



## Discussion

## Comment on “The Imbert–Fedorov shift of paraxial light beams”

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## ABSTRACT

Here I argue that Liu and Li [B.-Y. Liu, C.-F. Li, Opt. Commun. 281 (2008) 3427] reproduce calculations of the Imbert–Fedorov transverse shift previously made in a number of other works. However, it has recently been shown that these results are not valid for standard uniformly polarized beams. The corrected values of the Imbert–Fedorov shift were derived in papers [K.Y. Bliokh, Y.P. Bliokh, Phys. Rev. Lett. 96 (2006) 073903; Phys. Rev. E 75 (2007) 066609] and confirmed by recent measurements [O. Hosten, P. Kwiat, Science 319 (2008) 787] with a great accuracy.

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The Imbert–Fedorov effect [1,2] is a polarization-dependent transverse shift of the center of gravity of a wave beam scattered at a plane dielectric interface. Although the central wave vectors of the incident and scattered (i.e., refracted and reflected) beams lie in the same plane according to the Snell's law, the centers of gravity of the beams may be slightly shifted out of the plane of incidence because of complex interference of partial plane waves forming the confined beams.<sup>1</sup> This tiny effect is of a fundamental interest because of its relation to conservation of the angular momentum and spin-Hall effect of light.

The direct method of calculation of the Imbert–Fedorov shift is a Fourier representation of the incident beam which enables one to apply Fresnel formulas to each partial plane wave in the beam spectrum. This yields exact Fourier spectra of the scattered beams, and their spatial shape can be retrieved analytically using the paraxial approximation. Such a procedure has been realized in a number of papers [3–11] starting from the Schilling's paper in 1965, and essentially the same approach is used by Liu and Li [12].

The problem of scattering of a paraxial beam at a planar interface between two dielectric media is described by the following parameters: the wave number  $k$ , angle of the incidence  $\theta$ , and Fresnel reflection and transmission coefficients  $R_{\parallel,\perp}$  and  $T_{\parallel,\perp}$  of the central plane wave in the beam. In the case of the total internal reflection, the reflection coefficients are complex:  $R_{\parallel,\perp} = \exp(i\varphi_{\parallel,\perp})$ , and induce the phase difference  $\delta\varphi = \varphi_{\perp} - \varphi_{\parallel}$ . The polariza-

tion of the incident beam can be characterized by the normalized Jones vector of the central plane wave in the basis of  $p$  and  $s$  modes:  $\mathbf{e} = \begin{pmatrix} e_{\parallel} \\ e_{\perp} \end{pmatrix}$ ,  $\mathbf{e}^* \cdot \mathbf{e} = 1$ . Note that  $\sigma = 2\text{Im}(e_{\parallel}^* e_{\perp})$  is the helicity of the incident wave.

The Schilling's formula for the transverse shift of the reflected beam can be written as

$$\Delta^{(totr)} = -\frac{2 \cot \theta}{k} \left[ \text{Im}(e_{\parallel}^* e_{\perp}) (1 + \cos \delta\varphi) + \text{Re}(e_{\parallel}^* e_{\perp}) \sin \delta\varphi \right], \quad (1)$$

in the case of the total internal reflection, and

$$\Delta^{(r)} = -\frac{2 \cot \theta}{k} \text{Im}(e_{\parallel}^* e_{\perp}) \left[ 1 + \frac{R_{\parallel} R_{\perp}}{R_{\parallel}^2 |e_{\parallel}|^2 + R_{\perp}^2 |e_{\perp}|^2} \right], \quad (2)$$

in the case of partial reflection. Eqs. (1) and (2) have been obtained (up to some arithmetic inaccuracies) in papers [3–7,10]. The corresponding transverse shift of the transmitted beam under partial reflection has been first obtained by Fedoseev in [6] and subsequently by other authors [7,9,10]:

$$\Delta^{(t)} = -\frac{2 \cot \theta}{k} \text{Im}(e_{\parallel}^* e_{\perp}) \left[ 1 - \frac{T_{\parallel} T_{\perp} \cos \theta' / \cos \theta}{T_{\parallel}^2 |e_{\parallel}|^2 + T_{\perp}^2 |e_{\perp}|^2} \right], \quad (3)$$

where  $\theta'$  is the angle of propagation of the transmitted beam. It can be readily seen that Eqs. (24) and (26) derived in [12] coincide with Eqs. (1)–(3) previously obtained in [3–7,9,10].

Despite a number of independent calculations of the Imbert–Fedorov shift leading to Eqs. (1)–(3), recent calculations made by Bliokh and Bliokh [11] led to the Eq. (1) for the total-reflection case but distinct results in the partial-reflection regime:

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<sup>1</sup> Originally, the Imbert–Fedorov effect was mistakenly explained by the transverse energy flow in the evanescent waves generated under total internal reflection [1,2].

$$\Delta^{(r)'} = -\frac{\cot \theta}{k} \operatorname{Im}(e_{\parallel}^* e_{\perp}) \frac{(R_{\parallel} + R_{\perp})^2}{R_{\parallel}^2 |e_{\parallel}|^2 + R_{\perp}^2 |e_{\perp}|^2}, \quad (4)$$

$$\Delta^{(t)'} = -\frac{\cot \theta}{k} \operatorname{Im}(e_{\parallel}^* e_{\perp}) \frac{T_{\parallel}^2 + T_{\perp}^2 - 2T_{\parallel}T_{\perp} \cos \theta' / \cos \theta}{T_{\parallel}^2 |e_{\parallel}|^2 + T_{\perp}^2 |e_{\perp}|^2}, \quad (5)$$

As it was shown in [11], the reason of discrepancy between Eqs. (2), (3) and (4), (5) is a high sensitivity of the Imbert–Fedorov effect to details of the *polarization structure* of the incident beam. Indeed, the effect is essentially connected with the finite spectral width of the beam and one should carefully define polarizations of *all* the plane waves composing the beam. All previous calculations [3–7,9,10] and paper [12] are based on an assumption that all the partial plane waves in the incident beam have the same polarization with respect to the interface between media. However, this is not the case for real beams uniformly polarized in the plane orthogonal to the central wave vector. On the contrary, partial waves having the same polarization in the beam coordinate frame have *different* polarizations with respect to the medium interface [11]. This is because each plane wave has its own plane of incidence with respect to the interface. Moreover, as it is shown in [11], polarization model used in [3–7,9,10,12] leads to the non-uniformly polarized beams whose polarization structure depends on the angle of incidence  $\theta$ , which is unsatisfactory from the physical point of view.

Eqs. (4) and (5) were recently verified by Hosten and Kwiat [13], who came to the same conclusion as in [11], and confirmed the details of the beam polarization evolution experimentally with a great accuracy. Their results are in a precise agreement with Eq. (5) for the partial transmission of a wave beam. In addition, after this comment have been submitted, a theoretical paper [14] appeared which fully confirms results of [11,13] and Eq. (4).

Thus, contrary to the calculations of paper [12], which reproduce the known expressions (1)–(3) for the Imbert–Fedorov shift, the transverse shifts of real uniformly polarized beams are described by Eqs. (4) and (5) in the partial-reflection regime. It should also be noted that references on previous experiments [2,15–17] cannot be involved to discriminate between Eqs. (2), (3) and (4),

(5) since those experiments dealt with the total-reflection case where both the approaches result in the same Eq. (1). Finally, it is worth noting that the most general expression for the Imbert–Fedorov shift which can be applied to beams with various polarization structures have been obtained in [8], while the relation of the effect to the angular-momentum conservation was first revealed in [7] and further discussed in [10,11].

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