in Fig. 1(b): (i) a homogeneous coupling $C_{n,n+1} = C = \text{const}$ (ideally periodic structure with fixed waveguide separation), (ii) a square potential well, and (iii) a reflectionless modulation according to Eqs. (3) and (4). To facilitate a comparison, we have chosen the same maximum coefficient modulation for a square well and for a RP. These modulations of the coupling constants were realized through a variation of the spacing between adjacent waveguides as shown in Fig. 1(c), which was calculated using the experimental data reported in Ref. [18]. In experiments, we directly monitor the beam propagation through the waveguide array by using a fluorescence microscopy technique [14].

We first investigate the propagation of a broad beam. The incident beam with a Gaussian envelope is approximately 5 waveguides wide. We choose the incident angle to be half the Bragg angle (0.4° in our structure). Under such conditions the beam travels without significant broadening in the homogeneous array [19], which is experimentally observed in our structure [see Fig. 2(a)]. This result was confirmed by numerically integrating Eq. (1), as shown in

Fig. 2(b). When a square potential well profile is introduced for the coupling constants, we observe strong reflection at both sides of the well, as expected from inhomogeneities in the waveguide spacing. The reflected light is clearly visible in Figs. 2(c) and 2(d). The situation completely changes when the profile of the coupling constants follows a RP. In this case, the beam penetrates the potential without any reflections and with unity transmission amplitude, which is exactly shown by our experimental result in Fig. 2(e), and confirmed by the simulations [Fig. 2(f)].

Importantly, RPs are associated with nonresonant suppression of reflection, which means that the reflection vanishes for a broad region of spectral components. We investigate this feature by launching a broad range of *transverse spatial* frequencies simultaneously by exciting the waveguide lattice only at a single site [20]. The results of these experiments are summarized in Fig. 3. In a homogeneous array, an excitation of a single lattice site results in the well-known “discrete” propagation pattern, exhibiting two distinct side lobes and a strong intensity modulation in between [11,21] [see Figs. 3(a) and 3(b)]. This diffraction

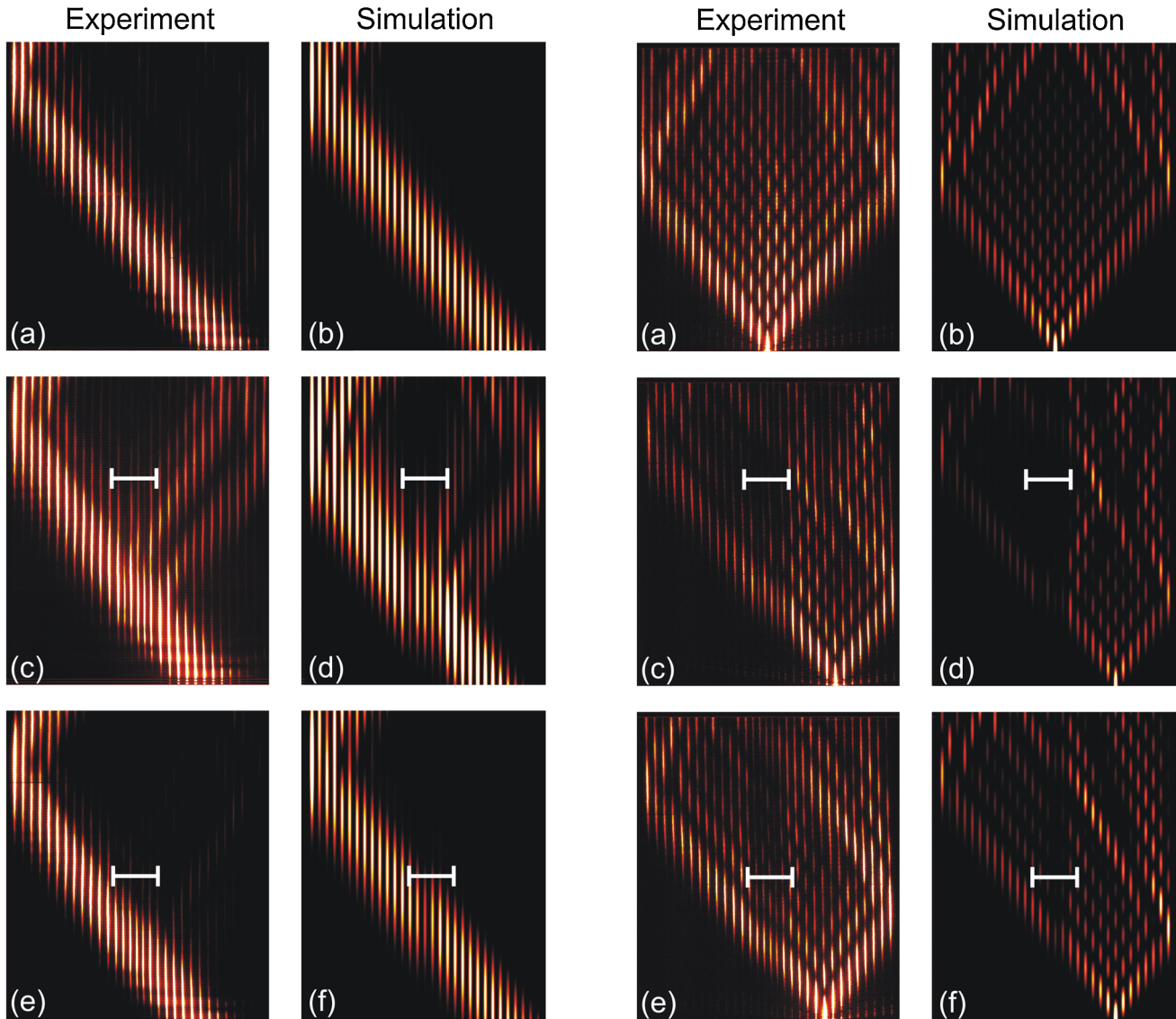


FIG. 2 (color online). (a) Evolution of a broad beam in a homogeneous array, as seen in the experiment. (b) Respective simulation. (c) Experimental observation of a broad beam in a lattice with a square potential well. Light is strongly reflected from the wells. (d) Numerical confirmation. (e) Experimental image for a broad beam penetrating a RP with unity transmission. (f) Respective simulation. In all panels, the propagation direction is bottom-top. The region of the respective potential is marked with a white bar.

FIG. 3 (color online). (a) Evolution of a narrow excitation in a homogeneous array, as seen in the experiment. (b) Respective simulation. (c) Experimental observation of the scattering of the discrete diffraction pattern at a square potential well. (d) Numerical confirmation. (e) Experimental image of how the discrete diffraction pattern “tunnels” through RPs with unity transmission. (f) Respective simulation. In all panels, the propagation direction is bottom-top. The region of the respective potential is marked with a white bar.

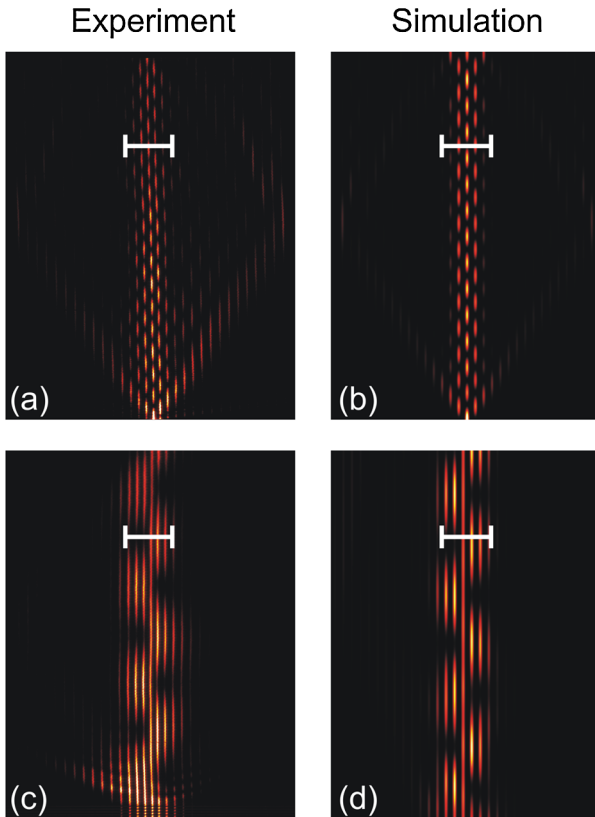


FIG. 4 (color online). (a) Trapping of a narrow excitation in the RP. (b) Numerical confirmation. (c) Trapping of a broad beam in the potential. (d) Respective simulation. In all panels, the propagation direction is bottom-top. The region of the respective potential is marked with a white bar.

pattern is highly distorted when a square well potential is introduced, due to a strong reflection at the potential boundaries [Figs. 3(c) and 3(d)]. Note that the fraction of the transmitted light is clearly smaller than the reflected light. In contrast, when a RP is introduced, the reflections are negligible. The light essentially “tunnels” through the potential, and the discrete diffraction pattern is continued on the other side of the potential [see Figs. 3(e) and 3(f)]. This illustrates the nonresonant character of the RP, which allows a broad range of spatial frequencies to pass with unity transmission.

Interestingly, even when the profile of the coupling modulation does not reflect incident waves, it can still act as a cavity supporting localized modes. It was recently shown theoretically that in general RPs support bound states [7]. In our optical system, two localized modes exist [10] whose profiles can be expressed as $E_n = \psi_n \sqrt{C + |\psi_n|^2}$ and $E_n = (-1)^n \psi_n^* \sqrt{C + |\psi_n|^2}$, where ψ_n is the AL soliton profile according to Eq. (3). We studied this property by launching a broad and a narrow beam into the cavity formed by the RP. The result for the narrow excitation, which covers a single lattice site only, is shown in Fig. 4(a). The beam remains localized within the defect region and exhibits a periodic breathing.

Our simulations confirm this observation [Fig. 4(b)]. The origin of breathing is due to interference of the two simultaneously excited modes. When a broad beam is launched into the cavity, this beam follows an undulated trajectory within the central region and cannot leave the RP well [see Figs. 4(c) and 4(d)]. These observations illustrate the property of a RP as an optical trap that does not perturb transversely propagating waves.

In conclusion, we experimentally realized a true RP, by applying the concept to arrays of evanescently coupled waveguides. We demonstrated that the transmission characteristics of such potentials are nonresonant and we proved the existence of bound states in such potentials. These results also constitute the first physical realization of Ablowitz-Ladik soliton potentials. We propose that, in a next step, these ideas can be applied to even more sophisticated settings, e.g., reflectionless potentials supporting multiple bound states [6] and systems with non-Hermitian Hamiltonians [22]. Considering the general applicability of the coupled-mode Eq. (1), our work has implications on a variety of areas in optics and beyond. Examples range from tunable light beams, over all-optical switching devices and beam shaping elements, to the control of pulses in coupled-resonator optical waveguides and Bose-Einstein condensates.

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