Solidification dynamics in channeled viscoplastic lava flows

J. C. Robertson1,2 and R. C. Kerr1

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[1] The influence of a viscoplastic lava rheology on the dynamics of channeled lava flows is analyzed using analogue experiments. The experiments used slurries of polyethylene glycol and kaolin, which flowed with a constant flux down an inclined channel under water. Three sets of complementary experiments are presented: isothermal, cooling, and solidifying flows which quantified the effects of the viscoplastic rheology on shear, internal convection and surface crust formation. The isothermal and cooling experiments showed the formation of unyielded central plug regions which were not broken up by the convective overturning. In the solidifying experiments flows fell into one of three regimes: a tube regime, in which crust covered the entire flow surface; a shear-controlled regime, with a mobile raft of crust in the channel center and open shear zones near the walls; and a plug-controlled regime where the width of the central crust was determined by the width of the central plug region. The crust coverage is parameterized in terms of two dimensionless parameters: the ratio of the central plug region width to channel width and a parameter \( \psi \) which characterizes the relative importance of the strain and solidification rates. Finally the dynamics of typical lava flows on Mt Etna and the 1984 Mauna Loa lava flow are examined. We show that our parameterization agrees with lava flow crust widths observed in the field and find that even small yield strengths have a major effect on crust coverage.


1. Introduction

[2] Molten lava from large volcanic eruptions often self-organizes into localized channels, which can transport lava large distances from an erupting vent (Figure 1). For basaltic volcanoes such as those in Hawaii these channels are commonly 10–100 m wide and of the order of 10 km in length [Lipman and Banks, 1987; Cashman et al., 1999], with flow 2–10 m deep during active periods, while for alkaline volcanoes such as Mt Etna the channels are typically shorter and narrower [Pinkerton and Sparks, 1976; Calvari and Pinkerton, 1998]. Much longer channels, up to 1000 km in length, were important in transporting lavas from large prehistoric flood basalt eruptions [Keszthelyi and Self, 1998; Self et al., 2008].

[3] The dynamic evolution of lava flows is strongly influenced by the nature and timing of cooling and solidification during eruption and flow emplacement [Griffiths, 2000]. Heat loss through the flow surface leads to the formation of a surface crust whose presence inhibits radiative heat transfer from the flow to the environment, dramatically reducing heat loss from the interior of the flow. A lava flow may travel much further under these conditions than in situations where the surface crust is continuously disrupted [e.g., Kauahikaua et al., 1998; Cashman et al., 1999]. Alternatively, surface crust can cause blockages within an established channel flow, causing overflows which starve the flow downstream.

[4] Since it can both help and hinder flow propagation, an effective model of lava flow dynamics must therefore include the formation and distribution of surface crust. It is not feasible to explicitly include surface solidification in lava emplacement models, since complex crustal behavior such as surface folding, rifting and fracturing are difficult to capture numerically. However, analogue laboratory experiments using polyethylene glycol (PEG, a clear wax with a freezing point close to room temperature) have proved useful in understanding this complex problem [e.g., Fink and Griffiths, 1990; Griffiths and Fink, 1997; Fink and Griffiths, 1998; Griffiths et al., 2003; Lyman et al., 2004; Cashman et al., 2006; Kerr et al., 2006].

[5] Fink and Griffiths [1990] carried out quantitative experiments studying the horizontal spreading of point- and line sourced viscous flows. They found a number of crust morphology regimes, and parameterized the conditions for each using a dimensionless group \( \psi \). This parameter is the
Figure 1. A well-developed lava channel fed from a rift eruption of Pu‘u Ō‘ō on 23 September 2011. The channel is approximately 4 meters wide, 2 meters deep, and is moving at speed of about 3 meters per second. The erupting rift can be seen in the background, in front of the cone of Pu‘u Ō‘ō. View is to the southwest. Photo: Tim Orr, Hawai‘ian Volcanological Observatory.

The ratio of characteristic advection and solidification timescales:

\[ \psi = \frac{U_m t_s}{H} \]  

(1)

where \( U_m \) and \( H \) are the maximum velocity at the surface of the flow and the flow depth respectively (so that \( H/U_m \) is a characteristic advection time), and \( t_s \) is the time taken for the flow surface to cool from the eruption temperature to the solidification temperature. Low values of \( \psi \) lead to solidification-dominated morphologies, while higher values lead to viscosity-dominated regimes.

[6] Griffiths et al. [2003] investigated the processes controlling flow and solidification of viscous flows in a sloping rectangular channel. They found that the dimensionless parameter:

\[ \vartheta = \psi Nu, \]  

(2)

(where \( Nu \) is the Nusselt number describing the dimensionless heat flux to the surface of the flow due to internal convection) ordered the flows into either a tube regime (where the crust entirely covers the flow surface), or a mobile crust regime (where the surface crust is restricted to a central raft of solidified material), separated by a critical value \( \vartheta = \vartheta^* \). Tube regime flows have \( \vartheta < \vartheta^* \), while flows in the mobile crust regime have \( \vartheta > \vartheta^* \).

[7] The study of Griffiths et al. [2003] helps to understand the solidification dynamics of crystal-poor lavas, which have Newtonian rheologies. However, partial crystallization of a lava can generate a touching network of crystals which can bear a stress in addition to the viscous response of the lava’s melt fraction. When this occurs the lava develops a viscoplastic rheology [Pinkerton and Sparks, 1978; Kerr and Lister, 1991]. In this study, a series of analogue laboratory experiments extend the work of Griffiths et al. [2003] to flows with viscoplastic rheologies, to examine the effects this rheology has on the solidification dynamics.

[8] Three sets of experiments are described in this paper: isothermal flows, cooling flows (without solidification), and solidifying flows; these move progressively toward the full problem of lava flow solidification. The results of the isothermal experiments are summarized in section 4, and compared with numerical solutions given by Robertson and Kerr [2012]. Qualitative observations of the non-solidifying and solidifying flows are given in section 5 and section 6.1 respectively. A detailed analysis of the solidifying flow experiments is provided in sections 6.2–6.3, while section 7 gives an outline of the applicability and implications of these results in volcanological studies.

2. Viscoplastic Fluid Rheology

[9] A viscoplastic material behaves as a solid up to a critical magnitude of an applied shear stress, called the yield strength \( \tau_y \). Above the yield stress, the material deforms like a fluid, with a differential viscosity \( \mu \) [Ancey, 2007]. The simplest viscoplastic fluid model is the Bingham fluid, which consists of a combination of plastic and Newtonian rheologies. The simplest viscoplastic rheology is the Bingham fluid, which has the following constitutive equation relating the deviatoric stress tensor, \( \tau \), to the strain rate tensor, \( \dot{e} \), within the fluid:

\[
\tau = \begin{cases} 
2\mu + \sqrt{2}\frac{\tau_y}{\|\dot{e}\|} & \text{if } \|\tau\| \geq \tau_y \\
\dot{e} & \text{otherwise}
\end{cases}
\]  

where \( \mu \) is the yielded dynamic differential viscosity, \( \tau_y \) the yield strength of the material and \( \|\cdot\| \) denotes the Frobenius tensor norm. This fluid rheology was first introduced by Bingham [1916] for one dimensional flows and later extended to a multidimensional tensorial formulation by Oldroyd [1947]. Unless stated explicitly otherwise, we will use the term viscosity to refer to the yielded differential viscosity \( \mu \), and define the effective viscosity as \( \eta = \tau/\dot{e} = 2\mu + \sqrt{2}\tau_y/\|\dot{e}\| \).

[10] The Bingham rheology divides a viscoplastic flow into two sets of domains: the first branch of equation (3)
3. Analogue Experiment Overview

[11] Consider a channel with width \( W \) which slopes at an angle \( \theta \) to the horizontal (Figure 2). The viscoplastic fluid has density \( \rho \), thermal diffusivity \( \kappa \), thermal expansion coefficient \( \alpha \), and a Bingham rheology with a dynamic differential viscosity \( \mu \) and yield strength \( \tau_y \); it is ‘erupted’ at a temperature \( T_s \), solidifies at \( T_r \) and flows with a constant flow rate \( Q \) under an ambient fluid of density \( \rho_a \) at temperature \( T_a \). The down-channel buoyancy force acting on the flow per unit mass is defined as \( g' = (1 - \rho_a/\rho)g\sin\theta \), where \( g \) is the vertical gravitational acceleration. For the rest of this paper we will use the term ‘viscous’ to refer to flows with a Newtonian rheology, and ‘viscoplastic’ to flows with a Bingham rheology.

3.1. Apparatus

[12] The experimental apparatus used is shown schematically in Figure 3. A large tank (with dimensions of \( 400 \times 200 \times 2000 \) mm) contained a sloping \( 1600 \) mm long acrylic channel, which was \( 80 \) or \( 150 \) mm wide. A vertically sliding gate \( 200 \) mm from the top of the channel formed a source lock which prevented the slurry from mixing with the water at the source and generated a uniform flow across the entire channel width. A tube fed the slurry through a gate valve (which was used to set the flow rate) and a stopcock (which was used to start and stop the flow independently of the gate valve) into the source lock from an overhead reservoir.

[13] After filling the large tank with water and the lock with slurry, the experiments were started by opening the stopcock and lifting the lock gate. This allowed the slurry to flow beneath the lock gate, down the channel and out into the larger tank. By raising and lowering the gate until the depth of slurry in the lock was steady the flow rates through the tube and underneath the gate could be matched. The loss of head over time from the overhead reservoir was used to determine the flow rate.

3.2. Slurry Preparation

[14] Slurry preparation involved mixing polyethylene glycol and kaolin to give a kaolin weight fraction of \( X_k = 0.25 \). The slurries were then stored in airtight containers overnight at a temperature of \( 25-28^\circ C \) (well above the solidification temperature of the polyethylene glycol at about \( 18^\circ C \)), which ensured that prepared slurries remained molten and that the kaolin had time to disperse throughout the polyethylene glycol. The slurry was then remixed and strained to remove any large clumps and ensure an even kaolin suspension.

3.3. Slurry Solidification Temperatures

[15] The solidification temperatures of the polyethylene glycol and slurries were determined by measuring the cooling history of a sample of polyethylene glycol cooled below its solidification temperature using a thermistor immersed in the sample. The release of latent heat generates a plateau in the temperature time series as the sample solidifies, allowing the solidification temperature to be read directly off the cooling history. This method will under-estimate the actual solidification temperature as some undercooling of the fluid must occur before the released latent heat is enough to balance the rate of heat loss to generate a signal in the cooling history.

[16] The rate of cooling as the sample passes through the solidification temperature determines the amount of undercooling, so the same initial and ambient temperatures of \( 26^\circ C \) and \( 5^\circ C \) for the slurry and cooler bath respectively were used for each solidification temperature measurement. Heating runs (with ambient and initial temperatures swapped) were also carried out to test for hysteresis effects in the solidification temperature. The melting temperature in a heating run was approximately one degree warmer than on the equivalent solidifying run, suggesting that a \( 0.5^\circ C \) margin of error in the measured solidification temperature is appropriate.
Thermal conductivity measurements are at most 5\% of the measured value.

\[ c = (1 - X_k) c_p + X_k c_f, \]  

and

\[ \alpha = (1 - \phi_k) \alpha_p + \phi_k \alpha_f. \]  

[20] Direct and accurate determination of the thermal conductivity \( k \) is difficult, so it was calculated based on the properties of the slurry end-members (\( k_k \) and \( k_p \), for kaolin and polyethylene glycol respectively). Maxwell [1892] gives the thermal conductivity of a composite substance with spheres of conductivity \( k_k \) dispersed evenly with volume fraction \( \phi_k \) in a host of conductivity \( k_p \):

\[ k = k_p \frac{2(1 - \phi_k) k_p + (1 + 2 \phi_k) k_k}{2 + \phi_k k_p + (1 - \phi_k) k_k}. \]  

Given \( k, \rho \) and \( c \), the expression \( \kappa = k/\rho c \) gives the thermal diffusivity of the slurry.

[21] The dynamic yielded differential viscosity \( \mu \) of the slurry was taken from Lyman et al. [2005], who measured this using a rotary viscometer for a range of different slurry compositions (see their Figure 3). Given \( \mu \) and \( \rho \), the kinematic viscosity is \( \nu = \mu/\rho \). The yield strength \( \tau_y \) of the slurries have been taken from the data of Lyman [2006], who determined it (to within 2\%) from the run-out thickness of a given volume of slurry released down a rectangular channel.

4. Isothermal Flows

[22] The isothermal experiments were carried out primarily for comparison with later experiments with cooling and solidification, but also as a test of the rheological measurements of Lyman et al. [2005] and Lyman [2006]. This section discusses the results of three representative experiments, all of which were performed in a channel 80 mm wide, inclined at 3.5\(^\circ\), with flow rates of 52.1, 24.8 and 12.6 \( \times \) 10\(^{-6} \) m\(^3\) s\(^{-1}\) (labeled Flow A, Flow B and Flow C respectively in Table 2). The water and slurry temperatures were

### Table 1. Physical Properties of the Materials Used in the Experiments

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>PEG(^b)</th>
<th>Kaolin(^c)</th>
<th>Slurry(^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>kg m(^{-3})</td>
<td>1122</td>
<td>2580</td>
</tr>
<tr>
<td>Specific heat</td>
<td>( c )</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>2490</td>
<td>920</td>
</tr>
<tr>
<td>Thermal expansion</td>
<td>( \alpha )</td>
<td>K(^{-1})</td>
<td>0.856</td>
<td>8 ( \times ) 10(^{-6})</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>( k )</td>
<td>W m(^{-1}) K(^{-1})</td>
<td>2.24 ( \times ) 10(^{-2})</td>
<td>1.97</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>( \kappa )</td>
<td>m(^2) s(^{-1})</td>
<td>8 ( \times ) 10(^{-7})</td>
<td>8.3 ( \times ) 10(^{-7})</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>( \mu )</td>
<td>Pa s</td>
<td>0.13</td>
<td>—</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>( \nu )</td>
<td>m(^2) s(^{-1})</td>
<td>1.2 ( \times ) 10(^{-4})</td>
<td>—</td>
</tr>
<tr>
<td>Yield strength</td>
<td>( \tau_y )</td>
<td>Pa</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

\(^a\)All properties given for substances at 25\(^\circ\)C. Slurry properties are given for a slurry with a kaolin weight fraction of \( X_k = 0.25 \). The margins of error on measurements are at most 5\% of the measured value.

\(^b\)Density, kinematic viscosity and thermal expansion from Fink and Griffiths [1990], thermal diffusivity from Kerr [2001].

\(^c\)All values from Ferrigno and Flores [1987].

\(^d\)Density as measured, dynamic viscosity from Lyman et al. [2005], yield strength from Lyman [2006]. All other properties are calculated using the expressions in section 3.4.
both 26°C, chosen to give isothermal conditions well above the solidification temperature of the slurry (about 18°C).

[23] Surface velocity profiles were determined by tracking small black plastic particles dropped through the water onto the flow surface. Particle locations were identified using a connected-component analysis applied to each frame from an overhead video recording, and particles were tracked between frames by minimizing a distance based on a particle’s prior velocity. Manual measurements of particles near the flow centerline were also made to confirm the velocities obtained by the particle tracker. Table 2 summarizes the measurements of the volumetric flow rate $Q$, surface velocity $U_m$ and plug width fraction $w_p = W_p / W$.

[24] Figure 4 shows the mapped velocity over the flow surface as determined from the particle tracking. The mapped area was approximately mid-way between the channel lock and the overflow, and the fact that there is little change in the velocity profiles with distance downstream shows that the flow is stable and fully developed. The velocity maps do not reach zero at the walls owing to the finite size of the particles used for the tracking. With decreasing flow rate (from flow A to flow C) the central plug region grows from approximately a quarter to around half the channel width.

[25] The method in Robertson and Kerr [2012, section 6.4] was used to estimate the flow depth, viscosity and yield strength for each flow; Table 3 contains the results. The estimated rheologies agree (within error) with the measurements made by Lyman et al. [2005] and Lyman [2006]. As a further comparison, Figure 5 shows a collapsed surface velocity profile for each flow, with the dark brown line showing the predicted velocity profile assuming the estimated rheologies in Table 3 (with expected error denoted by

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**Table 2. Measured Experimental Parameters for the Three Isothermal Slurry Flows**

<table>
<thead>
<tr>
<th>Flow</th>
<th>$Q$ ($10^{-6}$ m$^3$ s$^{-1}$)</th>
<th>$U_m$ ($10^{-3}$ m s$^{-1}$)</th>
<th>$w_p/W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>52 ± 2</td>
<td>62 ± 3</td>
<td>0.28 ± 0.02</td>
</tr>
<tr>
<td>B</td>
<td>24.8 ± 0.8</td>
<td>37 ± 2</td>
<td>0.37 ± 0.05</td>
</tr>
<tr>
<td>C</td>
<td>12.6 ± 0.8</td>
<td>21 ± 3</td>
<td>0.49 ± 0.03</td>
</tr>
</tbody>
</table>

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**Table 3. Fluid Rheologies Estimated From the Maximum Surface Velocity, Flow Rate and Plug Width Measurements**

<table>
<thead>
<tr>
<th>Flow</th>
<th>$\nu$ ($10^{-3}$ m$^2$ s$^{-1}$)</th>
<th>$\tau_y$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.2 ± 0.3</td>
<td>0.51 ± 0.06</td>
</tr>
<tr>
<td>B</td>
<td>1.9 ± 0.4</td>
<td>0.51 ± 0.09</td>
</tr>
<tr>
<td>C</td>
<td>1.8 ± 0.6</td>
<td>0.59 ± 0.14</td>
</tr>
</tbody>
</table>

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**Figure 4.** Maps of three isothermal surface velocity profiles obtained from overhead video analysis. Flow rates are (a) 51.2, (b) 24.8 and (c) $12.5 \times 10^{-6}$ m$^3$ s$^{-1}$ (1 mL = $10^{-6}$ m$^3$). Surface velocities are averages from one-second particle tracks. The contours show the velocity magnitude map, while the velocity vectors show one third of all the velocity measurements. The contour values are the same for all three figures.

**Figure 5.** Velocity profiles (collapsed onto a cross-sectional slice of the channel) for the three flows shown in Figure 4. Each dot represents a one second velocity measurement, dots are slightly transparent so that darker regions indicate a higher concentration of velocity measurements. The predicted velocity profiles (shown as the solid tan curves) use the rheologies given in Table 3 (with expected error denoted by
turning flow was much slower than the downstream flow, as dominating the flow (see Figure 6). The convective over-section, with laminar sinking plumes at either sidewall thickest toward the walls. This unstable thermal structure which is thinnest in the centerline of the channel and cooling flows generates a laterally varying thermal boundary temperatures of about 35°C. Several experiments with surface cooling but no solidification were conducted to observe the main features of the internal convective flow. These experiments had initial temperatures of about 35°C and ambient water temperatures of about 24°C (well above the solidification temperature of the PEG). Neutrally buoyant dye streams were released under the lock gate to observe the flow (Figure 6). However, the opacity of the slurries restricted direct observation of the dye streams to the walls and surface of the flow. The horizontal shearing at the free surface of these cooling flows generates a laterally varying thermal boundary layer which is thinnest in the centerline of the channel and thickest toward the walls. This unstable thermal structure drives overturning cells in each half of the channel cross section, with laminar sinking plumes at either sidewall dominating the flow (see Figure 6). The convective overturning flow was much slower than the downstream flow, as was previously observed by Griffiths et al. [2003] for viscous flows.

[26] The dye observations showed two differences between convection in viscous and viscoplastic flows. First, dye streams placed near the center of the channel showed no lateral movement and did not sink, showing that the central plug does not participate in thermal convection. Secondly, the corner plugs prevented dye streams in the sidewall shear zones from reaching full depth before they moved into the flow interior.

6. Flows With Surface Solidification

[29] The data set for solidifying flows consists of 146 viscoplastic slurry experiments with ambient water temperatures cold enough to allow the surface to solidify. These experiments build on a set of 80 experiments carried out using purely viscous polyethylene glycol, presented in Griffiths et al. [2003]. A range of flow dynamics was obtained by varying the source flow rate (4–100 × 10⁻⁴ m s⁻¹), channel inclination (3.5°, 6.5° and 7.3°) and channel width (80 mm and 150 mm), while the cooling rates were varied primarily via the ambient water temperatures (which were 5–15°C).

6.1. Flow Appearance

[30] For all experiments, crust cover reached a quasi-stable distribution on the surface of the flow, although the time taken to reach this state varied depending on the rate of surface cooling. The flows fell into two distinct crust distribution regimes: ‘tube’ and ‘mobile crust’ flows (following Griffiths et al. [2003]). In the tube regime, solidification of the flow surface was rapid enough to form a stationary roof, beneath which molten material continued to flow, insulated from further heat loss to the water. Rafts of solidified slurry characterized the mobile crust regime; these covered the center of the channel to varying degrees, separated from the walls by an open shear zone near each wall.

[31] Large source flow rates or high water temperatures produced the mobile crust regime, with a typical mobile crust regime flow shown in Figure 7. Under these conditions the shear near the walls of the channel was large enough to break up any solidified crust during formation, while the absence of shear in the center of the channel allowed rafts of crust to form which floated down-channel with the flow. The shear zones separating the central crust from the wall had little or no solidified crust on the surface of the flow, thus exposing molten material directly to the ambient water. The central rafts were largely contiguous for the length of the channel, however when the source flow rate increased the surface crust would respond by breaking into discontinuous sections. The rafts also commonly developed a folded surface morphology near the beginning of the channel due to compressive stresses associated with increased viscous drag as the crust widened into the regions of higher shear near the walls.

[32] Small flow rates or low water temperatures led to the tube regime. These conditions caused the flow surface to cool rapidly relative to the surface strain rate, thus allowing the whole surface to solidify. Tubing of the flow was often accompanied by inflation of the crust as the flow adjusted to the change in boundary conditions from no-shear to no-slip at its surface.
Some flows fell into a reproducible transitional regime between mobile crust and tube. In these regimes, the surface crust covered the entire channel width but continued to slip along the channel walls. This regime was unstable; eventually the crust would jam near the middle of the channel, the slip along the walls would cease and the flow would switch into the tube regime.

The transition between the tube and mobile crust regime could only be achieved by starting with a mobile crust flow and dropping the flow rate until the flow tubed. Due to the strength of the solidified crust it was not possible to return from tubes to mobile crust flows. If the surface flow rate increased after a tube formed, the surface crust would either inflate with new material, or in the case of large increases, the previous crust would be overrun by a new flow which propagated over the old surface. This complicated behavior, although difficult to quantify and not analyzed in this paper, is similar to conditions observed in lava flows, where the surface crust ruptures in response to surges in flow rate, forming tumuli or breakouts [Calvari and Pinkerton, 1998].

6.2. Dimensionless Parameters

Dimensionless parameters based on the isothermal flow height $H$ and maximum surface velocity $U_m$ are used to quantify and scale the solidifying flow results. For each experiment $H$ and $U$ were calculated using the method presented in Robertson and Kerr [2012, section 6.4], given the slurry properties in Table 1 and the measured flow rate $Q$.

The first dimensionless parameter is the cross-sectional aspect ratio

$$\beta = W/H.$$  \hspace{1cm} (10)

The second dimensionless parameter is the Bingham number

$$B = \frac{\tau_y}{\rho g H}$$  \hspace{1cm} (11)

which is the ratio of the yield strength of the fluid $\tau_y$ to a characteristic viscous stress $\rho g H$. Flows of purely viscous fluids have $B = 0$, while the yield strength is large enough to prevent flow when $B = B^*$. The value of the critical Bingham number $B^*$ is dependent on the channel geometry [Mosolov and Mjasnikov, 1965; Robertson and Kerr, 2012].

The third dimensionless parameter, introduced by Griffiths et al. [2003], is the thermal parameter

$$\vartheta = \psi Nu = \frac{U_m t_c}{H} Nu.$$  \hspace{1cm} (12)

where $Nu$ is the Nusselt number. The Nusselt number describes heat transfer to the flow surface due to convection, and is given by [Turner, 1973]

$$Nu = \gamma Ra^{1/3}$$  \hspace{1cm} (13)

where the Rayleigh number for the internal convection is

$$Ra = \frac{\rho g \alpha (T_e - T_s) H^3}{\kappa \mu}$$  \hspace{1cm} (14)

and $\gamma \approx 0.1$ is an empirical constant [Denton and Wood, 1979]. The solidification time $t_s$ comes from a one-dimensional thermal model of the flow surface (given in Appendix A, based on models in Huppert and Sparks [1988] and Fink and Griffiths [1990]), where $\lambda_c$ is the timescale over which convection in the water reduces the surface temperature from $T_e$ to $T_o$, given by

$$\lambda_c = \left(\frac{\rho c}{\rho_c c W_o}\right)^{2/3} \frac{\kappa}{\Delta T^{2/3}}.$$  \hspace{1cm} (15)

Figure 7. Overhead view of an experimental flow in the mobile crust regime, with a flow width of 150 mm. The field of view is approximately one meter long. The line drawing on the right shows the quasi-stable central crust width, shown as the solid lines in the drawings, and the incipient folds formed during crust formation (shown as thin lines). The dimensionless parameter values for this flow are $B = 0.32$, $\vartheta = 30$ and $\beta = 7.9$. 
Figure 8. Crust width ratio $w_c$ as a function of the thermal parameter $\vartheta$ for 80 mm wide flows. (top) Viscoplastic flows (red symbols) with aspect ratios of $2.3 < \beta < 11.3$ and Bingham numbers of $0.20 < B < 0.32$, giving plug width fractions of $w_p = 0.43 \pm 0.05$. (bottom) The 80 mm wide viscous flows (blue symbols) taken from the data set of Griffiths et al. [2003] for comparison. Symbols denote the crust regime: circles for experiments in the tube regime, squares for the mobile crust regime, with $w_c < 1$. The crust width ratio $w_c$ is shown as a function of $\vartheta$ in Figure 8, for experiments carried out in the 80 mm wide channel. In Figure 8 (top) the viscoplastic flows have aspect ratios of $2.3 < \beta < 11.3$ and Bingham numbers of $0.20 < B < 0.32$. Viscous flows from Griffiths et al. [2003] are shown in Figure 8 (bottom) for comparison; these have aspect ratios of $2.2 < \beta < 17.6$.

6.3. Analysis of Solidifying Flows

[40] At low values of $\vartheta$, both viscous and viscoplastic flows fall into the tube regime, with a crust cover ratio $w_c = 1$. At high values of $\vartheta$ both flows fall into the mobile crust regime, with $w_c < 1$. The crust width ratio $w_c$ is shown as a function of $\vartheta$ in Figure 8, for experiments carried out in the 80 mm wide channel. In Figure 8 (top) the viscoplastic flows have aspect ratios of $2.3 < \beta < 11.3$ and Bingham numbers of $0.20 < B < 0.32$. Viscous flows from Griffiths et al. [2003] are shown in Figure 8 (bottom) for comparison; these have aspect ratios of $2.2 < \beta < 17.6$.

[41] The transition between the tube and mobile crust regime, $\vartheta^*$ is dependent on flow rheology; it is $\vartheta^* \approx 6 \pm 1$ for the viscous flows but $\vartheta^* \approx 3 \pm 1$ for the viscoplastic flows in Figure 8. It is noted that in Griffiths et al. [2003], and subsequently in Cashman et al. [2006] and Kerr et al. [2006], the reported values of $\vartheta$ are too large by a factor of about 4, as the water viscosity, rather than that of PEG, was incorrectly used to calculate the Rayleigh number for the internal convection.

[42] Above $\vartheta^*$, the data from the viscous experiments describe a power law relationship between $w_c$ and $\vartheta$, shown as a straight line in the bottom plot in Figure 8. The viscoplastic data exhibit a similar power law trend in $w_c$ for $3 < \vartheta < 20$. However, the viscoplastic crust width ratios reach a minimum value of $w_c \approx 0.4$ for $\vartheta > 20$, beyond which there is no dependence on $\vartheta$. This minimum value is in good agreement with the calculated isothermal plug fraction for these flows, whose mean is $w_p = 0.43 \pm 0.05$. We therefore interpret $w_p$ as the lower bound on $w_c$ for viscoplastic flows, and separate the mobile-crust viscoplastic flows into shear-controlled and plug-controlled regimes.

[43] For viscoplastic flows in the shear-controlled regime, the width of the central crust is set by a balance between the rate at which the crust can solidify, and the shear rate near the walls. In contrast, solidification rates for viscoplastic flows in the plug-controlled regime are so weak that the only region of the flow surface to solidify is the plug region, where there is no strain. Viscous flows have no plug regions, and therefore do not have a plug-controlled regime.

[44] The shear-controlled to plug-controlled transition occurs at lower $\vartheta$ with increasing $w_p$, since the wider plug region can affect the growth of the central crust at wider crust widths. Figure 9 shows the experiments colored by the ratio of the isothermal plug width to crust width, $w_p/w_c$. The black through dark gray symbols indicate experiments which have crust widths much larger than their isothermal plug widths. For these flows the crust width will be controlled by the rate of shear. The red diamond symbols indicate flows in which $w_p/w_c \geq 0.9$, whose crust width is expected to be controlled by the plug width. The dotted line indicates the approximate location of the transition $\vartheta^{**}$ between these two regimes.

[45] Interestingly, viscoplastic flows in the shear-controlled regime tend to have lower values for $w_c$ when compared to viscous flows (with $w_p = 0$) at the same value of $\vartheta$. Figure 10 shows $w_c$ as a function of $\vartheta$ and $w_p$ for the complete data set of 226 viscous and viscoplastic experiments. The tube-mobile crust transition $\vartheta^*$ (plotted as a dashed line) also exhibits a
dependence on the rheology of the flow; i.e. tubing occurs at lower $J$ with increasing viscoplasticity. This may be due to increased strain partitioning into the shear zones near the walls (for a flow of a given depth).

To summarize these results, viscoplastic flows have three crust cover regimes: a tube regime in which the crust completely covers the channel, a shear-controlled mobile crust regime in which the central raft of crust is dependent on the parameter $J$, and a plug-controlled mobile crust regime in which the central plug region width sets the crust coverage. The transitions between these regimes are dependent on the rheology of the fluid, and both $\vartheta^*$ and $\vartheta^{**}$ occur at lower values of $\vartheta$ with increasing viscoplasticity.

### 6.4. Crust Cover Parameterization

[47] As a useful parameterization of our crust cover data set, we suggest an empirical model with (a) a critical value $\vartheta^*$ denoting the transition between tube and mobile crust flows, (b) a shear-controlled mobile crust region $w_c \sim \vartheta^n$ for some exponent $n$, and (c) a minimum crust width value given by the isothermal plug width. This gives the following model:

$$w_c = \begin{cases} \frac{1}{\max[(\vartheta^*/\vartheta)^n, w_p]} & \vartheta \leq \vartheta^* \\ \vartheta > \vartheta^* & \end{cases}$$

where $\vartheta^* = a + b w_p (w_p - 2)$, and $a = 6 \pm 1$, $b = 2.7 \pm 0.1$ and $n = 0.51 \pm 0.01$ are empirical constants which were determined using orthogonal distance regression on the entire data set of viscous and viscoplastic flows. Figure 11 shows a contour figure of this fit with the data from Figure 10 overlaid for comparison.

### 7. Discussion and Applications

[48] In this section we first show how the solidification time can be determined for sub-aerial lava flows, where radiative cooling is the dominant heat loss mechanism. We then examine where some typical Etna flows fall in relation to the three flow regimes found in section 6.3, based on published lava rheology and channel geometry measurements for Etna lava [Pinkerton and Sparks, 1976, 1978; Pinkerton and Norton, 1995; Calvari and Pinkerton, 1998]. Finally we examine the dynamics of the large lava flow emplaced during the March–April 1984 eruption of Mauna Loa, using measurements of channel geometry, maximum

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**Figure 9.** Plug-crust ratio $w_p/w_c$ plotted as a function of the thermal parameter $\vartheta$ and the plug width fraction $w_p$. The symbols denote crust regimes and are the same as in Figure 8 with the addition of a diamond which denotes a flow with $w_p/w_c \geq 0.9$. These flows are also plotted in red. The crust-plug transition $\vartheta^{**}$ is shown as a dotted line.

**Figure 10.** Crust width fraction $w_c$ for each experiment plotted as a function of the thermal parameter $\vartheta$ and the plug width fraction $w_p$. Point colors denote $w_c/w$, as shown in the color bar beneath the plot. The tube-mobile crust transition $\vartheta^*$ is shown as a dotted line.

**Figure 11.** Modeled fit for crust width fraction $w_c$ as a function of the thermal parameter $\vartheta$ and the plug width fraction $w_p$. The experimental data are overlain for reference; the symbols denote crust regimes (and are the same as Figure 9, and the colors show the measured crust width. The color scalings are the same for the contours and the experimental data points.
velocity and lava density and temperature from Lipman and Banks [1987].

### 7.1. Solidification Times With Radiative Cooling

[40] In Appendix A we give an approximation for $t_s$ of a sub-aerial lava surface (with initial temperature $T_a$) cooled by a combination of convective and radiative heat transfer to an ambient fluid at temperature $T_a$ and obtain:

$$
\frac{t_s}{\lambda_c} = \frac{1}{2} \left[ \frac{1 - \theta_s + \theta_a}{(\theta_s - \theta_a)^{1/3} + \Lambda^{1/2}(\theta_s - \theta_a)} \right]^2
$$

(20)

where $\Delta T = T_a - T_s$ is the difference between the internal lava temperature $T_a$ and the ambient temperature $T_a$, the dimensionless ambient temperature is $\theta_a = T_a/\Delta T$, the dimensionless solidification temperature is $\theta_s = T_s/\Delta T$, $\lambda_c$ is the convective cooling timescale given in equation (15), and $\lambda_r$ is the timescale for radiative cooling:

$$
\lambda_r = \left( \frac{\rho c}{\sigma} \right)^{2/3} \frac{\kappa}{\Delta T^6}
$$

(21)

with the Stefan-Boltzman constant $\sigma = 5.67 \times 10^{-8}$ J K$^{-4}$ m$^{-2}$ s$^{-1}$ and a lava emissivity $\varepsilon$, taken to be 0.98 [Harris and Rowland, 2001]. The parameter $\Lambda = \lambda_r/\lambda_c$ is the ratio of convective to radiative cooling timescales. This model could be extended to include forced convection. However, given that radiative heat transfer timescale $\lambda_r$ is about 64 times shorter than the convective cooling timescale $\lambda_c$, in the cases considered below forced convection can be safely ignored when calculating values of $t_s$.

[51] For typical Etnean temperatures and vesicularities ($T_e = 1100^\circ$C and a vesicularity of $\phi_v = 0.15 \pm 0.05$), the solidification time is $t_s \approx 67 \pm 7$ s, while this drops to as low as $\sim 10-20$ s for Hawaiian lavas with high vesicularities ($\phi_v = 0.6-0.8$) which have been observed in the 1984 Mauna Loa eruption [Lipman and Banks, 1987]. The solidification times for Hawaii agree well with the field observations reported in Gregg and Keszthelyi [2004], in which the strength of the forming crust was measured by its ability to support a field hammer. The values obtained in that study ranged from $t_s \sim 11-12$ s for pahoehoe toes, while faster moving pahoehoe lobes with increased stretching had $t_s \sim 30$ s.

### 7.2. Modeled Flows Using Etna Lava Rheology

[52] In this section we examine where a typical viscoplastic lava flow will plot in $\theta - \nu_p$ space. Consider some flows on Mt Etna with lava temperatures less than 1130$^\circ$C and crystallinities greater than 25–30%. Table 6 lists the rheological values assumed in our calculations, which are based on the measurements reported by Pinkerton and Sparks [1978] and Pinkerton and Norton [1995]. The yield strength is taken to be zero at temperatures above 1110$^\circ$C, but the yield strength grows as the temperature decreases to 1086$^\circ$C. The values used for the other physical properties of the lava (with an assumed vesicularity of $\phi_v = 0.15$) are given in Table 4.
geometry and rheology, such as might be observed at a single point along a well-developed channel over short timescales; (b) the effect of varying lava temperature, which corresponds to changes at a single point on a channel over longer (eruptive) timescales associated with changes in the erupted lava; and (c) the effect of varying channel geometry (i.e. flow width and inclination), which corresponds to the effect of varying topography along the length of a lava channel.

7.2.1. Effect of Varying Flow Rate

We first examine the consequences of a change in flow rate while holding channel geometry and lava rheology constant. In this case changing the flow rate will change the flow depth and hence the aspect ratio of the flow — with higher flow rates corresponding to deeper flows and hence lower aspect ratios. We will consider three channel geometries: a 1 m wide flow on a 20° slope, a 3 m wide flow on a 10° slope and a 10 m wide flow on a 5° slope. These channels are representative of those reported in Pinkerton and Sparks [1976] and Calvari and Pinkerton [1998], for the 1975 and 1991–93 eruptions of Mt Etna respectively.

Figure 13 shows the effects of varying the aspect ratio, for three lava temperatures of 1100, 1090 and 1086°C. Figure 13 (top) shows the trajectories obtained in $\theta - w_p$ space as the aspect ratio is varied. We can interpret these trajectories in terms of flow rate: during a surge the flow position will move downward along one of these trajectories (toward lower $w_p$ and higher $\theta$), while this will be reversed during periods of low flow rate. This is reflected in the crust cover (shown in Figure 13 (middle))

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$\mu$ (Pa s)</th>
<th>$\tau_v$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1125</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>1115</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>1110</td>
<td>470</td>
<td>5</td>
</tr>
<tr>
<td>1105</td>
<td>690</td>
<td>10</td>
</tr>
<tr>
<td>1100</td>
<td>1000</td>
<td>20</td>
</tr>
<tr>
<td>1095</td>
<td>1700</td>
<td>40</td>
</tr>
<tr>
<td>1090</td>
<td>3000</td>
<td>10</td>
</tr>
<tr>
<td>1086</td>
<td>9400</td>
<td>370</td>
</tr>
</tbody>
</table>

7.2.1. Effect of Varying Flow Rate

We first examine the consequences of a change in flow rate while holding channel geometry and lava rheology constant. In this case changing the flow rate will change the flow depth and hence the aspect ratio of the flow — with higher flow rates corresponding to deeper flows and hence lower aspect ratios. We will consider three channel geometries: a 1 m wide flow on a 20° slope, a 3 m wide flow on a 10° slope and a 10 m wide flow on a 5° slope. These channels are representative of those reported in Pinkerton and Sparks [1976] and Calvari and Pinkerton [1998], for the 1975 and 1991–93 eruptions of Mt Etna respectively.

Figure 13 shows the effects of varying the aspect ratio, for three lava temperatures of 1100, 1090 and 1086°C. Figure 13 (top) shows the trajectories obtained in $\theta - w_p$ space as the aspect ratio is varied. We can interpret these trajectories in terms of flow rate: during a surge the flow position will move downward along one of these trajectories (toward lower $w_p$ and higher $\theta$), while this will be reversed during periods of low flow rate. This is reflected in the crust cover (shown in Figure 13 (middle)).

Figure 13. (top) The effect of varying $\beta$ in $\theta - w_p$ space, for three typical Etna channel geometries at three different lava temperatures: $T_e = 1086, 1090$ and 1100°C. The colors correspond to channel geometries as shown in the legend. Symbols denote the crust regime predicted for each flow configuration and are the same as in Figure 11. The crust regime labels are ‘PCR’ — plug-controlled regime, ‘SCR’ — shear-controlled regime, and ‘Tube’ — tube regime. (middle) The variation in crust width $w_c$ calculated from equation (19) as a function of aspect ratio along the trajectories plotted in the upper row. (bottom) The variation in Bingham number $B$ with respect to aspect ratio. The dotted line in the Bingham number plots represents the maximum Bingham number $B^*(\beta)$ [see Robertson and Kerr, 2012].
which will generally be narrower during surges and larger during ebbs.

The trajectories show that a given flow can change regime with changes in flow rate/aspect ratio. At 1086°C both the 1 m and 3 m wide channels fall into the shear-controlled regime for all aspect ratios. Conversely, the 10 m wide channel lies completely in the plug dominated regime, where its stable crust coverage is determined by the plug width. At 1100°C only the 1 m and thickest 3 m wide flows fall into the shear-controlled regime, while the wider, thinner flows lie within the plug dominant regime. Figure 13 (middle) shows that crust coverage for flows in the plug-controlled regime is larger than flows in the shear-controlled regime, and much larger than might be expected for an equivalent purely viscous flow (since equation (19) with \( w_p = 0 \) predicts \( 0.2 \leq \frac{w_c}{\theta} \leq 0.1 \) for \( 100 \leq \vartheta \leq 1000 \)).

### 7.2.2. Effect of Varying Temperature

The fact that the plug is so important to the crust growth in the hottest flows considered in the previous section is perhaps unintuitive since the yield strength is very small (\( \tau_y = 20 \) Pa) and the Bingham numbers (shown in Figure 13 (bottom)) are also small. The reason for this is that the central plug region initially grows rapidly with the onset of a yield strength in high aspect ratio flows, as shown in Robertson and Kerr [2012, Figure 11]. To highlight the complex effects that this rapid plug development has on crust coverage in high \( \beta \)-low \( B \) flows we next consider the effects of varying eruption temperature, while holding flow rate and channel geometry constant.

We again consider three channel geometries which are slightly larger than the last section: a 3 m wide flow on a 30° slope, a 5 m wide flow on a 10° slope and a 10 m wide flow on a 3° slope. All geometries have the same fixed flow rate of \( Q = 2 \) m\(^3\) s\(^{-1}\); however since the flow rate and channel geometry are fixed, varying the temperature also changes the flow depth and aspect ratio.

Figure 14 shows the calculated \( \vartheta - w_p \) trajectories obtained by varying the rheology of the lava, while Figure 15 shows the variations in crust width ratio, plug width ratio, Bingham number and aspect ratio along these trajectories as a function of temperature. The trajectories all begin with \( w_p = 0 \) at temperatures above 1115°C, where the lava rheology is viscous, and have nonzero plug widths as the temperature decreases and the yield strength develops.

The narrow, steeply inclined flow (in red in Figure 14) lies within the shear-controlled regime for all the temperatures considered here. There is a gradual monotonic increase in crust width from \( \sim 0.02 \) to \( \sim 0.1 \) as the temperature decreases. However, the wider flows on gentler slopes (in light and dark blue) experience a rapid increase in the central plug region width to a local maximum around 1105°C, with a corresponding increase in the stable crust width, as the yield strength increases.

The peak in crust coverage around 1105°C would reduce the rate of radiative heat loss for wide flows of this temperature, and would have a buffering effect which may allow the flow to travel further before freezing. The channel geometry also has the strongest control on the crust width at these lowest yield strengths, which results in the widest spread in crust widths over the three geometries at 1105°C.

Below 1105°C the crust and plug widths decrease as the increasing viscosity and yield strength cause the flow to inflate (seen as the lower aspect ratios in the bottom plot in Figure 15). Inflation decreases the dynamic importance of the yield strength by increasing the viscous stresses in the flow. Interestingly, despite the factor of two decrease in the aspect ratio and factor of two increase in the Bingham number, the plug width ratio remains stable at around 0.2 for the whole range of temperatures considered.

### Figure 14

The trajectories obtained by varying the eruption temperature \( T_e \) in \( \vartheta - w_p \) space, for three different channel geometries shown in the legend. Symbols denote the crust regime predicted for each flow configuration and are the same as in Figure 11.

### Figure 15

Variations in crust width ratio, plug width ratio, Bingham number and aspect ratio with eruption temperature for the three channel geometries considered in Figure 14 (symbols and colors are the same for both figures).
number between 1190 and 1086°C there is little change in either the plug or crust width ratios.

### 7.2.3. Effect of Varying Geometry

[63] Finally we examine the consequences of changes in slope and channel width along a typical channel on Mt Etna. Kerr et al. [2006] have shown that channel width is linked to the underlying slope, and we have chosen the channel geometries from section 7.2.2 to vary consistently with the scaling theory of that study, so that the 3 m wide flow (on a 30° slope) is similar to that which would be expected near the summit of Etna, while the 5 and 10 m wide flows would be expected on the gentler slopes further down the volcano. Again all modeled flows have a constant flow rate of \( Q = 2 \text{ m}^3\text{s}^{-1} \), and a constant interior lava temperature down the channel.

[64] The results are shown in Figure 16. Each line represents a different lava temperature. The purely viscous flows (with \( T_r \geq 1115°C \)) fall on the \( \vartheta \) axis in the shear dominant regime, while flows with yield strength (\( T_r \leq 1110°C \)) start in the shear dominant regime for the steepest, narrowest portion of the channel, and then track into the plug-controlled regime as the flow widens on gentler slopes.

[65] Again the flow geometry has the strongest effect immediately at the onset of yield strength development, with the widest range in plug widths at 1105°C. This is because the Bingham number (and hence plug width) is most sensitive to changes in the driving gravitational force \( \rho g H \) when the yield strength is small. The effect of increased viscosity and yield strengths at lower temperatures is to increase the portion of the channel which is in the shear-controlled regime, pushing the transition to plug-dominance further down the hill.

### 7.3. Application to the Mauna Loa Eruption of 1984

[66] We now apply our study to a lava flow erupted on Mauna Loa in March and April of 1984. An excellent data set of velocities, channel geometries, lava densities and temperatures is reported for this flow by Lipman and Banks [1987, Table 57.3, pp. 1558–9], which allows us to examine where this flow falls in terms of our crust coverage regimes. We focus on a set of selected measurements were made on 3–4 April 1984, at Stations 11, 8, 4 and 1 (where Station 11 is closest to the erupting vents at 2800–2900 m above sea level, and Station 1 is furthest down-flow).

[67] The rheology of the lava is calculated using the isothermal data from the calculations in Robertson and Kerr [2012]. The use of this isothermal data is valid provided the effects of solidification on the surface velocity profiles are minimal, so we can extend the applicability of the rheology estimators presented in Robertson and Kerr [2012, section 6] to flows in the plug-controlled regime. Here we calculate flow rates and lava rheology simultaneously by assuming that the flows lie within the plug-controlled regime. This assumption can then be validated by finding where the flow falls in \( \vartheta - w_p \) space using the calculated rheology and flow rate.

[68] There are not enough flow parameters reported in the Lipman and Banks [1987] data set to constrain the rheology of each flow measurement by itself using any of the methods of Robertson and Kerr [2012, section 6]. However, since Lipman and Banks [1987] provide multiple readings from the same location with differing aspect ratios and maximum velocities, we use a routine based on Broyden’s method to determine the viscosity and yield strength for a set of flows. The routine minimizes the objective function

\[
S(v, \tau_y) = \left( \frac{1 - g H^2}{\nu U_m} u_m(\beta, B) \right)^2
\]

for the set of \( N \) measurements \( M = \{ m_i : m_i = (\theta_i, H_i, W_i, U_{mi}) \}, 1 \leq i \leq N \) over which \( v \) and \( \tau_y \) are assumed to be constant. Here the dimensionless velocity function \( u_m \) is taken from the numerical computations in Robertson and Kerr [2012]. To initialize the routine we set the initial yield strength \( \tau_y \) to zero and supply an initial viscosity evaluated by assuming two-dimensional Newtonian sheet flow, \( \nu = g H^2/2U_m \), where the \( H \) and \( U_m \) values are the averages for the observed values \( H \) and \( U_m \).

[69] Table 7 shows the values obtained for the viscosity and yield strength of the flow for the four stations. We also

### Table 7. Physical Properties Calculated for Measurements Given by Lipman and Banks [1987] for Four Stations on 3–4 April, During the 1984 Mauna Loa Eruption

<table>
<thead>
<tr>
<th>Station</th>
<th>Elevation (m)</th>
<th>( \rho ) (kg m(^{-3}))</th>
<th>( T_r ) (°C)</th>
<th>( \nu ) (m(^2) s(^{-1}))</th>
<th>( \tau_y ) (Pa)</th>
<th>( \phi_{0,b} )</th>
<th>( t_s ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2850</td>
<td>530</td>
<td>1140</td>
<td>0.29 ± 0.01</td>
<td>80 ± 30</td>
<td>0.80</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>2500</td>
<td>1300</td>
<td>1135</td>
<td>0.91 ± 0.04</td>
<td>410 ± 45</td>
<td>0.50</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>1900</td>
<td>1700</td>
<td>1130</td>
<td>3.9 ± 0.1</td>
<td>670 ± 70</td>
<td>0.35</td>
<td>41</td>
</tr>
<tr>
<td>1</td>
<td>1700</td>
<td>2200</td>
<td>1125</td>
<td>12.7 ± 0.6</td>
<td>1300 ± 130</td>
<td>0.15</td>
<td>76</td>
</tr>
</tbody>
</table>

\(^4\text{Elevations are given as meters above sea level, stations are ordered by proximity to the erupting vents at 2800–2900 m.}\)

\(^5\text{The vescularity of the flows is calculated from the measured sample densities assuming a bulk rock density of 2600 kg m\(^{-3}\).}\)

\(^6\text{Solidification times are calculated using the expressions given in section 7.1.}\)
show the measured densities from Lipman and Banks [1987], an inferred vesicularity for an assumed bulk rock density of 2600 kg m\(^{-3}\), and a solidification time calculated using section 7.1. The rheological properties are comparable to those obtained in a contemporaneous study by Moore [1987] of the Lipman and Banks [1987] data set (plotted in his Figure 58.15, p. 1582). We expect that these large yield strengths are a result of both the growth of microlite crystals, which have highly acicular morphologies [see Lipman and Banks, 1987, Figure 57.24–25], and the highly vesicular nature of this flow, which is known to facilitate the formation of touching crystal frameworks and the development of a yield strength [Walsh and Saar, 2008].

The results of our dynamical analysis for the 1984 Mauna Loa flow are summarized in Figure 17. Figure 17 (top) shows the four stations of the flow in \(\vartheta - w_p\) space, all of which fall into the plug-controlled regime (which validates the use of Robertson and Kerr [2012] to evaluate the lava rheology). Figure 17 (bottom) shows the Bingham numbers calculated for all stations. Despite the factor of \(\sim 16\) increase in yield strength downstream, the Bingham number increases by only a factor of \(\sim 4\), since this yield strength increase is accompanied by an corresponding increase in both lava viscosity and density. Figure 17 (middle) shows the stable crust coverage predicted using equation (19) as a function of station elevation. The crust coverage predicted for a purely viscous flow (with \(B = 0\)) at the same value of \(\vartheta\) is included for comparison. For all stations, we predict that the influence of the central plug region adds approximately 20% of the channel width to the stable area of crust on the surface of the flow.

Lipman and Banks [1987] include aerial photographs of the lava flow at locations corresponding to Station 1 and Station 8 (their Figures 57.8A and 57.8C respectively, reproduced in Figure 18). These photos show that the channel at Station 1 has a crust coverage \(w_c = 0.58 \pm 0.08\), while the channel at Station 8 has \(w_c = 0.19 \pm 0.06\). These values are in good agreement with the viscoplastic crust widths, \(w_c = 0.20 \pm 0.02\) for Station 8 and \(w_c = 0.45 \pm 0.10\).
for Station 1, predicted from equation (19). In contrast, Figures 17–18 show that the predictions assuming a purely viscous rheology for the flow significantly underestimate the amount of surface crust at both these stations.

[72] We can determine the resulting radiative heat loss from the channel for both viscous and viscoplastic flows as:

\[ f_{\text{rad}} = \sigma v W \left[ w_c T_{\text{c}}^4 + (1 - w_c) T_{\text{op}}^4 - T_a^4 \right], \tag{23} \]

where \( T_{\text{c}} \) and \( T_{\text{op}} \) are the average temperatures for the stable surface and the shear regions respectively. Using \( T_{\text{c}} = 425^\circ C \) and \( T_{\text{op}} = 1050^\circ C \) [Harris and Rowland, 2001] with an ambient temperature of \( T_a = 20^\circ C \) we find that the radiative heat transfer for the viscoplastic flow is only 75–80% of the radiative heat flux for the equivalent viscous flow for all stations considered here. Given that the radiative heat flux is the predominant cooling term in the heat budget of an open channel flow, this insulation effect due to the development of viscoplastic plugs may help (in addition to the frothy nature of the lava which helps to insulate the flow core, and latent heat release accompanying crystal growth) to explain why the temperatures of the 1984 Mauna Loa lava flow were so stable throughout the first fourteen kilometers of the flow [Lipman and Banks, 1987, Figure 57.19, p. 1549].

8. Conclusions

[73] The analogue experiments used in this study demonstrate the complex interplay between rheology and surface solidification in channelized lavas. The presence of a yield strength in the fluid rheology leads to the development of unyielded plug regions in the flow, which causes strain to localize into increasingly smaller sheared layers near the walls of the channel. In cooling flows the plug regions are robust enough to alter the distribution of convective overturning within the flow. Our experiments showed that thermal convection occurs in organized rolls within the sheared regions of the flow, aligned with the direction of shear, but that unyielded regions are not broken up by the convective overturning. In solidifying flows a quasi-stable width of crust forms on the surface of the flow. All flows, regardless of rheology, fall into either a tube regime, with solidification across the entire flow surface, or a mobile crust regime with open shear zones between a central raft of crust and the channel walls. Solidification occurs within the shear zones, but only forms dispersed fragments of crust.

[74] The central plug region changes the dynamics controlling surface solidification — a flow with a viscoplastic rheology has a lower bound on crust width set by the isothermal plug width at the surface of the flow [Robertson and Kerr, 2012]. Higher yield strengths lead to larger plug widths and thus to higher minimum crust widths. Mobile crust flows with a viscoplastic rheology have two mobile crust sub-regimes: one regime where a balance between shearing and solidification rate determines the degree of surface crust coverage, and one regime where the solidification rate is slow enough that the stable plug region at the flow surface sets the crust width.

[75] The degree of surface crust coverage and the transition between crust cover regimes can be parameterized in terms of two dimensionless parameters: \( \vartheta \), which characterizes the relative importance of the strain and crust growth rates; and \( w_p \), which is the ratio of the surface plug width to the channel width. Section 6 showed that these parameters can accurately predict the crust distribution for both viscous and viscoplastic flows in the mobile crust regime, as well as the transition between tubes and mobile crust flows. Equation (19) provides a simple empirical model, found by regressing the combined data set from this study and Griffiths et al. [2003], which can be used to predict the surface crust coverage of a given flow.

[76] Modeling of the dynamics of viscoplastic lava flows under typical conditions on Mt Etna showed that the plug-controlled regime can lead to complex relationships between crust coverage and flow conditions. Channel flows are most likely to be plug-controlled at temperatures where the lava has a low yield strength, since the relatively lower viscosities at these temperatures favor wide central plug region growth. We suggest that there may be significant feedbacks between the rheology and heat budget of a lava flow as a developing yield strength has an insulating effect through the growth of a surface plug region.

[77] Finally we examined some field data for the main lava channel from the Mauna Loa eruption of 1984. The bulk rheology of the lava was determined using the rheological estimators of Robertson and Kerr [2012] and the measurements of Lipman and Banks [1987] which classified the flow dynamics and their variation down-channel. Perhaps surprisingly, we find that crust coverage is dominated by the behavior of the central surface plug for all the stations considered. Aerial photographs taken at the time of the eruption were used to validate the crust coverage predictions of equation (19). These comparisons show that the stabilizing effect of the surface plug on the crust growth in this flow would have reduced the radiative heat loss by up to 25% compared to a dynamically equivalent viscous flow.

Appendix A: Calculating Solidification Times

[78] This appendix outlines the model used to calculate the surface solidification time \( t_s \). Previous studies have used a fitted polynomial provided by Fink and Griffiths [1990] for the solidification time in terms of a solidification temperature, however some of the viscoplastic experiments lie outside the valid range of this fit. Additionally, section 7 considers field situations in which radiative heat transfer is a dominant effect, which is not included in the fitted polynomial results. This means it is useful to have an approximate solution to the problem of cooling of the surface of the flow which allows for easy inclusion of radiative effects.

[79] The basic problem is flow surface cooling via two heat fluxes: radiative and convective heat transfer into the interior of an ambient fluid which is at a temperature \( T_a \). A heat flux from the flow interior balances these cooling fluxes. The flux from the flow interior is driven primarily by thermal diffusion from an interior temperature \( T_e \) through a thin thermal boundary layer near the flow surface. The changing contact temperature at the surface, \( T_{\text{c}} \), couples these fluxes and maintains the heat flow balance. We consider only short timescales (i.e. the timescale of the growing thermal boundary layer at the flow surface) so that the interior temperature \( T_e \) of the flow is constant. These solutions are valid when thermal convection in the interior of the flow is unable to significantly lower the internal temperature
from $T_e$. Huppert and Sparks [1988] analyze the long time case (where $T_e$ varies) in the analogous context of melting of the roof of a magma chamber.

[80] The contribution to the heat flux from the flow surface into the ambient fluid due to radiation is that given by Stefan’s law:

$$f_{\text{rad}}(T_e) = \varepsilon\sigma(T_e^4 - T_a^4), \quad (A1)$$

where $T_e$ is the contact temperature at the flow surface, $\sigma = 5.67 \times 10^{-8}$ J K$^{-4}$ m$^{-2}$ s$^{-1}$ is the Stefan-Boltzman constant and $\varepsilon = 0.98$ is the emissivity of the surface [Harris et al., 2010]. The contribution from a turbulent convective heat flux is

$$f_{\text{conv}}(T_e) = \rho_a c_p a J_a (T_e - T_a)^{4/3}, \quad (A2)$$

where $J_a = (g a T_a^2)/(\kappa a)$, variables with subscripted $a$ refer to the properties of the ambient fluid and $\gamma \approx 0.1$ is a constant [Turner, 1973].

[81] Substituting these values into equation (A4) gives the following result for the time $t$ taken to reach a given contact temperature $\theta_c$:

$$t = \frac{1}{2} \left(1 - \theta_c + \theta_a \right)^\frac{2}{\lambda_c} \left(\theta_c^4 - \theta_a^4 + \Lambda \theta_c^2 - \theta_a^2\right)^\frac{1}{2} \quad (A9)$$

where $\Lambda = \lambda_a/\lambda_c$. The solidification time $t_s$ is found by setting $\theta_c = \theta_a = T_s/\Delta T$.

[84] Note the role of the ratio $\Lambda$ which denotes the relative importance of radiative and convective heat fluxes in cooling the flow surface. For flows in which radiation is important (for example sub-aerial flows on Earth, or basalt flows on planet with thin atmospheres), $\lambda_a \ll \lambda_c$ and $\Lambda$ is large. Then the radiative term in the denominator dominates. Conversely, for flows where the heat capacity of the ambient fluid is large (as is the case in submarine environments), or for flows in which the radiative temperature difference $\theta_c^4 - \theta_a^4$ is small (as in the experiments), convective cooling is much faster than radiative cooling, i.e. $\lambda_c \gg \lambda_a$. For these cases the radiative term in the denominator is negligible, since $\Lambda \rightarrow 0$. In this case we recover the convective model used in section 6.

### Notation

- $B$: Bingham number.
- $B^*$: Critical Bingham number.
- $c, c_{\text{fr}}, c_{\text{pc}}, c_q$: Heat capacity of fluid/ambient fluid/PEG/kaolin fluid, J °C$^{-1}$.
- $f_{\text{conv}}$: Convective heat flux from flow surface to ambient fluid, W m$^{-2}$.
- $f_{\text{diff}}$: Diffusive heat flux from flow interior to surface, W m$^{-2}$.
- $f_{\text{rad}}$: Radiative heat flux from flow surface to ambient fluid, W m$^{-2}$.
- $g, g'$: Gravitational acceleration (prime denotes buoyancy-compensated downslope component), m s$^{-2}$.
- $\gamma$: Empirical convection constant from Turner [1973] and Denton and Wood [1979], $\approx0.1$.
- $H$: Flow depth, m.
- $J_a$: Buoyancy parameter ($=(g a T_a^2)/(\kappa a)^{1/3}$), m s$^{-1/3}$.
- $k, k_{\text{fr}}, k_{\text{pc}}, k_k$: Thermal conductivity of fluid/ambient fluid/PEG/kaolin fluid W m$^{-1}$°C$^{-1}$.
- $N_{\text{ir}}$: Nusselt number describing heat transfer to flow surface due to internal convection.
- $Q$: Total down-channel volumetric flow rate, m$^3$ s$^{-1}$.
- $Ra$: Rayleigh number describing internal convection.
- $t_s$: Solidification time for flow surface, s.
- $T_a$: Temperature of ambient fluid, °C, K.
- $T_{ce}$: Average temperature of stable surface crust, °C, K.
- $T_e$: Eruption/initial temperature of fluid, °C, K.
- $T_{op}$: Average temperature of surface shear zones, °C, K.
- $T_s$: Solidification temperature of fluid, °C, K.
\[ \Delta T \text{ Temperature difference between the eruption and ambient temperatures, } ^\circ \text{C, K.} \]
\[ U_m \text{ Maximum velocity in flow, m s}^{-1}. \]
\[ W \text{ Channel width, m.} \]
\[ W_p \text{ Width of central plug region at flow surface, m.} \]
\[ W_c \text{ Width of central crust at flow surface, m.} \]
\[ \gamma_c \text{ Ratio of crust width to channel width.} \]
\[ \omega_p \text{ Ratio of central plug region width to channel width.} \]
\[ \alpha_k \text{ Mass fraction of kaolin in slurry.} \]
\[ \alpha_r, \alpha_{kr}, \alpha_{pk} \text{ Thermal expansion coefficient of fluid/ambient fluid/PEG/kaolin, } ^\circ \text{C}^{-1}. \]
\[ \beta \text{ Channel aspect ratio.} \]
\[ \epsilon \text{ Lava emissivity.} \]
\[ \xi \text{ Shear strain rate tensor s}^{-1}. \]
\[ \gamma \text{ Numerical factor from Denton and Wood [1979] used to calculate heat flux due to turbulent convection from a surface.} \]
\[ \phi_k \text{ Volume fraction of kaolin in slurry.} \]
\[ \phi_l \text{ Surface radiative heat flux, W m}^{-2}. \]
\[ \kappa, \kappa_a \text{ Thermal diffusivity of fluid/ambient fluid, m}^2 \text{ s}^{-1}. \]
\[ \lambda_c, \lambda_f \text{ Timescale over which surface cools from the initial to the solidification temperature of the fluid via convection or radiation respectively, s.} \]
\[ \mu_r, \mu_a \text{ Differential dynamic viscosity of yielded fluid/ambient fluid, Pa s.} \]
\[ \nu_r, \nu_a \text{ Differential kinematic viscosity of yielded fluid/ambient fluid, m}^2 \text{ s}^{-1}. \]
\[ \psi \text{ Ratio of advection to solidification timescales in flow.} \]
\[ \rho, \rho_r, \rho_p, \rho_k \text{ Density of fluid/ambient fluid/PEG/kaolin, kg m}^{-3}. \]
\[ \sigma \text{ Stefan-Boltzmann constant, } =5.67 \times 10^{-8} \text{ J K}^{-4} \text{ m}^{-2} \text{ s}^{-1}. \]
\[ \tau \text{ Deviatoric stress tensor, Pa.} \]
\[ \tau_y \text{ Yield strength of fluid, Pa.} \]
\[ \tau_s \text{ Dimensionless solidification time.} \]
\[ \theta \text{ Channel inclination.} \]
\[ \theta_s \text{ Dimensionless solidification temperature.} \]
\[ \theta_a \text{ Dimensionless ambient temperature.} \]
\[ \vartheta \text{ Thermal parameter describing the relative importance of internal convection and advection versus solidification.} \]
\[ \vartheta^* \text{ Critical value of } \vartheta \text{ denoting the transition between tube and mobile crust regimes.} \]
\[ \vartheta^{**} \text{ Critical value of } \vartheta \text{ denoting the transition between shear-dominated and plug-dominated mobile crust regimes for viscoplastic flows.} \]

References


