Matter waves with orbital angular momentum: collapse suppression and bistability.

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Abstract: We explore the influence of the orbital angular momentum on the dynamics of vortex-free elliptic clouds of Bose-Einstein condensates with attractive interatomic interactions. We show that the number of atoms corresponding to the collapse threshold can be radically increased for such rotating nonlinear matter waves trapped in a radially symmetric harmonic potential. Moreover, rotating anisotropic traps can simultaneously support two differently aligned elliptical matter wave states stabilized against the collapse.

1. Introduction

Since the early days of ultracold atomic physics, the phenomena of matter wave collapse, i.e. unrestricted contraction of the Bose-Einstein condensate (BEC) with attractive interatomic interactions, has been studied both experimentally [1] and theoretically [2]. In particular, it has been shown that a dynamically stable BEC can exist both in 3D and 2D (strongly anisotropic, "pancake") harmonic traps, as long as the number of particles in the condensate is below a certain critical value. The initial dynamics of collapsing coherent matter waves under harmonic confinement is well described by the mean-field Gross-Pitaevskii theory, and is analogous to the collapsing light waves in self-focusing optical fibers.

Naturally, the problem of collapse control and possible suppression in such systems is of a great fundamental and practical importance. In particular, it was suggested that for anisotropic (i.e. elliptic) two-dimensional condensate clouds carrying angular momentum the collapse can be completely arrested [3]. Similar problem was recently analyzed in the context of nonlinear optics, where it was shown [4] that imprinting twisted phase front and thereby transferring an orbital angular momentum (OAM) onto light beams with elliptic cross-section allows to suppress their collapse. Although the rotating BEC with attractive interaction was studied in context of vortex formation, the fundamental role of OAM in the collapse dynamics of trapped BECs remains an open problem.

In this work we analyze, theoretically and numerically, the influence of the OAM on the dynamics and suppression of collapse of trapped matter waves with attractive interatomic interaction.

2. Model

We consider a BEC with negative scattering length trapped in a quasi-two-dimensional radially symmetric "pancake" trap with a tight transverse (axial) confinement. By using the appropriate dimensionality reduction procedure [6], the mean-field Gross-Pitaevskii model for the condensate wavefunction can be written in the following dimensionless form:

$$i\partial_t \psi + \Delta\psi - w^{-4} r^2 \psi + |\psi|^2 \psi = 0, \quad (1)$$

here $\Delta = \partial_x^2 + \partial_y^2$, and w defines the depth of the radially-symmetric trapping potential with $r^2 = x^2 + y^2$.
The radially symmetry of (1) implies conservation of OAM, the normalized number of atoms, and the Hamiltonian.

Approximate solutions of (1) in the form of such nonlinear rotating modes can be derived using the variational method, by way of standard Ritz minimization [3] with the Lagrangian. In order to create an anisotropic rotating matter wave, that would mimic the collapse resistant light beam described in [4], a phase twist should be imprinted onto the BEC cloud. We use the ansatz suggested for the elliptic spiraling soliton (i.e. a nonlinear self-focused state) in [4]:

$$\psi(x,y,t) = A(t)G[X / b(t)]G[Y / c(t)]\exp(i\chi), \quad (2)$$

with the Gaussian envelope $G[\xi] = \exp(-\xi^2 / 2)$ and phase $\chi = B(t)X^2 + \Theta(t)XY + C(t)Y^2 + \varphi(t)$, in the rotating frame.

3. Results

We demonstrate that a radially-symmetric harmonic trap can support elliptically shaped rotating clouds of ultracold atoms which exhibit a collapse-free dynamically stable evolution *with the number of particles greatly exceeding the threshold for radially-symmetric fundamental state of the trap* [5]. To achieve the suppression of collapse, one needs to increase ellipticity of the trapped state and its rotation (spiraling) velocity, by transferring orbital angular momentum to the matter wave, e.g. imprinting a twisted phase front onto an elliptic BEC cloud. As a result, the critical number of atoms for the collapse raises [see Fig. 1(a)], and the BEC cloud exhibits long-term dynamically stable behavior [Fig. 1(b)]. The results of numerical simulations are in agreement with the modified collapse threshold for the elliptic BEC states with OAM, predicted by variational analysis [dashed line in Fig. 1(a)].

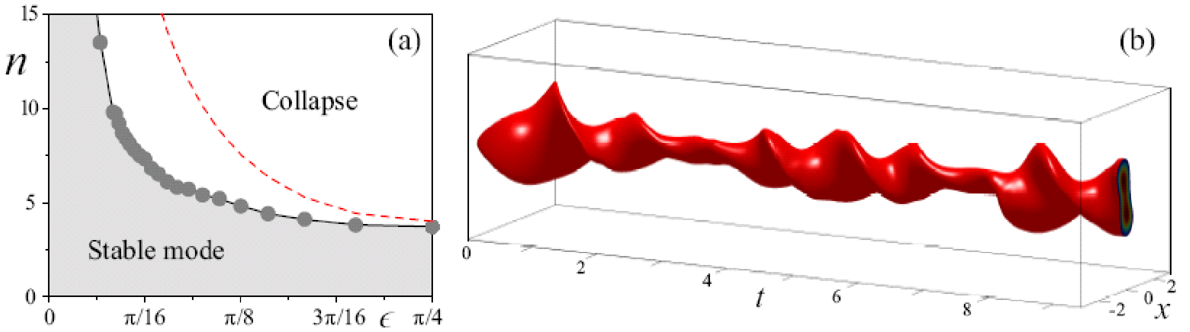

Fig. 1 (a) Existence and stability diagram for the 2D rotating elliptic matter waves; shown is the normalized number of atoms, n, vs. ellipticity. The region of stability against collapse identified via dynamical simulations of the mean-field Gross-Pitaevskii model is shaded. The boundary of the trapped mode existence and stability region predicted by variational approach is marked by dashed line. (b) Non-stationary dynamics of rotating elliptic cloud with n = 6.5 near the collapse threshold for the ellipticity $\varepsilon = \pi / 15$, but well above the critical atom number ($n_c = 4$) for the fundamental radially-symmetric mode with zero OAM.

While below the collapse threshold in Fig. 1(a) the elliptic clouds exhibit complex dynamics with rotation and oscillations, as seen in Fig. 1(b), above this threshold the BEC cloud collapses in finite time, defined by initial state and the trap frequency. Remarkably, even for an initially strongly anisotropic condensate cloud, the central region of collapsing condensate becomes radially symmetric [not shown], displaying the self-similar collapse dynamics, well described in the context of radially symmetric geometries of self-focusing nonlinear optical beams. Due to the formal similarity between the behavior of the coherent matter and light waves, our results could also be readily applied to the dynamics of elliptic “spiraling” modes of nonlinear optical waveguides.

Furthermore, we demonstrate that, in contrast to radially-symmetric traps, rotating *anisotropic elliptic traps* can simultaneously support two elliptical BEC states stabilized against the collapse. Most unusually, for the same angular velocity these modes will have different orientations of the major axis, i.e. aligned with or orthogonal to the major axis of the trap.

4. References

[1] J. M. Gerton, D. Strekalov, I. Prodan, and R. G. Hulet, “Direct observation of growth and collapse of a Bose-Einstein condensate with attractive interactions,” Nature **408**, 692 (2000).

[2] M. Wadati and T. Tsurumi, “Ground State of a Bose-Einstein Condensate under Axially Symmetric Magnetic Trap,” J. Phys. Soc. Jpn **68**, 3840 (1999).

[3] F. Haas, “Anisotropic Bose-Einstein condensates and completely integrable dynamical systems,” Phys. Rev. A **65**, 033603 (2002).

[4] A. S. Desyatnikov, D. Buccoliero, M. R. Dennis, and Yu. S. Kivshar, “Suppression of Collapse for Spiraling Elliptic Solitons,” Phys. Rev. Lett. **104** 053902 (2010).

[5] J. Abdullaev, A. S. Desyatnikov, and E. A. Ostrovskaya, “Suppression of collapse for matter waves with orbital angular momentum”, J. Opt. **13**, 064023 (2011)