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Fuzzy Rough Signatures

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***Abstract*— We extend the idea of Fuzzy Signature to Fuzzy Rough Signature (FRS). The proposed Fuzzy Rough Signature is capable of handling most kind of uncertainty: epistemic and random uncertainty, vagueness due to indiscernibility, and linguistic vagueness that exists in both large as well as small sample data sets. Additionally, this system is capable of hierarchical organization of inputs and use of flexible aggregation selection will simplify the combinations of inputs from different sources.**

***Index Terms*—Fuzzy Signatures, Rough sets, Mathematical Theory of Evidence, Polymorphic Fuzzy Signatures (PFS), Rough Fuzzy Signatures, Aggregation Operators, generalized Weighted Relevance Aggregation Operator (WRAO), possibility, probability, probability of fuzzy events, fuzzy probability.**

I. Introduction

In Computational Intelligence, there are many methods to find a synthesis or coherent view to a complex set of available information. In medical diagnosis, this presents as a decision problem where practitioners may need to ascertain possible diseases with only partially available information needed for a proper diagnosis and use knowledge of the uncertainty about the symptoms of the disease. In economics, this presents as a prediction of a new trend without many samples and possibly many inconsistencies in that data. In document classification in an urgent setting such as security investigation, systems need to deal with small sample data which is imprecise and contains incomplete data. Also, in our experience of HCI applications, highly inconsistent data can be found from user to user because of problems of the available technologies for data acquisition and the diversity of humans [1]. In petroleum and mining engineering, we find small sample data due to high cost of data collection [2], [3], [4], [5]. Some existing solutions, such as Dempster-Shafer model [6], transferable belief model (TBM) [7], Choquet Integral [8], belief revision [9], and subjective logic [10] consider all possible combinations from the power set of the input space and are thus extremely computationally complex and usually infeasible to implement practically. And most of the other solutions are mainly good at only for specific task, such as fuzzy rule base systems, neural networks, etc. Additionally, none of them try to capture the uncertainty caused by vagueness and ambiguity of insufficient, incomplete, and small sample information at the same time as trying to be computationally efficient.

We propose, the concept of fuzzy rough signatures (FRS) to handle the imprecise and insufficient information in real world systems in a way natural to humans, and reduce the computational complexity using a hierarchical structure as much as possible. In order to develop hierarchical mathematical model for knowledge representation and reasoning. We will combine the methods of uncertainty calculations in mathematical theory of evidence [6] with fuzzy signature concept. However, in practice, people often experience that both the nature of a real world event and the evidence of the event are uncertain (eg. "About 5 tall people attacked the bus driver" has uncertain information in the evidence of the event "about 5", and uncertainty in the nature of the event "tall" is how many cm exactly?). In order to reason about such a scenario precisely, we will further extend the methods of uncertainty calculations in mathematical theory of evidence [6] with advantages of uncertainty and vagueness modeling of generalized fuzzy rough sets [11], [12], [13] and fuzzy probability [14], [15], [16].

The theory of rough sets [17] is an extension to classical set theory that models the vagueness due to indiscernibility of objects in insufficient and incomplete information. On the other hand fuzzy sets theory and possibility theory mathematically model the partial belonging-ness of elements in a set, and the distribution of uncertainty which exists in such belonging-ness [18] respectively. In the literature [11], it has shown that rough sets and fuzzy sets can be combined and such a generalization [11], [12], [19], [13] would treat the ambiguity which exists in both the input and the nature of the input, and the vagueness of available information due to incomplete and inconsistent data. Hence, our aim is to modify the fundamentals of fuzzy signatures to be able to use rough fuzzy events [20], [12], [19].

In [21], Mendis has shown that the hierarchical organization of subgroups of data simplifies the approximation of aggregation functions to achieve the underlying global preference relation of a problem. That is, a hierarchically structured decision making system using a set of usually non-homogeneous hierarchically organized aggregation functions (local aggregation functions) easily and precisely approximates the desired global preference relation of the system. Thus the proposed method extends hierarchically structured polymorphic fuzzy signature to acquire these advantages. The polymorphic fuzzy signature (PFS) [21] concept has been introduced as a generalization to the fuzzy signature concept [22], [23]. Fuzzy signatures can be expressed as a fuzzy hierarchical multi aggregative descriptor of an object [21]. Simply, a polymorphic fuzzy signature represents sets of fuzzy signatures, which belong to same granule, and it replaces the atomic events in leaf nodes of fuzzy signatures from fuzzy events.

II. Fuzzy Signature Concept

Fuzzy signatures [22] can describe, compare and classify objects with complex structures and interdependent features.

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The hierarchical organizations of fuzzy signatures express the structural complexity of a problem.

A. Fuzzy Signatures

Fuzzy signatures are fuzzy descriptors of real world objects. The syntactic and semantic of an object will be represented using the hierarchical structure, usually non-homogenous set of weighted aggregation functions, and quantities as a possibility of a linguistic category. Thus, fuzzy signatures are capable of handling problems that are complex and inherently hierarchical.

Initially, the fuzzy signature concept was proposed as a good solution to the rule explosion problem in fuzzy logic [24], as fuzzy signatures are hierarchically structured and inherently sparse.

Definition 1: Fuzzy Signature is a recursive vector valued fuzzy set (VVFS), where each vector component either embeds another VVFS (branch) or a atomic possibilistic value (leaf), and denoted by,

$$A : X \to [a_i]_{i=1}^{k} \left(\equiv \prod_{i=1}^{k} a_i \right). \quad (1)$$

$$where\ a_i = \begin{cases} [a_{ij}]_{j=1}^{k_i} & ; if\ branch\ (k^i > 1) \\ [0,1] & ; if\ leaf \end{cases}$$

and Π denotes the Cartesian product.

B. Polymorphic Fuzzy Signature (PFS) Concept

The basic idea of Hierarchical Fuzzy Signatures is to identify a single Fuzzy Signature for each object or data point. In real world decision-making applications, people may not be interested or able to invest a large amount of time or funding in achieving the best possible solution. Instead, they may only consider simpler solutions, which are sufficiently efficient and more comprehensible to apply. This part introduces the concept of Polymorphic Fuzzy Signatures that was developed via extended experiments with the application of the Fuzzy Signature concept.

We observe that in some situations we may be able to find a single Hierarchical Fuzzy Signature for a set of individual data points (objects), by reducing the number of Fuzzy Signatures required to implement a model. We call such a Fuzzy Signature a *Polymorphic Fuzzy Signature* for the set of data points (objects) it represents. We discuss the results of our experiments in [25] as a form of evidence that the Polymorphic Fuzzy Signature concept is practical.

Below, we formulate the concept of Polymorphic Fuzzy Signature as an optimization problem, which is a conclusion of our experiment work in [25].

Definition 2: Let $A = \{S_1, S_2, \ldots, S_n\}$ be a collection of fuzzy signatures for a certain problem and let $\{d_1, d_2, \ldots, d_n\}$ be the collection of data points[1] they represent respectively. Now, let S_i be the corresponding fuzzy signature of the data point d_i. Further, let $S_i(d_i)$ be the degree[2] of match of the data point d_i with fuzzy signature S_i. Then S said to be the polymorphic fuzzy signature of the set A if

$$\sum_{i=1}^{n} |S(d_i) - S_i(d_i)| \le \delta \quad (2)$$

where δ is a small number close to zero and S must satisfy the following conditions

(i) $\bigcup_{i=1}^{n} V(S_i) \subseteq V(S)$

(ii) $\bigcup_{i=1}^{n} L(S_i) = L(S)$

where $V(S)$, and $L(S)$ denote sets of vertices, and leaf vertices (fuzzy sets) of fuzzy signature S respectively.

The major difference between PFS and fuzzy signatures is that PFS uses fuzzy sets as the leaf nodes and in fuzzy signatures they are atomic values (possibilities). Figure 1, shows a PFS for a real world problem called SARS patient classification. Medical practitioners SARS PFS is constructed based on domain expert knowledge. Each symptom check has been divided into a number of doctors diagnosis levels, such as *slight*, *moderate*, and *high* for body temperature (fever), *low*, *normal*, and *high* for both measurements of blood pressure, *slight*, *medium*, and *high* for nausea, and *slight*, and *high* for abdominal pain. The notations a_{ij}, $@_{ij}$, and W_{ij} represent the fuzzified input value (possibility), aggregation function, and weight for the branch ij of the SARS PFS in Figure 1.

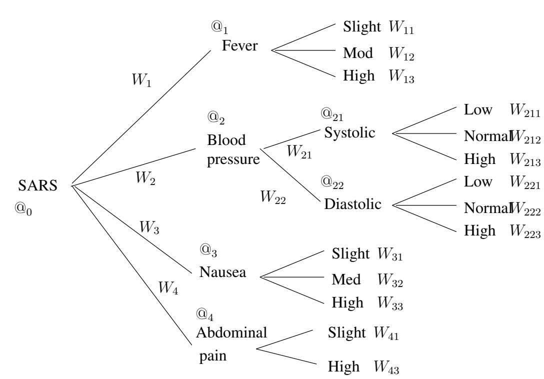

Fig. 1: SARS Patient Classification Fuzzy Signature

III. Fuzzy Rough Signatures

Traditional probabilistic decision analysis techniques as applied to the evaluation of random events (such as gambling) have difficulty in modeling the epistemic uncertainty associated with medical diagnosis, economic diagnosis, analysis of terrorists acts, intelligent document analysis, human computer interaction, etc. The specificity of the theory of evidence is its capability to capture epistemic as well as

[1] *In fuzzy signature concept, a data point means a collection of data which represent an event, e.g. in medical applications, patient's data record of a whole day can be considered as a data point [23].*

[2] *Degree of match is the final result of the input data point d_i aggregating using the fuzzy signature S_i*

random uncertainty [26]. At the same time, in the literature it has been shown that rough sets are excellent to model vagueness [27], [17], and fuzzy sets and possibility theory perform the mathematical modeling of linguistic ambiguity of information [18]. Consequently, the generalized fuzzy rough set concept [12], [19], [13] is an excellent mathematical tool to model both vagueness caused by the indiscernibility of objects, and linguistic uncertainties such as the "About 5 tall people" example. Finally, hierarchical organization of Fuzzy Rough Signatures will help reduce the computational complexity and also help simplify the aggregation approximation and aid better adoption [21]. Following subsections explain major theories which are the building blocks of the concept of Fuzzy Rough Signatures (FRS).

A. Mathematical Theory of Evidence

Shafer's seminal work on the subject is [6], which is an expansion of Dempster's idea of interval value probability [28]. In a discrete case, the theory of evidence can be interpreted as a generalization of probability theory, where probabilities are assigned to mutually exclusive sets. In conventional probability theory, evidence is associated with only one possible event. Unlike in conventional probability theory, in the theory of evidence atomic evidence can be associated with a set overlapping events. Therefore, the theory of evidence models the representation of uncertainty of system inputs where an imprecise input can be characterized by a set or an interval and the resulting output is a set or an interval. The most important features of the theory of evidence is the capability of modeling ignorance. The basic probability assignment (*bpa)* [6] is the fundamental building block of the theory of evidence. Basic probability assignment, represented by m, defines a mapping of the power set of the frame of discernment to $[0, 1]$,

$$m : 2^{\theta} \rightarrow [0, 1] \quad (3)$$

$$m(\emptyset) = 0 \quad (4)$$

$$\sum_{A \in 2^{\theta}} m(A) = 1 \quad (5)$$

θ is called the frame of discernment. The value of the *bpa* for a given set A, denoted $m(A)$, expresses the degree of support of available evidence that supports the claim that an element of θ is in the set A but not in any particular subsets of A [29]. Now, using the *bpa*, the upper and lower bounds of an interval A can be calculated. This interval contains the basic probability of a set of interest (in the classical sense) and is bounded by two non-additive measures called *Belief* and *Plausibility*.

$$Bel(A) = \sum_{B | B \subseteq A} m(B) \quad (6)$$

$$Pl(A) = \sum_{B | B \cap A \neq \emptyset} m(B) \quad (7)$$

With belief and plausibility distributions, a random variable X has an expected value interval $[E^*(X), E_*(X)]$ given by:

$$E^*(X) = \sum_{\forall A_i \subseteq X} \inf(A_i) * m(A_i) \quad (8)$$

$$E_*(X) = \sum_{\forall A_i \subseteq X} \sup(A_i) * m(A_i) \quad (9)$$

In the following example ([26]) in figure 2, consider $X = \{a, b, c\}$ as the frame of discernment and body of evidence given in the figure 2 for belief and plausibility calculations. The existing body of evidence in figure 2 are: $m(a) = 0.2$, $m(a, b) = 0.7$, and $m(b, c) = 0.1$. Using the above equations (6) and (7), *Bel* and *Pl* can be evaluated for any element in $Pow(X)$,as an eg. $Bel(a, b) = 0.2 + 0 + 0.7$ and $Pl(a, b) = 0.2 + 0 + 0.7 + 0.1$.

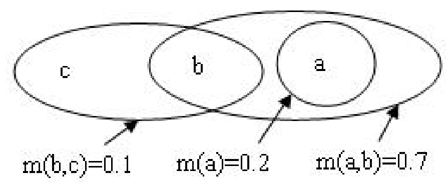

Fig. 2: Example Body of Evidence

In Shafer's theory crisp events are used to categorize real world events and thus such systems could not model the ambiguity existing in scenarios similar to the "About 5 tall people" example. Several attempts have been made to generalize the theory of evidence to use fuzzy events, e.g. [30]. In Fuzzy Rough Signatures, the crisp events of the body of evidence will be replaced with fuzzy events. In such a situation, the element b in figure 2 belongs to fuzzy evidence event $\{b, c\}$ only to a degree of $\mu_{\{b,c\}}(b)$. Thus events are weighted, $m(b, c) = m(b) \times \mu_{\{b,c\}}(b) + m(c) \times \mu_{\{b,c\}}(c)$ [16]. This means that in fuzzy rough signatures the fuzzy information granulation is considered. As explained in the "About 5 tall people" example, in most situations $m(b)$ and $\mu_{\{b,c\}}(b)$ are not explicitly available. Thus, we will further expand $m(b)$ and $\mu_{\{b,c\}}(b)$, so that $m(b)$ will be approximated using a possibility-probability distribution [14], [15], [16] of b in the corresponding fuzzy event, and $\mu_{\{b,c\}}(b)$ will be approximated using upper and lower approximations of rough fuzzy sets.

B. Generalized Fuzzy Rough Sets

Only employing fuzzy evidence of events is not enough to model most real world situations. One persons' linguistic expression of *tallness* may be different to a second persons' linguistic expression of *tallness*. Therefore, in this study we consider the concept of rough fuzzy events (later generalized fuzzy rough sets) [11], [12], [13] to express combining measures such as different peoples' *tallness* into one event. Using rough fuzzy events the new theory will be capable of providing upper and lower approximations to fuzzy evidence of a tall event. Accordingly, we need to modify the calculation of expected value of these upper and lower rough fuzzy events, to cope with information from fuzzy evidence events.

TABLE I: Example Information Table

x	a	b	c	d	e
0	1	0	2	2	0
1	0	1	1	1	2
2	2	0	0	1	1
3	1	1	0	2	2
4	1	0	2	0	1
5	2	2	0	1	1
6	2	1	1	1	2
7	0	1	1	0	1

Rough set [17] assumes that with every object of the universe of discourse we associate some information (data, knowledge). E.g., if objects are patients suffering from a certain disease, symptoms of the disease form information about patients. Objects characterized by the same information are indiscernible (similar) from the view of available information about them. The indiscernibility relation generated in this way is the mathematical basis of rough set theory.

Formally (briefly), any subset P of A determines a binary relation $I(P)$ on U, called an indiscernibility relation, and can be defined as follows [12]: $xI(P)y$ if and only if $a(x) = a(y)$ for every $a \in P$, where $a(x)$ denotes the value of attribute a for object x. Let U (universe) be a set of finite objects, and $I(P)$ be an equivalence relation on U. Let $U/I(P)$ denote the quotient set of equivalence classes, which form a partition in U. Let S be a subset of U. The main question addressed by rough sets [17] is how to represent S by means of the equivalence relation $U/I(P)$. A rough set is a pair of subsets $P^*(S)$ and $P_*(S)$ that approach as close as possible to S from outside and inside respectively.

$$P^*(S) = \{x \mid [x]_p \cap S \neq \emptyset, x \in U\} \quad (10)$$

$$P_*(S) = \{x \mid [x]_p \subseteq, x \in U\} \quad (11)$$

$P^*(S)$ and $P_*(S)$ are called the upper and lower approximations of S with respect to P. If $P^*(S) \neq P_*(S)$, it means that due to the indiscernibility of elements in U, S cannot be described using crisp sets but only using a rough set. Let P and Q be equivalence relations over U, then the positive, negative, and boundary regions can be defined as [12]:

$$POS_P(Q) = \bigcup_{X \in U/Q} \underline{P}X \quad (12)$$

$$NEG_P(Q) = U - \bigcup_{X \in U/Q} \overline{P}X \quad (13)$$

$$BND_P(Q) = \bigcup_{X \in U/Q} \underline{P}X - \bigcup_{X \in U/Q} \overline{P}X \quad (14)$$

The information in table I can be partitioned according to $P = \{b, c\}$ and $Q = \{e\}$ as follows [12]: $U/I(P) = \{\{2\}, \{0, 4\}, \{3\}, \{1, 6, 7\}, \{5\}\}$ and $U/I(Q) = \{\{0\}, \{1, 3, 6\}, \{2, 4, 5, 7\}\}$. Now, using above information table ***POS, NEG,*** and ***BND*** can be calculated as, $POS_P(Q) = \bigcup\{\emptyset, \{2, 5\}, \{3\}\} = \{2, 3, 5\}$, $NEG_P(Q) = U - \bigcup\{\{0, 4\}, \{2, 0, 4, 1, 6, 7, 5\}, \{3, 1, 6, 7\}\} = \emptyset$, and $BND_P(Q) = U - \{2, 3, 5\} = \{0, 1, 4, 6, 7\}$.

This original idea of rough sets can be extended into rough fuzzy sets and fuzzy rough sets [20], [11], [13].

C. Basic Probability Assessment of Rough Fuzzy Events

In this part of the new theory, we will extend the basic probability calculation (ie. $m(b)$) of evidential events into a fuzzy probability distribution [16], [31]. This will enhance the acquisition of quantitative evidence, such as "About 5", in the form of humans would like to express rather than they forcibly categorize into crisp event.

1) Possibility Measures: We first aim to provide some basic notions used in the possibility theory [32]. Let x be an unknown variable, taking its values from the variable X, attached to some attribute or entity. A possibility distribution π_x, is a mapping from X to $[0, 1]$. Here, π_x characterizes the set of values u of X in agreement with the variable x. That is, $\pi_x(u_1) = 0$ means that it is practically impossible that x takes a value u_1. While $\pi_x(u_1) = 1$ means that u_1 is a completely possible value, not necessarily unique, for the variable x. A possibility distribution π_x such that $\pi_x(u) < 1 | \forall u$; which means that there is no value u of the X, which is in full agreement with x.

A possibility measure Π_x can also be generated through the possibility distribution π_x as follows:

$$\Pi_x(A) = \max_{u \in A | A \subseteq X} \pi_x(u) \quad (15)$$

Note that Π_x is a mapping from the 2^X to $[0, 1]$. A dual measure of a possibility Π_x is called a necessity measure N_x characterizing the impossibility of the contrary event,

$$N_x(A) = 1 - \Pi_x(\overline{A}) \quad (16)$$

2) Fuzzy Probability Calculations: Now, we define the fuzzy probability (possible probability) as a possibility distribution of a probability of a fuzzy event.

Let $(\mathbb{R}^n, \varphi, P)$ be a probability space, let φ is the σ-field of Borel sets in $\mathbb{R}^n$, and P is a probability measure over $\mathbb{R}^n$. Let X be a variable which takes values from $\mathbb{R}^n$. Further, take A as a fuzzy subset of $\mathbb{R}^n$. Now, the possibility that the probability of "X is A" takes a value p be $\pi_A(p)$, and can be written as,

$$\Pi_{\Omega,P} = \{\pi_A(p) \mid P(XisA) = p\} \quad (17)$$

This form is called the possibility probability distribution of "X is A". This can also be written in the following form,

$$FProb\{XisA\} = \{\pi_A(p) \mid P(XisA) = p\} \quad (18)$$

As an example, one can find a fuzzy probability distribution for an expression: what is the possibility of the probability of *tall* students are *high*". Huang [15], and Huang and Gedeon [16] have developed methods to calculate fuzzy probabilities, especially when only small samples of data are available.

D. Hierarchical Structure of Fuzzy Rough Signatures:

The implementation of Dempster's rule is sometimes impractical because of its computational complexity but in [33], Shafer argues that this is not the case for hierarchical evidence. We propose the following mathematical guidelines for organizing hierarchical structure of Fuzzy Rough Signatures.

Hierarchical structure can be expressed as a triple $(N, X^{'}, \succeq)$, where

(i) $N = \{1, \ldots, n\}$ is the set of atomic evidences,
(ii) $X^{'}$ is the Cartesian product, $X^{'} = X^{'}_1 \times X^{'}_2 \times \ldots \times X^{'}_r$, where $r \leq 2^n$ and each event of evidence $X^{'}_i \in \rho(X)$, such that $X = X_1 \times \ldots \times X_i$,
(iii) $\succeq$ is a preference relation on $X^{'}$.

Now let $x^{'} = (x^{'}_1, \ldots, x^{'}_k) \in X^{'}$, where $k \leq 2^n$ and $x = (x_1, \ldots, x_n) \in X$.

(a) U^v_0 is global preference function such that $U^v_0 : X^{'} \rightarrow L$.
(b) M^v_0 is an aggregation function such that $M^v_0 : L^k \rightarrow L$.
(c) $\{u_i\}$ be the belief or plausibility at a particular time.

Such that, $\bar{U}^v_0(x) := M^v_0[u^{'}_1(x^{'}_1), \ldots, u^{'}_k(x^{'}_k)]$

where $u^{'}_j(x^{'}_j) = \begin{cases} u_i(x_i) & ; if\ x^{'}_j \in x \\ U^v_j(x^{'}_j) & ; else \end{cases}$

Note that, $U^v_j(x^{'}_j)$ recursively follows this definition to define the next hierarchy, if necessary.

Here, $i \in N$, $k \leq 2^n$, $j \in [1 \ldots k]$, and v is a monotonic measure.

1) Aggregation of Fuzzy Rough Signatures: The generalized Weighted Relevance Aggregation Operator (WRAO) (Mendis, 2008) would be a good choice for aggregation Fuzzy Rough Signatures in the first instance. WRAO outperformed OWA and fuzzy integrals for classification and decision making [21], [34], [35]. WRAO is monotonic with respect to preferential ordering, and is an aggregation function which can be defined in the following way.

The generalized Weighted Relevance Aggregation Operator (WRAO) of an arbitrary branch $a_{q \ldots i}$ with n subbranches, $a_{q \ldots i1}, a_{q \ldots i2}, \ldots, a_{q \ldots in} \in [0, 1]$, and weighted relevancies, $w_{q \ldots i1}, w_{q \ldots i2}, \ldots, w_{q \ldots in} \in [0, 1]$, for a hierarchical structure is a function $g : [0, 1]^{2n} \rightarrow [0, 1]$ such that,

$$a_{q \ldots i} = \left[\frac{1}{n} \sum_{j=1}^{n} (a_{q \ldots ij} w_{q \ldots ij})^{p_{q \ldots i}} \right]^{\frac{1}{p_{q \ldots i}}} \quad (19)$$

The WRAO must satisfy the following three properties,

1) $w_{q \ldots ij} \in [0, 1]$
2) $\vee^n_{j=1} w_{q \ldots ij} \leq 1$
3) $p_{q \ldots i} \neq 0$

IV. Conclusions

The proposed Fuzzy Rough Signature will embed mathematical guidelines for organizing a hierarchical structure for reasoning as well as it unifies the advantages of four best existing mathematical tools for decision making and classification: mathematical theory of evidence, rough sets, and fuzzy probability. We shown the way of combining the best features in above models, thus Fuzzy Rough Signatures are advantageous compared to today's methods and applications that are mostly based on one or two of the above methods. Also, in this new theory probability and possibility will be treated as complementary to each other, thus it is capable of handling large as well as small sample data. Additionally, Fuzzy Rough Signatures inherits high transparency from fuzzy systems due to use of rough fuzzy sets at the data acquisition level.

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