Analysis and Applications of
Smoothed Particle Magnetohydrodynamics

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I hereby state that this thesis is my own original work.
To my parents

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ABSTRACT. Smoothed Particle Hydrodynamics (SPH) is analysed as the weighted residual method. In particular the analysis focuses on the collocation aspect of the method. Using Monte Carlo experiments we demonstrate that SPH is highly sensitive to node disorder, especially in its symmetrised energy and momentum conserving form. This aspect of the method is related to low $\beta$ MHD instabilities observed by other authors. A remedy in the form of the Weighted Differences Method is suggested, which addresses this problem to some extent, but at a cost of losing automatic conservation of energy and momentum.

The Weighted Differences Method is used to simulate propagation of Alfvén and magnetosonic wave fronts in $\beta = 0$ plasma, and the results are compared with data obtained with the NCSA Zeus3D code with the Method of Characteristics (MOC) module.

SPH is then applied to two interesting astrophysical situations: accretion onto a white dwarf in a compact binary system, which results in a formation of an accretion disk, and gravitational collapse of a magnetised vortex. Both models are 3 dimensional.

The accretion disk which forms in the binary star model is characterised by turbulent flow: the Kármán vortex street is observed behind the stream-disk interaction region. The shock that forms at the point of stream-disk interaction is controlled by the means of particle merges, whereas Monaghan–Lattanzio artificial viscosity is used to simulate Smagorinsky closure.

The evolution of the collapsing magnetised vortex ends up in the formation of an expanding ring in the symmetry plane of the system. We observe the presence of spiraling inward motion towards the centre of attraction. That final state compares favourably with the observed qualitative and quantitative characteristics of the circumnuclear disk in the Galactic Centre. That simulation has also been verified with the NCSA Zeus3D run.

In conclusions we contrast the results of our Monte Carlo experiments with the results delivered by our production runs. We also compare SPH and Weighted Differences against the new generation of conservative finite differences methods, such as the Godunov method and the Piecewise Parabolic Method. We conclude that although SPH cannot match the accuracy and performance of those methods, it appears to have some advantage in simulation of rotating flows, which are of special interest to astrophysics.
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