Dual-energy iterative reconstruction for material characterisation

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ABSTRACT

In this paper, we develop a dual-energy ordered subsets convex method for transmission tomography based on material matching with a material dictionary. This reconstruction includes a constrained update forcing material characteristics of reconstructed atomic number (Z) and density (ρ) volumes to follow a distribution according to the material database provided. We also propose a probabilistic classification technique in order to manage this material distribution. The overall process produces a chemically segmented volume data and outperforms sequential labelling computed after tomographic reconstruction.

Keywords: Dual-Energy, X-Ray Tomography, Material Characterisation, Iterative Reconstructions

1. INTRODUCTION

In the field of material characterisation, dual-energy tomography can be used to recover the distribution of density, ρ , and atomic number, Z (or average atomic number Z_{eff}), of a specimen by combining data acquired at two energies E_1 and E_2 . Such a characterisation is based on an attenuation $\mu_E(x)$ model at each point x of the acquired sample depending on both ρ and Z according to the acquisition energy E :¹

$$\begin{aligned}\mu_E(x) &= \frac{K_1}{E^k} Z^n(x) \rho(x) + K_2 f_{KN}(E) \rho(x) \\ &= f_{e^-}^E(Z(x), \rho(x)) + f_{CS}^E(\rho(x))\end{aligned}\quad (1)$$

where K_1 and K_2 are two constants. First part of Eq. (1) refers to the photoelectric effect, denoted $f_{e^-}^E(\cdot, \cdot)$, appearing for low energy E and becoming negligible for high energies. The second part refers to the Compton scattering $f_{CS}^E(\cdot)$, depending on the Klein-Nishina formula $f_{KN}(E)$ defined by:

$$f_{KN}(E) = \frac{1 + \alpha}{\alpha^2} \left[\frac{2(1 + \alpha)}{1 + 2\alpha} - \frac{1}{\alpha} \ln(1 + 2\alpha) \right] + \frac{1}{2\alpha} \ln(1 + 2\alpha) - \frac{(1 + 3\alpha)}{(1 + 2\alpha)^2} \text{ where } \alpha = \frac{E}{510.975} \text{ keV} \quad (2)$$

The first proposed algorithm¹ consists of computing $[Z^n \rho]$ -projections and ρ -projections from the both acquisitions to reconstruct $[Z^n \rho]$ and ρ volumes directly using a standard tomographic algorithm. Since this approach does not take beam-hardening into account, a more comprehensive technique has been proposed^{2,3} to estimate $[Z^n \rho](x)$ and $\rho(x)$ volumes by a 2-step process i) reconstructing the attenuation volumes $\mu(x, E_1)$ and $\mu(x, E_2)$, and ii) recovering material characteristic volumes from attenuation volumes by solving an equation system based on Eq. 1 for each energy. $Z(x)$ is often provided since it defines more accurately the chemical elements composing the acquired sample. However, extracting $Z(x)$ from $[Z^n \rho](x)$ is subject to the noise in $\rho(x)$, leading to errors in material characterisation.

In this paper, we propose a dual-energy iterative reconstruction method based on the ordered subsets convex (OSC) algorithm for transmission tomography using the attenuation function Eq. (1) directly in the forward model. During reconstruction, we estimate the specimen physical characteristics by matching each voxel with a material dictionary. This estimation is based on a probabilistic classification allowing us to allocate the most likely material in the dictionary for each voxel. The classification is verified during subsequent iterations of the algorithm, allowing false matchings to be corrected. The algorithm results in a reconstruction which is already segmented according to the physical properties of the scanned object.

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Name	Chemical Formula	ρ (g/cc)	Z_{eff}	
Air	Air	0.001	7.2240	
Iron Pure	Fe	5.600	26.0000	
Rutile	TiO ₂	4.200	19.0006	
Siderite	FeCO ₃	3.960	21.0932	
Na-Montmorillonite	NaAl ₅ MgSi ₁₂ O ₃₀ (OH) ₆	2.650	11.4620	
Water	H ₂ O	1.000	7.5195	
Graphite	C	2.300	6.0000	
Calcite	CaCO ₃	2.710	15.7100	
Dolomite	CaMg(CO ₃) ₂	2.870	13.7438	
Quartz Mineral	SiO ₂	2.650	11.5800	
Aluminium Pure	Al	2.700	13.0000	
Anhydrite	CaSO ₄	2.950	15.6847	
Fused Quartz	SiO ₂	2.200	11.7842	
Pyrite	FeS ₂	5.020	21.9588	
Barite	BaSO ₄	4.500	47.2008	
Na-Feldspar	NaAlSi ₃ O ₈	2.610	11.5534	
K-Feldspar	KAlSi ₃ O ₈	2.530	13.3895	
Kaolinite	Al ₂ Si ₂ O ₅ (OH) ₄	2.600	11.1622	
Illite	KAl ₃ Si ₃ O ₁₀ (OH) ₂	2.800	9.6058	
Ca-Montmorillonite	Ca _{0.5} Al ₅ MgSi ₁₂ O ₃₀ (OH) ₆	2.650	11.8277	
Chlorite	Fe ₂ Mg ₂ Al ₂ Si ₂ Al ₂ O ₁₀ (OH) ₈	2.900	11.6449	
Celestite	SrSO ₄	3.900	30.4686	
Talc	Mg ₃ Si ₄ O ₁₀ (OH) ₂	2.750	8.4538	
Halite	NaCl	2.350	15.3295	

Table 1. Density ρ and average atomic number Z_{eff} values for common materials. Colors correspond to the classification labels used in simulated/reconstructed volumes.

This iterative process avoids the independence of the successive steps in the existing sequence of segmentation and labelling. Indeed approximations during reconstruction can act dramatically on the rest of the sequence. Conversely iterative processing can correct for the labelling errors during the subsequent iterations. Additionally, several acquisition optimizations and dynamic tomography methods are developed by our group for the high cone-angle helical-scanning micro-CT facility developed at ANU. These methods, proposed in companion papers,^{4,5} are based on iterative schemes which could ultimately include material characterisation in addition to their own processing aim.

Thus the remainder of the paper is organized as follows: In section 3 we investigate several material characterisation schemes based on numerical simulation. Our aim here is to define the most likely functions performing an efficient and noise-robust voxel material labelling with respect to $Z(x)$ and $\rho(x)$ volumes; or $[Z^n\rho](x)$ and $\rho(x)$ volumes. Then we introduce in section 4 the dual-energy iterative reconstruction algorithm including material characterisation during the iterations.

2. MATERIAL AND METHODS

2.1 Material dictionary and numerical simulation

This study is based on a numerical analysis of a synthetic object simulating a rock sample. This sample is modelled using the material database provided on Tab. 1. The numerical sample is composed of a cylindrical container filled randomly with different spherical mineral grains. The mineral composition we have chosen is 60% of quartz, 20% kaolinite and 20% calcite. The analytic object is rasterized into an N^3 voxel volume for comparison with reconstructed data (i.e. N^3 corresponds to reconstructed volume size). Fig. 1 shows the 2D central slices of $Z(x)$ and $\rho(x)$ volumes and a 3D multi-planar rendering coloured with the labels given in Tab. 1.

The synthetic object is acquired by a simulated acquisition using a detector 4 times with the number of pixels to prevent the inverse crime.⁶ X-ray lines (from source to each pixel of the detector) are traced with respect to the scanner geometry. Distances crossed by each X-ray line in each material composing the object are measured and total absorption is computed for each sub-pixel ii by $A(ii) = \sum_{m \in M_s} \int \mu_{E,m}(x) dx$, where M_s is the material set composing the sample and $\mu_{E,m}$ corresponds to μ_E (Eq. 1) for material m . We obtain the final projection value on the detector by averaging the four sub-pixel values and applying Poisson noise model:

$$R(i) = \text{Poisson} \left\{ \sum_{ii \in i} \gamma_0 \exp(-A(ii)) \right\} \quad (3)$$

where γ_0 is the average photon counts on detector (blank scan) and $ii \in i$ denotes sub-pixel indexes contained in the detector pixel i . For example, Fig. 2 shows a projection computed at 60keV and 120keV from the

Figure 1. Simulated object: (a) Central slice of $Z(x)$ volume. (b) Central slice of $\rho(x)$ volume. (c) 3D visualisation of the object imaged into a 512^3 voxel volume.

simulated object. In this study, reconstructed volume size is $N^3 = 256^3$ voxels and detector size is 256×192 pixels. Thus acquisitions have been performed using 512×384 detector size and downsampled to reconstruct 256^3 volumes.

Figure 2. Projection of synthetic sample at (a) 60 keV and (b) 120 keV.

2.2 Ordered subsets convex reconstruction

The well-established iterative expectation maximization (EM) algorithm refers to a wide class of tomographic reconstructions providing 3D structural volume of the sample from a set of radiographs. The expectation to observe the measurements, R , according to a given sample, μ , denoted $p(R|\mu)$ follows a Poisson distribution which can be defined by:

$$p(R|\mu) = \prod_i \frac{e^{-\hat{R}(i)} (\hat{R}(i))^{R(i)}}{R(i)!}, \quad (4)$$

where i is a projection line index and $\hat{R}$, the expected number of counted photons at detector position i is:

$$\hat{R}(i) = \gamma_0 e^{-\sum_j w_{ij} \mu(j)}, \quad (5)$$

where w_{ij} is a weight coefficient defining the voxel j contribution on the detector measurement i . w_{ij} is usually proportional to the distance travelled by the line i in the voxel j . Based on the likelihood $p(R|\mu)$ and the transmission model $\hat{R}$, the method consists of maximizing the log likelihood $L = \log p(R|\mu)$, ie. finding the solution μ that minimizes the partial derivative $\partial_\mu L$. The solution can be achieved by the Newton-Raphson method iterating in t :⁷⁻¹⁰

$$\mu_{t+1} = \mu_t - \frac{\partial_{\mu_t} L}{\partial_{\mu_t^2}^2 L} \quad (6)$$

A penalization $p(\mu)$ is introduced in addition to $p(R|\mu)$ to stabilize the solution and avoid divergence during the iterations. $p(\mu)$ is defined with respect to the Gaussian distribution of reconstructed data by:¹¹

$$p(\mu) = e^{-\beta \sum_j \sum_{k \in \mathcal{N}(j)} y_{jk} \Phi(\mu(j) - \mu(k))}, \quad (7)$$

where β is a relaxation parameter, $\mathcal{N}(j)$ denotes the neighbourhood of pixel j , $\Phi(\cdot)$ is a potential function and y_{jk} a weight factor (usually inverse proportional to the distance between voxels j and k). In this paper, we use the following LogCosh function as the potential of the regularization:¹²

$$\Phi(x) = 2gt^2 \log \cosh \left(\frac{|x|}{t} \right), \quad (8)$$

where g is a sensitivity parameter. This function attenuates the small variations leading to a better homogeneity of the regions and preserves the edges beyond the given threshold t . By replacing $L = \log p(R|\mu)p(\mu)$ in Eq. (6), the overall penalized ordered subsets convex (OSC) algorithm consists of iterating in t and subsets $s+1$ in order to update each voxel j of the volume μ until convergence of the solution:

$$\begin{aligned} \mu_{s+1}^t(j) &= \mu_s^t(j) + \alpha^t \Delta \mu_s^t(j), \\ \Delta \mu_s^t(j) &= \frac{\sum_{i \in S(s)} w_{ij} \left(\hat{R}_s^t(i) - R(i) \right) - \beta \sum_{k \in \mathcal{N}(j)} y_{jk} \Phi'(\mu(j) - \mu(k))}{\sum_{i \in S(s)} w_{ij} \left[\sum_l w_{il} \right] \hat{R}_s^t(i) + \beta \sum_{k \in \mathcal{N}(j)} y_{jk} \Phi''(\mu(j) - \mu(k))} \end{aligned} \quad (9)$$

where $\hat{R}_s^t(i)$ is the expected photon counts computed from μ_s^t using Eq. (5), α^t is a relaxation parameter, $S(s)$ are the radiographs in the subset s and $\Phi'(\cdot)$ and $\Phi''(\cdot)$ are respectively the first and second derivative of $\Phi(\cdot)$. The volume obtained by the update using the subset s is used as starting volume of the next subset $s+1$. A main iteration t is completed when all subsets have been processed.

3. MATERIAL CHARACTERISATION: A PRELIMINARY SIMULATION STUDY

Let $\mu_{E_1}(x)$ and $\mu_{E_2}(x)$ be the tomograms reconstructed respectively from two acquisitions at energies E_1 and E_2 with the standard OSC algorithm. One can recover the volume pairs (Z, ρ) or $([Z^n \rho], \rho)$ by solving one of the following equation systems at each voxel x :

$$\begin{cases} \mu_{E_1}(x) = A_1 Z^n(x) \rho(x) + B_1 \rho(x) \\ \mu_{E_2}(x) = A_2 Z^n(x) \rho(x) + B_2 \rho(x) \end{cases} \quad \text{or} \quad \begin{cases} \mu_{E_1}(x) = A_1 [Z^n \rho](x) + B_1 \rho(x) \\ \mu_{E_2}(x) = A_2 [Z^n \rho](x) + B_2 \rho(x) \end{cases} \quad (10)$$

where $A_1 = \frac{K_1}{E_1^k}$, $B_1 = K_2 f_{KN}(E_1)$, $A_2 = \frac{K_1}{E_2^k}$ and $B_2 = K_2 f_{KN}(E_2)$. Our aim in this section is to define and compare several probabilist material matching functions estimating that a reconstructed voxel corresponds to a material in the material dictionary. Since one can determine ρ , Z or $[Z^n \rho]$ using Eq. 10, a matching function will be based on a comparison of one or several reconstructed material characteristics with their inherent values in the material database.

We consider that reconstructed data $Z(x), \rho(x)$ and $[Z^n \rho](x)$ each follow a Gaussian distribution around an effective material value. Then we can define the probability that a voxel x corresponds to a material $m \in M$ by the following functions:

$$\begin{cases} p_\rho = p(\rho(x) = \rho_m) = \exp \left(-(\rho(x) - \rho_m)^2 / \sigma_{\rho_m}^2 \right) \\ p_Z = p(Z(x) = Z_m) = \exp \left(-(Z(x) - Z_m)^2 / \sigma_{Z_m}^2 \right) \\ p_{[Z^n \rho]} = p([Z^n \rho](x) = [Z^n \rho]_m) = \exp \left(-([Z^n \rho](x) - [Z^n \rho]_m)^2 / \sigma_{[Z^n \rho]_m}^2 \right) \end{cases} \quad (11)$$

p_ρ	p_Z	$p_{Z^n\rho}$
$p(\rho(x) = \rho_m)$	$p(Z(x) = Z_m)$	$p([Z^n\rho](x) = [Z^n\rho]_m)$
$e^{\left(-\frac{(\rho(x)-\rho_m)^2}{\sigma_{\rho_m}^2}\right)}$	$e^{\left(-\frac{(Z(x)-Z_m)^2}{\sigma_{Z_m}^2}\right)}$	$e^{\left(-\frac{([Z^n\rho](x)-[Z^n\rho]_m)^2}{\sigma_{[Z^n\rho]_m}^2}\right)}$
$\sigma_{\rho_m}^2 = \frac{(\rho_m - \rho_n)^2}{4 \ln(p_t)}$	$\sigma_{Z_m}^2 = \frac{(Z_m - Z_n)^2}{4 \ln(p_t)}$	$\sigma_{[Z^n\rho]_m}^2 = \frac{([Z^n\rho]_m - [Z^n\rho]_n)^2}{4 \ln(p_t)}$ $\sigma_{\rho_m}^2 = \frac{(\rho_m - \rho_n)^2}{2 \ln(p_t)}$
$\rho_n = \operatorname{argmin}\{\rho_m - \rho_{m_i}\}$ $m_i \in M \wedge m_i \neq m$	$Z_n = \operatorname{argmin}\{Z_m - Z_{m_i}\}$ $m_i \in M \wedge m_i \neq m$	$[Z^n\rho]_n = \operatorname{argmin}\{[Z^n\rho]_m - [Z^n\rho]_{m_i}\}$ $\rho_n = \operatorname{argmin}\{\rho_m - \rho_{m_i}\}$ $m_i \in M \wedge m_i \neq m$

Table 2. Singular matching functions investigated according to the reconstructed $\rho(x)$, Z and/or $[Z^n\rho]$ volumes. Complex matching functions (Eq. 12) are obtained by combination of p_ρ with p_Z or $p_{Z^n\rho}$.

or a combination of them such that:

$$\begin{cases} p_{Z\rho} = p(Z(x) = Z_m)p(\rho(x) = \rho_m) \\ p_{[Z^n\rho]\rho} = p([Z^n\rho](x) = [Z^n\rho]_m)p(\rho(x) = \rho_m) \end{cases} \quad (12)$$

where ρ_m , Z_m and $[Z^n\rho]_m$ are the chemical characteristics of material m in the dictionary M and σ_{ρ_m} , σ_{Z_m} and $\sigma_{[Z^n\rho]_m}$ are its respective standard deviations. Large deviation will result in a non-accurate estimation since all materials in the dictionary will achieve a good probability (smooth estimate). Conversely, too small deviation will lead to too-discriminant estimates. Let a_m be one of the comparison value for material m (i.e. $a_m = \{\rho_m, Z_m \text{ or } [Z^n\rho]_m\}$). We denote a_n the corresponding value for the material $n \in M$ such that $n \neq m$ and $a_n = \operatorname{argmin}\{a_m - a_{m_i}, m_i \in M \wedge m_i \neq m\}$. We define the standard deviation σ_a of m for the material characteristic a by:

$$\sigma_a^2 = \frac{(a_m - a_n)^2}{4 \ln(p_t)} \quad (13)$$

where $p_t \in \{0, 1\}$ define the distinguishability between m and its closer neighbourhood n in the material dictionary. Setting such a standard deviation for each material allows to locally adjust the difficulty of the probabilist matching function with respect to the chemicals composing the database. A summary of the three single best-matching functions (Eq. 11) is provided in Tab. 2. Fig. 3 show the probability material distributions of the database Tab. 1 according to the five matching functions (Eq. 11 and Eq. 12) investigated in this paper ($p_t = 0.8$ in this example). Particularly, Fig. 3(f) shows the impact on the distribution when using a restricted material database (containing only the chemicals which are used in our numerical sample) compared to a full database Fig. 3(d), for the same probability function.

In order to quantify the efficiency of each matching function, we first compute numerical acquisitions at two energies using the simulation explained in section 2.1. Then, we reconstruct the volumes (μ_1, μ_2) using the OSC algorithm detailed in section 2.2. We estimate the volumes (Z, ρ) and $([Z^n\rho], \rho)$ by solving Eq. 10 for each voxel. Finally, we apply the classification labelling using each matching function. This classification labels each voxel with the material $m \in M$ such that m maximizes the used probability function. Colored images on Fig. 4(b-f) show the results for each method compared to the reference image (central slice of the simulated object). The number of correct labelling (Corr. in % on Fig. 4) demonstrates that each function has correctly recovered at least 50% of the voxels composing the sample from full dictionary. Best result is obtained using $p_{Z\rho}$ even if we can globally denote that differences between different results are not significant. Fig. 5 shows most-likely material results when the material dictionary is restricted to the chemicals composing the acquired sample. The labelling process achieves a better result whatever the matching function. However we can note the high results obtained by p_Z and $p_{[Z^n\rho]}$ compared to the others. Thus one can expect to perform an accurate matching from (Z, ρ) volumes as well as than from $([Z^n\rho], \rho)$ volumes.

Figure 3. Material distribution of probabilities according to Z and ρ with respect to the probabilist function: (a) p_ρ , (b) p_z , (c) $p_{z\rho}$, (d) $p_{[Z^n\rho]}$ and (e) $p_{[Z^n\rho]\rho}$ for the complete material dictionary. (f) $p_{[Z^n\rho]}$ distribution of the material database restricted to the chemicals used in the numerical sample.

4. DUAL ENERGY RECONSTRUCTION AND MATERIAL MATCHING

We have observed in section 3 that a material labelling can be performed from (Z, ρ) as well as from $([Z^n\rho], \rho)$ volumes. The latter has the advantage to be not limited by the noise in ρ volume. However the matching process developed in the previous section is performed after tomographic reconstruction whereas it could be done during an iterative reconstruction technique, allowing false matching to be corrected during subsequent iterations of the algorithm. Thus we develop now a modified dual-energy OSC algorithm based on the reconstruction of $([Z^n\rho], \rho)$ volumes and including a constrained update of the voxels at each iteration. This update enforces each voxel under-reconstruction to be closer to the most likely material in the database.

The forward model of the algorithm is based on the previous estimate of $([Z^n\rho]_{s-1}^t, \rho_{s-1}^t)$. Thus $\mu_{E_1}(j)$ and

Figure 4. Estimate results compared to the original labelling (a). (b) p_ρ . (c) p_Z . (d) $p_{Z\rho}$. (e) $p_{[Z^n\rho]}$. (f) $p_{[Z^n\rho]\rho}$, using the full dictionary. Number of correct (Corr.) matching (in %).

Figure 5. Estimate results compared to the original labelling (a). (b) p_ρ . (c) p_Z . (d) $p_{Z\rho}$. (e) $p_{[Z^n\rho]}$. (f) $p_{[Z^n\rho]\rho}$, using a material dictionary restricted to the chemicals composing the numerical sample. Number of correct (Corr.) matching (in %).

$\mu_{E_2}(j)$ in Eq. 5 are given by:

$$\begin{cases} \mu_{E_1}(x) = A_1[Z^n\rho]_{s-1}^t(x) + B_1\rho_{s-1}^t(x) \\ \mu_{E_2}(x) = A_2[Z^n\rho]_{s-1}^t(x) + B_2\rho_{s-1}^t(x) \end{cases} \quad (14)$$

Then we obtain the update values $\Delta\mu_s^t(x, E_1)$ and $\Delta\mu_s^t(x, E_2)$ by applying the OSC update step Eq. 9 separately for E_1 and E_2 . Let $\Delta[Z^n\rho]_s^t(x)$ and $\Delta\rho_s^t(x)$ the update values of $([Z^n\rho]_{s-1}^t, \rho_{s-1}^t)$. They are recovered by solving for each x :

$$\begin{cases} \Delta\mu_s^t(x, E_1) = A_1\Delta[Z^n\rho]_s^t(x) + B_1\Delta\rho_s^t(x) \\ \Delta\mu_s^t(x, E_2) = A_2\Delta[Z^n\rho]_s^t(x) + B_2\Delta\rho_s^t(x) \end{cases} \quad (15)$$

We add a constrained update at the end of each super-iteration t based on best-matching functions. This update consists of modifying $[Z^n\rho]^t(x)$ and $\rho^t(x)$ if the most likely material m is such that $p_\alpha = p(x = m) > p_\beta$, where p_β is a probability threshold. In such a case, the constrained update performs as follows:

$$\begin{cases} [Z^n\rho]^t(x) \leftarrow p_\alpha[Z^n\rho]^t(x) + (1 - p_\alpha)[Z^n\rho]_m \\ \rho^t(x) \leftarrow p_\alpha\rho^t(x) + (1 - p_\alpha)\rho_m \end{cases} \quad (16)$$

where $[Z^n\rho]_m$ and ρ_m are the chemical characteristics of m . This constrained update enforces reconstructed data to be closer to the theoretical values of a material m in the dictionary if m is the best likelihood and $p(x = m)$ already provides a high probability (greater than threshold p_α). Matching providing a best estimate lesser than p_α are not modified by the constrained update. This constraint has the advantage to make the solution closer to the material in the dictionary.

This algorithm is now applied using the same set of acquisitions at E_1 and E_2 than on previous section. We have set in this paper $p_\beta = p_t = 0.8$, and we have used the probability function $p_{[Z^n\rho]}$ in the constrained update since it has provided the average best results in the preliminary study. Central slice of the reconstructed/labelled data are shown on Fig. 6(b-c) using respectively full-database and restricted-database. This technique has correctly labelled more than 66% (*resp.* 87%) of the voxels containing the sample from the full (*resp.* restricted) material database. Thus we observe a gain of more than 10% for the same probability function when using

the iterative reconstruction with constraint. These results demonstrate the efficiency to combine tomographic reconstruction with material labelling. However result still suffers from material labelling errors, due to the noise and partial volume approximations.

Figure 6. Estimate results compared to the original labelling (a) using $p_{[Z^n\rho]}$ in the iterative reconstruction: (b) with full material dictionary, (c) using material dictionary restricted to the chemicals composing the sample. Number of correct (Corr.) matching (in %) among the voxels containing chemicals only. Previous results obtained by sequential processing sequence using the same probability function are repeated for comparison.

5. CONCLUSION

In this paper we have investigated several probabilist techniques to label each voxel of a tomogram according to the physical characteristics contained in a material dictionary. These matching techniques are based on a local distinguishability of each material in the database with respect to its closest neighbourhood. Thus one can achieve a material labelling from (Z, ρ) or $([Z^n\rho], \rho)$ volumes with equivalent quantitative results. However since it is performed after tomographic reconstruction, the material labelling can not be corrected according to physical models. Since iterative reconstructions are based on such physical properties of X-Ray radiations according to material characteristics, we have proposed a dual-energy OSC algorithm, based on the reconstruction of $([Z^n\rho], \rho)$ volumes, including a constrained update at the end of each iteration. This additional update enforces the values of $([Z^n\rho], \rho)$ to be closer to the theoretical values of the material m which provides the best probability. Such a process achieves both a better reconstruction of the object and a more accurate labelling compared to a process performing tomographic reconstruction and material classification sequentially. However we have neglected the beam-hardening effects in our simulated study, while it is investigated by our group in another paper.¹³ These techniques can extract $([Z^n\rho], \rho)$ projections to reconstruct $([Z^n\rho], \rho)$ volumes, avoiding intermediates $\Delta\mu_s^t(x, E_1)$ and $\Delta\mu_s^t(x, E_2)$ and correcting for polychromatic spectra at once. A combination of these techniques with the dual reconstruction/labelling proposed in this paper will be investigated in our future works.

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