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A descriptor system approach to robust H_∞ control for linear systems with time-varying uncertainties

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This article addresses the problems of robust H_∞ performance analysis and synthesis for continuous-time linear systems with time-varying uncertainties using parameter-dependent Lyapunov functions. An H_∞ performance criterion in terms of linear matrix inequalities is derived via employing a descriptor model transformation of linear systems with polytopic uncertainties (polytopic systems). This H_∞ performance criterion can be applied to robust controller synthesis and filtering design. The descriptor system approach is also taken to explore robust H_∞ performance analysis and controller synthesis of affine parameter systems. A flight control system and two numerical examples are given to illustrate the effectiveness of the proposed analysis and synthesis procedures.

Keywords: H_∞ control; descriptor system approach; affine parameter systems; polytopic systems; parameter-dependent Lyapunov functions

1. Introduction

When designing a control system, it is often desirable to obtain the guarantees of stability and performance against uncertainties on the physical parameters of the system. Various types of parametric uncertainties including norm-bounded uncertainty, positive real uncertainty and polytopic uncertainty can be found in literature. Polytopic uncertainty is an important class of uncertainties with explicit physical meaning, and many uncertain control systems can be modelled as systems with polytopic uncertainty. Polytopic uncertainty can usually be expressed as a convex polytope whose vertices are given (Boyd, El Ghaoui, Feron and Balakrishnan 1994).

H_∞ control is an effective control strategy to deal with disturbances from the input–output point of view. A traditional approach of robust H_∞ control to deal with parametric uncertainties is based on the notion of quadratic stability, see, e.g. Khargonekar, Petersen and Zhou (1990), Boyd et al. (1994), Palhares, Takahashi and Peres (1997). This approach uses a single Lyapunov matrix which satisfies the bounded real lemma (BRL) (Boyd et al. 1994). It is well known that the use of the single Lyapunov matrix method usually leads to conservative results, especially in the case where

the uncertainty is time-invariant or slowly time-varying (Gahinet, Apkarian and Chilali 1996).

To improve quadratic stability-based results, parameter-dependent Lyapunov functions have extensively been used to assess robust stability and to compute guaranteed H_∞ performance index for continuous-time systems, see for instance Gahinet et al. (1996), Shaked (2001), Jia (2003), de Oliveira, Oliveira, Leite, Montagner and Peres (2004), Ebihara, Peaucelle, Arzelier and Hagiwara (2005), Goncalves, Palhares and Takahashi (2005), He, Wu and She (2005), Lin, Wang and Lee (2006), and Gao, Shi and Wang (2007). In Gahinet et al. (1996), linear matrix inequality (LMI) sufficient conditions for affine quadratic stability and H_∞ performance for a class of uncertain continuous-time systems, including time-varying parameters, were proposed by employing Lyapunov functions that varied depending on the uncertain parameters. In de Oliveira et al. (2004), LMI sufficient conditions for convex optimisation of H_∞ performance were proposed for both discrete- and continuous-time systems with polytopic uncertainties. In Ebihara et al. (2005), LMI sufficient conditions for robust stability and H_∞ performance analysis were given by employing Lyapunov functions that can be associated with

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higher-order derivatives of the state vectors. These LMI sufficient conditions are successful tools for performance analysis but might be hard to apply in the robust controller synthesis and filtering design. An efficient solution for the H_∞ control problem was presented by Shaked (2001) for continuous-time linear systems with polytopic uncertainties, and Jia (2003) found alternative proofs to the theorems in Shaked (2001). A further improvement of BRL representation in terms of LMIs was proposed by He et al. (2005). However, these uncertain parameters considered in Shaked (2001), de Oliveira et al. (2004), Ebihara et al. (2005), Goncalves et al. (2005) and He et al. (2005), are assumed to be time-invariant.

On the other hand, the H_∞ filtering problems have been extensively studied and applied to many practical systems. In the earlier work, the robust H_∞ filtering problems for systems with polytopic uncertainties were solved by Palhares and Peres (1999) and Jin and Park (2001), which were based on the notion of quadratic stability. Recently, based on the parameter-dependent Lyapunov function approach, the robust H_∞ filtering problems were solved for discrete-time systems (Gao and Wang 2004; Xie, Lu, Zhang and Zhang 2004; de Souza, Barbosa and Trofino 2006; Goncalves, Palhares and Takahashi 2006) and continuous-time systems with polytopic uncertainties (de Souza, Fu and Trofino 2000; Gao, Lam, Shi and Wang 2005a; Wang and Wang 2005b, Duan, Zhang, Zhang and Mosca 2006). In Duan et al. (2006), the methods of robust H_2 and H_∞ filtering design were developed for both discrete- and continuous-time systems with polytopic uncertainties. In Gao et al. (2005a), Montagner, Oliveira, Leite and Peres (2005) and Montagner and Peres (2005), these approaches of designing parameter-dependent H_∞ filters or controllers were suggested for continuous-time systems with polytopic uncertainties. These design approaches are based on the assumption that system parameters can be measured online. However, this assumption may be difficult to be satisfied in practice. Thus, it is necessary to design an effective filter or controller to achieve robust H_∞ performance for systems with non-measurable parameter uncertainties.

More recently, a new descriptor system approach was introduced by Fridman and Shaked (2002), for time-delayed systems. This approach was developed by Cao and Lin (2004), for deriving LMI conditions for robust stability and robust stabilisability of linear systems with polytopic uncertainties and affine parameter systems. The basic idea of the approach is as follows: first the considered system is equivalently transformed into a descriptor form, and then stability analysis is carried out for the descriptor system by employing a special form of Lyapunov function. As a result, robust stability conditions were obtained, which

did not involve the product between the Lyapunov matrix and the system matrix.

The objective of this article is to develop an H_∞ performance criterion for uncertain continuous-time systems with time-varying parameters using the descriptor system approach. This criterion can reduce the conservativeness of quadratic stability-based results. Furthermore, the criterion is formulated in the form that can be easily modified for robust controller synthesis and filtering design. Motivated by the results in Cao and Lin (2004), we adopt the descriptor system approach to obtain the H_∞ performance criterion in terms of LMIs for linear systems with polytopic time-varying uncertainties. This criterion can be viewed as an extension of the polyquadratic stability (Cao and Lin 2004). We also show how the criterion is employed for robust controller synthesis and filtering design. Moreover, the descriptor system approach is also taken to explore robust H_∞ performance analysis and controller synthesis of affine parameter systems. Finally, the H_∞ state-feedback controller design method is applied to the longitudinal dynamics of the *F-18* aircraft. We solve the robust H_∞ tracking control problem for the flight control system.

The rest of this article is organised as follows. Section 2 formulates the problem and gives preliminaries. Section 3 presents an H_∞ performance criterion for linear systems with polytopic time-varying uncertainties by employing the descriptor system approach. Then, a robust state-feedback controller is designed, which guarantees robust stability and H_∞ performance for the resulting closed-loop system in Section 4. Furthermore, we apply the results of Section 3 to discuss the robust H_∞ filtering problem for the system in Section 5. We also apply the descriptor system approach to explore robust H_∞ performance analysis and controller synthesis of affine parameter systems in Section 6. A flight control system and two numerical examples are given to illustrate our analysis and design procedures and demonstrate their effectiveness in Section 7. Finally, Section 8 concludes this article.

Notation: Throughout this article, $\text{col}\{a, b\}$ implies $[a^T b^T]^T$, the symbol $*$ within a matrix represents symmetric elements of the matrix, and the matrix I denotes the identity matrix with appropriate dimension.

2. Problem formulation and preliminaries

Consider the continuous-time linear uncertain system described by the following state space equations:

$$\dot{x}(t) = A(\theta)x(t) + B_1(\theta)\omega(t), \quad (1a)$$

$$z_1(t) = C_1(\theta)x(t) + D_1(\theta)\omega(t), \quad (1b)$$

where $x(t) \in R^n$ is the state vector, $\omega(t) \in L_2[0, \infty)$ is the exogenous disturbance signal, and $z_1(t) \in R^p$ is the output to be regulated. Suppose that the system matrices $A(\theta)$, $B_1(\theta)$, $C_1(\theta)$ and $D_1(\theta)$ are not precisely known, but belong to the convex polytopic set

$$\begin{aligned}\Omega_1 = & \{[A(\theta) \ B_1(\theta) \ C_1(\theta) \ D_1(\theta)] : [A(\theta) \ B_1(\theta) \ C_1(\theta) \ D_1(\theta)] \\ & = \sum_{i=1}^N \theta_i [A_i \ B_{1i} \ C_{1i} \ D_{1i}], \theta_i \geq 0, \sum_{i=1}^N \theta_i = 1\},\end{aligned}\quad (2)$$

where $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_N]^T \in R^N$ is a vector of uncertain and possibly time-varying real parameters. Suppose that $\dot{\theta}_i$ satisfy

$$\dot{\theta}_i \leq v_i \theta_i \quad (3)$$

with constants v_i . For a prescribed scalar $\gamma > 0$, we define the performance index

$$J(\omega) = \int_0^\infty (z_1^T z_1 - \gamma^2 \omega^T \omega) dt. \quad (4)$$

If the performance index $J(\omega) < 0$ holds, then γ represents the disturbance attenuation level.

The disturbance attenuation level γ of the uncertain system (1) with (2) can be computed through the following well-known BRL.

Proposition 2.1: (Palhares et al. 1997) *For a prescribed scalar $\gamma > 0$, the uncertain system (1) is robustly stable and the performance index $J(\omega) < 0$ is satisfied in the uncertainty domain (2) if there exists a matrix $P_1 > 0$ such that*

$$\begin{bmatrix} A_i^T P_1 + P_1 A_i & P_1 B_{1i} & C_{1i}^T \\ * & -\gamma^2 I & D_{1i}^T \\ * & * & -I \end{bmatrix} < 0, \quad i = 1, \dots, N. \quad (5)$$

Obviously, the system stability and performance index $J(\omega) < 0$ hold even when the parameters change arbitrarily fast. It is long known that such a quadratic stability approach often yields overly conservative results for the systems with constant or slowly varying parameters. To reduce the conservativeness of the quadratic stability approach, we resort to the following parameter-dependent Lyapunov function

$$V(x, \theta) = x^T P_1(\theta) x \quad (6)$$

with $P_1(\theta) = P_1^T(\theta) > 0$. For polytopic system (1), we simply select $P_1(\theta)$ of the following form

$$P_1(\theta) = \sum_{i=1}^N \theta_i P_{1i}, \quad P_{1i} > 0. \quad (7)$$

3. Robust H_∞ performance analysis

This section is concerned with the problem of robust H_∞ performance analysis for system (1). We first give the notion of polyquadratic H_∞ performance.

Definition 3.1: (Polyquadratic H_∞ performance). For a prescribed scalar $\gamma > 0$, the uncertain system (1) has polyquadratic H_∞ performance γ if there exists a parameter-dependent Lyapunov function $V(x, \theta) = x^T P_1(\theta) x$ with $P_1(\theta)$ given in (7), such that:

- (i) The uncertain system (1a) with $\omega(t) = 0$ is polyquadratically stable (Cao and Lin 2004), i.e. $dV/dt < 0$, $\forall x \neq 0$;
- (ii) Under the initial condition $x(0) = 0$, the performance index $J(\omega) < 0$ holds for all non-zero $\omega(t) \in L_2[0, \infty)$ and all admissible trajectories of the parameter vector θ satisfying (2) and (3).

Using the method in Fridman and Shaked (2002), we represent the uncertain system (1a) in the equivalent descriptor form:

$$\begin{aligned}\dot{x}(t) &= y(t), \\ y(t) &= A(\theta)x(t) + B_1(\theta)\omega(t),\end{aligned} \quad (8)$$

which is equivalent to the following descriptor system

$$E \begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ A(\theta) & -I \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_1(\theta) \end{bmatrix} \omega(t) \quad (9)$$

with $E = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}$.

According to Lin, Wang and Lee (2005), the regular and impulse-free property of the descriptor system (9) is ensured since $\text{degree}(\det(sE - \begin{bmatrix} 0 & I \\ A(\theta) & -I \end{bmatrix})) = \text{rank}(E) = n$. Thus, there is no impulse behaviour in system (9). Therefore, we can conclude that the stability of system (1a) is equivalent to the regular, impulse-free and stable property of system (9).

Let us choose a Lyapunov function candidate as

$$V(x, \theta) = \begin{bmatrix} x^T(t) & y^T(t) \end{bmatrix} E P(\theta) \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = x^T(t) P_1(\theta) x(t) \quad (10)$$

with

$$P(\theta) = \begin{bmatrix} P_1(\theta) & 0 \\ P_2 & P_3 \end{bmatrix} = \sum_{i=1}^N \theta_i \begin{bmatrix} P_{1i} & 0 \\ P_2 & P_3 \end{bmatrix},$$

$0 < P_{1i} \in R^{n \times n}$, and $P_2, P_3 \in R^{n \times n}$.

We have the following polyquadratic H_∞ performance criterion.

Theorem 3.2: *The uncertain system (1) has polyquadratic H_∞ performance γ in the uncertainty domain (2) if there exist matrices $P_{1i} > 0$, P_2 and P_3 such that*

$$\begin{bmatrix} A_i^T P_2 + P_2^T A_i + v_i P_{1i} & P_{1i} - P_2^T + A_i^T P_3 & P_2^T B_{1i} & C_{1i}^T \\ * & -P_3 - P_3^T & P_3^T B_{1i} & 0 \\ * & * & -\gamma^2 I & D_{1i}^T \\ * & * & * & -I \end{bmatrix} < 0, \quad i = 1, \dots, N. \quad (11)$$

Proof: Note that (11) implies

$$\begin{bmatrix} A_i^T P_2 + P_2^T A_i + v_i P_{1i} & P_{1i} - P_2^T + A_i^T P_3 \\ * & -P_3 - P_3^T \end{bmatrix} < 0,$$

thus, polyquadratic stability of the uncertain system (1a) with $\omega(t) = 0$ follows from Theorem 1 in Cao and Lin (2004).

Since $P_{1i} > 0$, according to (3) and (7), we have

$$\frac{dP_1(\theta)}{dt} = P_1(\dot{\theta}) \leq \sum_{i=1}^N v_i \theta_i P_{1i}. \quad (12)$$

Multiplying each inequality (11) by the non-negative scalar θ_i and then summing up the results for all $i = 1, \dots, N$ give

by Schur complement formula (Boyd et al. 1994). Differentiating $V(x, \theta)$ given by (10) with respect to t gives

$$\begin{aligned} \frac{dV(x, \theta)}{dt} &= 2x^T(t)P_1(\theta)\dot{x}(t) + x^T(t) \\ &\quad \times \frac{dP_1(\theta)}{dt}x(t) = 2 \begin{bmatrix} x^T(t) & y^T(t) \end{bmatrix} \\ &\quad P^T(\theta) \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} + x^T(t)P_1(\dot{\theta})x(t). \end{aligned} \quad (15)$$

Then, substituting (8) into (15) and taking (14) into account yield

$$\begin{aligned} \frac{dV(x, \theta)}{dt} &+ z_1^T(t)z_1(t) - \gamma^2 \omega^T(t)\omega(t) \\ &= 2 \begin{bmatrix} x^T(t) & y^T(t) \end{bmatrix} P^T(\theta) \begin{bmatrix} y(t) \\ -y(t) + A(\theta)x(t) + B_1(\theta)\omega(t) \end{bmatrix} \\ &\quad + x^T(t)P_1(\dot{\theta})x(t) + [C_1(\theta)x(t) + D_1(\theta)\omega(t)]^T [C_1(\theta)x(t) \\ &\quad + D_1(\theta)\omega(t)] - \gamma^2 \omega^T(t)\omega(t) \\ &= \xi^T \bar{\Psi}(\theta) \xi < 0, \quad \forall \xi \neq 0 \end{aligned} \quad (16)$$

with $\xi \triangleq \text{col}\{x(t), y(t), \omega(t)\}$.

Since $V(0) = 0$ and $V(\infty) \geq 0$, integrating (16) in t from 0 to ∞ produces

$$\begin{aligned} J(\omega) &= \int_0^\infty \left\{ \frac{dV(x, \theta)}{dt} + z_1^T(t)z_1(t) - \gamma^2 \omega^T(t)\omega(t) \right\} dt \\ &\quad - (V(\infty) - V(0)) < 0, \end{aligned} \quad (17)$$

which means that the uncertain system (1) has polyquadratic H_∞ performance γ . $\square$

$$\Psi(\theta) \triangleq \begin{bmatrix} A^T(\theta)P_2 + P_2^T A(\theta) + \sum_{i=1}^N v_i \theta_i P_{1i} & P_1(\theta) - P_2^T + A^T(\theta)P_3 & P_2^T B_1(\theta) & C_1^T(\theta) \\ * & -P_3 - P_3^T & P_3^T B_1(\theta) & 0 \\ * & * & -\gamma^2 I & D_1^T(\theta) \\ * & * & * & -I \end{bmatrix} < 0. \quad (13)$$

From (12) and (13), we have

$$\begin{bmatrix} A^T(\theta)P_2 + P_2^T A(\theta) + P_1(\dot{\theta}) & P_1(\theta) - P_2^T + A^T(\theta)P_3 & P_2^T B_1(\theta) & C_1^T(\theta) \\ * & -P_3 - P_3^T & P_3^T B_1(\theta) & 0 \\ * & * & -\gamma^2 I & D_1^T(\theta) \\ * & * & * & -I \end{bmatrix} \leq \Psi(\theta) < 0,$$

which is equivalent to

$$\begin{aligned} \bar{\Psi}(\theta) &\triangleq \begin{bmatrix} A^T(\theta)P_2 + P_2^T A(\theta) + P_1(\dot{\theta}) & P_1(\theta) - P_2^T + A^T(\theta)P_3 & P_2^T B_1(\theta) \\ * & -P_3 - P_3^T & P_3^T B_1(\theta) \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C_1^T(\theta) \\ 0 \\ D_1^T(\theta) \end{bmatrix} \\ &\quad \times \begin{bmatrix} C_1(\theta) & 0 & D_1(\theta) \end{bmatrix} < 0 \end{aligned} \quad (14)$$

Remark 1: In Proposition 2.1 which is based on quadratic stability approach, the Lyapunov matrices P_{1i} corresponding to each polytopic uncertainty vertexes i are required to be the same, i.e. $P_{1i} = P$. For polyquadratic H_∞ performance criterion (11), P_{1i} are not required to be the same.

The term $\frac{dP_1(\theta)}{dt} = P_1(\dot{\theta}) = 0$ in (12) holds provided that the uncertain parameters in (1) are time-invariant such that $\dot{\theta}_i = 0$, $i = 1, \dots, N$. From the proof of Theorem 3.2, we thus have the following polyquadratic H_∞ performance criterion for time-invariant polytopic systems.

Proposition 3.3: Suppose that the uncertain parameters in (1) are time-invariant. The uncertain system (1) has polyquadratic H_∞ performance γ in the uncertainty domain (2) if there exist matrices $P_{1i} > 0$, P_2 and P_3 such that

$$\begin{bmatrix} A_i^T P_2 + P_2^T A_i & P_{1i} - P_2^T + A_i^T P_3 & P_2^T B_{1i} & C_{1i}^T \\ * & -P_3 - P_3^T & P_3^T B_{1i} & 0 \\ * & * & -\gamma^2 I & D_{1i}^T \\ * & * & * & -I \end{bmatrix} < 0. \quad (18)$$

Remark 2: In fact, it can be easily verified that Proposition 3.3 is equivalent to Theorem 2 in He et al. (2005).

Corollary 3.4: For a prescribed scalar $\gamma > 0$, if there exists a matrix $P_1 > 0$ such that (5) holds, then there exist $P_{1i} > 0$, P_2 and P_3 such that (18) holds. But the converse is not true.

Proof: If there exists a matrix $P_1 > 0$ such that (5) holds, then there exists a sufficiently small scalar $\alpha_1 > 0$ such that

$$\begin{bmatrix} A_i^T P_1 + P_1 A_i & P_1 B_{1i} & C_{1i}^T \\ * & -\gamma^2 I & D_{1i}^T \\ * & * & -I \end{bmatrix} + \frac{\alpha_1}{2} \begin{bmatrix} A_i^T P_1 \\ B_{1i}^T P_1 \\ 0 \end{bmatrix} \times P_1^{-1} \begin{bmatrix} A_i^T P_1 \\ B_{1i}^T P_1 \\ 0 \end{bmatrix}^T < 0, \quad (19)$$

which is equivalent to

$$\begin{bmatrix} A_i^T P_1 + P_1 A_i & P_1 - P_1 + \alpha_1 A_i^T P_1 & P_1 B_{1i} & C_{1i}^T \\ * & -2\alpha_1 P_1 & \alpha_1 P_1 B_{1i} & 0 \\ * & * & -\gamma^2 I & D_{1i}^T \\ * & * & * & -I \end{bmatrix} < 0 \quad (20)$$

by Schur complement formula. Hence, there exist $P_{1i} = P_2 = P_1 > 0$ and $P_3 = \alpha_1 P_2 = \alpha_1 P_1$, such that (18) holds. Example 7.1 in Section 7 will show that the converse is not true. $\square$

Remark 3: Corollary 3.4 shows that the conditions in Proposition 3.3 are less conservative than those in Proposition 2.1. This can be seen further by Example 7.1 in Section 7.

4. Robust H_∞ control synthesis

In this section, we apply Theorem 3.2 presented in Section 3 to address the robust state-feedback H_∞ control problem.

Consider the continuous-time linear uncertain system described by

$$\dot{x}(t) = A(\theta)x(t) + B_1(\theta)\omega(t) + B_2(\theta)u(t), \quad (21a)$$

$$z_1(t) = C_1(\theta)x(t) + D_1(\theta)\omega(t) + D_2(\theta)u(t), \quad (21b)$$

where $x(t)$, $\omega(t)$ and $z_1(t)$ are defined in Section 2, $u(t) \in R^q$ is the control input and θ might be time-varying and satisfies (3). Suppose that the system matrices $A(\theta)$, $B_1(\theta)$, $B_2(\theta)$, $C_1(\theta)$, $D_1(\theta)$ and $D_2(\theta)$ belong to the convex polytopic set

$$\begin{aligned} \Omega_2 = & \{ [A(\theta) \ B_1(\theta) \ B_2(\theta) \ C_1(\theta) \ D_1(\theta) \ D_2(\theta)] \\ & : [A(\theta) \ B_1(\theta) \ B_2(\theta) \ C_1(\theta) \ D_1(\theta) \ D_2(\theta)] \\ & = \sum_{i=1}^N \theta_i [A_i \ B_{1i} \ B_{2i} \ C_{1i} \ D_{1i} \ D_{2i}], \theta_i \geq 0, \sum_{i=1}^N \theta_i = 1 \}. \end{aligned} \quad (22)$$

For system (21), we consider a state-feedback controller

$$u(t) = Kx(t), \quad (23)$$

where $K \in R^{q \times n}$ is a constant matrix gain to be designed. The resulting closed-loop system is given by

$$\dot{x}(t) = (A(\theta) + B_2(\theta)K)x(t) + B_1(\theta)\omega(t), \quad (24a)$$

$$z_1(t) = (C_1(\theta) + D_2(\theta)K)x(t) + D_1(\theta)\omega(t). \quad (24b)$$

We look for the state-feedback gain matrix K , such that the closed-loop system (24) satisfies the sufficient conditions for polyquadratic H_∞ performance.

Theorem 4.1: For a prescribed scalar $\gamma > 0$, the closed-loop system (24) has polyquadratic H_∞ performance γ in the uncertainty domain (22), if there exist matrices $X_{1i} > 0$, X_2 , Y and scalar α_2 such that

$$\begin{bmatrix} \Pi_i \left\{ \begin{array}{l} X_{1i} - X_2 + \alpha_2 X_2^T A_i^T \\ + \alpha_2 Y^T B_{2i}^T \end{array} \right\} & B_{1i} & X_2^T C_{1i}^T + Y^T D_{2i}^T \\ * & -\alpha_2 X_2 - \alpha_2 X_2^T & \alpha_2 B_{1i} & 0 \\ * & * & -\gamma^2 I & D_{1i}^T \\ * & * & * & -I \end{bmatrix} < 0, \quad i = 1, \dots, N, \quad (25)$$

where $\Pi_i = X_2^T A_i^T + A_i X_2 + B_{2i} Y + Y^T B_{2i}^T + v_i X_{1i}$. The state-feedback gain is then given by $K = YX_2^{-1}$.

Proof: Note that (25) implies $\alpha_2 X_2 + \alpha_2 X_2^T > 0$, and X_2 is obviously non-singular. We define $P_2 = X_2^{-1}$, $P_3 = \alpha_2 P_2 = \alpha_2 X_2^{-1}$, $P_{1i} = P_2^T X_{1i} P_2 > 0$ and $Y = KX_2$. Multiplying (25) by $\text{diag}\{P_2^T, P_2^T, I, I\}$ and $\text{diag}\{P_2, P_2, I, I\}$ on the left and the right, respectively, we obtain

$$\begin{bmatrix} \left\{ \begin{array}{c} A_{cli}^T P_2 + P_2^T A_{cli} \\ + v_i P_{1i} \end{array} \right\} & \left\{ \begin{array}{c} P_{1i} - P_2^T \\ + A_{cli}^T P_3 \end{array} \right\} & P_2^T B_{1i} & C_{cli}^T \\ * & -P_3 - P_3^T & P_3^T B_{1i} & 0 \\ * & * & -\gamma^2 I & D_{1i}^T \\ * & * & * & -I \end{bmatrix} < 0, \quad (26)$$

where $A_{cli} = A_i + B_{2i}K$ and $C_{cli} = C_{1i} + D_{2i}K$. For the closed-loop system (24), applying Theorem 3.2 completes the proof. $\square$

Remark 4: Theorem 4.1 contains the tuning scalar parameter α_2 . In order to obtain the minimum value of the disturbance attenuation level γ , the optimum value α_2 can be ascertained by the approach in Remark 4 of Xie et al. (2004). Then applying a numerical optimisation algorithm, such as the program *fminsearch* in the optimisation toolbox of Matlab (Coleman, Branch and Grace 1999), a numerical solution to the optimisation problem is obtained.

5. Robust H_∞ filtering design

In this section, we apply the results in Section 3 to discuss the robust H_∞ filtering problem for polytopic systems. First, the robust H_∞ filtering problem is formulated. Next, we apply the results in Section 3 to give sufficient conditions for the existence of robust H_∞ filters. We then apply the conditions to the design of the robust H_∞ filter.

Consider the following polytopic system:

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B_1(\theta)\omega(t), \\ y_2(t) &= C_2(\theta)x(t) + D_2(\theta)\omega(t), \\ z_2(t) &= L_1(\theta)x(t) + L_2(\theta)\omega(t), \end{aligned} \quad (27)$$

where $x(t)$ and $\omega(t)$ are defined in Section 2, $y_2(t) \in R^m$ is the measurement output, and $z_2(t) \in R^l$ is the estimated signal. Similar to the polytopic system (1), we also assume that θ might be time-varying and satisfies (3), and the system matrices $A(\theta)$, $B_1(\theta)$, $C_2(\theta)$, $D_2(\theta)$, $L_1(\theta)$ and $L_2(\theta)$ belong to the convex polytopic set

$$\begin{aligned} \Omega_3 = & \left\{ [A(\theta) \ B_1(\theta) \ C_2(\theta) \ D_2(\theta) \ L_1(\theta) \ L_2(\theta)] \right. \\ & : [A(\theta) \ B_1(\theta) \ C_2(\theta) \ D_2(\theta) \ L_1(\theta) \ L_2(\theta)] \\ & = \sum_{i=1}^N \theta_i [A_i \ B_{1i} \ C_{2i} \ D_{2i} \ L_{1i} \ L_{2i}], \theta_i \geq 0, \sum_{i=1}^N \theta_i = 1 \left. \right\}. \end{aligned} \quad (28)$$

We consider a filter of the same order as system (27) described by

$$\begin{aligned} \dot{\hat{x}}(t) &= A_f \hat{x}(t) + B_f y_2(t), \\ \hat{z}_2(t) &= C_f \hat{x}(t) + D_f y_2(t), \end{aligned} \quad (29)$$

where the matrices (A_f, B_f, C_f, D_f) are to be designed.

We denote $\zeta = \text{col}\{x(t), \hat{x}(t)\}$ and $e = z_2(t) - \hat{z}_2(t)$. It follows from (27) and (29) that the following system (called the filtering error system) is given by

$$\begin{aligned} \dot{\zeta}(t) &= \bar{A}\zeta + \bar{B}\omega, \\ e &= \bar{C}\zeta + \bar{D}\omega, \end{aligned} \quad (30)$$

where:

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A(\theta) & 0 \\ B_f C_2(\theta) & A_f \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_1(\theta) \\ B_f D_2(\theta) \end{bmatrix}, \\ \bar{C} &= [L_1(\theta) - D_f C_2(\theta) \quad -C_f], \quad \bar{D} = L_2(\theta) - D_f D_2(\theta). \end{aligned}$$

The robust H_∞ filtering problem to be investigated is stated as follows.

Polyquadratic H_∞ filtering problem: Find a filter of form (29) such that:

- The error system (30) with $\omega(t)=0$ is polyquadratically stable;
- Given a scalar $\gamma > 0$, system (30) satisfies the performance index $J_e(\omega) = \int_0^\infty (e^T e - \gamma^2 \omega^T \omega) dt < 0$, for all non-zero $\omega(t) \in L_2[0, \infty)$ and all admissible trajectories of the parameter vector θ satisfying (28) and (3).

Next, we apply Theorem 3.2 in Section 3 to give sufficient conditions for the existence of polyquadratical H_∞ filters for system (30).

Lemma 5.1: Given a filter (A_f, B_f, C_f, D_f) and a scalar $\gamma > 0$, the error system (30) is polyquadratically stable and satisfies $J_e(\omega) < 0$ for all non-zero $\omega(t) \in L_2[0, \infty)$ if there exist matrices $\bar{P}_{1i} > 0$, $\bar{P}_2$ and $\bar{P}_3$ such that

$$\begin{bmatrix} \bar{A}_i^T \bar{P}_2 + \bar{P}_2^T \bar{A}_i + v_i \bar{P}_{1i} & \bar{P}_{1i} - \bar{P}_2^T + \bar{A}_i^T \bar{P}_3 & \bar{P}_2^T \bar{B}_i & \bar{C}_i^T \\ * & -\bar{P}_3 - \bar{P}_3^T & \bar{P}_3^T \bar{B}_i & 0 \\ * & * & -\gamma^2 I & \bar{D}_i^T \\ * & * & * & -I \end{bmatrix} < 0, \quad i = 1, \dots, N, \quad (31)$$

where:

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ B_f C_{2i} & A_f \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_{1i} \\ B_f D_{2i} \end{bmatrix}, \\ \bar{C}_i &= [L_{1i} - D_f C_{2i} \quad -C_f], \quad \bar{D}_i = L_{2i} - D_f D_{2i}. \end{aligned}$$

Although Lemma 5.1 can be used to test the disturbance attenuation level γ of system (30) provided

that a filter (29) is given, it may not be directly applicable to the polyquadratic H_∞ filtering design problem due to the presence of the products of $\bar{P}_2$ with $\bar{A}_i$ and $\bar{P}_3$ with $\bar{A}_i$. As discussed in Xie et al. (2004), to facilitate the H_∞ filtering design, we specify the matrix $\bar{P}_2$ as

$$\bar{P}_2 = \Lambda \bar{P}_3, \quad (32)$$

where $\Lambda = \text{diag}\{\lambda_1 I_n, \lambda_2 I_n\}$ with λ_1 and λ_2 being any real scalars. Substituting (32) into (31) leads to

$$\begin{bmatrix} \left\{ \begin{array}{c} \bar{A}_i^T \Lambda \bar{P}_3 + \bar{P}_3^T \Lambda \bar{A}_i \\ + v_i \bar{P}_{1i} \end{array} \right\} & \left\{ \begin{array}{c} \bar{P}_{1i} - \bar{P}_3^T \Lambda \\ + \bar{A}_i^T \bar{P}_3 \end{array} \right\} & \bar{P}_3^T \Lambda \bar{B}_i & \bar{C}_i^T \\ * & -\bar{P}_3 - \bar{P}_3^T & \bar{P}_3^T \bar{B}_i & 0 \\ * & * & -\gamma^2 I & \bar{D}_i^T \\ * & * & * & -I \end{bmatrix} < 0. \quad (33)$$

Note that the conditions in (33) are LMIs over the matrix variables $\bar{P}_{1i}$, $\bar{P}_3$ and scalar variable γ^2 if the filter matrices (A_f, B_f, C_f, D_f) are given. However, since our purpose is to determine the filter matrices (A_f, B_f, C_f, D_f), conditions in (33) are actually non-LMIs. Here, our main purpose is, if possible, to convert the non-LMIs in (33) into LMIs. If this purpose is accomplished, the H_∞ filtering design problem turns out to be a convex programming problem that can be solved with recent standard numerical tools. To achieve the purpose, we give the following result whose conditions are equivalent to that of (33).

Proposition 5.2: Consider the polytopic system (27). For a prescribed scalar $\gamma > 0$, there exist filter matrices (A_f, B_f, C_f, D_f), matrices $\bar{P}_{1i} > 0$ and $\bar{P}_3$ satisfying (33), if and only if there exist matrices $\bar{P}_{11i} > 0$, $\bar{P}_{11i} \in R^{n \times n}$, $\bar{P}_{22i} > 0$, $\bar{P}_{22i} \in R^{n \times n}$, $\bar{P}_{12i} \in R^{n \times n}$, $R \in R^{n \times n}$, $F \in R^{n \times n}$, $U \in R^{n \times n}$, $\bar{A}_f \in R^{n \times n}$, $\bar{B}_f \in R^{n \times m}$, $\bar{C}_f \in R^{l \times n}$, $\bar{D}_f \in R^{l \times m}$, and scalars λ_1, λ_2 such that

$$\begin{bmatrix} \Xi_{11i} & \Xi_{12i} & \Xi_{13i} & \Xi_{14i} & \Xi_{15i} & \Xi_{16i} \\ * & \Xi_{22i} & \Xi_{23i} & \Xi_{24i} & \Xi_{25i} & \Xi_{26i} \\ * & * & \Xi_{33i} & \Xi_{34i} & \Xi_{35i} & 0 \\ * & * & * & \Xi_{44i} & R^T B_{1i} & 0 \\ * & * & * & * & -\gamma^2 I & \Xi_{56i} \\ * & * & * & * & * & -I \end{bmatrix} < 0, \quad i = 1, 2, \dots, N, \quad (34)$$

where:

$$\begin{aligned} \Xi_{11i} &= \lambda_1(A_i^T F + F^T A_i) + \lambda_2(\bar{B}_f C_{2i} + C_{2i}^T \bar{B}_f^T) + v_i \bar{P}_{11i}, \\ \Xi_{12i} &= \lambda_1(A_i^T R + F^T A_i) + \lambda_2(\bar{B}_f C_{2i} + A_f) + v_i \bar{P}_{12i}, \\ \Xi_{13i} &= \bar{P}_{11i} - \lambda_1 F^T + A_i^T F + C_{2i}^T \bar{B}_f^T, \\ \Xi_{14i} &= \bar{P}_{12i} - \lambda_1 F^T - \lambda_2 U + A_i^T R, \end{aligned}$$

$$\begin{aligned} \Xi_{15i} &= \lambda_1 F^T B_{1i} + \lambda_2 \bar{B}_f D_{2i}, \\ \Xi_{16i} &= L_{1i}^T - C_{2i}^T \bar{D}_f, \\ \Xi_{22i} &= \lambda_1(A_i^T R + R^T A_i) + v_i \bar{P}_{22i}, \\ \Xi_{23i} &= \bar{P}_{12i}^T - \lambda_1 R^T + A_i^T F + C_{2i}^T \bar{B}_f^T + \bar{A}_f^T, \\ \Xi_{24i} &= \bar{P}_{22i} - \lambda_1 R^T + A_i^T R, \\ \Xi_{25i} &= \lambda_1 R^T B_{1i} \\ \Xi_{26i} &= L_{1i}^T - C_{2i}^T \bar{D}_f - \bar{C}_f, \\ \Xi_{33i} &= -F - F^T, \\ \Xi_{34i} &= -F^T - U - R, \\ \Xi_{35i} &= F^T B_{1i} + \bar{B}_f D_{2i}, \\ \Xi_{44i} &= -R - R^T, \\ \Xi_{56i} &= L_{2i}^T - D_{2i}^T \bar{D}_f^T. \end{aligned}$$

Proof Necessity. Note that (33) implies

$$\bar{P}_3 + \bar{P}_3^T > 0, \quad (35)$$

which means $\bar{P}_3$ is non-singular. Let $\bar{P}_3$ and $\bar{P}_3^{-1}$ be partitioned as

$$\bar{P}_3 = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \quad \bar{P}_3^{-1} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}.$$

Without loss of generality, we assume that G_{21} and S_{21} are all square and non-singular matrices (Apkarian, Tuan and Bernussou 2001). Define the following matrices:

$$J_G \triangleq \begin{bmatrix} G_{11} & I \\ G_{21} & 0 \end{bmatrix}, \quad J_S \triangleq \begin{bmatrix} I & S_{11} \\ 0 & S_{21} \end{bmatrix}. \quad (36)$$

Therefore, J_G and J_S are all non-singular and we have $\bar{P}_3^{-1} J_G = J_S$, $\bar{P}_3 J_S = J_G$. Multiplying (35) by J_S^T and J_S , on the left and the right, respectively, yields

$$J_S^T (\bar{P}_3 + \bar{P}_3^T) J_S = \begin{bmatrix} G_{11} + G_{11}^T & G_{11}^T S_{11} + G_{21}^T S_{21} + I \\ * & S_{11} + S_{11}^T \end{bmatrix} > 0.$$

Thus, S_{11} is non-singular. Define the following matrices

$$J_3 \triangleq J_S \begin{bmatrix} I & 0 \\ 0 & S_{11}^{-1} \end{bmatrix} = \begin{bmatrix} I & I \\ 0 & S_{21} S_{11}^{-1} \end{bmatrix}. \quad (37)$$

Therefore, J_3 is also non-singular. Multiplying (33) by $\text{diag}\{J_3^T, J_3^T, I, I\}$ and $\text{diag}\{J_3, J_3, I, I\}$, on the left and the right, respectively, gives rise to

$$\begin{bmatrix} \left\{ \begin{array}{c} J_3^T (\bar{A}_i^T \Lambda \bar{P}_3 \\ + \bar{P}_3^T \Lambda \bar{A}_i \\ + v_i \bar{P}_{11i}) J_3 \end{array} \right\} & \left\{ \begin{array}{c} J_3^T (\bar{P}_{1i} \\ - \bar{P}_3^T \Lambda \\ + \bar{A}_i^T \bar{P}_3) J_3 \end{array} \right\} & \left\{ \begin{array}{c} J_3^T \bar{P}_3^T \\ \times \Lambda \bar{B}_i \end{array} \right\} & J_3^T \bar{C}_i^T \\ * & J_3^T (-\bar{P}_3 - \bar{P}_3^T) J_3 & J_3^T \bar{P}_3^T \bar{B}_i & 0 \\ * & * & -\gamma^2 I & \bar{D}_i^T \\ * & * & * & -I \end{bmatrix} < 0, \quad (38)$$

where:

$$J_3^T \bar{P}_{1i} J_3 \triangleq \begin{bmatrix} \bar{P}_{11i} & \bar{P}_{12i} \\ \bar{P}_{12i}^T & \bar{P}_{22i} \end{bmatrix}, \quad (39)$$

$$J_3^T \bar{P}_3 J_3 = \begin{bmatrix} G_{11}^T & G_{11}^T + G_{21}^T S_{21}^{-1} S_{11}^{-1} \\ S_{11}^{-T} & S_{11}^{-T} \end{bmatrix}, \quad (40)$$

$$J_3^T \bar{A}_i^T \bar{P}_3 J_3 = \begin{bmatrix} A_i^T G_{11} + C_{2i}^T B_f^T G_{21} & A_i^T S_{11}^{-1} \\ \left\{ \begin{array}{c} A_i^T G_{11} + C_{2i}^T B_f^T G_{21} \\ + S_{11}^{-T} S_{21}^T A_i^T G_{21} \end{array} \right\} & A_i^T S_{11}^{-1} \end{bmatrix}, \quad (41)$$

$$J_3^T \bar{P}_3 \bar{B}_i = \begin{bmatrix} G_{11}^T B_{1i} + G_{21}^T B_f D_{2i} \\ S_{11}^{-T} B_{1i} \end{bmatrix}, \quad (42)$$

$$J_3^T \bar{C}_i^T = \begin{bmatrix} L_{1i}^T - C_{2i}^T D_f^T \\ L_{1i}^T - C_{2i}^T D_f^T - S_{11}^{-T} S_{21}^T C_f^T \end{bmatrix}. \quad (43)$$

Define the following matrix variables:

$$\begin{aligned} R &\triangleq S_{11}^{-1}, & F &\triangleq G_{11}, & U &\triangleq G_{21}^T S_{21} R, \\ \bar{A}_f &\triangleq G_{21}^T A_f S_{21} R, & \bar{B}_f &\triangleq G_{21}^T B_f, \\ \bar{C}_f &\triangleq C_f S_{21} R, & \bar{D}_f &\triangleq D_f. \end{aligned} \quad (44)$$

Substituting (39)–(43) into (38) and applying (44), we obtain (34), and the necessity is proved.

Sufficiency. Note that (34) implies

$$\begin{bmatrix} F + F^T & F^T + U + R \\ * & R + R^T \end{bmatrix} > 0, \quad (45)$$

F and R are all non-singular matrices. Then U must be a non-singular matrix, because otherwise there exists a vector $x \neq 0$ satisfying $Ux = 0$ and we have

$$\begin{bmatrix} x^T & -x^T \end{bmatrix} \begin{bmatrix} F + F^T & F^T + U + R \\ * & R + R^T \end{bmatrix} \begin{bmatrix} x \\ -x \end{bmatrix} = 0,$$

which contradicts to (45). Hence, one can always find square and non-singular matrices S_{21} and G_{21} satisfying $UR^{-1} = G_{21}^T S_{21}$ (Scherer, Scherer and Chilali 1997). Now construct the uniquely-defined filter matrices (A_f, B_f, C_f, D_f) as

$$\begin{aligned} A_f &= G_{21}^{-T} \bar{A}_f R^{-1} S_{21}^{-1}, & B_f &= G_{21}^{-T} \bar{B}_f, \\ C_f &= \bar{C}_f R^{-1} S_{21}^{-1}, & D_f &= \bar{D}_f. \end{aligned} \quad (46)$$

Next, select the following matrix

$$G_{11} = F, \quad S_{11} = R^{-1}, \quad U = G_{21}^T S_{21} R, \quad (47)$$

and define J_G and J_S as in (36), J_3 as in (37) and the non-singular matrix $\bar{P}_3 \triangleq J_G J_S^{-1}$. Then, it can be verified that (39)–(43) hold. Substituting (39)–(44) into (34) yields (38). Then, (33) follows by multiplying (38) by $\text{diag}\{J_3^{-T}, J_3^{-T}, I, I\}$ and $\text{diag}\{J_3^{-1}, J_3^{-1}, I, I\}$, on the left and the right, respectively, and the sufficiency proof is completed. $\square$

From the earlier proof, we know that an admissible filter can be constructed from (46). However, there seems to be no systematic way to determine the matrices S_{21} and G_{21} needed for the filter matrices. To deal with this problem, we consider the filter transfer function from measured output $y_2(t)$ to estimated signal $\hat{z}_2(t)$

$$T_{\hat{z}_2 y_2} = C_f (sI - A_f)^{-1} B_f + D_f.$$

Substituting the filter matrices (46) into the above transfer function and considering the relationship $U = G_{21}^T S_{21} R$ yield

$$\begin{aligned} T_{\hat{z}_2 y_2} &= \bar{C}_f R^{-1} S_{21}^{-1} (sI - G_{21}^{-T} \bar{A}_f R^{-1} S_{21}^{-1})^{-1} G_{21}^{-T} \bar{B}_f + \bar{D}_f \\ &= \bar{C}_f (sU - \bar{A}_f)^{-1} \bar{B}_f + \bar{D}_f \\ &= \bar{C}_f (sI - U^{-1} \bar{A}_f)^{-1} U^{-1} \bar{B}_f + \bar{D}_f. \end{aligned} \quad (48)$$

Therefore, we can conclude from (48) that the parameters of the filter to be specified in (29) can be constructed by

$$A_f = U^{-1} \bar{A}_f, \quad B_f = U^{-1} \bar{B}_f, \quad C_f = \bar{C}_f, \quad D_f = \bar{D}_f. \quad (49)$$

Note that for given scalars λ_1 and λ_2 , conditions in (34) are not only LMI over the matrix variables, but also over the scalar γ^2 . According to Lemma 5.1 and Proposition 5.2, we can cast the polyquadratic H_∞ filtering problem into an LMI feasibility test, and any feasible solution to (34) will yield a suitable filter. Hence, the suboptimal polyquadratic H_∞ filter of the form (29) can be determined by solving the following convex optimisation problem.

Theorem 5.3: Consider the polytopic system (27). The suboptimal polyquadratic H_∞ filter of the form (29) can be determined by solving the following convex optimisation problem:

$$\begin{aligned} &\min_{\bar{P}_{11i}, \bar{P}_{12i}, \bar{P}_{22i}, R, F, U, \bar{A}_f, \bar{B}_f, \bar{C}_f, \bar{D}_f, \lambda_1, \lambda_2} \gamma^2 \\ &\text{subject to } \bar{P}_{11i} > 0, \bar{P}_{22i} > 0, \text{ and (34)}. \end{aligned}$$

The suboptimal filter is given by (49).

Remark 5: Theorem 5.3 contains the tuning parameters λ_1 and λ_2 . As for the problem of searching for optimal scaling parameters λ_1 and λ_2 , a similar procedure as mentioned in Remark 4 can be applied.

6. Robust H_∞ control of affine parameter systems

In this section, we also apply the descriptor system approach presented to explore robust H_∞ performance analysis and controller synthesis of affine parameter systems.

Consider the continuous-time linear uncertain system

$$\dot{x}(t) = A(\theta)x(t) + B_u(\theta)u(t) + B_\omega(\theta)\omega(t), \quad (50a)$$

$$z_1(t) = C_1(\theta)x(t) + D_u(\theta)u(t) + D_\omega(\theta)\omega(t), \quad (50b)$$

where $x(t)$, $u(t)$, $\omega(t)$ and $z_1(t)$ are defined as in Section 4. We assume that the system matrices $A(\theta)$, $B_u(\theta)$, $B_\omega(\theta)$, $C_1(\theta)$, $D_u(\theta)$ and $D_\omega(\theta)$ depend affinely on the parameters θ_i . That is,

$$\begin{bmatrix} A(\theta) & B_u(\theta) & B_\omega(\theta) \\ C_1(\theta) & D_u(\theta) & D_\omega(\theta) \end{bmatrix} = \begin{bmatrix} A_0 & B_{u0} & B_{\omega0} \\ C_{10} & D_{u0} & D_{\omega0} \end{bmatrix} + \theta_1 \begin{bmatrix} A_1 & B_{u1} & B_{\omega1} \\ C_{11} & D_{u1} & D_{\omega1} \end{bmatrix} + \dots + \theta_N \begin{bmatrix} A_N & B_{uN} & B_{\omega N} \\ C_{1N} & D_{uN} & D_{\omega N} \end{bmatrix}, \quad (51)$$

where

$$\begin{bmatrix} A_0 & B_{u0} & B_{\omega0} \\ C_{10} & D_{u0} & D_{\omega0} \end{bmatrix}, \begin{bmatrix} A_1 & B_{u1} & B_{\omega1} \\ C_{11} & D_{u1} & D_{\omega1} \end{bmatrix}, \dots, \begin{bmatrix} A_N & B_{uN} & B_{\omega N} \\ C_{1N} & D_{uN} & D_{\omega N} \end{bmatrix}$$

are known constant matrices.

As discussed in Gahinet et al. (1996), we also assume that the lower and upper bounds of the parameter values and their rates of variation are known. Specifically, parameters θ_i and their derivatives $\dot{\theta}_i$ satisfy:

- (i) Parameter θ_i range between known external values $\underline{\theta}_i$ and $\bar{\theta}_i$, i.e.

$$\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]; \quad (52)$$

- (ii) The derivatives $\dot{\theta}_i$ are well defined at all times and satisfy

$$\dot{\theta}_i \in [\underline{v}_i, \bar{v}_i], \quad (53)$$

where $\underline{v}_i \leq 0 \leq \bar{v}_i$ are known lower and upper bounds on $\dot{\theta}_i$.

Condition (i) means that the parameter vector θ takes values in a hyper-rectangle called the parameter box. Condition (ii) means that the derivative vector $\dot{\theta}$ also takes values in a hyper-rectangle called the variation rate box. Next, we denote

$$\Gamma \triangleq \{(\delta_1, \delta_2, \dots, \delta_N)^T : \delta_i \in [\underline{\theta}_i, \bar{\theta}_i]\},$$

$$\Upsilon \triangleq \{(\tau_1, \tau_2, \dots, \tau_N)^T : \tau_i \in [\underline{v}_i, \bar{v}_i]\}$$

the sets of the 2^N vertices of the parameter box and the variation rate box, respectively.

As in Gahinet et al. (1996), for the affine parameter system (50), we choose $P_1(\theta)$ in the Lyapunov candidate (6) to have equivalently affine form

$$P_1(\theta) = P_{10} + \theta_1 P_{11} + \dots + \theta_N P_{1N}. \quad (54)$$

Based on this expression, the time derivative of $P_1(\theta)$ can be derived as

$$dP_1(\theta)/dt = \dot{\theta}_1 P_{11} + \dots + \dot{\theta}_N P_{1N} = P_1(\dot{\theta}) - P_{10}. \quad (55)$$

We now recall the notion of affine quadratic H_∞ performance.

Definition 6.1: (Affine quadratic H_∞ performance (Gahinet et al. 1996)). For a prescribed scalar $\gamma > 0$, the unforced uncertain system (50) with $u(t) = 0$ has affine quadratic H_∞ performance γ if there exists a parameter-dependent quadratic Lyapunov function $V(x, \theta) = x^T P_1(\theta)x$ with $P_1(\theta)$ given in (54), such that:

- The unforced uncertain system (50a) with $\omega(t) = 0$ is affinely quadratically stable, i.e. $dV/dt < 0$, $\forall x \neq 0$;
- Under the initial condition $x(0) = 0$, the performance index $J(\omega) < 0$ holds for all non-zero $\omega(t) \in L_2[0, \infty)$ and all admissible trajectories of the parameter vector θ satisfying (52) and (53).

The following theorem gives an affine quadratic H_∞ performance criterion based on LMIs.

Theorem 6.2: The unforced uncertain system (50) has affine quadratic H_∞ performance γ in the uncertainty domain (52)–(53) if there exist $N+1$ symmetric matrices $P_{10}, P_{11}, \dots, P_{1N}$, matrices P_2 and P_3 such that:

$$P_{10} + \delta_1 P_{11} + \dots + \delta_N P_{1N} > 0,$$

$$\begin{bmatrix} \begin{bmatrix} A^T(\delta)P_2 \\ + P_2^T A(\delta) \\ + P_1(\tau) - P_{10} \end{bmatrix} & \begin{bmatrix} P_1(\delta) - P_2^T \\ + A^T(\delta)P_3 \end{bmatrix} & P_2^T B_\omega(\delta) & C_1^T(\delta) \\ * & -P_3 - P_3^T & P_3^T B_\omega(\delta) & 0 \\ * & * & -\gamma^2 I & D_\omega^T(\delta) \\ * & * & * & -I \end{bmatrix} < 0$$

for all $(\delta, \tau) \in \Gamma \times \Upsilon$.

Proof: The proof follows the similar approach as Theorem 3.2, and will be omitted for conciseness. $\square$

Remark 6: LMI sufficient conditions for affine quadratic H_∞ performance were also proposed in Theorem 4.2 of Gahinet et al. (1996). As discussed in Gahinet et al. (1996), controller design is a very difficult problem because of the presence of the quadratic function of the parameters and their rates

in the LMIs. In contrast, based on Theorem 6.2, it is very easy to obtain an LMI design method for the state feedback controller as follows.

Now, we apply Theorem 6.2 to solve the problem of designing a robust H_∞ state-feedback controller.

Theorem 6.3: *For a prescribed scalar $\gamma > 0$, the uncertain system (50) is affinely quadratically stabilisable and the performance index $J(\omega) < 0$ is satisfied in the uncertainty domain (52)–(53) if there exist $N+1$ symmetric matrices $X_{10}, X_{11}, \dots, X_{1N}$, matrices X_2 and Y , and scalar α_3 such that:*

$$X_{10} + \delta_1 X_{11} + \dots + \delta_N X_{1N} > 0,$$

$$\begin{bmatrix} \begin{Bmatrix} \bar{M}_0(\delta) \\ +X_1(\tau) \\ -X_{10} \end{Bmatrix} & \bar{M}_1(\delta) & B_\omega(\delta) & \begin{Bmatrix} X_2^T C_1^T(\delta) \\ +Y^T D_u^T(\delta) \end{Bmatrix} \\ * & \begin{Bmatrix} -\alpha_3 X_2 \\ -\alpha_3 X_2^T \end{Bmatrix} & \alpha_3 B_\omega(\delta) & 0 \\ * & * & -\gamma^2 I & D_\omega^T(\delta) \\ * & * & * & -I \end{bmatrix} < 0,$$

for all $(\delta, \tau) \in \Gamma \times \Upsilon$, where: $\bar{M}_0(\delta) = X_2^T A^T(\delta) + A(\delta)X_2 + B_u(\delta)Y + Y^T B_u^T(\delta)$, $\bar{M}_1(\delta) = X_1(\delta) - X_2 + \alpha_3 \times X_2^T A^T(\delta) + \alpha_3 Y^T B_u^T(\delta)$.

The state-feedback gain is then given by $K = YX_2^{-1}$.

Proof: The proof is similar to that of Theorem 4.1, and is thus omitted for conciseness. $\square$

7. Examples

Example 7.1: This example is to demonstrate the effectiveness of Proposition 3.3 and Theorem 3.2. We consider the system discussed in Shaked (2001), which is of the form (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -1+g & -1-g \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = [1 \quad -2], \quad D_1 = 0,$$

where g is an uncertain parameter that is known to reside in the interval $[-g_1, g_1]$. The system is stable for $0 \leq g_1 \leq 1$, and quadratically stable for $g_1 < 0.681$ (Shaked 2001).

It is clear that system matrix A resides with the matrix polytope whose vertices are given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1+g_1 & -1-g_1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -1-g_1 & -1+g_1 \end{bmatrix}.$$

First, we suppose that the uncertain parameter g is time-invariant. For comparison, we optimise the disturbance attenuation level γ using the three different approaches: the approach in Palhares et al. (1997),

which is based on quadratic stability; the approaches by Corollary 2.4 in Shaked (2001) and Proposition 3.3, which are both based on the parameter-dependent Lyapunov function method. In the time-invariant case, Proposition 3.3 yields the minimum value of γ , which is the same as the one by He et al. (2005) according to Remark 2. The minimum values of γ are given in Table 1.

For $g_1 \neq 0$, Table 1 indicates that the minimum values of γ using parameter-dependent Lyapunov functions are smaller than those by parameter-independent Lyapunov function (the approach in Palhares et al. (1997)). When the uncertain parameter upper bound reaches $g_1 = 0.681$, the uncertain system is not quadratically stable. In contrast, the conditions in Proposition 3.3 can guarantee robust stability and achieve the disturbance attenuation level $\gamma = 6.7207$. Hence, this example also shows that the conditions in Proposition 3.3 are less conservative than those in Proposition 2.1.

For the nominal system ($g_1 = 0$), when an extra scalar parameter ε is given small enough by Shaked (2001), the minimum value of γ is about the same as the one by Proposition 3.3. The simulation results have demonstrated that Proposition 3.3 gives the best result in any case, as seen from Table 1.

Now, we suppose that g is a time-varying real parameter satisfying $0 \leq \dot{g} \leq a(g + g_1)$, where a is a constant. It describes that $\dot{g}$ resides in the interval $[0, 2ag_1]$. In this case, Corollary 2.4 in Shaked (2001) and Theorem 2 in He et al. (2005) are not directly applicable. Obviously, the minimum values of γ of Palhares et al. (1997) remain unchanged under arbitrary parameter variations. According to $g = (2\theta_1 - 1)g_1$ and $\dot{\theta}_1 \leq \nu_1 \theta_1$ (recall $\theta_2 = 1 - \theta_1$), we have $\nu_1 = a$ and $\nu_2 = 0$. Setting $a = 0.2$, for $g_1 = 0.3777$, $g_1 = 0.681$, and $g_1 = 0.746$, Theorem 3.2 gives values of 3.4965, 7.4580 and 13.4228, respectively, which are clearly much better than those given by Palhares et al. (1997).

Example 7.2: This example is concerned with the robust H_∞ tracking control of *F-18* aircraft model (Hyde 1995; Yang and Lum 2003).

Table 1. The minimum values of γ by the three approaches.

The values of g_1	0	0.3777	0.681	0.746
Palhares et al. (1997)	2.2483	5.0016	–	–
Shaked (2001) $\varepsilon = 0.1$	2.6551	5.0052	32.8751	–
$\varepsilon = 0.03$	2.3618	4.4878	24.6550	4265
$\varepsilon = 0.001$	2.2520	4.8055	86.4505	–
Proposition 3.3	2.2483	3.4963	6.7207	8.4195

Note: Symbol ‘–’ indicates that the method does not give any feasible solution.

A. Aircraft model and problem statement

The dynamical equations of longitudinal motion of the F-18 aircraft are described by

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta E} & Z_{\delta PTV} \\ M_{\delta E} & M_{\delta PTV} \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_{PTV} \end{bmatrix}. \quad (56)$$

In this model, α and q represent angle of attack and pitch rate, respectively; δ_E and δ_{PTV} represent symmetric elevator position and symmetric pitch thrust velocity nozzle position, respectively. Jet aircrafts typically have multiple operating flight conditions that correspond to the convex combination of the given operating points. For the three operating points under consideration, the values of the system matrices parameters are given in the Appendix.

Denoting $x(t) = \text{col}\{\alpha, q\}$, $u(t) = \text{col}\{\delta_E, \delta_{PTV}\}$, we can rewrite the system as

$$\dot{x}(t) = A(\theta)x(t) + B_2(\theta)u(t) + B_1\omega(t), \quad (57)$$

where $A(\theta)$ and $B_2(\theta)$ are the system matrices defined in (22), $N=3$, $B_1=I$, $\omega(t)$ represents the vertical gust disturbance, θ satisfies (3), and $\dot{\theta}_i$ satisfy $\dot{\theta}_i \leq 0.01\theta_i$, $i=1,2,3$. For more detail, refer to Yang and Lum (2003).

We choose the measured output

$$y_1(t) = C_2x(t) \quad (58)$$

with $C_2 = [1 \ 0]$.

The design problem considered in this example is to find a state feedback controller such that:

- (i) The closed-loop system is robustly stable;
- (ii) The output $y_1(t)$ tracks the reference signal $r(t)$ without steady-state error, i.e.

$$\lim_{t \rightarrow \infty} e(t) = 0, \quad e(t) = r(t) - y_1(t);$$

- (iii) For the resulting closed-loop system, given a scalar $\gamma > 0$, the performance index $J(\omega) < 0$ holds for all nonzero $\omega(t) \in L_2[0, \infty)$.

It is well known that the tracking error integral action of a controller can effectively eliminate the steady-state tracking error (Liao, Wang and Yang 2002). In order to obtain a robust tracking controller with state feedback plus tracking error integral, we introduce the following augmented state-space description of the uncertain system (57)

$$\dot{\eta}(t) = \tilde{A}(\theta)\eta(t) + \tilde{B}_2(\theta)u(t) + \tilde{B}_1\tilde{\omega}(t), \quad (59)$$

where $\eta(t) = \text{col}\{\int_0^\infty e(t)dt, x(t)\}$, $\tilde{\omega}(t) = \text{col}\{r(t), \omega(t)\}$, and

$$\tilde{A}(\theta) = \begin{bmatrix} 0 & -C_2 \\ 0 & A(\theta) \end{bmatrix}, \quad \tilde{B}_2(\theta) = \begin{bmatrix} 0 \\ B_2(\theta) \end{bmatrix}, \\ \tilde{B}_1 = \begin{bmatrix} I & 0 \\ 0 & B_1 \end{bmatrix}.$$

It is obvious that if we obtain a controller robustly stabilising the augmented system (59), the controller also stabilises the original system (57) and achieves the output regulation $\lim_{t \rightarrow \infty} e(t) = 0$ (Liao et al. 2002).

Choose the performance output $z_1(t)$ as

$$z_1(t) = \tilde{C}_1\eta(t) + \tilde{D}_2u(t), \quad (60)$$

where $\tilde{C}_1 = [Q^{1/2} \ 0]^T$ and $\tilde{D}_2 = [0 \ R^{1/2}]^T$ with $Q > 0$ and $R \geq 0$. For the flight tracking controller design problem, the performance index (4) will be used to characterise the closed-loop performance. Thus, the design problem can be described as follows.

Robust H_∞ tracking control problem: Find a state feedback controller $u(t) = K\eta(t)$ such that:

- (i) The closed-loop system

$$\dot{\eta}(t) = (\tilde{A}(\theta) + \tilde{B}_2(\theta)K)\eta(t) + \tilde{B}_1\tilde{\omega}(t), \\ z_1(t) = (\tilde{C}_1 + \tilde{D}_2K)\eta(t)$$

is robustly stable;

- (ii) Given a scalar $\gamma > 0$, the system described by (59) and (60) satisfies the performance index $J(\tilde{\omega}) < 0$, for all non-zero $\tilde{\omega}(t) \in L_2[0, \infty)$ and all admissible trajectories of the parameter vector θ satisfying (3).

B. Simulation results

For the sake of comparison, we design two different robust tracking controllers: (i) the optimal quadratic controller obtained by a quadratically stabilising solution (Palhares et al. 1997); and (ii) the optimal polyquadratic controller obtained by Theorem 4.1 with $\alpha_2 = 0.7179$. Setting $Q = 0.02I_3$ and $R = 10^2I_2$, the corresponding state feedback gains are given by

$$K_q = \begin{bmatrix} -61.9746 & 10.6022 & 3.1540 \\ -12.1519 & 2.0811 & 0.6185 \end{bmatrix}, \\ K_p = \begin{bmatrix} -6.5293 & 1.7747 & 0.5392 \\ -1.2837 & 0.3515 & 0.1062 \end{bmatrix}.$$

Table 2 presents the actually achieved values of γ for the earlier mentioned three operating points.

From Table 2, the worst value of γ for the three given operating points is 10.9226, just slightly lower than the design value 10.9268. Two different optimal controllers yield the values of γ to be almost the same in the second operating point, as well as in the third

Table 2. Actually achieved values of γ for the three operating points.

Operating point	1	2	3
Robust-quadratic	10.2197	8.5823	10.9226
Theorem 4.1	5.6602	8.5706	10.9118

operating point. However, the first operating point value of γ is much higher by the optimal quadratic controller.

Example 7.3: This numerical example is to demonstrate the robust H_∞ filtering design procedures in Theorem 5.3. Consider the following uncertain continuous-time system which has been widely studied in the literature (Shaked and de Souza 1995; Geromel 1999).

$$\dot{x}(t) = \begin{bmatrix} 0 & -1 + 0.3\beta \\ 1 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \omega(t),$$

$$y_2(t) = [-100 \ 100]x(t) + [0 \ 1]\omega(t), \quad z_2(t) = [1 \ 0]x(t),$$

where β is an uncertain parameter satisfying $-1 \leq \beta \leq 1$. The above uncertainty matrix resides with the matrix polytope whose vertices are given by

$$A_1 = \begin{bmatrix} 0 & -0.7 \\ -1 & -0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1.3 \\ -1 & -0.5 \end{bmatrix}.$$

By the approach proposed in Palhares and Peres (1999) and Jin and Park (2001), the obtained minimum disturbance attenuation level is $\gamma^* = 5.4611$. Applying the approach of parameter search and setting the initial parameter values as $(\lambda_1^0, \lambda_2^0) = (1, 1)$, Theorem 4 in Duan et al. (2006) gives the minimum disturbance attenuation level $\gamma^* = 2.7066$ with the optimal searching parameter $(\lambda_1^*, \lambda_2^*) = (0.9129, 1.1210)$ and the corresponding filter parameters:

$$A_f = \begin{bmatrix} 28.6784 & -24.9224 \\ 93.3414 & -76.9917 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.2375 \\ -0.7688 \end{bmatrix},$$

$$C_f = [-0.9030 \quad -0.4077].$$

While Theorem 5.3 of this article yields $\gamma^* = 2.6583$ with the corresponding searching parameter $(\lambda_1^*, \lambda_2^*) = (3.2055, 4.1743)$ and filter parameters:

$$A_f = 10^3 \times \begin{bmatrix} 0.9458 & -0.9469 \\ 1.1822 & -1.1819 \end{bmatrix}, \quad B_f = \begin{bmatrix} 9.4575 \\ 11.8131 \end{bmatrix},$$

$$C_f = [1.0345 \quad 0.0475].$$

Therefore, it is clear that Theorem 5.3 provides a smaller value of γ^* than that obtained by the approach in Duan et al. (2006).

8. Conclusion

LMI-based optimisation procedures were proposed to design robust H_∞ controllers for polytopic time-varying systems and affine parameter systems by means of parameter-dependent Lyapunov functions. A descriptor system approach was taken to derive the polyquadratic H_∞ performance criterion for polytopic systems. Furthermore, the criterion was formulated in the form that can be easily modified for robust controller synthesis and filtering design. The approaches of robust H_∞ performance analysis and controller synthesis were also developed for affine parameter systems.

A flight robust H_∞ tracking control problem was solved which demonstrated the applicability of the polyquadratic H_∞ performance criterion. In the flight control example, it was found that the criterion, which provided a solution over the entire domain of uncertainties, achieved in the robust H_∞ control case a bound on the disturbance attenuation level which was smaller than the one based on quadratic stability approach.

The results obtained in this article can be regarded as an improvement on previous results of the quadratic stability. How to further reduce the conservativeness constitutes an important problem for future investigation.

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Appendix

System matrices parameters of the three operating points are given in Table A1.

Table A1. System matrices parameters.

Operating point	1	2	3
Mach number	0.7	0.85	0.9
Altitude (ft)	14000	5000	10000
Z_α	−1.175	−2.328	−2.452
Z_q	0.9871	0.9831	0.9856
M_α	−0.8458	−30.44	−38.61
M_q	−0.8776	−1.493	−1.34
$Z_{\delta E}$	−0.194	−0.3012	−0.2757
$Z_{\delta PTV}$	−0.0359	−0.0587	−0.0523
$M_{\delta E}$	−19.29	−38.43	−37.36
$M_{\delta PTV}$	−3.803	−7.815	−7.247