Holistic Inversion
of
Airborne Electromagnetic Data

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Declaration

This thesis is the result of research undertaken while I was a student in the Research School of Earth Sciences at the Australian National University. Except as otherwise stated in the text, the work described is original and my own. The thesis has never been submitted to another university or similar institution.

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Abstract

A holistic method for simultaneously calibrating, processing, and inverting frequency-domain airborne electromagnetic data has been developed. A spline-based, 3D, layered conductivity model covering a complete survey area is recovered through inversion of an entire raw airborne data set and available independent geoelectric and interface-depth data. The holistic inversion formulation includes a mathematical model to account for systematic calibration errors such as incorrect gain, phase and zero-level. By taking these elements into account in the inversion, the need to pre-process the airborne data prior to inversion is eliminated.

Conventional processing schemes involve the sequential application of a number of calibration corrections, with data from each frequency being treated separately. This is followed by inversion of each multi-frequency airborne sample in isolation from other samples. By simultaneously considering all of the available information in a holistic inversion, the inter-frequency and spatial coherency characteristics of the data are able to be exploited. The formulation ensures that the conductivity and calibration models are optimal with respect to the airborne data and prior information. Introduction of inter-frequency inconsistency and multistage error propagation stemming from the sequential nature of conventional processing schemes is also avoided.

It is confirmed that accurate conductivity and calibration parameter values are recovered from holistic inversion of synthetic data sets. It is also demonstrated that the results from holistic inversion of raw survey data are superior to the output of conventional 1D inversion of final processed contractor delivered data. In addition to the technical benefits, it is expected that holistic inversion will reduce costs by avoiding the expensive calibration→processing→recalibration paradigm. Furthermore, savings may
also be made because specific high altitude zero-level observations, needed for conventional processing, may not be required.

The same philosophy is also applied to the inversion of time-domain data acquired by fixed-wing towed-bird systems. A spline-based, 2D, layered conductivity model covering a complete survey line is recovered along with a calibrations model. In this instance, the calibration model is a spline based representation of three unmeasured elements of the system geometry. By inverting the less processed total field data, the procedure is able to prevent incorrect assumptions made in conventional primary field removal from being propagated into the inversion stage. Furthermore, by inverting a complete line of data at once the along-line spatial coherency of the geology and the geometry variations is exploited.

Using real survey data, it was demonstrated that all components of the data could be simultaneously and satisfactorily fitted and that the resulting conductivity model was consistent with independent prior information. This was an improvement over the conventional approach, in which the data could not be satisfactorily fitted, nor was the conductivity model consistent with prior information. It was further established that by using the holistic inversion spline parameterization, the resulting conductivity model was more continuous and interpretable than if the conventional style discrete parameterization was used.

If adopted, the holistic approach, could reduce survey costs, reduce data processing turnaround times, and improve the quantitative information that can be extracted from data, and hence, increase the value of airborne electromagnetics for mineral exploration and environmental mapping applications.
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