Timing and Frequency Synchronization in Practical OFDM Systems

Ming (Matt) Ruan
Submitted July 2008
Revised April 2009

A Thesis submitted in complete fulfilment of the Degree of Doctor of Philosophy in the Research School of Information Sciences and Engineering of the Australian National University
Declaration

I declare that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge it does not contain any material previously published or written by any other person except where due reference is made in the text.

Ming (Matt) Ruan
July 10, 2008
Acknowledgements

This thesis and the work it embodies was only possible through the continual help and support of a large number of people.

This research was funded through an Australian Postgraduate Award (APA) and through National ICT Australia. Without these financial supports this work wouldn’t have happened.

Leaving our homeland and striving for everything in this country was a big decision we made, not only me, but also my wife Janlin. Without her support to the immigration plan and help on every aspect of my life, from start to finish, none of this work would have been possible.

I would like to thank my supervisor Dr. Mark C. Reed, the first and only person I knew when came to Australia, for guiding me into the academic world and encouraging me to find real practical problems that can be solved with statistical signal processing approaches. I am also grateful for the help from my co-supervisor Dr. Zhenning Shi for the constructive technical comments on my research work.

Other people who have also been important in the making of this thesis are Prof. Rodney A. Kennedy for teaching me how to think abstractly, Dr. Leif W. Hanlen for introducing me to the spatial dimension of wireless communications, Dr. Thushara Abhayapala and Dr. Tharaka Lamahewa for providing me the thesis template, Prof. Li Ping of the Hong Kong City University for sharing with me his views on modern communications technologies.

Finally, a very large thankyou must go to my family in China. No matter where I am, I can feel their eyes staring at me so eagerly to see the success. This hidden force keeps pushing me forward in face of all kinds of challenges.
Abstract

Orthogonal frequency-division multiplexing (OFDM) has been adopted by many broadband wireless communication systems for the simplicity of the receiver technique to support high data rates and user mobility. However, studies also show that the advantage of OFDM over the single-carrier modulation schemes could be substantially compromised by timing or frequency estimation errors at the receiver. In this thesis we investigate the synchronization problem for practical OFDM systems using a system model generalized from the IEEE 802.11 and IEEE 802.16 standards.

For preamble based synchronization schemes, which are most common in the downlink of wireless communication systems, we propose a novel timing acquisition algorithm which minimizes false alarm probability and indirectly improves correct detection probability. We then introduce a universal fractional carrier frequency offset (CFO) estimator that outperforms conventional methods at low signal to noise ratio with lower complexity. More accurate timing and frequency estimates can be obtained by our proposed frequency-domain algorithms incorporating channel knowledge. We derive four joint frequency, timing, and channel estimators with different approximations, and then propose a hybrid integer CFO estimation scheme to provide flexible performance and complexity tradeoffs. When the exact channel delay profile is unknown at the receiver, we present a successive timing estimation algorithm to solve the timing ambiguity. Both analytical and simulation results are presented to confirm the performance of the proposed methods in various realistic channel conditions.

The ranging based synchronization scheme is most commonly used in the uplink of wireless communication systems. Here we propose a successive multiuser detection algorithm to mitigate multiple access interference and achieve better performance than that of conventional single-user based methods. A reduced-complexity version of the successive algorithm feasible for hardware real-time implementation is also presented in the thesis. To better understand the performance of a ranging
detector from a system point of view, we develop a technique that can directly translate a detector’s missed detection probability into the maximum number of users that the method can support in one cell with a given number of ranging opportunities. The analytical results match the simulations reasonably well and show that the proposed successive algorithms allow a base station to serve more than double the number of users supported by the conventional methods.

Finally, we investigate inter-carrier interference which is caused by the time-varying communication channels. We derive the bounds on the power of residual inter-carrier interference that cannot be mitigated by a frequency-domain equalizer with a given number of taps. We also propose a Turbo equalization scheme using the novel grouped Particle filter, which approaches the performance of the Maximum A Posterior algorithm with much lower complexity.
List of Publications

Journal Papers


• Ming (Matt) Ruan, Mark C. Reed, and Zhenning Shi, “Successive multiuser detection and interference cancellation for contention based OFDMA ranging channel,” third revision submitted to IEEE Transactions on Wireless Communications, 2009.

Conference Papers

• Ming (Matt) Ruan, Mark C. Reed, and Zhenning Shi, “A universal frequency offset estimator for OFDM applications,” submitted to IEEE Global Telecommunications Conference (GLOBECOM), 2009.

• Ming (Matt) Ruan, Mark C. Reed, and Zhenning Shi, “A hybrid integer carrier frequency offset estimator for practical OFDM systems,” submitted to IEEE Global Telecommunications Conference (GLOBECOM), 2009.

• Ming (Matt) Ruan, Mark C. Reed, and Zhenning Shi, “Turbo equalization using Particle filtering with grouping,” in Proc. International Symposium on Communications and Information Theory (ISCIT), Oct. 2007, pp. 1197-1200.1

• Ming (Matt) Ruan, Mark C. Reed, and Zhenning Shi, “Approximated maximum likelihood estimation of carrier frequency offset in practical OFDM

1The paper received the Best Student Paper Prize of that conference.


**Patent Applications**


Contents

List of Figures xv

List of Tables xix

1 Introduction 1
  1.1 Practical OFDM Systems 1
  1.2 Timing and Frequency Synchronization 4
  1.3 OFDM Receiver Structure 6
  1.4 Literature Review 8
    1.4.1 OFDM 8
    1.4.2 Coarse Timing Estimation 8
    1.4.3 Coarse Frequency Estimation 9
    1.4.4 Refined Frequency Estimation 10
    1.4.5 Refined Timing Estimation 12
    1.4.6 OFDMA Uplink Synchronization 13
    1.4.7 OFDMA Ranging Channel Detection 14
    1.4.8 ICI Analysis 15
    1.4.9 Turbo Equalization 15
  1.5 Motivation of the Research 16
  1.6 Thesis Outline 18
  1.7 Thesis Contributions 20

2 System Model 25
  2.1 Introduction 25
  2.2 Channel Model 26
    2.2.1 Propagation Loss 26
    2.2.2 Shadowing 26
    2.2.3 Multipath Fading 27
  2.3 Signal Model 30
4.6.2 OFDMA Physical Layer: \( L_1 = 3 \) .................................................. 86
4.7 Summary ................................................................. 88

5 Refined Timing and Frequency Estimation 89

5.1 Introduction .......................................................... 89
5.2 Matrix Representation of the Signal ............................... 90
5.3 Approximated Maximum Likelihood Joint Estimator .......... 91
5.4 Simplified Joint Estimation Algorithms ......................... 93
  5.4.1 Method A ......................................................... 93
  5.4.2 Method B ......................................................... 94
  5.4.3 Method C ......................................................... 95
  5.4.4 Method D ......................................................... 96
  5.4.5 Complexity Analysis .......................................... 97
  5.4.6 Comparison between the Method A and B .................... 97
  5.4.7 Timing Ambiguity ............................................. 98
  5.4.8 Performance Analysis of the Method D ..................... 100
  5.4.9 Numerical Results ............................................. 102
5.5 A Hybrid CFO Estimation Scheme ................................ 104
  5.5.1 The Method .................................................... 105
  5.5.2 Complexity Analysis .......................................... 105
  5.5.3 Implementation ................................................ 105
  5.5.4 Performance Analysis ........................................ 106
  5.5.5 Numerical Results ............................................. 109
5.6 Successive Joint Channel and Timing Estimation ............... 111
  5.6.1 Overview ....................................................... 111
  5.6.2 Path Detector .................................................. 113
  5.6.3 Channel Re-estimator ........................................ 113
  5.6.4 An Interpretation ............................................. 114
  5.6.5 The Simplified Algorithm ................................... 115
  5.6.6 Performance Analysis ....................................... 116
  5.6.7 Numerical Results ............................................. 119
5.7 Summary ............................................................. 122

6 Successive Ranging Detection 123

6.1 Introduction ........................................................ 123
6.2 Successive Multiuser Detection and Interference Cancellation .... 124
  6.2.1 Derivation of the SMUD ....................................... 125
6.2.2 Interference Cancellation for DSS ........................................ 128
6.2.3 Implementation ......................................................... 129
6.3 Reduced-Complexity SMUD ............................................... 130
6.4 Performance Analysis .................................................... 132
  6.4.1 Interference Analysis .................................................. 133
  6.4.2 Comparison with the SAGE .......................................... 134
  6.4.3 Comparison with the Single-User Detector ....................... 135
  6.4.4 Determining the Thresholds ........................................ 136
6.5 Complexity Analysis .................................................... 138
6.6 Numerical Results ....................................................... 139
  6.6.1 User Detection Performance ........................................ 140
  6.6.2 Complexity .......................................................... 144
  6.6.3 CFO Estimation Performance ....................................... 146
  6.6.4 Data Subscriber Station’s Bit Error Rate ....................... 146
6.7 Summary ...................................................................... 146

7 System Performance of Ranging Detectors ............................ 149
  7.1 Introduction .................................................................. 149
  7.2 Assumptions And Definitions ......................................... 150
  7.3 Performance Analysis .................................................... 152
    7.3.1 Maximum Number of New Ranging Users .................... 152
    7.3.2 Maximum Number of Users in a Cell ......................... 153
  7.4 Numerical Results ....................................................... 155
    7.4.1 Periodic Ranging .................................................... 156
    7.4.2 Initial/Handover Ranging ........................................ 156
  7.5 Summary ...................................................................... 160

8 Bounds for Frequency-Domain Equalization ......................... 161
  8.1 Introduction .................................................................. 161
  8.2 OFDM over Time-varying Channel .................................. 161
  8.3 ICI Analysis .................................................................. 163
  8.4 Closed-Form Bounds ..................................................... 164
  8.5 Polynomial Bounds ....................................................... 166
  8.6 Comparison of the Bounds ............................................. 168
  8.7 Summary ...................................................................... 169
## List of Figures

1.1 Received waveform of one subcarrier with and without cyclic prefix. 2  
1.2 Simplified clock distribution tree. ................................. 5  
1.3 Impact of carrier frequency offset. ................................. 5  
1.4 OFDM receiver structure. ............................................ 7  
1.5 Thesis outline. ....................................................... 18  

2.1 Tapped delay line channel model. ................................. 27  
2.2 The fluctuation of channel gains in CH-A and CH-B. ............ 29  
2.3 Diagram of one OFDM burst. ........................................ 31  
2.4 Diagram of a training symbol with $N = 8$, $L_1 = 3$. ............ 32  
2.5 One ranging opportunity in a TDD OFDMA system. .............. 34  

3.1 Two scenarios in the unequal power case. ......................... 45  
3.2 Variation of $g(\eta)$ over the ratio of power $\eta$. ................ 49  
3.3 Coarse timing estimates for a given timing metric. .......... 54  
3.4 Performance in AWGN channel (Case A). ......................... 60  
3.5 Performance in AWGN channel (Case B). ......................... 64  
3.6 Performance in AWGN channel (Case C). ......................... 66  
3.7 False alarm probability per frame. ............................... 68  
3.8 Missed detection probability per frame. ......................... 70  
3.9 Distribution of timing offsets. ................................... 71  

4.1 The performance as a function of SNR. ......................... 83  
4.2 The performance as a function of $L_1$ at SNR=10dB. ............ 84  
4.3 Fractional CFO estimation performance. ......................... 87  

5.1 Channel estimates under different integer CFO hypotheses. .... 96  
5.2 Implementation of the function $\Lambda_A(\epsilon, \tau)$. ............. 98  
5.3 An example of the timing ambiguity problem. ................... 99  
5.4 Performance analysis for the Method D. ......................... 102
5.5 Imperfect channel knowledge used by the Method A. ............... 103
5.6 Performance of the simplified algorithms. ......................... 104
5.7 Implementation of the hybrid algorithm. ........................ 106
5.8 Most likely error patterns considered in performance analysis. .. 107
5.9 Performance analysis of the hybrid method. ...................... 108
5.10 Performance of the hybrid integer CFO estimators. ............ 109
5.11 Complexity vs performance of joint integer CFO estimators .... 110
5.12 Diagram of the successive joint channel and timing estimator. 112
5.13 First channel tap correct detection probability. ............... 118
5.14 Simulation results for the successive timing estimators. ....... 121

6.1 Diagram of the proposed SMUD. ................................. 127
6.2 Examples of \( S_i \) in different cases. ............................. 131
6.3 False alarm probability. ......................................... 141
6.4 Missed detection probability. .................................... 141
6.5 Expected number of ranging attempts for low-power RSSs ...... 142
6.6 Timing estimation performance. ................................ 143
6.7 Power estimation performance. ................................ 143
6.8 Complexity as a function of the number of active RSSs. ........ 145
6.9 CFO estimation performance. .................................. 145
6.10 Data subscriber station bit error rate. .......................... 147

7.1 Simplified diagram of one ranging process. ...................... 151
7.2 Maximum number of new RSSs in each ranging opportunity. ... 154
7.3 Position of a RSS entering the cell. ............................ 155
7.4 Periodic ranging performance. ................................... 157
7.5 Initial/handover ranging (\( \nu = 120\text{km/h}, T_{op} = 20\text{ms} \)). 158
7.6 Initial/handover ranging (\( \nu = 120\text{km/h}, T_{op} = 40\text{ms} \)). 159

8.1 Bounds for classical channel model. ............................ 167
8.2 Bounds for uniform channel model. ............................. 167
8.3 Bounds for two-path channel model. ........................... 168

9.1 Diagram of Turbo equalization. ................................. 173
9.2 Equalizer output histogram in Turbo schemes (SNR=7dB). .... 174
9.3 Diagram of GPF with an example (\( L_g = 4, \mathcal{A} = \{-1, +1\} \)). 176
9.4 GPF equalizer output histogram in Turbo schemes. ............ 178
9.5 EXIT chart analysis. ............................................. 179
9.6 Simulation results for GPF in Proakis C channel. . . . . . . . . . . 181
9.7 Simulation results for GPF using the early-stopping criteria. . . . . 182
9.8 Complexity of GPF using the early-stopping criteria. . . . . . . . 183
List of Tables

2.1 Power delay profiles for CH-A and CH-B. . . . . . . . . . . . . . . . 29
2.2 Interference in OFDM symbols of a ranging opportunity. . . . . . . 37
4.1 Complexity of the estimators used in simulations (OFDM). . . . . 86
4.2 Complexity of the estimators used in simulations (OFDMA). . . . . 88
5.1 Complexity of the simplified algorithms. . . . . . . . . . . . . . . . 97
6.1 Ranging detector complexity comparison. . . . . . . . . . . . . . . . 138
7.1 Complexity of the ranging channel detectors. . . . . . . . . . . . . 150
7.2 System parameters in simulation and analysis. . . . . . . . . . . . . 152
7.3 Maximum average number of new RSSs. . . . . . . . . . . . . . . . 154
7.4 Maximum number of SSs in a cell (periodic ranging). . . . . . . . 156
7.5 Maximum number of SSs in a cell (initial/handover ranging). . . . 156
8.1 Constants in the polynomial bounds for the three channel models. . 166
Chapter 1

Introduction

1.1 Practical OFDM Systems

Orthogonal frequency-division multiplexing (OFDM) is a multi-carrier modulation scheme that uses a large number of closely-spaced orthogonal subcarriers for data transmission. The subcarriers can be efficiently separated by the fast Fourier transform (FFT) algorithm at the receiver. On each of the subcarriers, the data is modulated with a conventional modulation scheme such as quadrature amplitude modulation (QAM). Because of the large number of subcarriers, very high data rate can be achieved with much lower data rate on each subcarrier.

The primary advantage of OFDM over single-carrier schemes is the simplicity of equalizer design at the receiver to cope with multipath fading channels, which are prevalent in wireless communications. The multipath fading causes frequency-selective distortion to the transmit signal and incurs inter-symbol interference (ISI). These adverse impacts need to be mitigated by an equalizer that is more complex as the system bandwidth increases. For OFDM systems, every subcarrier can be seen as a narrow-band low-rate signal. With the insertion and removal of guard intervals, equalization in OFDM systems can be performed by a single-tap equalizer on each subcarrier, which is much simpler than that in single-carrier systems. This advantage motivated the adoption of OFDM in many commercial wireless communication systems such as the digital audio broadcasting (DAB) [1], the terrestrial digital video broadcasting (DVB-T) [2], the IEEE 802.11a wireless local area network (WiFi) [3], and the IEEE 802.16 wireless local and metropolitan area networks (WiMAX) [4], etc.

To fully take advantage of OFDM in realistic communication environments, many useful measures are widely incorporated in practical OFDM systems [1–4] to
combat the channel impairments. We review some of them as follows.

**Cyclic Prefix**

The cyclic prefix (CP) protects the OFDM symbols from ISI. The name of cyclic prefix indicates what it is and how it is generated. Firstly, as a prefix, it needs to be placed in the beginning of an OFDM symbol. Secondly, being a cyclic extension to the original signal, the signal within the CP needs to be the same as the last portion of the OFDM symbol.

Although the use of the CP prolongs the duration of OFDM symbols and results in a loss in data throughput, it has become a common practice in most commercial OFDM systems because of the effectiveness in ISI mitigation. Figure 1.1 illustrates the benefit of the CP with an exemplary received waveform of one subcarrier in an OFDM signal. The solid and dashed lines represent two replica of the transmit signal arriving at the receiver via two channel paths with different delays. Without CP, it is inevitable for the FFT window to contain the signals from two OFDM symbols. As a result, the sudden phase change on the subcarrier spreads its spectrum into others’ after FFT and causes inter-carrier interference (ICI) that could significantly degrade the system performance. With the insertion of CP, which is assumed to be longer than the time interval between the first and last channel paths, the phase discontinuity can be avoided by a properly located FFT window.

![Figure 1.1: Received waveform of one subcarrier with and without cyclic prefix.](image)

Another scheme called “zero-padding” was also proposed for ISI mitigation in OFDM systems. Instead of making the OFDM symbols cyclic, that scheme simply
turns off the transmitter for a certain period of time before the transmission of every OFDM symbol. A comprehensive analysis of the zero-padding method can be found in [5] where perfect timing and frequency knowledge at the receiver was assumed. Further investigation on the robustness of the zero-padding scheme in the presence of synchronization error is however still required.

**Spectrum blanking**

In the time domain, the CP protects OFDM symbols from interfering with each other; in the frequency domain, a similar approach called spectrum blanking is often used to protect the signal from the interference of other communication channels. The spectrum blanking method leaves a set of unused subcarriers at two ends of the spectrum, forming a guard band between adjacent communication channels. Although spectrum blanking introduces a loss in data throughput, it offers dual protection to the communication systems. Firstly, it relaxes the requirements for filter design and reduces the out-of-band interference to adjacent channels or communication systems. Secondly, in the presence of significant frequency offset (numerous subcarriers of offset) between the transmitter and receiver, the guard band allows the received signal to pass through the baseband filters without severe distortion or attenuation. It is also possible to make use of the guard band to achieve frequency synchronization without any knowledge of the channel and data [6–9].

The middle point of the spectrum, which becomes a direct current in the baseband signal, usually is also blanked in practical wireless systems. This improves the efficiency of amplifier and up/down conversion circuitries, and suppresses the interference resulted from carrier leakage.

**Pilots and training symbols**

Coherent detection of OFDM data subcarriers requires the knowledge of the channel, which can be estimated from certain subcarriers modulated by known data symbols. Those subcarriers are known as pilots. The positions of the pilots and the number of them usually are designed for certain time and frequency selectivity of the channel. For different channel conditions, the positions and number of pilots can either vary over different OFDM symbols [2], or be fixed for all symbols [3].

An OFDM symbol that only consists of pilots is known as a training symbol. For packet orientated OFDM systems, one or multiple training symbols known as
the preamble are usually placed at the beginning of every OFDM burst. Because the whole training symbol is pre-determined, it is possible to embed certain signal structure in the training symbol to facilitate timing and frequency estimation. For instance, if the pilots are all placed on the even subcarriers, the time-domain training symbol consists of two identical halves that can be exploited for timing and frequency synchronization [10].

The pilots are also designed to have low peak to average power ratio (PAPR) so that they can be transmitted at higher power without suffering from power amplifier distortion. The higher power of the pilots also allows reliable timing and frequency reference to be obtained even in harsh communication environments where the signal quality is too poor for data detection.

1.2 Timing and Frequency Synchronization

The initial objective of timing synchronization in an OFDM system is to find the proper timing positions of FFT windows. As illustrated in Figure 1.1, the use of the CP avoids the ISI and ICI only when a FFT window is properly located, and if the timing window is selected incorrectly then ISI is present. It should be noted that in OFDM systems, ISI directly produces spectral leakage which manifests as ICI because the phase discontinuity on a subcarrier spreads its spectrum into others’.

An ISI-free FFT window is all one needs to perform the FFT on the received signal, however, the further processing of the signal in the frequency domain often requires higher accuracy in the timing estimates. This is because any timing offset from the ideal timing position causes a linear phase shift on the subcarriers, which has the same effect as a prolonged channel impulse response that gives the channel extra frequency selectivity [11]. As we mentioned earlier, the pilot allocation is designed for a presumed maximum length of channel impulse response. If the effective channel length is beyond the supported range, the channel estimation performance will be reduced. In most practical OFDM systems where pilot allocation scheme are predefined, higher accuracy in timing estimates means a smaller chance for the effective channel length to exceed the supported range, and a lower risk of data detection error.

OFDM is very sensitive to the frequency offset between the oscillators in the transmitter and receiver as will be illustrated in the following example. A simplified diagram of the clock distribution tree in a typical OFDM system with 3.5GHz carrier, 5MHz bandwidth and 512 subcarriers is shown in Figure 1.2. It follows the
1.2 Timing and Frequency Synchronization

Figure 1.2: Simplified clock distribution tree.

recommendation [4] that both the carrier and sample clock are derived from the same crystal oscillator. We assume typical crystal oscillators, e.g., Maxim DS4100H [12], are used in the system with frequency stability ±39ppm. Then, using the parameters shown in the figure, we can compute the maximum carrier frequency offset (CFO) between the transmitter and receiver can be $3.5\text{GHz} \times (39+39) \times 10^{-6} = 273\text{kHz}$, which is about 28 times of the subcarrier spacing. In the mean time, the sample clock causes a time error of 0.008% for each OFDM symbol, which has much less impact as that of the CFO [13].

A CFO can be partitioned into two parts: the integer part is an integer multiple of the subcarrier spacing, and the rest gives the fractional part which is a fraction

Figure 1.3: Impact of carrier frequency offset.

Delta function is produced by a discrete frequency Fourier Transform

Sinc function is produced by a continuous frequency Fourier Transform
of one subcarrier spacing. The different impacts of the integer and fractional CFO on the received signal are illustrated in Figure 1.3. The fractional CFO causes the frequency domain signal on different subcarriers to be no longer orthogonal to each other, and as the fractional CFO increases, we can see the wanted signal suffers from more severe attenuation and stronger interference from adjacent subcarriers. This has a similar effect to the ISI in a single-carrier system, which can significantly degrade the system performance if the adverse impact is not properly mitigated. The integer CFO circularly shifts the subcarriers away from their original locations. If the integer CFO is not estimated and accounted for, the whole OFDM burst cannot be properly decoded and all the data in the burst will be lost. Therefore, frequency synchronization is critical to the performance of OFDM systems.

One receiver compensation technique for frequency synchronization error is to adjust the receiver clock synthesizer according to the CFO estimate. Once this has been performed, the sample clock error is automatically reduced to a negligible level. For instance, assume the CFO is controlled within 2% of subcarrier spacing as required by the IEEE 802.16 standard [4], the crystal accuracy is higher than $\frac{2\pi}{20} \times 2 \times 39\text{ppm} \approx 55.7\text{ppb}$, and the sample clock error becomes negligible.

1.3 OFDM Receiver Structure

A simplified diagram of a typical OFDM receiver is shown in Figure 1.4. The received signal is down-converted to baseband and passed through the low-pass filter before being sampled and buffered in a memory. A coarse timing position is estimated from the baseband signal and the data samples for FFT are extracted. The fractional CFO needs to be estimated and corrected in the time domain before the FFT so that the ICI is minimized. In the frequency domain, the integer CFO should be first estimated to determine the mapping from the logical to physical subcarriers and extract the used subcarriers. Then, refined timing and channel estimation are performed. After equalization and decoding, the transmitted data can be fully recovered. The integer and fractional parts of the CFO are also sent to the crystal controller module to adjust the local clock synthesizer so that the CFO and sample clock error are both minimized; the coarse and refined timing estimates are also given to the FFT data extraction module for more accurate control of FFT windows.

It should be noted that this OFDM receiver structure has taken into account the tradeoff between the performance and complexity. For instance, it is possible
to refine the fractional CFO estimate in the frequency domain, however, as shown in [9,14–16], its marginal performance improvement over the time-domain methods cannot justify the extra complexity. It is also possible to estimate the integer CFO in the time domain using the methods of [17–20], however, they only support a relatively small estimation range and the remaining integer CFO ambiguity still needs to be solved in the frequency domain. For instance, to support an estimation range of ±28 subcarrier spacing, the methods of [17–20] require the training symbol to consist of at least 28 identical segments, which is simply unrealistic for practical OFDM systems. Hence, we take a more cost-effective approach that the fractional and integer CFOs are estimated in the time and frequency domains respectively rather than the other way around. More detailed descriptions of this receiver structure are given in Chapter 3, 4, and 5 of this thesis.

It is also worth noting that the receiver structure in Figure 1.4 is only suitable for the reception of the signal from a single transmitter. In other words, it is not applicable to the uplink of orthogonal frequency-division multiple access (OFDMA) systems. In those systems, timing and frequency adjustment at the receiver is far more difficult, so the synchronization is usually achieved in a closed-loop-control manner such that the transmitters adjust their timing and frequency according to the receiver’s instructions. The signal modeling of this synchronization scheme is presented in Section 2.3.3, and more detailed discussion is given in Chapter 6 and 7 of this thesis.
1.4 Literature Review

1.4.1 OFDM

A brief review on the history of OFDM research and the development of practical OFDM systems was presented by Zou and Wu [21] with many examples. Bingham predicted the time for multicarrier modulation had come in [22] with detailed explanation on the basic theories behind the multicarrier modulation schemes. In [23], Wang and Giannakis discussed the potential multiple access schemes based on OFDM modulation for future-generation broadband wireless systems. Some other works on the general aspects of OFDM include, but are not limited to, [24–26].

The sensitivity of OFDM systems to timing offset and CFO was studied in a number of papers [13, 27–30]. These results motivated a significant amount of research on OFDM synchronization techniques in the past one and a half decades. Recently, Morelli et al summarized the most important advances in this area in a comprehensive survey [31].

1.4.2 Coarse Timing Estimation

In OFDM systems coarse timing estimation is required to determine the positions of FFT windows. This is realized by exploiting the auto-correlation of OFDM signals in the time domain.

When the OFDM transmission is symbol based and continuous, e.g., in digital broadcasting systems, the high correlation between the CP and the OFDM symbol that follows it can be exploited for timing estimation. The methods in [32–34] were based on this idea, and some refinements were proposed in [35–38]. However, in multipath fading channels, the correlation between the CP and OFDM symbol is compromised by the ISI, and the performance of CP-based methods largely depends on the channel condition. This undermines their reliability in harsh wireless communication environments.

For packet based OFDM systems, usually one or more known training symbols are transmitted at the beginning of each OFDM burst to facilitate timing estimation. Schmidl and Cox [10] proposed to construct the training symbol with two identical halves in the time domain and estimate the timing from the auto-correlation of the received signal. Coulson refined that scheme in [39], and gave detailed performance analysis in [40]. Minn et al [41] proposed to use a training symbol with more than two identical segments, and suggested to flip the signs
of the segments with certain pattern to give the timing metric a steeper roll-off trajectory at the ideal timing position. Shi and Serpedin analyzed Minn’s synchronization scheme in [42], and proposed a more advanced timing metric based on the maximum likelihood (ML) criterion. As an extension to van de Beek’s method [33], Cheng and Chou [43] proposed a timing estimator using the training symbols composed of identical segments. However, the extension failed to address the original method’s vulnerability to multipath fading.

If a training symbol is produced by transmitting a real-valued pseudo-random sequence on the even subcarriers and leaving all the odd subcarriers unused, e.g., using the binary phase shift keying (BPSK) modulation for all even subcarriers, the training symbol will have a conjugate-symmetric structure in the time domain. The conjugate-symmetric structure can be detected by the timing metric proposed in [44–46]. In an additive white Gaussian noise (AWGN) channel, those timing metrics can generate a sharp peak to indicate the ideal timing position of the training symbol. However, the conjugate-symmetric structure can be violated by various channel impairments including CFO and multipath fading, so the performance of those methods can be compromised in practical wireless communication channels where those impairments are prevalent. Moreover, since the conjugate-symmetric structure based methods cannot be simplified by the integrate-and-dump algorithm, their complexity is much higher than that of autocorrelation-based methods.

The IEEE 802.16 (WiMAX) OFDMA physical layer standard [4] has specified a training symbol that consists of three highly correlated but not identical segments. This scenario has not been investigated by the aforementioned literature and a heuristic solution was proposed in [47] without rigorous justification.

1.4.3 Coarse Frequency Estimation

The correlators constructed for timing estimation usually also provide frequency offset information. A number of CFO estimators were developed in conjunction with the timing estimation algorithms. The CP-based frequency estimation methods [33, 37, 38] suffer from the timing estimation error and multipath fading, so their performance is usually inferior to that of preamble-based methods in realistic wireless communication environments. Some well-known preamble-based CFO estimation methods are briefly reviewed as follows.

Moose [48] proposed an estimator based on the observation of two consecutive identical symbols. Schmidl and Cox [10] constructed a training symbol with two identical halves to double the estimation range. Later, Morelli and Mengali [17]
extended that idea to a preamble composed of more than two identical segments, and called their method an Extended Schmidl and Cox Algorithm (ESCA). The ESCA constructs a number of component estimators based on different pairs of the identical segments in the training symbol, then combines them using the best linear unbiased estimator (BLUE) principle. When the signal to noise ratio (SNR) is known, Minn et al [18, 19] proposed several methods to take advantage of the SNR knowledge for further performance improvement. Yu and Su derived a ML estimator in [49], however, the complexity is much higher than that of the other methods due to the lack of closed-form expression for the estimate. The coarse CFO estimators of [50, 51] were able to combine multiple CFO estimates with different estimation ranges, however, the heuristic approaches did not take into account the correlations among the estimates and the performance was much inferior to that of the ESCA [17].

The estimation range of a time-domain CFO estimator is shown to be $\pm L/2$ subcarrier spacing [17], where $L$ is the number of identical segments in the training symbol. When the CFO is beyond this range, the integer CFO ambiguity has to be solved in the frequency domain [52]. However, the estimation and correction of the fractional CFO in the frequency domain is much more complex and less effective. Therefore, compared to a wide estimation range, higher fractional CFO estimation accuracy is more desirable for the time-domain coarse estimator so that the computationally expensive fractional CFO refinement in the frequency domain can be avoided.

1.4.4 Refined Frequency Estimation

A coarse CFO estimate can be refined in both the integer and fractional parts. The integer CFO refinement is needed to expand the estimation range of coarse CFO estimators and solve the ambiguity in the coarse estimates. Schmidl and Cox [10] proposed to construct two training symbols whose subcarriers are modulated with two differentially encoded sequences. This embeds a phase rotating pattern in the used subcarriers and the integer CFO can be estimated from the locations of the differentially encoded sequences. A ML integer CFO estimator based on Schmidl and Cox’s preamble structure was derived by Morelli et al [52], and the references of [53–55] investigated the methods for further performance improvement and complexity reduction. Unfortunately, in many practical OFDM systems like the IEEE 802.11a and IEEE 802.16 (WiMAX), the differentially encoded training symbols are not available. Applying the methods of [10, 52–55] to the training
symbols without differential coding gives much inferior performance in multipath fading channels.

The fractional CFO refinement, which is realized by jointly estimating the channel, can be performed after the integer CFO is determined. Morelli and Mengali derived a CFO estimator in [14] based on the ML criterion and implicitly took advantage of the knowledge of the length of channel impulse response. The authors kept their method general enough to be applicable to virtually any transmission schemes including both single and multiple carrier systems. Liu and Tureli [6] derived a blind CFO estimator that exploited the subspace of the transmit signal and channel. The subspace method was further investigated and extended by Chen [56]. Lin et al [57] developed a similar estimator that took into account phase noise. To avoid the highly complex grid search required by the aforementioned joint channel and CFO estimators, iterative algorithms were proposed to reduce the complexity. Lee et al [58] proposed an estimator based on the expectation maximization (EM) algorithm, and a non-linear recursive least square (NL-RLS) algorithm was proposed by Freda et al [59, 60]. Cui and Tellambura [9] combined both the pilot and null subcarriers in their iterative algorithm for further performance improvement. Actually, the iterative algorithms are expected to converge to the local minima of the cost function, so the performance largely depends on the initial coarse CFO estimate. Moreover, the complexity of those fractional CFO refinement algorithms is typically at the order of $O(N^2)$, where $N$ is the number of subcarriers. This complexity is beyond the capability of most practical OFDM systems. A more cost-effective approach is to improve the accuracy of coarse fractional CFO estimate and skip the fractional CFO refinement. We have introduced the receiver structure taking this approach in Section 1.3.

It should be noted that all the refined CFO estimation algorithms we mentioned above assume ideal timing synchronization, which is difficult to achieve before the integer CFO is determined. The residual timing offset leads to a linear phase shift on the subcarriers and compromises the performance of those joint CFO and channel estimators. A joint timing, CFO, and channel estimator was proposed in [61] by combining the ML principle with a sliding observation vector. Unfortunately, that algorithm has a limited CFO estimation range and only refines the fractional CFO. The joint estimation of timing, channel and integer CFO has not been discussed in the literature.
1.4.5 Refined Timing Estimation

An accurate timing estimate not only reduces the chance of ISI, but also improves the performance of pilot-based channel estimation methods [62–64]. As we explained in Section 1.2, a residual timing offset aggravates the frequency selectivity of the channel and degrades the performance of channel estimation. A thorough analysis of the impact of residual timing offset on system performance was presented in [11].

In the time domain, the boundaries between the OFDM symbols are blurred by the multipath channel, which makes it very difficult to accurately estimate the ideal timing positions of FFT windows. Although great efforts have been made by Minn et al [41] and Park et al [44] to construct certain patterns in the training symbol to improve the accuracy, the effectiveness of those methods can be compromised by the channel impairments like CFO, multipath fading, etc.

For the OFDM systems with continual transmission, e.g., DAB [1] and DVB-T [2], Yang et al [65] proposed a pilot based timing refinement algorithm which was able to detect the strongest channel path and take average over a number of symbols for further performance improvement. Minn and Bhargava [66] also suggested to use the estimated strongest channel path to refine the timing estimates. Speth et al [34] proposed to evaluate the likelihood of timing estimates by the energy in the truncated time-domain channel estimate, however, when the length of the channel was not exactly known, there would be a timing ambiguity. Recently, Mostofi and Cox [11, 67] proposed an iterative method that kept fine-tuning the timing position of the channel taps until the decision errors on the data subcarriers were minimized. However, the utilization of decision device in the cost function considerably increased the complexity of the iterative method, whose convergence behavior still needs to be carefully analyzed and verified.

For packet oriented OFDM systems, a preamble is usually placed at the beginning of every OFDM burst for timing, frequency and channel estimation. Larsson et al [68] proposed a joint channel and timing estimator based on the ML principle and the generalized Akaike information criterion (GAIC). The method has very high complexity, but works very well in indoor wireless communication environments where the time intervals between channel taps are usually small. Due to the greater distance to the scatterers in outdoor environments, the time intervals between channel taps are significantly increased. This compromises the efficiency and accuracy of the GAIC based method. A novel timing estimation method based on Bayesian principle was proposed by Pacheco et al [69], however, it also assumes
an indoor environment and the performance is not as good as that of the GAIC method.

1.4.6 OFDMA Uplink Synchronization

In an OFDMA system, multiple users are allowed to transmit in the uplink channel simultaneously. This reduces users' delays and improves the efficiency of bandwidth allocation. Because different users usually have different timing and frequency offsets, the synchronization problem in the uplink of an OFDMA system is more challenging than that in a single-user OFDM system and has attracted the interest of a lot of research in recent years.

van de Beek et al [70] extended their single-user synchronization algorithms to the multiuser scenario by assuming a block-wise subcarrier allocation to each user. Barbarossa et al [7] also extended their single-user null-subcarrier based method to the uplink of an OFDMA system where the blocks of user data subcarriers were separated by bands of unused subcarriers. Later, Yao et al [8] analyzed the null-subcarrier based method and proposed a small extension. Cao et al investigated the interleaved subcarrier allocation scheme for OFDMA systems in [71], and then proposed subspace based methods for user identification and CFO estimation in [72–74]. Morelli [75] proposed a low-complexity timing and CFO estimator for OFDMA systems with only one asynchronous user. That work was extended by his colleagues in [76, 77] using the space-alternating generalized expectation-maximization (SAGE) algorithm for more general cases where multiple asynchronous users existed in the system. Recently, Fu et al [78] modified the SAGE algorithm for reduced complexity and improved efficiency.

If perfect knowledge of CFO is assumed to be available at the receiver, then the method to correct CFO in a single-user system is straightforward; however, it is not the case in an OFDMA system where multiple users may have different CFOs. For instance, if two users’ CFOs are +0.1 and −0.1 subcarrier spacing respectively, directly correcting one user’s CFO in the time domain will enlarge the other user’s CFO and increase ICI. Huang and Letaief [79] investigated this problem and proposed to account for different users’ CFOs in the frequency domain with an equalizer. Unfortunately, this solution requires quite high complexity which compromises the advantage of OFDM over single-carrier modulation schemes.

All the aforementioned OFDMA synchronization schemes assume the transmission of a training symbol in the uplink. A prerequisite for those synchronization schemes is the base station (BS)’s knowledge of existing subscriber stations (SSs),
which is difficult to obtain before the uplink synchronization is established. In other words, the users need to be detected before synchronization can be achieved, while the synchronization is also expected to be achieved before the users can be detected, which leads to a deadlock.

One way to resolve the dilemma is to use a random access process. For instance, in the IEEE 802.16 standard [4], the terminals only transmit on a given set of subcarriers of the specified OFDM symbols with a specially designed signal before they are detected by the BS. Then, the terminals adjust their transmission parameters and repeat this random access process until the timing and frequency errors are within an acceptable range. This way, the interference between the SSs is effectively managed and minimized. And more importantly, the highly complex frequency-domain equalization at the receiver is not required for CFO correction.

1.4.7 OFDMA Ranging Channel Detection

The uplink synchronization in practical OFDMA systems like the IEEE 802.16 is realized by a mechanism called ranging. A subscriber station (SS) achieves initial timing and frequency synchronization using the preamble in the downlink before any uplink transmission starts. A ranging opportunity is a set of subcarriers in a number of OFDM symbols allocated by the BS for the transmission of a specially designed signal in the uplink channel. A SS can randomly select a ranging opportunity and transmit a randomly selected ranging signal from a given set. The BS estimates the timing and frequency offsets of the SSs and instruct them for adjustments until the offsets are within an acceptable range. Because all the SSs need to pass through this process before any data transmission starts, the interference among the SSs in the communication system is therefore managed and minimized.

To utilize the communication resource of the ranging channel in a more efficient manner, contention based multiple access schemes are usually employed to allow multiple SSs to share the same ranging opportunity without conflicting with each other. For instance, the ranging channel specified in the IEEE 802.16 standard is a multi-carrier code-division multiple access (MC-CDMA) channel. For those multiple access channels, it is the multiple access interference (MAI) rather than the noise that limits the detectors’ performance. The ranging detection methods of [80–84] are based on single user detection algorithms so the performance degrades rapidly with the increase in the number of ranging subscriber stations (RSSs).

Other ranging detection approaches [85–87] were proposed to mitigate the MAI
by specially designed ranging codes and modulation schemes. However, the specially designed codes work best only under certain channel conditions, and the effectiveness of a code design can be compromised by a large timing offset or long channel multipath spread. For instance, the methods in [85–87] require RSSs’ timing offsets to be shorter than the CP, which may be violated in broadband wireless communication systems.

1.4.8 ICI Analysis

In practical OFDM systems, a time-varying channel can cause ICI in the frequency domain signal. Russel and Stüber [88] analyzed this problem in a mobile wireless channel and derived a closed-form expression for the ICI power. This work was extended by Robertson and Kaiser [89] who applied their analytical results to an OFDMA uplink channel and obtained more insights into the ICI problem. Later, Li [90] suggested polynomial upper and lower bounds to facilitate the computation of the closed-form bounds. Recently, more advanced frequency domain equalization techniques were proposed in [91, 92] to collect the signal energy spread in neighboring subcarriers for better performance. This idea resembles the equalization technique that has been developed for single-carrier systems to combat ISI in the time domain. With the frequency domain equalizers, the ICI from neighboring subcarriers are mitigated and the residual ICI power becomes dependent on the equalizer design. An extension to the classic works incorporating the impact of frequency-domain equalization is needed to provide useful guidelines to the equalizer design.

1.4.9 Turbo Equalization

The concept of iterative equalization was first presented in [93] and named “Turbo equalization”. This idea was extended to channel equalization and multi-user detection in [94,95]. Later, Tüchler et al [96,97] proposed some linear complexity Turbo equalization methods based on minimum mean square error (MMSE) criterion, but the matrix inverse required by those algorithms compromised their low-complexity advantage.

The use of the Particle filter as a de-convolution device was introduced by Liu and Chen [98], and then became an important research topic for its parallel computational structure and ability to track dynamics of unknown parameters in the system [99]. Recently, Dong et al [100,101] reported Particle filter’s good perfor-
mance in iterative detection schemes for multiple-input-multiple-output (MIMO) channels. On the other hand, for severe ISI channels, the conventional Particle filter has never been reported as a good soft-input-soft-output (SISO) detector in an iterative scheme. Bertozzi et al [102, 103] mentioned the idea of grouping the particles for higher efficiency, however, the application of the grouping idea to the Turbo receiver has not been investigated.

1.5 Motivation of the Research

This research on synchronization techniques for practical OFDM systems was motivated by our development of an IEEE 802.16 compliant OFDM receiver on hardware. At early stage of the project, we mainly focused on the channel estimation and data detection algorithms, and the synchronization part largely followed Schmidl and Cox’s work [10]. Computer simulation results showed that our channel estimation and data detection algorithms could achieve 3-5dB gain over conventional methods under the assumption of perfect synchronization. However, in the presence of timing and frequency estimation errors, the advantage of our receiver simply disappeared because the synchronization modules could not give satisfactory performance at the operating SNR of our data detection algorithms, and the overall performance of the receiver was limited by the synchronization errors. This suggested an imbalance in existing research and motivated our study on novel synchronization algorithms practical for hardware real-time implementation.

Due to our practical work we were motivated to look at practical multi-user and multi-cell environments. The following paragraphs describe in more details the motivating scenarios for both downlink and uplink synchronization.

In the downlink of IEEE 802.16 (WiMAX) systems, a scheme called partial usage of subchannels (PUSC) was defined to mitigate the interference between the cells. That scheme allows a BS to pick up one out of three disjoint sets of subcarriers to reach the SSs at the edge of the cell. Because a SS can be at the edge of three cells simultaneously, it is possible for the BSs closest to a SS to use different subcarrier sets to minimize the interference. For the same reason, the WiMAX training symbol is defined in the frequency domain by using one out of every three subcarriers to form three disjoint sets of subcarriers. Unfortunately, three is not divisible by the total number of subcarriers which is a power of 2, so in the time-domain the training symbol is made up of three highly correlated but nonidentical segments. This scenario has not been systematically studied in the
1.5 Motivation of the Research

literature and only a heuristic solution for coarse timing synchronization was provided in [47] without rigorous justification. Furthermore, because the nonidentical segments in the training symbol violated the basic assumption of time-domain CFO estimators [10,17–19,49], it was not clear whether it was possible to use that training symbol for CFO estimation. These problems motivated our research on the practical synchronization solutions applicable to the generalized training symbol structure, and most of the outcomes are presented in Chapter 3, 4 and 5.

In IEEE 802.16 systems, the uplink synchronization between the SSs and BS is realized by a contention based MC-CDMA ranging channel. Same as other CDMA channels, multiple access interference (MAI) limits the performance of multiuser detectors. Existing literature [81–84] treated MAI as noise so the performance degraded rapidly with the increase in the number of RSSs. Another problem yet to be studied in the literature is the interference from the RSSs to the data subscriber stations (DSSs). When there are a large number of RSSs, the signal power in the ranging channel could be much higher than that on the data subcarriers, so the ICI and ISI caused by the asynchronous ranging users can be high enough to impair the signal quality of the DSSs who are sharing the same transmission time slots with the RSSs. Also, from a system point of view, the missed detection probability provided by the simulations does not give as much information as other performance measures, e.g., the maximum number of users a BS can serve, the average time needed to complete a ranging process, the successful rate of handover, etc. Directly measuring those system parameters via simulations is very complex and time consuming, it would be convenient if the simulated missed detection probabilities can be directly translated into system performance measures. These problems motivated our research on the ranging detection algorithms and the system performance analysis. Most of our research results on the OFDMA uplink synchronization are presented in Chapter 6 and 7.

The development of a frequency domain equalizer leads to our research results presented in Chapter 8 and 9. The existing residual ICI power analysis does not take into account frequency domain equalization, so we extend the classic works to provide useful guidelines for the performance and complexity tradeoff of the equalizers. We also improve the conventional Particle filtering technique to provide a low-complexity alternative to the Maximum A Posteriori (MAP) equalizer in Turbo receivers.
1.6 Thesis Outline

The flow of this thesis reflects, to a large part, the procedures of timing and frequency synchronization in a bi-directional wireless communication system. The synchronization needs to be established in the downlink first, then in the uplink via the ranging process. For the residual ICI caused by certain channel impairments, we analyze the performance bounds of frequency-domain equalization and propose a novel Particle filtering technique for Turbo receivers. The outline of this thesis is shown in Figure 1.5.

Figure 1.5: Thesis outline.

Chapter 2 introduces the mathematical modeling of the wireless communication channels and the practical OFDM signals. We use a tapped delay line with time-varying taps to represent the wireless channel. The preamble based synchronization system model is generalized from the IEEE 802.11 [3] and IEEE 802.16 [4] standards. We assume the OFDM symbols in one burst have different power levels to reach the users at different distances, and each training symbol consists of highly correlated but not necessarily identical segments. We also give the mathe-
matical description of an OFDMA uplink ranging channel which is compliant to the IEEE 802.16 standard. These realistic channel and system models ensure the direct applicability of the results presented in this thesis to practical OFDM systems.

Chapter 3 describes a novel coarse timing estimation algorithm. We derive the statistics of the correlation between highly correlated segments in the generalized training symbols, and propose a method to combine the outputs of multiple correlators in a linear manner to minimize the false alarm probability. Performance analysis and simulation results for different scenarios confirm the performance of the proposed method in various channel conditions.

Chapter 4 investigates a universal fractional CFO estimator applicable to the generalized training symbols no matter whether they are composed of identical segments or not. We show the universal CFO estimator outperforms the ESCA [17] at low SNR and approaches the Cramer-Rao bound (CRB) at high SNR. It is also possible to combine a number of such universal CFO estimators using the BLUE principle, however, we show that the BLUE is most cost-effective in the multiple-training-symbol case because the marginal performance gain in the single-training-symbol case may not justify the extra complexity.

Chapter 5 presents a range of refined joint timing and frequency estimation algorithms. We simplify the joint channel, timing and CFO estimator by different assumptions and approximations to obtain four refined estimators. A hybrid scheme is introduced to provide improved performance and complexity tradeoffs. Analysis and simulation results show that the novel hybrid method can achieve significant performance improvement over the autocorrelation based low-complexity algorithm at the cost of slightly increased complexity. When the true channel multipath spread is not exactly known, we propose a successive timing estimator and its simplified version to solve the timing ambiguity problem. The novel timing estimation algorithms work for both indoor and outdoor wireless communication channels. Performance analysis and simulation results show that the proposed successive timing estimators achieve quite high accuracy even at very low SNR.

Chapter 6 gives detailed information about the proposed successive multiuser detector (SMUD) for contention based ranging channels. The new method uses path-by-path iteration approach to jointly estimate the ranging users and their channels, then remove the interference from the detected paths to give low-power users a better chance of detection. A simplified algorithm was also proposed to reduce the complexity and improve the efficiency of the SMUD. Besides the improved user detection performance, the proposed algorithms are able to mitigate the in-
terference from the ranging users to the data subscriber stations and considerably improve their data throughput.

Chapter 7 analyzes the performance of the ranging detectors from a system point of view. More specifically, we translate the ranging detectors’ missed detection probabilities into the number of users a base station can serve with a given number of ranging opportunities, which is a more sensible performance measure for ranging channel bandwidth allocation. Our analytical results match the simulations reasonably well and reveal that the proposed reduced-complexity SMUD can allow more than double the number of users in a cell compared to the conventional methods.

Chapter 8 derives the bounds of residual ICI power that cannot be mitigated by a frequency-domain equalizer with a given number of taps. The results give useful guidelines for equalizer design and simplify performance analysis. It is indicated that the residual ICI power is inversely proportional to the number of equalizer taps, so the gain of employing additional equalization taps diminishes as the equalization window extends.

Chapter 9 studies a new Particle filter that can be used as the SISO deconvolution device in a Turbo equalization scheme. The method is developed for single-carrier systems but also applicable to OFDM systems without significant modification. It is shown that the proposed method approximates the MAP equalizer with lower complexity. The efficiency of the proposed method is further improved by an early-stopping algorithm.

Chapter 10 concludes this thesis by summarizing the original contributions. It reviews the new techniques we have proposed and their importance in solving OFDM synchronization problems. We also discuss future work and highlight the open areas in this subject.

1.7 Thesis Contributions

Below we list for each of the chapter where significant contributions have been made and what they are. Also given are references to the associated papers that have been published or submitted for publication as a result of this work.

Chapter 3

The following contributions have been included in our paper [104]:

1.7 Thesis Contributions

- The derivation of statistical properties of the autocorrelation of received signal at correct and incorrect timing positions.
- The construction of component timing metrics.
- The method to combine the component timing metrics in a linear manner to minimize the false alarm probability while keeping the missed detection probability almost the same.

Chapter 4

The following contributions have been included in our paper [105]:

- A universal fractional CFO estimator for the generalized training symbols composed of highly correlated but not necessarily identical segments.
- The derivation of the fractional CFO estimation error correlation matrix for the proposed universal CFO estimators.

The following contribution has not been included in any paper:

- A scheme that takes advantage of multiple training symbols with the least number of universal estimators.

Chapter 5

The following contributions have been included in our paper [106]:

- A joint timing, CFO and channel estimator based on the maximum likelihood (ML) channel estimation method and two simplified algorithms that can be implemented without matrix multiplications.
- A hybrid scheme that achieves significant performance gain over the autocorrelation based integer CFO estimator at the cost of slightly higher complexity consisting of a small number of inverse fast Fourier transforms (IFFTs).

The following contributions have extended our earlier work [15,16] but not yet been included in any paper:

- The derivation of joint timing, CFO and channel estimator using the minimum mean square error (MMSE) channel estimation method and the comparison to the joint estimator based on the ML method.
• A path-by-path successive refined joint timing and channel estimator.

• A recursive matrix update algorithm that simplifies the implementation of the proposed successive timing estimator.

• A simplified version of the successive timing estimator that can be implemented with a number of IFFTs.

• The guidelines for threshold settings of the successive timing estimator based on performance analysis.

Chapter 6

The following contributions have been included in our paper [107]:

• A path-by-path successive ranging channel detection algorithm and its efficient implementation.

• The method to mitigate the interference from RSSs to DSSs.

• The guidelines for threshold settings based on performance analysis.

The following contribution has not been included in any paper:

• A simplified version of the successive ranging channel detector that can be implemented with a number of IFFTs.

Chapter 7

The following contribution has not been included in any paper:

• A method to translate the ranging detectors’ missed detection probabilities to the number of SSs a BS can serve with a given number of ranging opportunities.

Chapter 8

The following contributions have been included in our paper [108]:

• The closed-form upper and lower bounds on the residual ICI power for three widely used channel models.

• Polynomial approximations to the bounds.
Chapter 9

The following contribution has been included in our paper [109]:

- The grouped Particle filtering (GPF) algorithm for Turbo equalization.

The following contribution has not been included in any paper:

- An early-stopping algorithm for the GPF.
Chapter 2

System Model

2.1 Introduction

In this chapter, we describe the mathematical modeling of the wireless channel and the OFDM system of interest. The models capture the nature of the problem, while the mathematical expressions enable us to accurately formulate and analyze the problem in the rest of the thesis.

We first introduce the wireless channel model. The classic models for propagation loss and shadowing effect presented in [110–112] are briefly reviewed. The wireless multipath channel can be modeled by a tapped delay line, and the methods to emulate time-varying Rayleigh fading on each channel tap are discussed. Two channel models recommended by the standards [113, 114] are selected for realistic performance evaluation of the algorithms in wireless communication environments.

We consider two kinds of synchronization scenarios typically seen in practical OFDM systems for downlink and uplink communications respectively. In the downlink, one or more training symbols are transmitted at the beginning of every OFDM burst to facilitate timing, frequency, and channel estimation. In the uplink, a number of subcarriers in certain OFDM symbols are allocated to the ranging channel via which the BS can estimate the timing and frequency offsets of the RSSs and send them the instructions for adjustment. Both the downlink preamble-based and the uplink ranging-based synchronization system models are generalized from the IEEE 802.11 [3] and IEEE 802.16 [4] standards so that the algorithms developed in this thesis are directly applicable to those practical OFDM systems. For the emerging OFDM systems, the chance of direct applicability is also quite high.
2.2 Channel Model

In this section, we briefly review the classic models for propagation loss and shadowing, then explain the models of multipath fading. We give the details of two realistic channel models, which represent stationary and mobile wireless communication channels respectively and are selected for the simulations in this thesis.

2.2.1 Propagation Loss

A radio signal becomes weaker and weaker with its propagation in the air. There are many propagation loss models developed in the literature, including the well-known COST-231 Hata model \[110,111\] recommended by the WiMAX Forum. The median propagation loss of that model is given by

\[
PL = 46.3 + 33.9 \log_{10} f_c - 13.82 \log_{10} h_b + (44.9 - 6.55 \log_{10} h_b) \log_{10} d_m - C_J(h_m) + C_F,
\]

(2.1)

where \( f_c \) is the carrier frequency, \( h_b \) and \( h_m \) are the antenna heights of the base station (BS) and subscriber station (SS) respectively, \( d_m \) is the distance between them. \( C_F \) is the correction factor, which equals to 3dB and 0dB for the urban and suburban areas respectively. \( C_J(h_m) \) is the SS antenna correction factor given by

\[
C_J(h_m) = (1.11 \log_{10} f_c - 0.7)h_m - (1.56 \log_{10} f_c - 0.8).
\]

(2.2)

2.2.2 Shadowing

Besides the distance, there are many other factors affecting the received signal strength. For instance, trees and buildings located between the transmitter and receiver could cause temporary signal degradation, while a temporary line-of-sight transmission path could lead to abnormally high received power. Due to the random nature of shadowing, usually it is modeled by a log-normal random variable \[112\]

\[10 \log_{10}(\chi) \sim \mathcal{N}(0, \sigma^2_{\chi}),\]

(2.3)

where \( \chi \) is the shadowing factor, \( \mathcal{N}(0, \sigma^2_{\chi}) \) denotes a Gaussian distribution with zero mean and variance \( \sigma^2_{\chi} \). According to \[112\], typical values for \( \sigma_{\chi} \) are in the range of 6-12dB, which means that the received signal power could fluctuate in a quite large dynamic range.
In practical wireless communication systems, the automatic gain control (AGC) module at the receiver will adjust the amplifier to compensate for the propagation loss and shadowing effect. This keeps the received signal strength almost constant during the communications and allows the use of full dynamic range of the analog-to-digital converter (ADC) to reduce quantization error. Unfortunately, not only the useful signal but also the noise is amplified by the AGC, and the ratio between the energy of signal and noise remains the same after the amplification. It means that the AGC cannot improve the signal to noise ratio (SNR) of the received signal, which can vary dramatically with the channel gain in practical mobile wireless communication systems. Therefore, the performance of algorithms developed for wireless communications need to be verified in a wide SNR range to understand their behaviors in realistic environments.

2.2.3 Multipath Fading

The multipath fading is a phenomenon caused by multiple copies of the transmit signal arriving at the receiver via different routes. The length of the route determines the delay of the replica of transmit signal. In outdoor environments, the distance to the scatterers is usually larger than that in indoor environments, so the channel’s multipath spread, which is the time interval between the first and last arrived channel paths, is also expected to be larger. For each route, the effect of the channel can be described by a multiplicative complex gain that represents the magnitude ratio and phase difference between the transmitted and received signals. Because the received signal is a summation of multiple copies of the transmit signal multiplied by independent complex gains, the multipath channel can either boost or attenuate the signal and introduce frequency-selective distortions.

The effect of a multipath fading channel can be well modeled by a tapped delay line as shown in Figure 2.1, where $L_h$ is the number of channel taps, $\Delta_t[l]$ and $h[l](t)$

![Tapped delay line channel model](image)
are the relative delay and complex gain of the $l^{th}$ channel tap respectively. Denote the transmit signal as $x(t)$. Ignoring the noise, we can represent the received signal from a multipath channel as

$$\hat{x}(t) = \sum_{l_1=1}^{L_h} h[l_1](t) x \left( t - \sum_{l_2=1}^{l_1} \Delta[l_2] \right).$$

(2.4)

The channel gain $h[l](t)$ is a function of time $t$, which indicates that the channel is varying over the time. The time-varying property of a channel tap can be described by the correlation of the channel gain at a given time interval, which can be further characterized by the power spectral density (PSD) function of the autocorrelation [115]. The classic model for time-varying Rayleigh fading was presented in [116] and typically referred to as Jakes’ model. It was shown in [117] that the channel taps generated by Jakes’ models are actually correlated, however, if the receiver does not take advantage of the a priori knowledge of the correlation, the impact on simulation results is negligible. Recently, a method to emulate the time-varying fading of fixed radio links was proposed by a team in the Stanford University [113] and known as the SUI model. In this thesis, these two methods are utilized in the simulations of mobile and stationary wireless communication channels respectively.

For the given wireless communication environment, it is feasible to measure the delays and gains of each channel taps to obtain a statistically averaged channel power delay profile. In the literature, Jeong et al [118] listed a number of typical channel power delay profiles measured in a Korean metropolitan area; a team in the Stanford University [113] suggested several wireless channel models for fixed WiMAX radio links; and the international telecommunication union (ITU) recommended a few channel power delay profiles for different communication environments [114]. These profiles reflect the channel conditions in real communication environments, and give realistic performance evaluation for the algorithms developed for practical wireless communication systems.

In this thesis, two channel models are selected for the simulations, namely “CH-A” and “CH-B”. Their power delay profiles are listed in Table 2.1, which follow the SUI-3 and Vehicular-A channel models specified in [113] and [114] respectively. CH-A has a maximum Doppler frequency of 0.4Hz, which is much smaller than the OFDM symbol rate typically at the magnitude of several kilohertz, hence the channel almost remains constant in one symbol period. For CH-B, we use Jakes’ models [116] to emulate the time-varying Rayleigh fading on each of the channel
taps for a SS at speed 120km/h. This corresponds to a maximum 388Hz Doppler frequency for an OFDM system with 3.5GHz carrier, almost one thousand times that of CH-A. This suggests much higher time-selectivity in CH-B than that in CH-A. Table 2.1 also indicates that CH-B has higher frequency selectivity as well because its multipath spread is much larger than that of CH-A.

<table>
<thead>
<tr>
<th>Tap</th>
<th>CH-A</th>
<th>CH-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative Delay (µs)</td>
<td>Average Power (dB)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>-5</td>
</tr>
<tr>
<td>3</td>
<td>0.9</td>
<td>-10</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

We run our channel models from a random initial state for 10 seconds and measure the average channel gain of the first symbol in each 5ms OFDM burst. The results are plotted in Figure 2.2. It shows that CH-A is more likely to have a deeper and longer fading, which can be explained by its fewer channel taps and lower spatial diversity.

![Figure 2.2: The fluctuation of channel gains in CH-A and CH-B.](image-url)
Since deep fading and severe time and frequency selectivity are the most prominent challenges to the wireless signal processing algorithms, CH-A and CH-B are expected to give realistic performance evaluations.

2.3 Signal Model

The system models of interest are introduced in this section. We first give the mathematical expression of a general OFDM signal, then describe the generalized training symbols and the transmit signals in the time slots of a ranging opportunity as two special cases of the general OFDM signal model.

2.3.1 OFDM Signal

An OFDM signal consists of a number of OFDM symbols concatenated in the time domain. A baseband OFDM signal can be represented by

\[ x(t) = \sum_{i} x_i(t - t_i) (s(t - t_i) - s(t - t_{i+1})) , \]

where \( x_i(t) \) and \( t_i \) are the waveform and start time instance of the \( i^{th} \) OFDM symbol respectively, \( s(t) \) is the unit step function defined by

\[ s(t) = \begin{cases} 1, & t \geq 0; \\ 0, & t < 0. \end{cases} \]

The baseband OFDM symbols share the same set of physical subcarriers indexed \( \{-N/2, -N/2 + 1, \cdots, N/2 - 1\} \), whose frequencies are equal to the indices multiplied by the subcarrier spacing \( \Delta f \). \( N \) is the total number of subcarriers, usually a power of 2 to facilitate the FFT and IFFT operations. Each OFDM symbol lasts for \( T_s \triangleq 1/\Delta f \), plus a certain period for the CP. The highest frequency of the baseband OFDM signal is \( N\Delta f/2 \), so according to the Nyquist-Shannon Theorem [119], each OFDM symbol can be fully recovered by \( N \) samples of the signal with an time interval \( t_s \triangleq 1/(N\Delta f) \). The sample time interval \( t_s \) is also known as the duration of one OFDM sample.

As we explained in Section 1.1, not necessarily all the subcarriers of OFDM symbols are used, and there can be a mapping between the original data sequence and the physical subcarriers. To keep the symbol model generic, for OFDM symbol \( i \), we define \( N_p[i] \) as the number of used subcarriers, and the mapping from the
logical to physical subcarriers is defined by a vector $o_i$ where $o_i[m]$ gives the physical subcarrier index of the $m^{th}$ data symbol. Write the data also into a $(N_p[i] \times 1)$ vector $X_i$, the time-domain signal for the $i^{th}$ OFDM symbol is given by

$$x_i(t) = \frac{1}{\sqrt{N_p[i]}} \sum_{m=1}^{N_p[i]} X_i[m] e^{j \frac{2\pi}{N} (t/t_s - N_g) o_i[m]},$$  

(2.7)

where $N_g$ is the CP length normalized by sample duration $t_s$.

### 2.3.2 Preamble

A diagram of one OFDM burst preceded by $I_p$ training symbols in the preamble is shown in Figure 2.3. The training symbols are defined in the frequency domain by using one out of every $L_i$ subcarriers, where $i$ is the training symbol index, and different symbols can have different $L_i$ values. For the example shown in Figure 2.3, $L_1 = 4$ and $L_{I_p} = 3$. The figure also shows that every OFDM symbol in the burst could have different power levels, which reflects the realistic situation in multiuser downlink channels where the BS uses different signal strengths to reach SSs at different distances. The training symbols in the preamble usually have the same power, however, we keep our signal model as general as possible and assume the power levels can be different. And, the equal power scenario can be seen as a special case of the generalized preamble definition.

![Figure 2.3: Diagram of one OFDM burst.](image)
In the time domain, using (2.7), we have

\[
    x_i(t + t_s N/L_i) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N_p[i]} X_i[m] e^{j \frac{2\pi}{N} (t/t_s + N/L_i - N_p) \alpha_i[m]} = \frac{1}{\sqrt{N}} \sum_{m=1}^{N_p[i]} X_i[m] e^{j \frac{2\pi}{N} (t/t_s - N_p) \alpha_i[m]} e^{j \frac{2\pi}{N} \frac{N}{L_i} (\alpha_i[1] + (m-1)L_i)} = x_i(t) e^{j \frac{2\pi}{L_i} \alpha_i[1]}.
\]

(2.8)

This indicates that the magnitude of the time domain waveform of the training symbol is repeated at a period of \((N/L_i)t_s\), while the phase rotates \((2\pi \alpha_i[1]/L_i)\) over each period.

Figure 2.4: Diagram of a training symbol with \(N = 8, L_1 = 3\).

Figure 2.4 shows an exemplary training symbol with \(N = 8\) and \(L_1 = 3\) such that its waveform \(x(t)\) consists of three periods of a sine wave in the time domain excluding the CP. The baseband signal \(x(t)\) is passed through the low-pass filter and sampled every \(t_s\). We obtain a sequence of samples

\[
    x[n] = \int_{-\infty}^{+\infty} x(t) \delta(t - nt_s) \, dt = x(nt_s),
\]

(2.9)

where \(\delta(\cdot)\) is the Dirac delta function. When \(N\) is not an integer multiple of \(L_i\), the period of \(x(t)\) is not sample-spaced, which causes the repetition structure not
preserved in the sampled sequence \( x[n] \). Figure 2.4 has illustrated this fact with a straightforward example. We know one period of the sine wave \( x(t) \) lasts for \( N/L_1 = 2\pi \) samples, so \( x[0] \) is not identical to either \( x[2] \) or \( x[3] \), although they are highly correlated. The analysis above suggests that the scenario studied in the literature where the training symbols consist of several identical segments is only a special case of the generalized training symbol defined in this thesis.

The received baseband signal, after low-pass filtering and sampling, is given by

\[
y[n] = e^{j2\pi n\epsilon_0} \sum_{l=1}^{L_h} h[l](nt_s) x[n - \tau[l]] + w[n],
\]

(2.10)

where \( \epsilon_0 \) is the true CFO normalized by subcarrier spacing \( \Delta f \); \( h[l](t) \) and \( \tau[l] \) are the complex gain and sample-spaced delay of the \( l^{th} \) channel tap; \( w[n] \) is the additive white Gaussian noise (AWGN) with variance \( \sigma^2_w \); and, \( \hat{x}[n] \) is the sampled transmit signal after traveling through the multipath channel,

\[
\hat{x}[n] = \sum_{l=1}^{L_h} h[l](nt_s) x[n - \tau[l]].
\]

(2.11)

It is worth noting that (2.10) has absorbed the initial phase difference between the transmitter and receiver in the complex channel gain, and assumed the channel taps’ delays are all OFDM-sample spaced. As shown in [76,91], these simplifications have little impact on the performance of system modeling.

### 2.3.3 Ranging Signal

Ranging is the process of establishing an initial link in the uplink of OFDMA systems. It initiates closed loop timing and frequency control such that the SSs can adjust their transmission parameters to minimize ISI and ICI in the uplink received signal. A diagram of one ranging opportunity in a Time Division Duplex (TDD) OFDMA system is shown in Figure 2.5. The allocation of the ranging channel, which contains one or multiple ranging opportunities, is broadcasted in the downlink burst. Each ranging opportunity consists of \( N_r \) subcarriers in \( I_r \) OFDM symbols. For the example shown in Figure 2.5, \( I_r = 2 \).

The maximum delay of a RSS in a cell, in terms of OFDM samples, is determined by the cell radius and signal bandwidth. For a typical 1024-subcarrier OFDM system with 10MHz bandwidth and 5-kilometer cell radius, the maximum
propagation delay \( d_{max} \) is given by

\[
d_{max} = 2 \times \frac{5 \times 1000m}{3 \times 10^8 m/s} \times 10MHz \approx 333 \text{ (samples).}
\]  

We can see it is about a third of one OFDM symbol, exceeding the maximum CP length specified in the IEEE 802.16 (WiMAX) standard [4].

Suppose there are \( K_d \) data subscriber stations (DSSs) sharing the ranging time slots with \( K_r \) RSSs. Because the DSSs have synchronized with the BS, we assume their timing offsets are within \( \pm \tau_{max} \) samples, where \( \tau_{max} \) is the maximum acceptable timing offset usually much smaller than \( d_{max} \). For instance, in the IEEE 802.16 standard [4], the maximum timing offset for a DSS is required to be less than 25% of the minimum CP length, which equals to 8 OFDM samples for a 1024-subcarrier system, about 2.67% of the maximum timing offset of a RSS given in (2.12).

In one ranging opportunity, the transmit signal of the \( k^{th} \) RSS can be written
as
\[
x_k(t) = (s(t) - s(t - I_r(N + N_p)t_s)) \\
\times \frac{1}{\sqrt{N}} \sum_{m=1}^{N_r} X_k[m] e^{j \frac{2\pi}{Ts}(t/N_g - N_g)o^{(r)}[m]},
\]
(2.13)

where \( o^{(r)} \) denotes the mapping from the logical to physical subcarriers allocated to the ranging opportunity; \( X_k \) is the BPSK modulated ranging code sent by RSS \( k \), whose elements are given by
\[
X_k[m] = \sqrt{a_k} C_{n_k}[m],
\]
(2.14)

where \( C_n \) denotes the BPSK modulated \( n \)th pseudo-random ranging code, \( n_k \) and \( a_k \) are the code index and power level for RSS \( k \) respectively.

Suppose there are totally \( N_c \) available ranging codes, each RSS randomly selects one in a ranging opportunity. Following the literature [81, 83, 84], we assume \( N_c \) is much larger than \( K_r \) so that the probability for two or more RSSs selecting the same ranging code is negligible.

Different from ordinary OFDM signals, the ranging signal has a continuous phase within \( I_r \) OFDM symbols. This design reduces the interference to other SSs even in the presence of large timing offsets. We will discuss this feature in details at the end of this section.

The transmit signal of a single DSS can be described by the general OFDM signal model given in (2.5) and (2.7). However, besides the symbol index \( i \), now the DSS index \( k \) should also be incorporated into the expressions as follows:
\[
x_k(t) = \sum_{i=1}^{I_r} (s(t - (i - 1)(N + N_g)t_s) - s(t - i(N + N_g)t_s)) \times x_{i,k}(t - (i - 1)(N + N_g)t_s)
\]
\[
x_{i,k}(t) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N_p[i,k]} X_{i,k}[m] e^{j2\pi(t/N_g - N_g)o_{i,k}[m]/N},
\]
(2.15) \hspace{1cm} (2.16)

where for DSS \( k \) and the \( i \)th symbol in the ranging opportunity, \( N_p[i,k] \) denotes the number of used subcarriers, \( o_{i,k} \) denotes the mapping from the logical to physical subcarriers allocated to that user, \( X_{i,k} \) is the vector of modulated data symbols in the frequency domain.

The signal observed at the receiver is a summation of all the \((K_r + K_d)\) transmit
signals with different delays and frequency offsets via various multipath channels. After down conversion and filtering, the samples of the received signal in baseband can be expressed by

$$y[n] = \sum_{k=1}^{K_r+K_d} e^{j(2\pi\epsilon_k n/N)} \left( \sum_{l=1}^{L_h[k]} h_k[l](n t_s) x_k[n - \tau_k[l]] \right) + w[n], \quad (2.17)$$

where the RSSs and DSSs are numbered from 1 to $K_r$ and $(K_r + 1)$ to $(K_r + K_d)$ respectively. For the $k^{th}$ SS, $\epsilon_k$ is the CFO normalized by the subcarrier spacing; $L_h[k]$ is the number of channel taps whose complex gain and sample-spaced delay are $h_k[l](t)$ and $\tau_k[l]$ respectively. Assume each SS’s tap delays are arranged in ascending order, then the delay of SS $k$ can be represented by the delay of the first channel tap $\tau_k[1]$.

It is shown in Figure 2.5 that the allocation of FFT windows should take into account the range of acceptable timing offsets to minimize ISI and ICI among the SSs. In this thesis, we suggest to set the FFT window for the $i^{th}$ OFDM symbol in the ranging opportunity to

$$[(i - 1)N + i N_g - \tau_{max}, i(N + N_g) - \tau_{max} - 1]. \quad (2.18)$$

This avoids any ISI or ICI incurred by the synchronized SSs whose timing offset is within $[-\tau_{max}, N_g - \tau_{max} - \tau_k[L_h[k]]]$ samples.

After FFT, we denote $Y_i$ as the observation of all the subcarriers in the $i^{th}$ OFDM symbol of the ranging opportunity, which is a $N \times 1$ vector whose elements are

$$Y_i[m] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y[(i - 1)N + i N_g - \tau_{max} + n]e^{-j\frac{2\pi}{N}nm}. \quad (2.19)$$

And the observed signal on the subcarriers allocated to the $i^{th}$ OFDM symbol of the ranging opportunity is denoted by $\tilde{Y}_i$, where $\tilde{Y}_i[m] \triangleq Y_i[\hat{o}(r)[m]]$.

As shown earlier, the maximum delay of a RSS could be as large as a third of one OFDM symbol in a typical OFDM system. This means that there can be many RSSs whose timing offsets are larger than $(N_g - \tau_{max} - \tau_k[L_h[k]])$ samples and incur ISI and ICI in $Y_1$; however, thanks to the phase continuity of the ranging signal, $Y_2, Y_3, \ldots, Y_I$, are not affected as long as the timing offset does not exceed one symbol period, or more precisely, $(N + 2N_g - \tau_{max})$ OFDM samples. For the example shown in Figure 2.5, we can see that the signals in the second FFT window all have continuous phases, which indicates much less ICI and ISI than those in
the first OFDM symbol. The interferences and their causes in different OFDM symbols are summarized in Table 2.2. It is shown that RSSs’ CFOs cause ICI to all the symbols in the ranging time slot, while only the first OFDM symbol suffers from the extra ISI and ICI introduced by RSSs with large timing offsets.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>CFO</th>
<th>Range of Timing Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ICI</td>
<td>$(-\tau_{\text{max}}, N_g - \tau_{\text{max}}]$</td>
</tr>
<tr>
<td>{2, 3, ..., \text{I}_r}</td>
<td>ICI</td>
<td>$(-, N_g - \tau_{\text{max}}, N + 2N_g - \tau_{\text{max}}]$</td>
</tr>
</tbody>
</table>

### Table 2.2: Interference in OFDM symbols of a ranging opportunity.

#### 2.4 Summary

In this chapter, we have described the mathematical modeling of the wireless channel and the OFDM system of interest. The classic methods to model the propagation loss and shadowing effect of wireless channels have been briefly reviewed. These models suggested a large dynamic range of SNR in outdoor wireless communication environments.

We modeled the multipath fading channel with a tapped delay line, and emulated the taps’ time-varying Rayleigh fading by the Jakes’ model [116] and the SUI model [113] respectively for mobile and stationary wireless channels. Two channel models were selected for the simulations in this thesis. Their parameters followed the recommendation of the standards and were representative for deep-fading and time-frequency-selective characteristics of wireless communication channels respectively.

The mathematical expressions of OFDM signals were given, followed by the generalized definition of the training symbols. It was shown that the generalized training symbols did not necessarily consist of identical segments in the time domain in spite of the high correlation between them.

The OFDMA signal observed at the receiver in the ranging time slot was described mathematically. The timing positions of FFT windows were suggested to minimize ISI and ICI. It was observed that the RSSs were more likely to incur ICI and ISI on the first OFDM symbol in the ranging opportunity, and thanks to the continuous phase of the ranging signal over the whole ranging opportunity, the following OFDM symbols were much less affected by the large timing offsets.
Chapter 3

Coarse Timing Estimation

3.1 Introduction

An OFDM signal contains multiple OFDM symbols concatenated in the time domain. At the receiver, the boundaries between the OFDM symbols need to be determined before the signal can be translated to the frequency domain using an FFT function. In this chapter, we present a novel algorithm to take advantage of the special structure in the training symbols and locate the start positions of OFDM bursts.

In realistic wireless communication environments, the boundaries between the OFDM symbols are often blurred by multipath channels, so the ideal start of the FFT window at the receiver is usually difficult to locate in the time domain. We know that as long as the samples in the FFT window belong to the same OFDM symbol, the ISI and ICI are avoided. Therefore, it suffices for the coarse timing estimator to find the ISI-free FFT window of the training symbol, then the residual timing offset can be estimated from the frequency domain signal using a refined joint channel and timing estimator discussed in Chapter 5. For instance, in an AWGN channel, all the timing positions from sample \(0\) to \((N_g - 1)\) are good coarse timing estimates, and we leave the accurate timing estimation to a latter signal processing stage.

A coarse timing estimator needs to detect the arrival of a new OFDM burst, and then give an estimate where the burst starts. Comparing the two steps of timing estimation, the former is much more challenging than the latter, therefore, the discussion on coarse timing estimation actually is mainly about OFDM burst detection. Nevertheless, detection is only an integral step of estimation, and the purpose of the coarse timing estimator is to locate the position where the burst
starts rather than determining whether a burst has started or not.

For the generalized training symbols we introduced in Chapter 2, the high correlation between the segments in the training symbol differentiates the preamble from the noise and other OFDM symbols in the burst. It means that the timing can be estimated by a correlator whose output is expected to be high when the samples are highly correlated, which indicates a good chance to be a training symbol. Once the training symbol is located, the timing for the whole burst can be derived. Although it is possible to use only one correlator for timing estimation, when there are multiple highly correlated segments in the training symbols, one would intuitively want to use a number of correlators and combine the outputs for enhanced performance. These basic ideas lay down the main procedures of our timing estimation method.

In this chapter, we propose to construct a series of component timing metrics, one for each pair of highly correlated segments in the training symbol. Then we linearly combine them to minimize the false alarm probability under the constraints on correct detection performance. The component timing metrics only require the correlation between the segments to be high, so the proposed method is applicable to generalized training symbols including those specified by the IEEE 802.16 OFDMA (WiMAX) standard which have three highly correlated but not identical segments. Moreover, the proposed method takes advantage of multiple training symbols for further performance improvement. It is also worth noting that the data symbols in one OFDM burst can have different power levels to reach the users at different distances. We take that into account and yield more realistic results yet found in the existing literature.

The performance of the proposed method is analyzed in three scenarios generalized from the IEEE 802.11 and IEEE 802.16 standards. When there is one training symbol with two and four identical segments, the proposed method is equivalent to that in [10] and [42] respectively. Our analytical results show that using more identical segments in the training symbol only reduces false alarms and has little impact on the missed detection probability. This justifies our false alarm probability based optimization criterion, and contradicts the existing timing metric design approach which is based on the detection probability. Simulations agree with our analytical results reasonably well and confirm the performance of the proposed method in various channel conditions.
3.2 The Method

In the time domain, the high autocorrelation between the segments of the training symbols distinguishes the preamble from the noise and other OFDM symbols in the burst. This means that timing can be estimated from any pair of correlated segments. When multiple pairs exist in one or multiple training symbols, one would intuitively want to combine them for enhanced performance. In this section, we first analyze the statistical properties of the correlators and component timing metrics, then derive the coefficients to combine them in a linear manner such that the false alarm probability is minimized under the constraint on missed detection performance. A brief summary concludes the section.

3.2.1 The Correlators

Since the high autocorrelation distinguishes the training symbol to others, we follow the traditional works [10, 39–42] and base our timing estimator on the correlators defined as

\[
R(\tau, d) \triangleq \sum_{n=0}^{N-d-1} y^*[\tau + n] y[\tau + n + d],
\]

(3.1)

where \(\tau\) is the timing hypothesis, \(d\) is the correlation interval. The correlators given by (3.1) can be easily implemented on hardware using the integrate-and-dump algorithm [40].

Denote \(\tilde{\tau}_i\) as the ideal timing position for the \(i^{th}\) training symbol in the preamble. As a special case, when \(d = 0\),

\[
E \{R[\tilde{\tau}_i, 0]\} = E \left\{ \sum_{n=0}^{N-1} |y[\tilde{\tau}_i + n]|^2 \right\} = N(\sigma_i^2 + \sigma_w^2),
\]

(3.2)

where \(\sigma_i^2\) is the average power of the samples in training symbol \(i\), which can be calculated by

\[
\sigma_i^2 \triangleq \frac{1}{N} \sum_{n=N_g}^{N+N_g-1} |\tilde{x}[\tilde{\tau}_i + n]|^2 = \frac{N_p[i]}{N} |X_i[1]|^2 \sum_{l=1}^{L_h} |h[l]|^2.
\]

(3.3)

The second equality of (3.3) follows Parseval’s Theorem that the received signal’s power in the time domain should equal that in the frequency domain. It should be noted that the received signal within the CP may come from two OFDM symbols in a multipath channel, so we exclude it from the signal power calculation. The
signal to noise ratio of that training symbol is defined as \( \text{SNR}_i \triangleq \frac{\sigma^2_i}{\sigma^2_w} \).

When \( d > 0 \) and is close to a multiple of \( (N/L_i) \), from (3.1), we can expand \( R[\tilde{\tau}_i, d] \) as

\[
R[\tilde{\tau}_i, d] = e^{j2\pi d\epsilon_0} \left( \sum_{n=0}^{N-d-1} \tilde{x}^*[\tilde{\tau}_i + n] \tilde{x}[\tilde{\tau}_i + n + d] + \sum_{n=0}^{N-d-1} \tilde{w}^*[\tilde{\tau}_i + n] \tilde{w}[\tilde{\tau}_i + n + d] + \Theta(\tilde{\tau}_i, d) \right),
\]

where \( \tilde{w}[n] \triangleq e^{-j2\pi n\epsilon_0/N} w[n] \) has the same statistical properties as \( w[n] \), and

\[
\Theta(\tau, d) \triangleq \sum_{n=0}^{N-d-1} \tilde{w}^*[\tau + n] \tilde{x}[\tau + n + d] + \sum_{n=d}^{N-1} \tilde{w}[\tau + n] \tilde{x}^*[\tau + n - d].
\]

The second term in the bracket of (3.4) is the summation of products of uncorrelated noise terms, its contribution to the value of \( R[\tilde{\tau}_i, d] \) is much smaller than the other terms when the signal is not weaker than the noise. The first term in the bracket is the summation of products of signal terms, which can be expanded as

\[
\tilde{x}^*[\tilde{\tau}_i + k] \tilde{x}[\tilde{\tau}_i + k + d] = \sum_{l_1=1}^{L_h} |h[l_1]|^2 x_i^*[k - \tau[l_1]] x_i[k + d - \tau[l_1]] + \sum_{l_1=1}^{L_h} \sum_{l_2 \neq l_1} h[l_1]^* h[l_2] x_i^*[k - \tau[l_1]] x_i[k + d - \tau[l_2]].
\]

The first summation in (3.6) contains the products of samples with fixed distance \( d \), which can be evaluated as

\[
x_i[k]^* x_i[k + d] = \frac{1}{N} \sum_{m_1=1}^{N_p[i]} X_i[m_1]^* e^{-j2\pi (\alpha_i[1] + (m_1-1)L_i)k} \sum_{m_2=1}^{N_p[i]} X_i[m_2] e^{j2\pi (\alpha_i[1] + (m_2-1)L_i)(k+d)}
\]

\[
= \frac{1}{N} \sum_{m_2=1}^{N_p[i]} e^{j2\pi (\alpha_i[1] + (m_2-1)L_i)d} \cdot \left( |X_i[m_2]|^2 + \sum_{m_1 \neq m_2} X_i[m_1]^* X_i[m_2] e^{j2\pi (m_2-m_1)L_i} \right).
\]

Because \( X_i[\cdot] \) is a pseudo-random phase shift keying (PSK) modulated training
sequence, the summation of the second term in the bracket of (3.7) is a summation of phase rotating terms with equal magnitude, the value is expected to be close to 0. However, with a properly selected \( d \), the first term accumulates over the summation and will dominate the value of the equation. It follows that

\[
x_i[k]^*x_i[k + d] \approx |X_i[1]|^2 \frac{1}{N} \sum_{m_2=1}^{N_p[i]} e^{j \frac{2\pi}{N} (o_i[1]+(m_2-1)L_i) d} = |X_i[1]|^2 \frac{N_p}{N} \rho_i(d) \phi_i(d),
\]

where we used the property of PSK modulation that \(|X_i[m_2]|^2 = |X_i[1]|^2\) and defined

\[
\rho_i(d) \triangleq \begin{cases} 
1, & d \text{ is multiple of } (N/L_i); \\
\sin\left(\frac{\pi}{N} L_i d N_p[i]\right), & \text{others},
\end{cases}
\]

\[
\phi_i(d) \triangleq \begin{cases} 
e^{j \frac{2\pi}{N} o_i[1]}, & d \text{ is multiple of } (N/L_i); \\
e^{j \frac{2\pi}{N} (L_i(N_p[i]-1) + 2o_i[1])}, & \text{others}.
\end{cases}
\]

This indicates that the correlation between the samples is high when their distance is close to a multiple of \((N/L_i)\), and it decays quickly as the distance to the multiple of \((N/L_i)\) increases. We know the samples whose distance is \((d - \tau[l_2] + \tau[l_1])\) where \(l_1 \neq l_2\) are weakly correlated, the summation of their products weighted by zero-mean uncorrelated channel gains is expected to be much smaller than the first term. Hence, we neglect the second summation in (3.6) and replace the first one with (3.8),

\[
\tilde{x}^*[\tilde{r}_i + k] \tilde{x}[\tilde{r}_i + k + d] \approx |X_i[1]|^2 \frac{N_p}{N} \rho_i(d) \phi_i(d) \sum_{l_1=1}^{L_i} |h[l_1]|^2
\]

\[
\approx \sigma_i^2 \rho_i(d) \phi_i(d),
\]

where the last equality follows the equivalence of signal power in the time and frequency domains. Substituting the summation of signal products in (3.4) with (3.11), one obtains

\[
R[\tilde{r}_i, d] \approx e^{j \frac{2\pi}{N} \epsilon_0} \left( (N - d) \sigma_i^2 \phi_i(d) \rho_i(d) + \Theta(\tilde{r}_i, d) \right).
\]

We can see only the first term in the bracket of the right hand side of (3.12) represents the useful signal, whose magnitude is given by \((N - d)|\rho_i(d)|\). Because
where $k \in [1, L_i - 1]$, and $[\cdot]$ denotes the function that outputs the integer closest to the argument. The existing papers [10,39–42] have only studied the case where $(N/L_i)$ is an integer, while this thesis investigates a more general scenario that covers the IEEE 802.16 OFDMA (WiMAX) training symbols where $(L_i = 3)$.

### 3.2.2 Component Timing Metric

A timing metric measures the likelihood of a given timing position to be the start of a training symbol. We know that the correlators can detect the highly correlated segments in a training symbol, so each one of them can provide a timing metric. However, as revealed by (3.12), the output of a correlator fluctuates with the received signal strength. To avoid the adverse impact of power fluctuation, we normalize the correlator so that at the ideal timing position $\hat{n}_i$ the expectation of the correlation equals 1. Ideally, the normalization can be performed as

$$
\frac{|R[\hat{\tau}_i, d_{i,k}]|}{E \{|R[\hat{\tau}_i, d_{i,k}]|\}} = \frac{N}{N - d_{i,k}} \cdot \frac{1}{|\rho_i(d_{i,k})|} \cdot \frac{|R[\hat{\tau}_i, d_{i,k}]|}{N\sigma_i^2}.
$$

(3.14)

Because the knowledge of signal power $\sigma_i^2$ is not available to the receiver, the ideal normalization is not practical. An approximation to that is to use $R[\hat{n}_i, 0]$ as an estimate of $N\sigma_i^2$, and the component timing metric can be defined as

$$
T_c[\tau, d_{i,k}] \triangleq \frac{N}{N - d_{i,k}} \cdot \frac{1}{|\rho_i(d_{i,k})|} \cdot \frac{|R[\tau, d_{i,k}]|}{R[\tau, 0]}.
$$

(3.15)

At perfect timing position $\hat{\tau}_i$, using (3.12), we have

$$
T_c[\hat{\tau}_i, d_{i,k}] = \frac{N}{N - d_{i,k}} \cdot \frac{1}{|\rho_i(d_{i,k})|} \cdot \frac{|R[\hat{\tau}_i, d_{i,k}]|}{R[\hat{\tau}_i, 0]}.
$$

(3.16)

This indicates that at high SNR the timing metric is very close to 1. We also note that the timing metrics in the references [10,39,41] are equivalent to the component timing metric $T_c[\tau, N/L_i]$ when $L_i$ is divisible by $N$.

Denote $\bar{\tau}_i$ as a timing position that is far away from the ideal one $\hat{\tau}_i$, i.e., none of the samples from $\bar{\tau}_i$ to $\bar{\tau}_i + N - 1$ belong to the $i^{th}$ training symbol. In the following,
3.2 The Method

Figure 3.1: Two scenarios in the unequal power case.

we derive the statistical properties of the component timing metric $T_c[\tau_i, d]$ in an AWGN channel. The ISI caused by a multipath channel violates our assumption about the probability distribution of OFDM samples, however, when the channel’s multipath spread is much shorter than the OFDM symbol duration, the impact of ISI is insignificant and the AWGN channel result gives a good approximation to that of the multipath channel. Following [33], we approximate the received samples at incorrect timing positions as independent complex-valued zero-mean Gaussian random variables. As pointed out in [120], the samples are correlated when there are unused subcarriers in the OFDM symbols, however, the correlation is negligible when the number of unused subcarriers is small, which is usually the case in practical OFDM systems. For instance, in the IEEE 802.16 (WiMAX) standard, the unused subcarriers (guard band) occupy about 16% of the total spectrum.

The samples in the correlation window, i.e., from sample $\tau_i$ to $(\tau_i + N - 1)$, can belong to two OFDM symbols of different power levels. Assume the boundary of the two symbols is at sample $(\tau_i + (N + N_g - l_b))$. From Figure (3.1), we can see the number of samples belonging to the latter OFDM symbol is given by $l_B = \max(0, l_b - N_g)$, and that of the former symbol is $l_A = (N - \max(0, l_b - N_g))$. Denote the variance of the samples of the former symbol as $\sigma^2_A$, and that of the latter as $\eta \cdot \sigma^2_A$. It is worth mentioning that substantial power fluctuation within one symbol period violates our assumptions and leads to certain performance degradation. This effect is analyzed in Section 3.5 with simulation results.

The denominator of the component timing metric defined by (3.15) can be
expanded as
\[
R[\bar{\tau}_i, 0] = \sum_{k=0}^{l_A-1} |y[\bar{\tau}_i + k]|^2 + \sum_{k=l_A}^{N-1} |y[\bar{\tau}_i + k]|^2 = \chi_A + \chi_B, \tag{3.17}
\]
where \(\chi_A\) and \(\chi_B\) are the summations of the squared magnitude of the samples in the former and latter symbols respectively, so they follow Chi-square distribution with \((2l_A)\) and \((2l_B)\) degrees of freedom scaled by \(\sigma_A^2/2\) and \(\eta \sigma_A^2/2\) respectively.

When \(l_A\) is sufficiently large, \(\chi_A\) can be further approximated by a Gaussian random variable whose mean and variance are given by
\[
E\{\chi_A\} = \sum_{k=0}^{l_A-1} E\{|y[\bar{\tau}_i + k]|^2\} = l_A \sigma_A^2, \tag{3.18}
\]
and
\[
Var\{\chi_A\} = E\{\chi_A^2\} - (E\{\chi_A\})^2
= \sum_{k_1=0}^{l_A-1} \sum_{k_2=0}^{l_A-1} E\{|y[\bar{\tau}_i + k_1]|^2|y[\bar{\tau}_i + k_2]|^2\} - (l_A \sigma_A^2)^2
= \sum_{k=0}^{l_A-1} E\{|y[\bar{\tau}_i + k]|^4\} - (l_A \sigma_A^2)^2
+ \sum_{k_1=0}^{l_A-1} \sum_{k_2 \neq k_1} E\{|y[\bar{\tau}_i + k_1]|^2\} E\{|y[\bar{\tau}_i + k_2]|^2\}
= 3 \cdot (2l_A) \left( \frac{\sigma_A^2}{2} \right)^2 + 2l_A(2l_A - 1) \left( \frac{\sigma_A^2}{2} \right)^2 - (l_A \sigma_A^2)^2
= l_A \sigma_A^4. \tag{3.19}
\]
It suggests that the mean of \(\chi_A\) equals its standard deviation multiplied by \(\sqrt{l_A}\).

When \(l_A\) is large, the deviation of \(\chi_A\) from its mean value is small. Similarly, \(\chi_B\) can also be approximated by its mean when \(l_B\) is large. It follows that
\[
R[\bar{\tau}_i, 0] \approx E\{\chi_A\} + E\{\chi_B\} = (l_A + \eta l_B) \sigma_A^2. \tag{3.20}
\]
If not both \(l_A\) and \(l_B\) are large, since \((l_A + l_B = N)\), at most one of them can be small. Without loss of generality, we assume that \(l_A\) is small. If \(\eta\) is large, \(\chi_B\) dominates the value of \(R[\bar{\tau}_i, 0]\), then (3.20) still holds. However, when \(\eta\) is much smaller than \(l_A/l_B\), \(\chi_A\) dominates the value of \(R[\bar{\tau}_i, 0]\), and (3.20) can be violated.
When this happens, we can show that the value of the component timing metric is proportional to $\sqrt{\eta}$, which must be very small and unlikely to cause any false alarms. This means that (3.20) is a good enough approximation, and we can write

$$ T_c[\bar{\tau}_i, d] \approx \frac{N}{N - d} \frac{1}{\rho_i(d)} |R[\bar{\tau}_i, d]| = \frac{N}{N - d} \frac{1}{\rho_i(d)} \frac{|R[\bar{\tau}_i, d]|}{(l_A + \eta l_B) \sigma_A^2}. \quad (3.21) $$

$|R[\bar{\tau}_i, d]|$ is the magnitude of the summation of a large number of zero-mean uncorrelated complex values, so it follows Rayleigh distribution. $T_c[\bar{\tau}_i, d]$ equals $|R[\bar{\tau}_i, d]|$ multiplied by a scaling factor, so the probability density function (pdf) of $T_c[\bar{\tau}_i, d_i,k]$ approximately follows that of Rayleigh distribution as well:

$$ f_{T_c[\bar{\tau}_i, d_i,k]}(u) \approx f_{(\cdot)}(u \mid \sigma_i^2) \triangleq \frac{u}{\bar{\sigma}_i^2} e^{-\frac{u^2}{2\bar{\sigma}_i^2}}, \quad (3.22) $$

where $f_Z(\cdot)$ denotes the pdf of random variable $Z$, $f_{(\cdot)}(\cdot \mid \sigma^2)$ denotes the pdf of Rayleigh random variables, and

$$ \bar{\sigma}_i^2 \triangleq \frac{1}{2} E \{ T_c^2[\bar{\tau}_i, d_i,k] \}. \quad (3.23) $$

As $T_c^2[\bar{\tau}_i, d]$ is a Rayleigh random variable, its statistical properties are determined by the second moment. In the following, we calculate $E \{ T_c^2[\bar{\tau}_i, d] \}$. Expanding the expectation conditionally on $l_b$, and approximating the summation with an integral, one obtains

$$ E \{ T_c^2[\bar{\tau}_i, d] \} = \sum_{l_b=0}^{N+N_g-1} E \{ T_c^2[\bar{\tau}_i, d] \mid l_b \} P(l_b) $$

$$ = \frac{1}{N+N_g} \sum_{l_b=0}^{N+N_g-1} E \{ T_c^2[\bar{\tau}_i, d] \mid l_b \} $$

$$ \approx \frac{1}{N+N_g} \int_{0}^{N+N_g} E \{ T_c^2[\bar{\tau}_i, d] \mid l_b \} \, dl_b $$

$$ \approx \frac{1}{N+N_g} \cdot \frac{N^2}{(N - d)^2} \cdot \frac{1}{\rho_i(d)} \cdot \frac{1}{\sigma_A^4} \cdot \int_{0}^{N+N_g} E \{ |R[\bar{\tau}_i, d]|^2 \mid l_b \} \, dl_b. \quad (3.24) $$
For a given $l_b$, we can write

$$E \left\{ |R[\tau_i, d]|^2 | l_b \right\}$$

$$= E \left\{ \left( \sum_{n_1 = \tau_i}^{\tau_i + N - d - 1} y^*[n_1] y[n_1 + d] \right) \left( \sum_{n_2 = \tau_i}^{\tau_i + N - d - 1} y^*[n_2] y[n_2 + d] \right) | l_b \right\}$$

$$= E \left\{ \sum_{n_1 = \tau_i}^{\tau_i + N - d - 1} |y^*[n_1] y[n_1 + d]|^2 | l_b \right\}$$

$$+ E \left\{ \sum_{n_1 = \tau_i}^{\tau_i + N - d - 1} \sum_{n_2 \neq n_1} y[n_1] y^*[n_2] y^*[n_1 + d] y[n_2 + d] | l_b \right\}$$

$$= \sum_{n_1 = \tau_i}^{\tau_i + N - d - 1} E \left\{ |y[n_1]|^2 | l_b \right\} E \left\{ |y[n_1 + d]|^2 | l_b \right\}, \quad (3.25)$$

where the last equality follows the assumption on the samples’ independent zero-mean Gaussian distribution. For each individual sample, $l_b$ determines whether it belongs to the former or latter OFDM symbol, where $E \{ |y[n_1]|^2 | l_b \}$ equals $\sigma_A^2$ and $\eta \sigma_A^2$ respectively. Define $l_{AA}$, $l_{AB}$, $l_{BB}$ as the numbers of terms in the summation of (3.25) that are the products of samples both from the former symbol, one from the former one from the latter, and both from the latter symbol, respectively. Then, (3.25) can be simplified to

$$E \left\{ |R[\tau_i, d]|^2 | l_b \right\} = (l_{AA} + \eta l_{AB} + \eta^2 l_{BB}) \sigma_A^4. \quad (3.26)$$

However, $l_{AA}$, $l_{AB}$, $l_{BB}$ change with $l_b$ differently in various scenarios. When $d \leq N/2$,

$$l_{AA} = \begin{cases} (N - d), & l_b \in [0, N_g); \\ (N - d + N_g - l_b), & l_b \in [N_g + d, N_g + N - d); \\ 0, & l_b \in [N_g + N - d, N_g + N). \end{cases} \quad (3.27)$$

$$l_{AB} = \begin{cases} 0, & l_b \in [0, N_g); \\ (l_b - N_g), & l_b \in [N_g, N_g + d); \\ d, & l_b \in [N_g + d, N_g + N - d); \\ \eta(N + N_g - l_b), & l_b \in [N_g + N - d, N_g + N). \end{cases} \quad (3.28)$$

$$l_{BB} = \begin{cases} 0, & l_b \in [0, N_g + d); \\ (l_b - d - N_g), & l_b \in [N_g + d, N_g + N - d); \\ (l_b - N_g - d), & l_b \in [N_g + N - d, N_g + N). \end{cases} \quad (3.29)$$
When \( d > (N/2) \),

\[
l_{AA} = \begin{cases} 
(N - d), & l_b \in [0, N_g); \\
(N - d + N_g - l_b), & l_b \in [N_g, N_g + N - d); \\
0, & l_b \in [N_g + N - d, N_g + N). 
\end{cases}
\] (3.30)

\[
l_{AB} = \begin{cases} 
0, & l_b \in [0, N_g); \\
(l_b - N_g), & l_b \in [N_g, N_g + N - d); \\
(N - d), & l_b \in [N_g + N - d, N_g + d); \\
(N + N_g - l_b), & l_b \in [N_g + d, N_g + N). 
\end{cases}
\] (3.31)

\[
l_{BB} = \begin{cases} 
0, & l_b \in [0, N_g + d); \\
(l_b - N_g - d), & l_b \in [N_g + d, N_g + N). 
\end{cases}
\] (3.32)

Substituting the expectation in the integral of (3.24) with (3.26), and after some algebraic manipulation, we obtain

\[
E \{ T_c^2[\tilde{r}_i, d] \} \approx \frac{1}{N + N_g} \cdot \frac{1}{\rho_i^2(d)} \cdot \frac{N^2}{(N - d)^2} \cdot \left( \frac{(N - d)N_g}{N^2} - \frac{N - d}{N} + g(\eta) \right),
\] (3.33)

Figure 3.2: Variation of \( g(\eta) \) over the ratio of power \( \eta \).
where

\[
g(\eta) \triangleq \log\left(\frac{N\eta}{(N - d) + \eta d}\right) + \frac{1}{\eta - 1} \log\left(\frac{\eta(\eta(N - d) + d)}{(N - d) + \eta d}\right).
\]  

Equation (3.33) holds for both \((d \leq N/2)\) and \((d > N/2)\) scenarios. It is easy to compute \(g(\eta) = 2(1 - d/N)\) as \(\eta \to 1\), and \(g(\eta) = \log(N/d)\) as \(\eta \to 0\) or \(+\infty\). Figure 3.2 plots \(g(\eta)\) for all \(d\) values in the cases of \(L_i \in [2, 8]\). The figure shows that \(g(\eta)\) is bounded by its limits at 0, 1, and \(+\infty\), and only fluctuates within a small range for all \(d \geq N/8\). Considering the fact that \(\eta\) is more likely to fluctuate around 1, it will not introduce much error to approximate \(g(\eta)\) with \(g(1)\) in (3.33), and then \(E\{T_2^c[\bar{\tau}_i, d]\}\) can be well approximated by \((\rho_i^2(d)(N - d)^{-1}\).

### 3.2.3 Combining the Timing Metrics

Multiple component timing metrics can be constructed for the training symbols. Although each one of them gives a timing estimate, it is possible to combine them for better performance. We propose to linearly combine the component timing metrics to minimize the false alarm probability while keeping the asymptotic missed detection probability low. The combined timing metric is defined as

\[
T[\tau] \triangleq \sum_{i=1}^{I_p} \sum_{k=1}^{K_i} \omega_{i,k} T_c[\tau + (i - 1)(N + N_g), d_{i,k}],
\]  

where \(K_i\) is the number of component metrics constructed for training symbol \(i\), and \(\{\omega_{i,k}\}\) are the coefficients that need to be determined according to two criteria:

- In a noiseless channel, at the ideal timing position \(\bar{\tau}_1\), \(T[\bar{\tau}_1] \approx 1\).

- At a wrong timing position \(\bar{\tau}_1\), the probability for \(T[\bar{\tau}_1]\) to reach the given threshold \(\lambda_c\) should be minimized.

To meet the first criterion, from (3.16) and (3.35), it requires that

\[
\sum_{i=1}^{I_p} \sum_{k=1}^{K_i} \omega_{i,k} = 1.
\]  

Under this constraint, we compute the coefficients \(\{\omega_{i,k}\}\) satisfying the second criterion as follows.
First, we need to express the false alarm probability as a function of the weighting coefficients. Although the exact expression is not available, a good approximation is given by the following theorem.

**Theorem 3.1.** At an incorrect timing position $\bar{\tau}_1$, assume the component timing metrics follow Rayleigh distribution and are uncorrelated from each other. The probability for the combined timing metric to reach the detection threshold $\lambda_c$ is approximately given by

$$P(T[\bar{\tau}_1] \geq \lambda_c) \approx (\sqrt{2\pi})^{K_c-1} \frac{\prod_{p=1}^{l_p} \prod_{k=1}^{K_p} \omega_{l_p,k}^2 \bar{\sigma}_{l_p,k}^2}{\left( \sum_{p=1}^{l_p} \prod_{k=1}^{K_p} \omega_{l_p,k}^2 \bar{\sigma}_{l_p,k}^2 \right)^{K_c-1/2}} \lambda_c^{K_c-1} e^{-\frac{\lambda_c^2}{2 \sum_{p=1}^{l_p} \sum_{k=1}^{K_p} \omega_{l_p,k}^2 \bar{\sigma}_{l_p,k}^2}},$$

(3.37)

where $K_c \triangleq \sum_{p=1}^{l_p} K_i$ is the total number of component timing metrics. This approximation is accurate for large thresholds.

**Proof.** Because the component timing metrics are uncorrelated from each other, there is no difference whether they are based on the same training symbol or different ones. This means that it suffices to prove (3.37) in the one-training-symbol case.

Define the false alarm probability of combining $n$ component timing metrics as a function of detection threshold $u$ as

$$q_n(u) \triangleq \int_u^{+\infty} f\left(\sum_{k=1}^{n} \omega_{1,k} T_c[\bar{\tau}_1, d_1,k]\right)(v) \, dv.$$  

(3.38)

The false alarm probability function must satisfy the initial condition that

$$q_1(u) = 1 - \int_0^u f(\omega_{1,1} T_c[\bar{\tau}_1, d_1,1])(v) \, dv = e^{-u^2/(2 \omega_{1,1}^2 \bar{\sigma}_{1,1}^2)}.$$  

(3.39)

The false alarm probability function should also satisfy the recursive condition that

$$q_n(u) = \int_0^u f(\omega_{1,n} T_c[\bar{\tau}_1, d_1,n])(v_1) \int_{u-v_1}^{+\infty} f(\sum_{k=1}^{n-1} \omega_{1,k} T_c[\bar{\tau}_1, d_1,k])(v_2) \, dv_2 \, dv_1 = \int_0^u q_{n-1}(u-v_1) f(v_1|\omega_{1,n}^2 \bar{\sigma}_{1,n}^2) \, dv_1.$$  

(3.40)

To simplify the notation, we define

$$A_n \triangleq (2\pi)^{n-1/2} \left( \sum_{k=1}^{n} \omega_{1,k}^2 \bar{\sigma}_{1,k}^2 \right)^{1/2-n} \sqrt{\prod_{k=1}^{n} \omega_{1,k}^2 \bar{\sigma}_{1,k}^2}.$$  

(3.41)
In the following we show
\[ q_n(u) \approx A_n u^{n-1} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2}. \tag{3.42} \]

When \( n = 1 \), the right hand side of (3.42) reduces to that of (3.39), so the equation holds. For all \( n \geq 1 \), assume (3.42) holds for \( q_n(u) \), we derive \( q_{n+1}(u) \) from the recursive equation (3.40) as follows.

Using (3.42), we can compute the integral in (3.40) as
\[
q_{n+1}(u) = \int_0^u q_n(u - v) f(v) \omega^2_{1,n+1} \bar{\sigma}_{1,n+1}^2 \, dv \\
\approx \frac{A_n}{\omega^2_{1,n+1} \bar{\sigma}_{1,n+1}^2} \int_0^u v(u - v)^{n-1} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} \, dv \\
= \frac{A_n}{\omega^2_{1,n+1} \bar{\sigma}_{1,n+1}^2} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} \int_0^u v(u - v)^{n-1} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} \, dv. \tag{3.43}
\]
The value of the integral is dominated by the exponential term, which approaches 0 very quickly out of the small region around \( v = \frac{\omega^2_{1,n+1} \bar{\sigma}_{1,n+1}^2}{\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} u \). So, we can approximate that as
\[
q_{n+1}(u) \approx \frac{A_n}{\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} u^n \left( \sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2 \right)^{-n-1} \\
\times \int_{-\infty}^{+\infty} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2} \, dv \\
= A_n u^n \left( \sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2 \right)^{-n-1} \sqrt{2\pi} \sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2 \\
= A_{n+1} u^{n} e^{-2\sum_{k=1}^{n+1} \omega_k^2 \bar{\sigma}_{1,k}^2}. \tag{3.44}
\]
Therefore, (3.42) holds for all \( k \). And, the approximation in (3.44) is accurate for large thresholds. From (3.42) and the definition of \( q_n(u) \) in (3.38), we have
\[
P(T[\bar{\tau}_1] \geq \lambda_c) = q_K(\lambda_c) \approx A_K \lambda_c^{K-1} e^{-2\sum_{k=1}^{K} \omega_k^2 \bar{\sigma}_{1,k}^2}, \tag{3.45}
\]
which is equivalent to (3.37) in the one-symbol case and is sufficient to establish (3.37) for the multiple-symbol case as well.

We note that the assumption about the component timing metrics’ statistics in Theorem 3.1 only holds in an AWGN channel without using the guard band. Nevertheless, the result is still approximately applicable to practical OFDM systems where both the guard band and channel multipath spread are small.

It is revealed by (3.37) that for high thresholds the false alarm probability is determined by the exponential term. To minimize the false alarm probability, \( \{\omega_{i,k}\} \) should be chosen to minimize the terms in the denominator of the exponent

\[
\varpi \triangleq \sum_{i=1}^{I_p} \sum_{k=1}^{K_i} \omega_{i,k}^2 \sigma_{i,k}^2.
\]  

(3.46)

Under the constraint in (3.36), we can show that the coefficients \( \{\omega_{i,k}\} \) that minimize (3.46) are given by

\[
\omega_{i,k} = \frac{\rho_i^2(d_{i,k})(N - d_{i,k})}{\sum_{i=1}^{I_p} \sum_{n=1}^{K_i} \rho_i^2(d_{i,n})(N - d_{i,n})}.
\]  

(3.47)

And, it follows that

\[
\varpi = \frac{1}{\sum_{i=1}^{I_p} \sum_{n=1}^{K_i} \rho_i^2(d_{i,n})(N - d_{i,n})}.
\]  

(3.48)

This means that the component metrics must have large \( |\rho_i(d_{i,k})| \) values to make significant contribution to the combined timing metric. This justifies our selection of \( \{d_{i,k}\} \) in (3.13).

### 3.2.4 Summary of the Method

We briefly summarize the proposed coarse timing estimator as follows.

1. Construct \( (L_i - 1) \) component timing metrics \( T_c[\tau, d_{i,k}] \) for each training symbol using (3.15) where \( \{d_{i,k}\} \) are given by (3.13).

2. Compute the combined timing metric \( T[\tau] \) using (3.35) where \( \{\omega_{i,k}\} \) are given by (3.47).

3. Once the combined timing metric \( T[\tau] \) reaches certain threshold \( \lambda_c \), the position corresponding to the maximum of the timing metric within the following
N OFDM samples is the coarse timing estimate.

It should be noted that choosing the maximum of the timing metric within certain window avoids the dependency of coarse timing estimates on the detection threshold $\lambda_c$. Shi and Serpedin [42] assumed the coarse timing estimate to be where the threshold is first reached. Figure 3.3 shows an example to illustrate the difference between their approach and ours. It is shown that our estimates are independent of the threshold settings and must fall in the correct timing window for any threshold lower than the peak. However, Shi and Serpedin’s estimates are determined by the threshold settings, which have to be precisely set close to the peak to obtain correct estimates. In practical OFDM systems where the timing metrics change dramatically from burst to burst due to the channel variation, our approach gives much more stable performance.

![Figure 3.3: Coarse timing estimates for a given timing metric.](image)

### 3.3 Comparison to Oversample Approach

It has been indicated in Section 2.3.2 that when the number of repetition segments in the training symbol is not divisible by the number of subcarriers, the training symbol loses the repetition structure after sampling. One method to maintain the repetition structure is to oversample the received signal. For the example shown in Figure 2.4, the sampled signal will be periodic if the sample rate is tripled. Then, it is possible to use conventional methods [10, 39–42] to achieve timing synchronization.

Before comparing the oversample approach to the proposed method, we mathematically describe the oversample approach to form the basis of discussion. Following the notations in Section 2.3.1, we denote the analog waveform of a training symbol by $x(t)$. Assume the waveform has $L_i$ identical segments, we sample the
signal at an interval $t_s/L_i$ and obtain a sequence of the oversampled signal

$$x'[k] = \int_{-\infty}^{+\infty} x(t) \delta(t - kt_s/L_i)dt. \quad (3.49)$$

To simplify the discussion, we ignore the channel effects and the noise, and assume the input to the timing detector is the perfectly oversampled sequence $\{x'[k]\}$. Since $x'[k] = x'[k+N]$, the sequence has a repetition structure and can use the conventional methods to detect the timing according to the correlation function

$$R'[\tau,d] \triangleq \sum_{k=0}^{L_iN-1} x''[\tau+k] x'[\tau+k+d]. \quad (3.50)$$

According to Nyquist-Shannon Theorem [119], the analog waveform can be recovered from the samples as

$$x(t) = \sum_n x[n]s(t - nt_s), \quad (3.51)$$

where $s(t)$ is the interpolation function. We can rewrite $(3.49)$ as

$$x'[k] = \int_{-\infty}^{+\infty} \sum_n x[n]s(t - nt_s) \delta(t - kt_s/L_i)dt$$

$$= \sum_n x[n]s[k - nL_i], \quad (3.52)$$

where

$$s[k - nL_i] = s((k - nL_i)t_s/L_i). \quad (3.53)$$

Using $(3.52)$, we can expand $(3.50)$ as

$$R'[\tau,d] = \sum_k \sum_n x^*[\tau+n]s^*[k - nL_i] \sum_m x[\tau+m]s[k + d - mL_i]$$

$$= \sum_n \sum_m \sum_k x^*[\tau+n]x[\tau+m]s^*[k - nL_i]s[k + d - mL_i]$$

$$= \sum_n \sum_m \sum_k x^*[\tau+n]s[\tau + m] \sum_{k'} s^*[k']s[k' + d + (n-m) L_i]$$

$$= \sum_n \sum_{d'} s[\tau + n + d'] \sum_{k'} s'[k']s[k' + d - d'L_i]$$

$$= \sum_{d'} ss'[d - d'L_i]R[\tau,d'], \quad (3.54)$$
where
\[ s_5[d] \triangleq \sum_{k'} s^*[k']s[k' + d], \] (3.55)
which is fixed for any given correlation intervals. It is revealed by (3.54) that
the auto-correlation of the oversampled sequence is a linear combination of the
auto-correlation of the samples without oversampling, and the combination weight
coefficients are determined by the interpolation function and correlation intervals.
Therefore, the oversample approach can be seen as a special case of the proposed
method that

- the length of interpolation window determines how many component timing
  metrics are employed;

- the weight coefficients of the component timing metrics are determined by
  the interpolation function.

The first property indicates much higher complexity for the oversample approach
because the interpolation window has to be sufficiently long to achieve satisfac-
tory performance, while the proposed algorithm allows the use of a small number
of component timing metrics without significant performance degradation. The
second property of the oversample approach suggests that the method only con-
siders correct detection probability without taking into account the false alarms.
As we explained earlier, the detection threshold is always a tradeoff between the
false alarms and missed detections. Only considering one aspect of the problem
may lead to suboptimal solutions. These facts suggest that the proposed timing
detection algorithm is more flexible and efficient than the oversample approach.

3.4 Case Study and Performance Analysis

To provide more insights into the proposed algorithm, we analyze the performance
in three special cases generalized from the IEEE 802.11 [3] and IEEE 802.16 [4]
standards. In Case A, we compare our estimator to those conventional methods
[10,42] and shed new light on them; in Case B, we investigate the gain of combining
the second training symbol into the timing metric; in Case C, we analyze the
performance of the proposed method for generic WiMAX training symbols.
3.4 Case Study and Performance Analysis

3.4.1 Case A: $I_p = 1$, $L_1$ is divisible by $N$

This case generalizes the uplink channel in IEEE 802.16 [4] OFDM systems where the data OFDM symbols are placed after the training symbol which is QPSK modulated and has four identical segments ($L_1 = 4$). From (3.47), we can compute

$$\omega_{1,k} = \frac{(N - kN/L_1)}{\sum_{n=1}^{L_1-1} (N - nN/L_1)} = \frac{2(L_1 - k)}{L_1(L_1 - 1)}. \quad (3.56)$$

The combined timing metric becomes

$$T[\tau] = \frac{2}{L_1 - 1} \sum_{k=1}^{L_1-1} \left| \sum_{n=0}^{N-k} R[\tau + n] y[(\tau + n) \frac{N}{L_1}] \right|^2 \sum_{n=0}^{N-1} |y[\tau + n]|^2. \quad (3.57)$$

Although derived from different optimization criteria, the proposed timing metric coincides with those in [10, 42] respectively for $L_1 = 2$ and $L_1 = 4$ cases. From (3.48), we can compute

$$\varpi = \frac{1}{\sum_{n=1}^{L_1-1} (N - nN/L_1)} = \frac{2}{N(L_1 - 1)}. \quad (3.58)$$

This indicates that the false alarm probability decreases as the number of identical segments increases.

For this simple case, the false alarm probability of the combined timing metric given by (3.37) can be simplified to

$$P(T[\bar{\tau}_1] > \lambda_c) \approx \sqrt{\prod_{k=1}^{L_1-2} \frac{2\pi N}{L_1} \lambda_c}^{L_1-2} e^{-N(L_1-1)\lambda_c^2 2}. \quad (3.59)$$

Next, we analyze the missed detection probability. Define

$$Z_A[\bar{\tau}_1] \triangleq \left( \frac{2}{L_1 - 1} \sum_{k=1}^{L_1-1} |R[\bar{\tau}_1, d_{1,k}]| \right) - \lambda R[\bar{\tau}_1, 0]. \quad (3.60)$$

The mean of $Z_A[\bar{\tau}_1]$ can be directly computed as

$$E \{Z_A[\bar{\tau}_1]\} = \frac{2}{L_1 - 1} \sum_{k=1}^{K_1} |E \{|R[\bar{\tau}_1, d_{1,k}]|\} - \lambda E \{R[\bar{\tau}_1, 0]\} = N\sigma_1^2 - \lambda N(\sigma_1^2 + \sigma_\omega^2). \quad (3.61)$$
The variance of \( Z_A[\tilde{r}_1] \) is given by

\[
Var \{ Z_A[\tilde{r}_1] \} = E \left\{ (Z_A[\tilde{r}_1] - E \{ Z_A[\tilde{r}_1] \})^2 \right\} = \left( \frac{2}{L_1} \right) \sum_{k_1=1}^{L_1-1} \sum_{k_2=1}^{L_1-1} E \left\{ \tilde{R}[\tilde{r}_1, d_{1,k_1}] \tilde{R}[\tilde{r}_1, d_{1,k_2}] \right\} + \lambda_c^2 E \left\{ \tilde{R}[\tilde{r}_1, 0]^2 \right\} - 2\lambda_c \left( \frac{2}{L_1} \right) \sum_{k=1}^{L_1-1} E \left\{ \tilde{R}[\tilde{r}_1, d_{1,k}] \tilde{R}[\tilde{r}_1, 0] \right\},
\]

(3.62)

where

\[
\tilde{R}[\tilde{r}_1, 0] \triangleq R[\tilde{r}_1, 0] - E \{ R[\tilde{r}_1, 0] \} \approx \Theta(\tilde{r}_1, 0),
\]

(3.63)

\[
\tilde{R}[\tilde{r}_1, d] \triangleq |R[\tilde{r}_1, d]| - E \{ |R[\tilde{r}_1, d]| \} \approx \gamma_1(d) \Re (\phi_1^*(d) \Theta(\tilde{r}_1, d)).
\]

(3.64)

In (3.64), \( \gamma_i(d) \) is the sign of \( \rho_i(d) \), and \( \Re(\cdot) \) takes the real part of the argument.

For all \( d_1 \leq d_2 \), using (3.5), we can compute

\[
E \{ \Theta^*(\tilde{r}_1, d_1) \Theta(\tilde{r}_1, d_2) \} = \sum_{n=0}^{N-d_2-1} E \{ |\tilde{w}[\tilde{r}_1 + n]|^2 \} E \{ \tilde{x}^*[\tilde{r}_1 + n + d_1] \tilde{x}[\tilde{r}_1 + n + d_2] \}
\]

\[
+ \sum_{k=d_2}^{N-1} E \{ |\tilde{w}[\tilde{r}_1 + n]|^2 \} E \{ \tilde{x}^*[\tilde{r}_1 + n - d_2] \tilde{x}[\tilde{r}_1 + n - d_1] \}
\]

\[
= 2 \phi_i(d_2 - d_1) \rho_i(d_2 - d_1) (N - d_2) \sigma_i^2 \sigma_w^2
\]

(3.65)

and

\[
E \{ \Theta(\tilde{r}_1, d_1) \Theta(\tilde{r}_1, d_2) \} = \sum_{n=d_2}^{N-d_1-1} E \{ |\tilde{w}[\tilde{r}_1 + n]|^2 \} E \{ \tilde{x}^*[\tilde{r}_1 + n - d_2] \tilde{x}[\tilde{r}_1 + n + d_1] \}
\]

\[
+ \sum_{n=d_1}^{N-d_2-1} E \{ |\tilde{w}[\tilde{r}_1 + n]|^2 \} E \{ \tilde{x}^*[\tilde{r}_1 + n - d_1] \tilde{x}[\tilde{r}_1 + n + d_2] \}
\]

\[
= 2 \phi_i(d_2 + d_1) \rho_i(d_2 + d_1) \max(N - d_1 - d_2, 0) \sigma_i^2 \sigma_w^2
\]

(3.66)

where \( \max(\cdot) \) denotes the function that takes the maximum of the arguments. It
3.4 Case Study and Performance Analysis

immediately follows that

\[ E \{ \tilde{R}[\tilde{\tau}, 0]^2 \} \approx E \{ \Theta(\tilde{\tau}, 0)^2 \} = 2N\sigma_1^2\sigma_w^2, \]  

(3.67)

\[ E \{ \tilde{R}[\tilde{\tau}, d]\tilde{R}[\tilde{\tau}, 0] \} \approx \frac{\gamma_i(d)}{2} E \{ \Theta(\tilde{\tau}, 0) (\phi_i(d)\Theta^*(\tilde{\tau}, d) + \phi_i^*(d)\Theta(\tilde{\tau}, d)) \} \]

\[ = 2(N - d) |\rho_i(d)| \sigma_1^2\sigma_w^2. \]  

(3.68)

Similarly,

\[ E \{ \tilde{R}[\tilde{\tau}, d_1]\tilde{R}[\tilde{\tau}, d_2] \} \]

\[ \approx \frac{\gamma_i(d_1)\gamma_i(d_2)}{4} (E \{ \phi_i(d_1)\Theta^*(\tilde{\tau}, d_1) (\phi_i(d_2)\Theta^*(\tilde{\tau}, d_2) + \phi_i^*(d_2)\Theta(\tilde{\tau}, d_2)) \} \]

\[ + E \{ \phi_i^*(d_1)\Theta(\tilde{\tau}, d_1) (\phi_i(d_2)\Theta^*(\tilde{\tau}, d_2) + \phi_i^*(d_2)\Theta(\tilde{\tau}, d_2)) \} \]

\[ = \gamma_i(d_1)\gamma_i(d_2) ((N - d_2)|\rho_i(d_2 - d_1) + \max(N - d_2 - d_1, 0)|\rho_i(d_2 + d_1)). \]  

(3.69)

Therefore, using (3.67), (3.68) and (3.69), the variance of \( Z_A[\tilde{\tau}_1] \) in (3.62) can be simplified to

\[ \text{Var} \{ Z_A[\tilde{\tau}_1] \} \approx 2N\sigma_1^2\sigma_w^2(1 - \lambda_c)^2. \]  

(3.70)

Following [40] and [42], we approximate \( Z_A[n] \) by a Gaussian random variable, and the missed detection probability is given by

\[ P(T[\tilde{\tau}_1] < \lambda_c) = P(Z_A[\tilde{\tau}_1] < 0) \]

\[ \approx \int_{-\infty}^{0} \frac{1}{\sqrt{4\pi N\sigma_1^2\sigma_w^2(1 - \lambda_c)^2}} e^{-\frac{(v - N\sigma_1^2(1 - \lambda_c) + N\sigma_w^2\lambda_c)^2}{4N\sigma_1^2\sigma_w^2(1 - \lambda_c)^2}} dv \]

\[ = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{N}}{2} \left( \sqrt{\frac{\sigma_1^2}{\sigma_w^2}} - \frac{\lambda_c}{1 - \lambda_c} \sqrt{\frac{\sigma_w^2}{\sigma_1^2}} \right) \right), \]  

(3.71)

where \( \text{erfc}(x)(u) \triangleq \frac{2}{\sqrt{\pi}} \int_{-u}^{+\infty} e^{-t^2} dt \). This indicates that the missed detection probability monotonously decreases as the SNR \( (\sigma_1^2/\sigma_w^2) \) increases. And, to limit the missed detection probability within 50%, it requires

\[ \lambda_c \leq \frac{\sigma_1^2}{\sigma_w^2 + \sigma_1^2}. \]  

(3.72)

This upper bound of \( \lambda_c \) varies from 0.5 to 0.9 as the SNR changes from 0dB to 10dB, which indicates that the missed detection probability is highly sensitive to the noise level, and optimizing the threshold for a large range of SNR is virtually impossible.
Another interesting observation from (3.71) is that the missed-detection probability does not depend on $L_1$. This means that using more identical segments in the training symbol will not lower the missed detection probability. In other words, Schmidl’s estimator [10] cannot be outperformed in terms of missed detection probability by more complex algorithms including Shi and Serpedin’s method [42].

We numerically evaluate the missed detection and false alarm probabilities of Case A for an unequal power OFDM system with 512 subcarriers in an AWGN channel. 5% of the subcarriers at each end of the spectrum are unused, and the CFO is modeled by a random variable uniformly distributed in $[-10,10]$ subcarrier spacing. When testing the false alarm probability, the relative power level of each OFDM symbol is independently selected from $\{-12\text{dB},-9\text{dB},\ldots,+9\text{dB}\}$ with equal probability and the noise power is fixed to $-10\text{dB}$. The modulation scheme for each data symbol is independently selected from QPSK, 16-QAM and 64-QAM also with equal probability. When testing the missed detection probability, the training symbol is QPSK modulated and randomly generated for each experiment, and we change the power of the training symbol to validate the analytical results.

In Figure 3.4, the solid lines represent the analytical results given by (3.59) and (3.71), and the various marks represent the simulation results averaged for at least

![Figure 3.4: Performance in AWGN channel (Case A).](image-url)
10^5 independent OFDM symbols. The figure shows that our analysis matches the simulation results very well. The Gaussian approximation in the derivation of missed detection probability causes slight discrepancy with the simulation results, however, the analysis is accurate enough to predict the positions of the steep rising edges of the missed detection curves and provide useful design guidelines for the threshold settings.

### 3.4.2 Case B: \( I_p = 2 \), \( L_1 \) and \( L_2 \) are both divisible by \( N \)

This case generalizes the short and long training symbol scheme specified in the IEEE 802.11 [3] and IEEE 802.16 [4] standards. In the former, \( N = 128 \), \( L_1 = 8 \), \( L_2 = 2 \); in the latter, \( N = 256 \), \( L_1 = 4 \), \( L_2 = 2 \).

From (3.47), we can calculate

\[
\omega_{i,k} = \frac{N - kN/L_i}{\sum_{n=1}^{L_1-1} (N - nN/L_1) + \sum_{n=1}^{L_2-1} (N - nN/L_2)} = \frac{2(L_i - k)}{L_i(L_1 + L_2 - 2)}. \tag{3.73}
\]

The false alarm probability decays exponentially with the inverse of

\[
\varpi = \sum_{i=1}^{2} \sum_{k=1}^{L_i-1} \omega_{i,k}^2 \sigma_{i,k}^2 = \frac{2}{N(L_1 + L_2 - 2)}. \tag{3.74}
\]

Comparing (3.74) to that of the Case A given by (3.58), we can see that the benefit of combining the long preamble is negligible if the number of identical segments in the short training symbol is large, i.e., in the IEEE 802.11 case. However, for smaller \( L_1 \) as in the IEEE 802.16 case, the combined metric gives much fewer false alarms.

Similar to the approach we used in Case A, define

\[
Z_B[\hat{\tau}_1] \triangleq R_{B,1}[\hat{\tau}_1]R[\hat{\tau}_2, 0] + R_{B,2}[\hat{\tau}_2]R[\hat{\tau}_1, 0] - \lambda_e R[\hat{\tau}_1, 0]R[\hat{\tau}_2, 0], \tag{3.75}
\]

where

\[
R_{B,i}[\hat{\tau}_i] \triangleq \frac{2}{L_1 + L_2 - 2} \sum_{k=1}^{L_i-1} |R[\hat{\tau}_i, d_{i,k}]|. \tag{3.76}
\]
The second moment of $Z$ can be directly evaluated as

$$
E \{ Z_B[\hat{\tau}_1] \} = \frac{2}{L_1 + L_2 - 2} \sum_{k=1}^{L_1-1} E \{ |R[\hat{\tau}_1, d_{1,k}]| \} E \{ R[\hat{\tau}_2, 0] \}
$$

$$
+ \frac{2}{L_1 + L_2 - 2} \sum_{k=1}^{M_2-1} E \{ |R[\hat{\tau}_2, d_{2,k}]| \} E \{ R[\hat{\tau}_1, 0] \}
$$

$$
- \lambda E \{ R[\hat{\tau}_1, 0] \} E \{ R[\hat{\tau}_2, 0] \}
$$

$$
= N^2 \sigma_i^2 (\sigma_1^2 + \sigma_w^2) - \lambda N^2 (\sigma_1^2 + \sigma_w^2)^2. \quad (3.77)
$$

The second moment of $Z_B[\hat{\eta}_1]$ can be expanded to

$$
E \{ Z_B[\hat{\eta}_1]^2 \} = E \{ (|R_{B,1}[\hat{\eta}_1]|R[\hat{\eta}_2, 0]|R_{B,2}[\hat{\eta}_2]|R[\hat{\eta}_1, 0])^2 \}
$$

$$
+ \lambda^2 E \{ R[\hat{\eta}_1, 0]^2 \} E \{ R[\hat{\eta}_2, 0]^2 \}
$$

$$
- \lambda E \{ |R_{B,1}[\hat{\eta}_1]|R[\hat{\eta}_1, 0] \} E \{ |R_{B,2}[\hat{\eta}_2]|^2 \}
$$

$$
- \lambda E \{ |R_{B,2}[\hat{\eta}_2]|R[\hat{\eta}_2, 0] \} E \{ |R_{B,1}[\hat{\eta}_1]|^2 \}. \quad (3.78)
$$

According to the definition of $R_{B,i}[\hat{\eta}_i]$, we have

$$
E \{ R_{B,i}[\hat{\eta}_i] R[\hat{\eta}_i, 0] \} = \frac{2}{L_1 + L_2 - 2} \sum_{k=1}^{L_1-1} E \{ |R[\hat{\eta}_i, d_{i,k}]| R[\hat{\eta}_i, 0] \}
$$

$$
= \frac{2}{L_1 + L_2 - 2} \sigma_i^2 \left( N (\sigma_i^2 + \sigma_w^2) + 2\sigma_w^2 \right) \sum_{k=1}^{L_1-1} (N - k N/L_i)
$$

$$
= N \sigma_i^2 \left( N (\sigma_i^2 + \sigma_w^2) + 2\sigma_w^2 \right) \frac{L_i - 1}{L_1 + L_2 - 2}. \quad (3.79)
$$

Also,

$$
E \{ |R_{B,i}[\hat{\eta}_i]|^2 \} = \left( \frac{2}{L_1 + L_2 - 2} \right)^2 \sum_{k_1=1}^{L_i-1} \sum_{k_2=1}^{L_i-1} E \{ |R[\hat{\eta}_i, d_{i,k_1}]| R[\hat{\eta}_i, d_{i,k_2}]| \}
$$

$$
= \left( \frac{2\sigma_i^2}{L_1 + L_2 - 1} \sum_{k=1}^{L_i-1} (N - k N/L_i) \right)^2 + 2N \sigma_i^2 \sigma_w^2 \left( \frac{L_i - 1}{L_1 + L_2 - 2} \right)^2
$$

$$
= N \sigma_i^2 \left( N (\sigma_i^2 + \sigma_w^2) \right) \left( \frac{L_i - 1}{L_1 + L_2 - 2} \right)^2. \quad (3.80)
$$

Thus, ignoring the high order noise terms, we can approximate the products of
3.4 Case Study and Performance Analysis

expectations as

\[
E \left\{ (R[\tilde{\tau}_1, 0]R[\tilde{\tau}_2, 0])^2 \right\} \\
\approx N^4 \sigma_1^6 (\sigma_1^2 + 4\sigma_w^2) + 4N^3 \sigma_1^6 \sigma_w^2
\]  
(3.81)

\[
E \left\{ (RB[\tilde{\tau}_1]R[\tilde{\tau}_2, 0])^2 \right\} \\
\approx N^4 \sigma_1^6 (\sigma_1^2 + 2\sigma_w^2) + 4N^3 \sigma_1^6 \sigma_w^2
\]  
(3.82)

\[
E \left\{ RB[\tilde{\tau}_1]R[\tilde{\tau}_2, 0] R^2[\tilde{\tau}_1, 0] \right\} \\
\approx \left( N^4 \sigma_1^6 (3\sigma_1^2 + 4N^3 \sigma_1^6 \sigma_w^2) \right) \frac{L_1 - 1}{L_1 + L_2 - 2}
\]  
(3.83)

\[
E \left\{ RB[\tilde{\tau}_2]R[\tilde{\tau}_1, 0] R^2[\tilde{\tau}_1, 0] \right\} \\
\approx \left( N^4 \sigma_1^6 (3\sigma_1^2 + 4N^3 \sigma_1^6 \sigma_w^2) \right) \frac{L_2 - 1}{L_1 + L_2 - 2}.
\]  
(3.84)

Combine them with (3.78), then minus the square of the mean given by (3.77), it can be calculated that

\[
Var \{Z_B[\tilde{\tau}_1]\} \approx 4N^3 \sigma_1^6 \sigma_w^2 (1 - \lambda_c)^2.
\]  
(3.85)

Thus,

\[
P(T[\tilde{\tau}_1] < \lambda_c) = P(Z_B[\tilde{\tau}_1] < 0)
\]

\[
\approx \int_{-\infty}^{0} \frac{1}{\sqrt{8\pi}N^3 \sigma_1^6 \sigma_w^2 (1 - \lambda_c)^2} e^{-\frac{(v-N^2 \sigma_1^2 (\sigma_1^2 - \lambda_c \sigma_w^2)^2}{8N^2 \sigma_1^6 \sigma_w^2 (1 - \lambda_c)^2}} \, dv
\]

\[
= \frac{1}{2} \text{erfc} \left( \sqrt{\frac{N}{2\sqrt{2}}} \left( \sqrt{\frac{\sigma_1^2}{\sigma_w^2}} - \frac{\lambda_c}{1 - \lambda_c} \sqrt{\frac{\sigma_w^2}{\sigma_1^2}} \right) \right).
\]  
(3.86)

Compared to (3.71),

- The 50% missed detection probability corresponds to \( \lambda_c = \sigma_1^2 / (\sigma_1^2 + \sigma_w^2) \), which is the same as that of Case A.

- Combining the timing metric of the second symbol slightly increases the missed detections at high SNR.

- Because the missed detection probability given by (3.86) is not a function of \( L_1 \) or \( L_2 \), using more than two identical segments in the training symbols does not reduce missed detections.

We verify the analytical results using the same simulation environment as that
of Case A and plot the results in Figure 3.5. The analytical and simulation results for Case B are represented by the solid lines and the various markers respectively. The dotted lines are the analytical results in the scenario where only the short training symbol is utilized (Case A). It is shown that our analysis agrees with the simulations reasonably well. The figure suggests that using both training symbols considerably reduces false alarms for the \((L_1 = 4)\) case, but the improvement is marginal for the \((L_1 = 8)\) case. The missed detection curves of Case A and B are very close to each other under the given SNRs, and all of them exhibit very steep rising edges. This means that combining two training symbols does not have practical impact on the detection performance, which is largely determined by the SNR.

### 3.4.3 Case C: \(I_p = 1, L_1 \text{ is not divisible by } N\)

This case generalizes the IEEE 802.16 [4] OFDMA downlink channel where \(L_1 = 3\). In this case,

\[
w_{1,k} = \frac{(N - d_{1,k}) \rho_1^2(d_{1,k})}{\sum_{m=1}^{L_1-1} \rho_1^2(d_{1,m})(N - d_{1,m})} = \frac{2(N - d_{1,k})}{N} \frac{\rho_1^2(d_{1,k})}{\sum_{m=1}^{L_1-1} \rho_1^2(d_{1,m})},
\]  

(3.87)
where the last equality follows the fact that $|\rho_i(d)| = |\rho_i(N - d)|$. The false alarm probability of the combined timing metric is determined by

$$w = \sum_{k=1}^{L_1-1} w_{1,k}^2 \sigma_{1,k}^2 = \frac{2}{N} \sum_{k=1}^{L_1-1} \frac{1}{\rho_1^2(d_{1,k})} .$$  \hfill (3.88)

Define

$$Z_C[\hat{\tau}_1] \triangleq \frac{2}{\sum_{k=1}^{K_1} \rho_1^2(d_{1,k})} \sum_{k=1}^{K_1} |\rho_1(d_{1,k})R[\hat{\tau}_1, d_{1,k}]| - \lambda_c R[\hat{\tau}_1, 0] .$$  \hfill (3.89)

It is easy to compute

$$E \{ Z_C[\hat{\tau}_1] \} = \frac{2}{\sum_{k=1}^{K_1} \rho_1^2(d_{1,k})} \sum_{k=1}^{K_1} |\rho_1(d_{1,k})E \{ |R[\hat{\tau}_1, d_{1,k}]| \} - \lambda_c E \{ R[\hat{\tau}_1, 0] \}$$

$$= N\sigma_1^2 - \lambda_c N(\sigma_1^2 + \sigma_w^2) .$$  \hfill (3.90)

From (3.90), it is revealed that the threshold corresponding to 50% missed detection probability of Case C is the same as those of Case A and B. Considering the steepness of the missed detection curves, we expect the detection performance of Case C to be similar to that in the former two cases. Using (3.67), (3.68), and (3.69),

$$Var \{ Z_C[\hat{\tau}_1] \} = E \{ Z_C^2[\hat{\tau}_1] \} - (E \{ Z_C[\hat{\tau}_1] \})^2$$

$$= 2\lambda_c^2 N\sigma_1^2 \sigma_w^2 - 4\lambda_c N\sigma_1^2 \sigma_w^2 + \left( \frac{2}{\sum_{k=1}^{L_1-1} \rho(d_{1,k})^2} \right)^2 \sum_{k_1=1}^{L_1-1} \sum_{k_2=1}^{L_1-1} \rho(d_{1,k_1})\rho(d_{1,k_2})$$

$$\times ((N - \max(d_{1,k_1}, d_{1,k_2}))\rho(d_{1,k_2} - d_{1,k_1})$$

$$+ \max(N - d_{1,k_1} - d_{1,k_2}, 0)\rho(d_{1,k_2} + d_{1,k_1})).$$  \hfill (3.91)

We verify the analytical results using the same simulation environment as that of Case A. The results are plotted in Figure 3.6 where we use solid and dashed lines to represent the analytical results, and various markers to represent the simulation results. Also, we use dotted lines to represent the analytical missed detection probabilities of Case A under the given SNRs. It is shown that our analysis agrees with the simulations reasonably well. The figure suggests that using more highly correlated segments in the training symbols reduces false alarms but has little impact on the missed detection probability, which is largely determined by the SNR. These results coincide with our findings in the former two cases. The analytical
results for missed detection probabilities are accurate enough to predict the positions of the steep rising edges of the curves despite of the slight discrepancy with the simulation results caused by the Gaussian approximation in the derivation of missed detection probability.

3.5 Numerical Results

In this section, we present the simulation results in realistic wireless communication scenarios. An IEEE 802.16 [4] (WiMAX) system is modeled in the simulations using two kinds of training symbols specified for the OFDM and OFDMA physical layers respectively. The OFDM physical layer has $N = 256$ subcarriers, and employs two training symbols with $L_1 = 4$ and $L_2 = 2$. The OFDMA physical layer has $N = 512$ subcarriers, and uses only one training symbol with $L_1 = 3$. The carrier frequency is set to 3.5GHz, the CP is 1/8 of one symbol duration. The true CFO is modeled by a uniformly distributed random variable within $\pm 20$ subcarrier spacing. Each OFDM burst lasts for 5ms, containing 47 OFDM symbols plus an idle period between the bursts. Every point in the figures is an average of at least $3 \times 10^4$ independent experiments, each of which contains at least one complete OFDM
burst, but the starting position of observation is randomly selected. The training symbols have equal power, which is 9dB higher than the average power level of data symbols. Each data symbol’s relative power level is independently selected from \{-12dB, -9dB, \cdots , +9dB\} with equal probability, and the modulation scheme is randomly selected from QPSK, 16-QAM and 64-QAM also with equal probability. The SNR is defined as the ratio between the total power of the received signal and noise in the training symbol.

Two channel models introduced in Chapter 2 are used in the simulations. CH-A is a stationary wireless communication channel with long deep fading, while CH-B is a mobile channel with up to \( f_d \approx 388.9 \text{Hz} \) Doppler frequency. In the simulations, each OFDM symbol lasts for 91\( \mu s \), which gives \(( f_d T_s \approx 0.0354 )\). We move the channel taps to the nearest sample-spaced position to simplify the channel emulation.

In the simulations, we take the maximum of the timing metric within the window \([ \hat{\tau}_i - N, \hat{\tau}_i + N ]\) as the timing estimate. As explained earlier, our coarse timing estimates do not depend on the threshold settings, and for all the thresholds that allow the detection of the OFDM burst, the timing estimates will be the same.

For the OFDM physical layer, we refer to the method of [42] as the conventional method, which only uses the short training symbol for timing estimation. In Figure 3.7(a), 3.8(a) and 3.9(a), we compare that to the proposed method which takes advantage of multiple training symbols. For the OFDMA physical layer, we refer to the method of [47] as the conventional method, which only uses the first component timing metric for timing estimation. In Figure 3.7(b), 3.8(b) and 3.9(b), we compare that to the proposed method which combines two timing metrics for reduced false alarms. In both the OFDM and OFDMA cases, the proposed methods require one more correlator than the conventional methods.

### 3.5.1 False Alarm Probability

Figure 3.7(a) and 3.7(b) plot the false alarm probabilities as a function of the threshold \( \lambda_c \) for OFDM and OFDMA physical layers respectively. It is shown that the proposed method achieves much better performance than that of the conventional methods [42, 47] in all the cases. The false alarm probability at 0dB is lower than that at 20dB. This can be explained by the fact that higher SNR causes larger power variation within each OFDM symbol, which violates our assumptions when deriving the variance of the component timing metrics. For the same reason, the time-selectivity in CH-B degrades the performance of the combined timing
Figure 3.7: False alarm probability per frame.

(a) OFDM physical layer ($L_1 = 4$, $L_2 = 2$)

(b) OFDMA physical layer ($L_1 = 3$)
metric. However, the figure shows that the conventional methods also suffer from the power fluctuation, and the advantage of the proposed method in terms of false alarm probability is quite evident.

### 3.5.2 Missed Detection Probability

The missed detection probabilities are plotted in Figure 3.8(a) and 3.8(b) respectively for OFDM and OFDMA physical layers. The figures show that virtually there is no difference between the missed detection probabilities of the proposed method and those of the conventional methods [42, 47]. However, the curves of different SNRs differ dramatically with each other. This confirms our analytical results that the missed detection probability is highly sensitive to SNR, which fluctuates with the channel condition in a wide range. Therefore, optimizing the detection performance at certain SNR cannot give stable timing estimation performance in various channel conditions. Nevertheless, for any given false alarm probability, our method allows the use of a lower threshold, which gives a better chance to detect the right timing position.

### 3.5.3 Distribution of Timing Offsets

As explained in the introduction, we define the correct timing positions to be those that incur little or no ISI on the samples in the FFT window. Because of the CP, even in an AWGN channel, the estimated coarse timing position can fluctuate between 0 and \( N_g \) from the time when the first sample of the training symbol arrives at the receiver. We take the middle point of the \( N_g \)-sample-long timing window as the reference point, and compute the probability for the coarse timing estimates falling out of certain distance from the reference point. In the OFDM and OFDMA cases, the lengths of the CP are 32 and 64 samples respectively, which means that the correct estimates need to be within 16 and 32 samples from the reference point for the OFDM and OFDMA cases respectively.

Using the criterion described above, in Figure 3.9(a) and 3.9(b), we see about 90% of the estimates fall in the window of correct estimates. Very steep falling edges are observed exactly at \( N_g/2 \). This indicates that the probability for large timing offsets or significant ISI would decay quickly out of the window of correct estimates. It is observed that the estimates of the proposed method distributed in a narrower region around the reference timing position than that of the conventional methods [42, 47]. This suggests superior performance of the proposed algorithm.
Figure 3.8: Missed detection probability per frame.
3.5 Numerical Results

Figure 3.9: Distribution of timing offsets.
For both proposed and conventional estimators, the coarse timing estimates are evenly distributed in the window of correct estimates, so one needs to use the refined timing estimation methods [61,68,69] to determine the ideal timing position of each OFDM burst. In Chapter 5, we discuss refined timing estimation in details and propose a successive joint channel and timing estimator to achieve very high timing estimation accuracy at low SNR.

3.6 Summary

In this chapter, we have proposed a universal coarse timing estimator that is robust to various channel conditions. The proposed method took advantage of multiple training symbols and worked for the training symbols consisting of highly correlated but not necessarily identical segments. The new method was able to work for all the preamble structures specified in the IEEE 802.11 [3] and IEEE 802.16 [4] standards, including the downlink of IEEE 802.16 OFDMA (WiMAX) system where the training symbol consisted of three non-identical segments. Our analysis provided more insights into the proposed method and shed new light on existing works. It was shown that a larger number of correlated segments in the training symbol could reduce false alarms but had little impact on missed detection probability. The simulation results under various channel conditions confirmed the performance of the proposed method.
Chapter 4

Coarse Frequency Estimation

4.1 Introduction

As part of OFDM acquisition two key parameters need to be determined in the time domain before moving to the frequency domain. In Chapter 3 we have discussed one of the key parameters, which is the timing positions of FFT windows. In this chapter, we investigate the estimation of the other key parameter, which is the fractional carrier frequency offset (CFO). A fractional CFO is the portion of CFO as a fraction of one subcarrier spacing. Without being correctly estimated and accounted for in the time domain, a fractional CFO introduces inter-carrier interference (ICI) to the frequency domain signal and dramatically degrades system performance.

References [52–55] have shown that the integer CFO, which is a multiple of the subcarrier spacing, can be determined with high accuracy in the frequency domain. However, other references [6, 9, 14, 56–60] show that fractional CFO refinement in the frequency domain is computationally expensive. These facts highlight the importance of the accuracy of a coarse CFO estimator. The goal of design is to keep the estimation error within an acceptable range so that there is no need for further frequency-domain refinement. Different from the approach of ESCA [17] which maximizes the CFO estimation range in the time domain, we give a higher priority to the estimation accuracy and expand the estimation range at a latter signal processing stage discussed in Chapter 5.

Existing CFO estimation algorithms require the training symbol to consist of a number of identical segments, and only make use of one training symbol. When there are multiple training symbols, the estimation ranges of the estimators based on those symbols can be different from each other, it is not clear how to combine
the estimates for higher accuracy. In [51], Zhang et al proposed a method to combine multiple CFO estimates assuming the true CFO is within the smallest estimation range of the estimators. That assumption do not hold for CFO initial acquisition where the CFO can be in a large range. Another method to combine CFO estimates with different estimation range was proposed in [50]. That method is only applicable to the scenario when the estimation range of one estimator is divisible by that of the other, which does not necessarily hold for the generalized training symbols studied in this thesis. It is also worth noting that the methods of [50, 51] do not take into account the correlation between the estimates when combining them, so the performance is inferior to that of the ESCA [17].

Another scenario not taken into account in the existing literature is when the segments in the training symbol are highly correlated but not identical. For example, in the IEEE 802.16 OFDMA (WiMAX) standard [4], the downlink training symbol consists of three highly correlated but not identical segments. This violates the basic assumption of conventional methods, and invalidates their application to the generalized training symbols.

In this Chapter, we propose a novel universal CFO estimator that works for the generalized training symbols consisting of highly correlated but not necessarily identical segments, which allows the fractional CFO to be accurately estimated from the training symbol in IEEE 802.16 OFDMA (WiMAX) systems. The universal estimators have a fixed estimation range of ±0.5 subcarrier spacing, so their estimates can be combined using the BLUE principle for higher accuracy. When there is only one training symbol consisting of identical segments, the proposed universal CFO estimator dramatically outperforms the ESCA at low SNR with lower complexity; when there are multiple training symbols, the proposed method takes advantage of them and achieves a linear decrease in the mean square error; when the segments in the training symbols are not identical, the conventional methods do not work, while the performance of the proposed universal estimator and the BLUE based on our universal estimator still approaches the CRB. Simulation results in various realistic multipath fading wireless channels are presented to confirm the performance of the proposed methods.

### 4.2 Brief Review of Correlators

We assume the correct coarse timing estimate $\hat{\tau}_i$ is available to the CFO estimators based on the $i^{th}$ training symbol in the preamble. In Chapter 3, we have derived
the approximated expression for the correlator output as

\[ R[\hat{\tau}_i, d] \approx e^{j \frac{2\pi d}{N}\hat{\epsilon}_0} \left( (N - d)\sigma_i^2 \phi_i(d) \rho_i(d) + \Theta(\hat{\tau}_i, d) \right), \tag{4.1} \]

where

\[
\Theta(\hat{\tau}_i) \triangleq \sum_{n=0}^{N-d-1} \hat{\Phi}^* [\tau + n] \hat{x}[\tau + n + d] + \sum_{n=d}^{N-1} \hat{\Phi} [\tau + n] \hat{x}^*[\tau + n - d], \tag{4.2} 
\]

\[
\rho_i(d) \triangleq \begin{cases} 1, & d \text{ is multiple of } (N/L_i); \\ \frac{\sin \left( \frac{\pi}{N} L_i d \right)}{N_i \sin \left( \frac{\pi}{N} L_i d \right)}, & \text{others}, \end{cases} \tag{4.3} 
\]

\[
\phi_i(d) \triangleq \begin{cases} e^{j \frac{2\pi d}{N}\hat{\epsilon}_0[1]}, & d \text{ is multiple of } (N/L_i); \\ e^{j \frac{2\pi d}{N}(L_i(N_i - 1)+2\hat{\epsilon}_0[1])}, & \text{others}. \end{cases} \tag{4.4} 
\]

This indicates that the correlator output is a function of CFO and therefore it is possible to estimate CFO from the correlator’s output.

Conventional methods \([10, 17, 18, 39, 40]\) assume both \(d\) and \(\vartheta_0\) are multiples of \((N/L_i)\) and estimate CFO from the phase of \(R[\hat{\tau}_i, d]\) as

\[
\hat{\epsilon}(\hat{\tau}_i, d) = \frac{N}{2\pi d} \arg \left( R[\hat{\tau}_i, d] \cdot e^{j 2\pi m} \right) = \frac{N}{2\pi d} \arg \left( R[\hat{\tau}_i, d] \right) + \frac{N}{d} m, \tag{4.5} 
\]

where \(m\) is an indistinguishable integer that causes the ambiguity in the CFO estimates. If \((N/d)\) is an integer, this ambiguity does not affect the fractional part of the CFO and can be solved in the frequency domain \([52]\). However, when \((N/d)\) is not an integer, the conventional methods cannot solve the ambiguity in the fractional CFO and the resulted substantial ICI will significantly compromise the performance of frequency-domain signal processing algorithms including the integer CFO estimator in \([52]\).

### 4.3 Universal CFO Estimators

From the analysis above, we know the CFO estimates based on a single correlator has an ambiguous fractional part when \((N/d)\) is not an integer. To solve the problem, we multiply \(R[\hat{\tau}_i, d]\) with another complementary term such that the term \(N/d\) cancels out without causing any ambiguity in the fractional CFO estimate. The complementary term universal to all \(d\) values is found to be \(R[\hat{\tau}_i, N - d]\), and
the CFO can be estimated by
\[
\hat{\varepsilon}_c(\hat{\tau}_i, d_{i,k}) = \frac{1}{2\pi} \arg(R[\hat{\tau}_i, d_{i,k}] R[\hat{\tau}_i, N - d_{i,k}]),
\]  
(4.6)

where \(\hat{\tau}_i\) is the estimated timing position for the \(i\)th training symbol, \(d_{i,k}\) is the correlation interval for the \(k\)th universal estimator based on training symbol \(i\). The estimation ranges for these estimators are all equal to \(\pm 0.5\) subcarrier spacing regardless of \(\hat{\tau}_i\) and \(d_{i,k}\).

Assume \(\hat{\tau}_i\) is one of the correct timing estimates for that symbol, and \(d_{i,k}\) is close to an integer multiple of \(\left(\frac{N}{L_i}\right)\) so that \(\rho_i(d_{i,k})\) is significantly larger than \(0\). When the fractional CFO estimation error is small, we can write
\[
\hat{\varepsilon}_c(\hat{\tau}_i, d_{i,k}) - [\varepsilon_0]_1 = \frac{1}{2\pi} \left( \arg(R[\hat{\tau}_i, d_{i,k}] R[\hat{\tau}_i, N - d_{i,k}]) - 2\pi\varepsilon_0 \right)
\]
\[
= \frac{1}{2\pi} \left( \arg(R[\hat{\tau}_i, d_{i,k}] \phi_i^*(d_{i,k}) \gamma_i(d_{i,k}))
\right.
\]
\[
+ \arg(R[\hat{\tau}_i, N - d_{i,k}] \phi_i^*(N - d_{i,k}) \gamma_i(N - d_{i,k})) - 2\pi\varepsilon_0 \right)
\]
\[
= \frac{1}{2\pi} \left( \psi_i(d_{i,k}) + \psi_i(N - d_{i,k}) \right),
\]  
(4.7)

where \(\varepsilon_0\) is the true CFO, \([\cdot]_1\) denotes the function that takes the fractional part of its argument, and we define
\[
\psi_i(d) \triangleq \arg \left( R[\hat{\tau}_i, d] \phi_i^*(d) \gamma_i(d) \exp(2\pi\varepsilon_0 d/N) \right).
\]  
(4.8)

Denote \(\Re(\cdot)\) and \(\Im(\cdot)\) as the functions that take the real and imaginary parts of their complex arguments respectively. Following the fact that \(\tan(u) \approx u\) holds for all small quantity \(u\), when the CFO estimation error is small, we can approximate \(\psi_i(d)\) by
\[
\psi_i(d) \approx \tan^{-1} \left( \frac{\Im(R[\hat{\tau}_i, d] \phi_i^*(d) \gamma_i(d) \exp(-j2\pi\varepsilon_0 d/N))}{\Re(R[\hat{\tau}_i, d] \phi_i^*(d) \gamma_i(d) \exp(-j2\pi\varepsilon_0 d/N))} \right)
\]
\[
\approx \frac{\Im(\phi_i^*(d) \gamma_i(d) \Theta(\hat{\tau}_i, d))}{(N - d)\rho_i(d) |\sigma_i^2|}.
\]  
(4.9)

Because \(\Theta(\hat{\tau}_i, d)\) has a random phase uniformly distributed in \([0, 2\pi]\), the expectation of \(\psi_i(d)\) is 0 and
\[
E \{\hat{\varepsilon}_c(\hat{\tau}_i, d_{i,k})\} - [\varepsilon_0]_1 = \frac{1}{2\pi} \left( E \{\psi_i(d_{i,k})\} + E \{\psi_i(N - d_{i,k})\} \right) = 0.
\]
This indicates that at high SNR or when the estimation error is small, the proposed universal CFO estimators are unbiased.

The estimators proposed in [17–19] will give ambiguous fractional CFO estimates when $L_i$ is not divisible by the number of subcarriers, so they are not universally applicable to all $L_i$ values. For instance, the training symbol specified by the IEEE 802.16 (WiMAX) standard [4] has $L_i = 3$ highly correlated segments, only the proposed method can provide unbiased fractional CFO estimates.

4.4 The BLUE Estimator

When multiple universal CFO estimators are constructed for the highly correlated segments in the training symbols, it is possible to combine the estimators for higher accuracy. Write all the universal CFO estimates $\{\hat{\epsilon}_c(\hat{\tau}_i, d_{i,k})\}$ into a vertical vector $\hat{\epsilon}_c$, and the dimension of $\hat{\epsilon}_c$ equal the number of universal CFO estimators. Denote $\mathbf{1}$ as the all-one vector having the same size as $\hat{\epsilon}_c$. The universal fractional CFO estimates can be combined by the best linear unbiased estimator (BLUE) as [121]

$$\hat{\epsilon}_f = \beta^T \hat{\epsilon}_c, \quad (4.10)$$

where

$$\beta = \frac{\Psi^{-1} \mathbf{1}}{\mathbf{1}^T \Psi^{-1} \mathbf{1}} \quad (4.11)$$

and the elements of the universal CFO estimation error correlation matrix $\Psi$ are defined to be

$$\Psi[m_1, m_2] \triangleq E \left\{ (\hat{\epsilon}_c(\hat{\tau}_{i_1}, d_{i_1,k_1}) - [\epsilon_0]_1) (\hat{\epsilon}_c(\hat{\tau}_{i_2}, d_{i_2,k_2}) - [\epsilon_0]_1) \right\}, \quad (4.12)$$

where the $m_1^{th}$ and $m_2^{th}$ elements of $\hat{\epsilon}_c$ are $\hat{\epsilon}_c(\hat{\tau}_{i_1}, d_{i_1,k_1})$ and $\hat{\epsilon}_c(\hat{\tau}_{i_2}, d_{i_2,k_2})$ respectively. Note that although ESCA [17] also combines multiple CFO estimates, they are not based on the proposed universal estimators, so their results are not applicable to the BLUE in this thesis.

In the rest of this section, we derive the expression of $\Psi[m_1, m_2]$, then discuss three $\{d_{i,k}\}$ selection schemes for different performance and complexity tradeoffs. The complexity analysis of the proposed methods is presented at the end.
For the universal estimates based on two different symbols \((i_1 \neq i_2)\), the estimation errors are uncorrelated, so

\[
E \{ (\hat{\epsilon}_c(\hat{\tau}_{i_1}, d_{i_1,k_1}) - [\epsilon_0]_1)(\hat{\epsilon}_c(\hat{\tau}_{i_2}, d_{i_2,k_2}) - [\epsilon_0]_1) \} = (E \{ \hat{\epsilon}_c(\hat{\tau}_{i_1}, d_{i_1,k_1}) \} - [\epsilon_0]_1)(E \{ \hat{\epsilon}_c(\hat{\tau}_{i_2}, d_{i_2,k_2}) \} - [\epsilon_0]_1) = 0.
\] (4.13)

For the universal estimators based on the same training symbol, using the property that \(\phi_i(d_1 + d_2) = \phi_i(d_1)\phi_i(d_2)\), we have

\[
\begin{align*}
E \{ \Im (\phi_i'(d_1)\gamma_i(d_1)\Theta(\hat{\tau}_i, d_1)) \} & = -\frac{\gamma_i(d_1)\gamma_i(d_2)}{4} E(\phi_i'(d_1)\Theta(\hat{\tau}_i, d_1) - \phi_i(d_1)\Theta^*(\hat{\tau}_i, d_1)) \\
& \times (\phi_i'(d_2)\Theta(\hat{\tau}_i, d_2) - \phi_i(d_2)\Theta^*(\hat{\tau}_i, d_2)) \\
& = \frac{\gamma_i(d_1)\gamma_i(d_2)}{2} \times E \{ \Re (\phi_i'(d_2 - d_1)\Theta^*(\hat{\tau}_i, d_1)\Theta(\hat{\tau}_i, d_2)) \} \\
& - \frac{\gamma_i(d_1)\gamma_i(d_2)}{2} \times E \{ \Re (\phi_i'(d_2 + d_1)\Theta(\hat{\tau}_i, d_2)\Theta(\hat{\tau}_i, d_1)) \} \\
& = \gamma_i(d_1)\gamma_i(d_2) \sigma_w^2 \sigma_i^2 (\rho_i(d_2 - d_1) - \rho_i(d_2 + d_1)) \max(N - d_1 - d_2, 0) \rho_i(d_2 + d_1)).
\end{align*}
\] (4.14)

Therefore,

\[
E \{ \psi_i(d_1)\psi_i(d_2) \} = \frac{\sigma_w^2}{\sigma_i^2} (\rho_i(d_2 - d_1) - \rho_i(d_2 + d_1)) \max(N - d_1 - d_2, 0) \frac{1}{(N - d_1)(N - d_2)} \rho_i(d_1) \rho_i(d_2).
\] (4.15)

Without loss of generality, we suppose \(0 < d_1 \leq d_2 \leq N/2\) and attain

\[
E \{ \psi_i(d_1)\psi_i(N - d_2) \} = \frac{\sigma_w^2}{\sigma_i^2} \frac{\rho_i(d_2 - d_1)(N - d_2) - \rho_i(d_2 + d_1)(N - d_1 - d_2)}{(N - d_1)(N - d_2)} \rho_i(d_1).\] (4.16)

\[
E \{ \psi_i(d_1)\psi_i(N - d_2) \} = \frac{\sigma_w^2}{\sigma_i^2} \frac{\rho_i(d_2 + d_1)(d_2 - d_1)(N - d_1 - d_2)}{d_2(N - d_1)\rho_i(d_1)\rho_i(d_2)}.
\] (4.17)

\[
E \{ \psi_i(N - d_1)\psi_i(d_2) \} = \frac{\sigma_w^2}{\sigma_i^2} \frac{\rho_i(d_2 + d_1)}{(N - d_2)\rho_i(d_1)\rho_i(d_2)}\]

\[
E \{ \psi_i(N - d_1)\psi_i(N - d_2) \} = \frac{\sigma_w^2}{\sigma_i^2} \frac{\rho_i(d_2 + d_1)}{d_2\rho_i(d_1)\rho_i(d_2)}.
\] (4.18)
Combining them together using (4.7), we have

$$
E\{ (\hat{\epsilon}_c(\hat{\tau}_i, d_1) - [\epsilon_0]_1)(\hat{\epsilon}_c(\hat{\tau}_i, d_2) - [\epsilon_0]_1) \}
= \frac{1}{4\pi^2} E\{ (\psi_i(d_1) + \psi_i(N-d_1))(\psi_i(d_2) + \psi_i(N-d_2)) \}
= \frac{N \sigma_w^2}{4\pi^2 \sigma_i^2} \rho_i(d_2-d_1)(N-d_2) + \rho_i(d_2+d_1)d_2,
$$

(4.20)

Thus, following (4.13) and (4.20),

$$
\Psi[m_1, m_2] = E\{ (\hat{\epsilon}_c(\hat{\tau}_{i_1}, d_{i_1,k_1}) - [\epsilon_0]_1)(\hat{\epsilon}_c(\hat{\tau}_{i_2}, d_{i_2,k_2}) - [\epsilon_0]_1) \}
= \delta_{i_1,i_2} \frac{N \sigma_w^2}{4\pi^2 \sigma_i^2} \rho_i(d_{i_1,k_1}) \rho_i(d_{i_2,k_2}) \times \left( \frac{\rho_i(d_{i_1,k_1} + d_{i_2,k_2})}{(N-d_{i_1,k_1})(N-d_{i_2,k_2})} + \frac{\rho_i(d_{i_1,k_1} - d_{i_2,k_2})}{(N - \min(d_{i_1,k_1}, d_{i_2,k_2})) \max(d_{i_1,k_1}, d_{i_2,k_2})} \right),
$$

(4.21)

where $\delta_{i_1,i_2}$ is the Kronecker delta function that equals 1 when $i_1 = i_2$; and 0, otherwise. From (4.11) and (4.21), we can see that the BLUE coefficient vector $\beta$ can be pre-calculated without SNR knowledge because the noise variance $\sigma_w^2$ in (4.21) does not change with $m_1$ and $m_2$, and will be cancelled out. When the power levels of the training symbols are different, the calculation of $\beta$ only needs to take into account the relative signal strengths of the training symbols, which are known in advance at the receiver.

### 4.4.2 Three Construction Schemes for the Proposed BLUE

Flexible tradeoffs between the performance and complexity can be made by different $\{d_{i,k}\}$ selection schemes. In this thesis, we study three of them to illustrate the flexibility of the proposed algorithm.

#### Scheme A

$$
d_{i,k} = [k \cdot N/L_i],
$$

where $[\cdot]$ denotes the rounding function that outputs the integer nearest to the argument; $k$ is an integer between 1 and $[L_i/2]$. Here $[\cdot]$ denotes the floor function that outputs the largest integer not larger than the argument. For this scheme, the number of universal estimators for each training symbol is fixed to $[L_i/2]$. 
Scheme B

\[ d_{i,2k-1} = \lceil k \cdot N / L_i \rceil, \quad d_{i,2k} = \lfloor k \cdot N / L_i \rfloor, \]

where \( \lceil \cdot \rceil \) denotes the ceiling function that outputs the smallest integer not smaller than the argument; \( k \) is an integer between 1 and \( \lfloor L_i / 2 \rfloor \). It is easy to see that there can be up to \( (L_i - 1) \) universal estimators for this scheme, however, when \( (k \cdot N / L_i) \) is an integer, the universal estimators \( \hat{e}_c(\hat{\tau}_i, d_{i,2k-1}) \) and \( \hat{e}_c(\hat{\tau}_i, d_{i,2k}) \) become the same, and the actual number of universal estimators can be smaller. For the case when \( N \) is a multiple of \( L_i \), this scheme becomes the same as the Scheme A.

Scheme C

\[ d_{i,1} = \hat{d}_i, \]

where \( \hat{d}_i \) is given by

\[
\hat{d}_i \triangleq \arg \min_d E \left\{ (\hat{e}_c(\hat{\tau}_i, d) - [\epsilon_0]_1)^2 \right\} = \arg \min_d \frac{N - d(1 - \rho_i(2d))}{\rho_i^2(d) d(N - d)^2}. \tag{4.22}
\]

The universal estimator \( \hat{e}_c(\hat{\tau}_i, \hat{d}_i) \) is referred to as the best universal estimator for training symbol \( i \). For this scheme, there is only one universal estimator for each training symbol. A special case is when \( L_i \) is divisible by \( N \), \( \rho_i(d) = \rho_i(2d) = 1 \), we can show \( \hat{d}_i = \lfloor L_i / N \rfloor \).

4.4.3 Complexity

The complexity of the schemes described above can be compared by the number of universal estimators required for each training symbol. It is easy to see that the Scheme A is \( \lfloor L_i / 2 \rfloor \) times as complex as the Scheme C, while the Scheme B is at most twice as complex as the Scheme A, depending on the value of \( L_i \).

Each universal estimator of the proposed method requires two correlators, except for the special case of \( (d_{i,k} = N/2) \) where the two correlators become the same. For the ESCA, each component estimator only requires one correlator, so the complexity will be half as that of the proposed method if the same number of component estimators are used. However, as shown in the following sections, it is possible for the proposed method to outperform the ESCA with lower computational complexity for certain training symbol structures.
4.5 Performance Analysis

In this section, we analyze the performance of the universal estimator, the BLUE in one training symbol case, and the BLUE for multiple training symbols. The results show that the universal CFO estimator approximates the Cramer-Rao bound (CRB) for all $L_i$ values no matter divisible by the number of subcarriers or not. The BLUE only has marginal performance gain over the best universal estimator in one training symbol case, however, considerable gain can be achieved when there are multiple training symbols.

4.5.1 Universal Estimators

The diagonal terms of the error correlation matrix $\Psi$ give the mean square error (MSE) of the universal estimators. From (4.21), we can write

$$E \left\{ (\hat{\epsilon}_c(\hat{\tau}_i, d_{i,k}) - [\epsilon_0]_1)^2 \right\} = \frac{N(N - d_{i,k}(1 - \rho_i(2d_{i,k})))}{4\pi^2 \rho_i^2(d_{i,k})d_{i,k}(N - d_{i,k})^2} \frac{\sigma_w^2}{\sigma_i^2}$$

$$= \frac{N}{4\pi^2d_{i,k}(N - d_{i,k})^2} \cdot \eta_i(d_{i,k}), \quad (4.23)$$

where

$$\eta_i(d_{i,k}) \triangleq \frac{(N - d_{i,k}) + d_{i,k}\rho_i(2d_{i,k})}{N \rho_i^2(d_{i,k})} \geq 1. \quad (4.24)$$

The equality holds when $\rho_i(d_{i,k}) = 1$, which requires $d_{i,k}$ to be a multiple of $(N/L_i)$. Otherwise, $\eta_i(d_{i,k})$ represents the performance loss caused by imperfect repetition and sampling.

When $N$ is a multiple of $L_i$, the CRB is given by [17]

$$\text{CRB}_{L_i} = \frac{3}{2\pi^2 N(1 - L_i^2)} \cdot \text{SNR}_i^{-1}. \quad (4.25)$$

As $L_i$ increases, the CRB approaches the limit $\text{CRB}_\infty = \frac{3}{2\pi^2 N} \cdot \text{SNR}_i^{-1}$. When $N$ is not divisible by $L_i$, ideally it is possible to oversample the received signal and achieve the performance given by (4.25), so the CRB is applicable to those cases as well. Compare the MSE of the universal estimator given by (4.23) and the CRBs, and neglect the imperfect sampling loss ($\eta(d_k) = 1$), we find

- If $N$ is a multiple of $L_i = 3$, the proposed universal estimator achieves CRB$_3$.
- As the number of identical segments increases, the performance of the best universal estimator asymptotically approaches its performance bound CRB$_3$, 

which is only 0.51dB away from CRB\(_\infty\).

- When \( N \) is a power of 2, the MSE of the best universal estimator is upper bounded by that in \((L_1 = 4)\) case, which is about 0.73dB away from CRB\(_\infty\).

### 4.5.2 BLUE

The MSE of the BLUE can be expressed by [121]

\[
E \left\{ (\hat{\epsilon}_f - [\epsilon_0])^2 \right\} = (1^T \Psi^{-1} 1)^{-1},
\]

(4.26)

where for the proposed estimator, the elements of matrix \( \Psi \) is given by (4.21).

The exact expression of \( \Psi \) depends on the \( \{d_{i,k}\} \) selection scheme. For the three schemes described in Section 4.4.2, we numerically verify the analytical results against the simulations to give more insights into the methods. A typical 512-subcarrier OFDM system in AWGN channel is used in the numerical calculations where the CFO is modeled by a uniformly distributed random variable within \( \pm 20 \) subcarrier spacing. We assume the timing estimates are perfect. The analytical performance for the proposed Scheme A and B are computed from (4.26), and that of the Scheme C is calculated from (4.23).

First, we plot the MSE of the estimators as a function of SNR in Figure 4.1. It is indicated that our analysis is quite accurate for SNRs above 0dB. For lower SNRs, Coulson’s analytical result in [40] for the \( L_1 = 2 \) case serves as a theoretical upper bound on the error variance. In the \( L_1 = 8 \) case, the proposed Scheme A and B become the same, and both of them outperform the ESCA of [17]. In this case, the ESCA needs 4 correlators, while the proposed Scheme C only requires 2. Nevertheless, it still achieves significantly better performance than that of the ESCA at SNRs below 0dB. In the \( L_1 = 5 \) case, the proposed estimators have quite stable performance throughout the interested SNR region. The performance of the Scheme B is slightly superior to that of the Scheme A at the cost of double complexity, while the Scheme C saves half of the complexity of the Scheme A at the cost of 0.45dB performance loss. It should be noted that at low SNR the performance of the ESCA is even worse than the \((M = 2)\) Coulson bound, which means that the linear combining procedure of the ESCA fails to offer any performance improvement in the presence of high noise. In the contrast, the proposed algorithms have consistent performance throughout the interested SNR range.

Next, we fix the SNR to 10dB and let the number of highly correlated segments \( L_1 \) change from 2 to 16. Because both the MSE and the CRB are proportional
Figure 4.1: The performance as a function of SNR.
to the SNR, we expect the curves remain the same except for an offset on the y-axis for other SNRs. Figure 4.2 shows that the simulation results match our analytical curves reasonably well. It is observed that the proposed Scheme A and B can achieve the CRB when $N$ is a multiple of $L_1$, but suffer from a slight performance degradation in the other cases due to the imperfect repetition and sampling. The performance loss is less than 0.62dB, 0.44dB and 1.10dB respectively for the Scheme A, B and C.

4.5.3 Multiple Training Symbol Gain

We analyze the performance gain that is achieved by combining the universal estimates from multiple training symbols. From (4.21) and (4.26),

$$
E \left\{ (\epsilon_f - [\epsilon_0]_1)^2 \right\} = \left( 1^T \begin{pmatrix} \Psi_1 & 0 \\ 0 & \Psi_p \end{pmatrix} \right) \left( \begin{pmatrix} \Psi_1 & 0 \\ 0 & \Psi_p \end{pmatrix} \right)^{-1} 1 = \left( \sum_{i=1}^{I_p} 1_i^T \Psi_i^{-1} 1_i \right)^{-1} \left( \sum_{i=1}^{I_p} \hat{\theta}_i^{-1} \right)^{-1},
$$

(4.27)
where for the \( i \)th training symbol, \( \Psi_i \) is the universal CFO estimation error correlation matrix, \( \mathbf{1}_i \) is a vector matching the size of \( \Psi_i \) with all one elements, and \( \hat{\theta}_i \) is the variance of the BLUE based on training symbol \( i \). Because the variation of \( \hat{\theta}_i \) is bounded by the performance gap between the best universal estimator and the CRB, which is relatively small, the MSE of the BLUE is approximately inversely proportional to the number of training symbols.

Comparing to the analytical results presented in Section 4.5.2, we find that combining the estimates from the same training symbol gives much smaller performance improvement than the multiple-training-symbol gain which is at least 3dB. So the Scheme C is often a cost-effective choice because it earns the multiple-symbol gain with the least number of universal estimators.

### 4.6 Numerical Results

In this section, we present the simulation results in realistic wireless communication scenarios. An IEEE 802.16 [4] system is modeled in the simulations using two kinds of training symbols specified for the OFDM and OFDMA physical layers respectively. The OFDM physical layer has \( N = 256 \) subcarriers, and employs two training symbols with \( L_1 = 4 \) and \( L_2 = 2 \) identical segments. The OFDMA physical layer has \( N = 512 \) subcarriers, and uses only one training symbol with \( L_1 = 3 \) highly correlated but not identical segments. The carrier frequency is set to 3.5GHz, the CP is 1/8 of one symbol duration. The coarse timing estimate is modeled by a random variable uniformly distributed in the ISI-free window \([\bar{\tau}_1 - N_g/2, \bar{\tau}_1]\).

Two channel models introduced in Chapter 2 are used in the simulations. CH-A is a stationary wireless communication channel with long deep fading, while CH-B is a mobile channel with up to \( f_d \approx 388.9\text{Hz} \) Doppler frequency. In the simulations, each OFDM symbol lasts for 91\( \mu \)s, which gives \( f_d T_s \approx 0.0354 \).

The true CFO is modeled by a uniformly distributed random variable within \( \pm 20 \) subcarrier spacing to emulate the scenario when a mobile station has just switched on and started searching the cell. The pseudo-random sequences modulated in the training symbols are specified in the IEEE 802.16 (WiMAX) standard [4], and for the OFDMA physical layer, we change the training symbol every 100 experiments by randomly selecting a new base station identification number and sector index. Every point in the figures is an average of at least \( 10^5 \) independent experiments.
4.6.1 OFDM Physical Layer: $L_1 = 4$, $L_2 = 2$

The simulation results for the OFDM physical layer downlink channel are shown in Figure 4.3(a). The complexity of the estimators used in the simulations is listed in Table 4.1 in terms of the number of required correlators. Because both $L_1$ and $L_2$ are divisible by the number of subcarriers, the proposed Scheme A and B become the same. For the ESCA, the estimates obtained from the two training symbols have different estimation ranges, and there is no literature on how to combine them with a BLUE.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Number of Correlators</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESCA (Symb.1)</td>
<td>2</td>
</tr>
<tr>
<td>ESCA (Symb.2)</td>
<td>1</td>
</tr>
<tr>
<td>Proposed Scheme A/B</td>
<td>4</td>
</tr>
<tr>
<td>Proposed Scheme C</td>
<td>3</td>
</tr>
</tbody>
</table>

The ESCA based on the first training symbol achieves 0.4dB gain over that based on the second at the cost of the extra complexity of combining one more estimate. The Scheme A/B also combines one more universal estimate than the Scheme C, and the performance gain is about 0.25dB in CH-A. These performance improvements are much smaller than the multiple-training-symbol gain, which is about 3dB and demonstrated by the gap between the curves of the Scheme C and ESCA in CH-A. These results coincide with our analysis in Section 4.5.3.

In CH-B, the estimators exhibit similar behaviors as those in CH-A, however, error floors occur to all the estimators and overlap with each other at high SNR due to the Doppler frequency of the time-varying channel.

4.6.2 OFDMA Physical Layer: $L_1 = 3$

The simulation results for the OFDMA physical layer downlink channel are shown in Figure 4.3(b). The complexity of the estimators used in the simulations is listed in Table 4.2. In this case, the proposed Scheme A and C become the same. And, because the training symbol does not consist of identical segments, the ESCA cannot work for this case, only the CP based method suggested in [47] can be used as the performance benchmark.

In CH-A, the proposed estimators have more than 4dB gain over the CP based method. The Scheme B is about 0.3dB better than the Scheme A/C, at the cost of double complexity. In CH-B, the estimators exhibit similar behaviors as those
4.6 Numerical Results

Figure 4.3: Fractional CFO estimation performance.
in CH-A, however, error floors occur to all the estimators and overlap with each other at high SNR due to the Doppler frequency of the time-varying channel.

Table 4.2: Complexity of the estimators used in simulations (OFDMA).

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Number of Correlators</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP Based</td>
<td>1</td>
</tr>
<tr>
<td>Proposed Scheme A/C</td>
<td>2</td>
</tr>
<tr>
<td>Proposed Scheme B</td>
<td>4</td>
</tr>
</tbody>
</table>

4.7 Summary

In OFDM systems, accurate fractional CFO estimates can be obtained from the training symbols consisting of highly correlated segments. In this chapter, we proposed a universal fractional CFO estimator that does not require the correlated segments to be identical, and takes advantage of multiple training symbols to achieve a linear decrease in the mean square error. The flexibility of the proposed method was illustrated by three schemes with different performance and complexity trade-offs. For some training symbol structures, the proposed estimator outperforms the existing methods [17, 18] with lower complexity. Both the analytical and numerical results confirmed the performance of the proposed method in various channel conditions.
Chapter 5

Refined Timing and Frequency Estimation

5.1 Introduction

The coarse timing estimates indicate the positions of ISI-free FFT windows, however, as shown in Chapter 3, they are not able to provide the ideal timing positions which are critical to the performance of pilot-based channel estimation and tracking algorithms [62–64]. The fractional CFO correction before the FFT significantly reduces ICI on the subcarriers, however, as we explained in Chapter 1, the integer CFO ambiguity remaining in the coarse CFO estimates can lead to a circular shift in the mapping between the true and detected subcarriers, making the data of the whole OFDM packet undeterminable.

In this chapter, we introduce the refined timing and frequency estimation algorithms which are designed to solve the ambiguities in the coarse estimates. We start with a matrix representation of the training symbol and derive an approximated maximum likelihood (ML) estimator that jointly estimates the channel, timing and frequency. Based on different assumptions on the channel and transmit signal, we obtain four simplified algorithms. A hybrid integer CFO estimation scheme is proposed to provide improved performance and complexity tradeoffs. In the SNR region of interest in most practical communication systems, the hybrid scheme achieves satisfactory performance with feasible complexity for hardware real-time implementation.

In practical OFDM systems where the actual channel length is unknown to the receiver, we show that the timing ambiguity is not distinguishable by the joint integer CFO and timing estimators. To solve that problem, we propose a succes-
sive joint channel and timing estimator that achieves excellent accuracy even at low SNR. A simplified version of the algorithm is also developed to avoid the matrix inverse operation and can be implemented with a number of IFFT functions. Compared to the method in [68] which is based on GAIC and requires channel dimension search, our algorithm is more versatile to various channel conditions and requires lower complexity to implement.

5.2 Matrix Representation of the Signal

Assume the coarse timing and frequency offset estimates are available. The true residual frequency offsets and the channel taps’ delays can be written as

\[
\begin{align*}
\hat{\tau}_0 &= \hat{\tau}_f, \\
\hat{\tau}_0[l] &= \hat{\tau}[l] + \hat{\tau}_1 - \hat{\tau}_1,
\end{align*}
\]

(5.1)

(5.2)

where \(\hat{\tau}_1\) and \(\hat{\tau}_0\) are the true timing and frequency offsets respectively, and \(\hat{\tau}_1\) and \(\hat{\tau}_f\) are the coarse estimates.

In this chapter, without loss of generality, we assume the refined timing and frequency estimation is performed on the first training symbol. Write the samples in the FFT window into a vector \(\mathbf{y}\), we can show

\[
\mathbf{y} = \sqrt{N} \text{diag}(F(\hat{\epsilon}_0)) \mathbf{W}^H \text{diag}(G(-\hat{\tau}_0[1])) \text{diag}(X_1) \mathbf{V}^H \mathbf{h} + \mathbf{w},
\]

(5.3)

where \((\cdot)^H\) denotes the matrix conjugate transpose operation and

\[
\begin{align*}
\mathbf{h} &\triangleq [h[1], h[2], \ldots, h[L_h]]^T \\
\mathbf{w} &\triangleq [w[0], w[1], \ldots, w[N - 1]]^T \\
G(\hat{\tau}) &\triangleq [e^{j \frac{2\pi}{N} o_1[1]\hat{\tau}}, e^{j \frac{2\pi}{N} (o_1[1]+L_1)\hat{\tau}}, \ldots, e^{j \frac{2\pi}{N} (o_1[1]+(N_p[1]-1)L_1)\hat{\tau}}]^T \\
F(\epsilon) &\triangleq [1, e^{j \frac{2\pi}{N} \epsilon}, \ldots, e^{j \frac{2\pi}{N} (N-1)\epsilon}]^T
\end{align*}
\]

(5.4)

(5.5)

(5.6)

(5.7)

and \(\mathbf{W}\) and \(\mathbf{V}\) are truncated Discrete Fourier Transform (DFT) matrices with dimensions \(N_p[1] \times N\) and \(N_p[1] \times L_h\), respectively. The elements of \(\mathbf{W}\) and \(\mathbf{V}\) are

\[
\begin{align*}
W[m, n] &\triangleq \frac{1}{\sqrt{N}} e^{-j \frac{2\pi}{N} (o_1[1]+mL_1)n} \quad \text{and} \quad V[m, l] &\triangleq \frac{1}{\sqrt{N}} e^{-j \frac{2\pi}{N} (o_1[1]+mL_1)(\hat{\tau}_0[l]-\hat{\tau}_0[1])},
\end{align*}
\]

respectively.

Because the signal strength can be absorbed into the channel gain, we define

\[
X_1^H X_1 = N,
\]

which is equivalent to normalizing the power of time-domain OFDM
samples to $\sigma^2_w = 1$, and consider the actual transmit signal power as a part of the channel fading.

We set the range of possible residual timing offsets to $\left[-\frac{N_p[1]}{2}, \frac{N_p[1]}{2}\right]$, and denote the number of possible timing offsets as $M_t$. We assume the fractional part of the residual CFO is negligible and $\dot{\epsilon}_0$ is an integer within the width of the guard band, i.e., $\epsilon_0 \in [-N/2 + o_1[1], N/2 - (o_1[1] + (N_p[1] - 1)L_1)]$. The number of possible residual CFOs is denoted by $M_f$.

### 5.3 Approximated Maximum Likelihood Joint Estimator

We assume the variance of noise power $\sigma^2_w$ and the channel statistical information are available at the receiver. The channel statistical information gives the estimated number of channel taps $\hat{L}_h$, relative tap delays $\Delta_\tau[l]$, and the channel gain correlation matrix $C_{hh} \equiv E\{\mathbf{h}\mathbf{h}^H\}$. The estimated channel information is usually different from the true channel in practical OFDM systems.

At the receiver, based on the observed signal vector $\mathbf{y}$, one needs to estimate the variables on the right hand side of (5.3), which are the residual timing offset $\tau_0$, CFO $\epsilon_0$ and channel $\mathbf{h}$. Strictly speaking, the matrix $\mathbf{V}$ is also unknown because the channel taps' relative delays are not available at the receiver. However, with the a priori statistical channel information, we can use the estimated relative delays $\{\Delta_\tau[l]\}$ to approximate $\mathbf{V}$ with $\hat{\mathbf{V}}$ whose elements are defined as $\hat{V}[m, l] \equiv \frac{1}{\sqrt{N}} e^{-j2\pi (o_1[1]+mL_1)\Delta_\tau[l]}$. Thus, from (5.3), the probability distribution of $\mathbf{y}$ for the given channel $\mathbf{h}$, CFO $\dot{\epsilon}$, and timing offset $\dot{\tau}$ is given by

$$f_y(u|h, \dot{\epsilon}, \dot{\tau}) = \frac{1}{(\pi\sigma^2_w)^N} e^{-\frac{1}{\sigma^2_w} \|u-A(\dot{\epsilon}, \dot{\tau})h\|^2}, \quad (5.8)$$

where

$$A(\dot{\epsilon}, \dot{\tau}) \equiv \sqrt{N} \text{diag}(\mathbf{F}(\dot{\epsilon})) \mathbf{W}^H \text{diag}(\mathbf{G}(-\dot{\tau})) \text{diag}(\mathbf{X}_1) \hat{\mathbf{V}}. \quad (5.9)$$

Because (5.9) uses $\hat{\mathbf{V}}$ instead of the true truncated DFT matrix $\mathbf{V}$, the probability distribution is not exact. From (5.8), we obtain an approximated maximum likelihood joint estimator:

$$\{\hat{h}, \hat{\epsilon}, \hat{\tau}\} = \arg \min_{h, \epsilon, \tau} \|\mathbf{y} - A(\epsilon, \tau)\mathbf{h}\|^2. \quad (5.10)$$
For the given residual timing and CFO, use the method in [122], the minimum mean square error (MMSE) channel estimate is given by

\[
\hat{h}(\dot{\epsilon}, \dot{\tau}) = (A(\dot{\epsilon}, \dot{\tau})^H A(\dot{\epsilon}, \dot{\tau}) + \sigma_w^2 C_{hh}^{-1})^{-1} A(\dot{\epsilon}, \dot{\tau})^H y
\]

\[
= \frac{1}{\sqrt{N}} \left( \frac{N}{N_p[1]} \hat{V}^H \hat{V} + \frac{\sigma_w^2}{N^2} C_{hh}^{-1} \right)^{-1} \hat{V}^H \text{diag}(X_1^*) \left( G(\dot{\tau}) \odot Y(\dot{\epsilon}) \right)
\]

(5.11)

where \( \odot \) denotes vector element-wise product, and the elements of \( Y(\dot{\epsilon}) \) are

\[
Y(\dot{\epsilon})[m] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y[n] e^{j\frac{2\pi}{N} (\dot{\epsilon} + \alpha_1[1] + (m-1)L_1)n},
\]

(5.12)

and we define the least-square frequency domain channel estimate as

\[
\hat{H}(\dot{\epsilon}) \triangleq \frac{N_p[1]}{N} X_1^* \odot Y(\dot{\epsilon}).
\]

(5.13)

Substituting the channel \( h \) in (5.10) with its MMSE estimate in (5.11), we obtain a cost function only about \( \dot{\epsilon} \) and \( \dot{\tau} \) as

\[
\Lambda(\dot{\epsilon}, \dot{\tau}) \triangleq \left( G(\dot{\tau}) \odot \hat{H}(\dot{\epsilon}) \right)^H D \left( G(\dot{\tau}) \odot \hat{H}(\dot{\epsilon}) \right),
\]

(5.14)

where

\[
D \triangleq \hat{V} \left( 2I - \left( \hat{V}^H \hat{V} + \frac{N_p[1] \sigma_w^2}{N^2} C_{hh}^{-1} \right)^{-1} \hat{V}^H \hat{V} \right)
\]

\[
\times \left( \hat{V}^H \hat{V} + \frac{N_p[1] \sigma_w^2}{N^2} C_{hh}^{-1} \right)^{-1} \hat{V}^H.
\]

(5.15)

And, the joint refined CFO and timing estimator becomes

\[
\left\{ \hat{\epsilon}, \hat{\tau} \right\} = \arg \max_{\epsilon, \tau} \Lambda(\epsilon, \tau).
\]

(5.16)

Some remarks on the estimator are as follows:

- When there is no residual timing offset, the joint timing and CFO estimator reduces to the CFO estimators proposed in [9, 16]. However, the estimator of [9] only has an estimation range within ±0.5 subcarrier spacing, while we are more interested in determining the integer part of the CFO relative to
5.4 Simplified Joint Estimation Algorithms

The subcarrier spacing.

- When the residual CFO is small, Minn et al [61] proposed to solve (5.10) alternatively for timing, CFO, and channel in a recursive manner. That method does not apply to the problem studied here because the residual timing offset is an integer number of OFDM samples and residual CFO is an integer multiple of subcarrier spacing, both of which are in large discrete spaces and Minn’s recursive algorithm may encounter difficulty in converging to the right estimates.

5.4 Simplified Joint Estimation Algorithms

In this section, we simplify the approximated maximum likelihood estimator by making different assumptions on the channel and transmit signal. The simplified algorithms represent different complexity and performance tradeoffs. In the Method A we assume the residual fractional CFO is negligible and use numerical techniques to simplify the algorithm. In the Method B we further assume the noise is negligible and the channel statistical information is not available. This allows the pre-calculation of the matrix inverse and reduces the complexity of the Method A. In the Method C we further reduce the complexity by ignoring the guard bands so that the matrix inverse and multiplication are simplified to an IFFT function. In the Method D we assume the channel is as long as the number of used subcarriers in the training symbol so that the algorithm reduces to a generalized ML algorithm for integer CFO estimation [52]. Because the Method D does not make use of the channel knowledge and the cost function is independent of the timing offset, the complexity of the Method D is much lower than that of the other methods.

5.4.1 Method A

We assume the residual fractional CFO is negligible and \( \dot{\epsilon} \) is an integer. This simplifies the evaluation of (5.12) to a single FFT for all possible integer CFOs. Then, for the given \( \dot{\epsilon} \), we write the cost function in (5.14) into a polynomial of \( e^{j\frac{2\pi}{N} \dot{\epsilon}} \) as

\[
\Lambda_A(\dot{\epsilon}, \dot{\tau}) = G(\dot{\tau})^H \text{diag} \left( \hat{H}(\dot{\epsilon})^* \right) D \text{ diag} \left( \hat{H}(\dot{\epsilon}) \right) G(\dot{\tau}) \\
= 2 \sum_{m_1=1}^{N_p-1} \Re \left( s[m_1; \dot{\epsilon}] e^{-j\frac{2\pi}{N} m_1 L_1 \dot{\tau}} \right),
\]

(5.17)
where

\[ s[m_1;  \hat{\epsilon}] \triangleq \sum_{m_2=m_1+1}^{N_p[1]} D[m_2, m_2 - m_1] \hat{H}(\hat{\epsilon})^\ast[m_2] \hat{H}(\hat{\epsilon})[m_2 - m_1]. \] (5.18)

Thus, using (5.17) to replace the cost function in (5.16), we obtain a simplified algorithm for joint timing, CFO and channel estimation. In (5.17), once \( s[m_1;  \hat{\epsilon}] \) is calculated, it only takes one FFT to compute the cost function for all \( \hat{\tau} \in \left[ -\frac{N_p[1]}{2}, \frac{N_p[1]}{2} \right] \). Nevertheless, to compute \( s[m_1;  \hat{\epsilon}] \) for all possible integer CFOs, one still needs to compute the matrix \( \hat{H}(\hat{\epsilon}) \hat{H}(\hat{\epsilon})^H \) for all \( \hat{\epsilon} \), which is \( O(N^2) \) complex.

Another part of the complexity comes from the calculation of the matrix \( D \), which needs \( O(\hat{L}_h^3) \) complexity and is required every time the channel statistical information is updated. Since the update occurs at a relatively low rate, the complexity of \( D \) calculation is insignificant compared to that of the two-dimension search for \( \hat{\epsilon} \) and \( \hat{\tau} \) in a large discrete space using (5.17).

### 5.4.2 Method B

When the SNR and channel statistical information are not available at the receiver, we assume the noise is very close to zero so that (5.15) can be simplified to

\[ D_B \triangleq \lim_{\sigma_w^2 \to 0} D = \hat{V} \left( \hat{V}^H \hat{V} \right)^{-1} \hat{V}^H. \] (5.19)

Although the statistical channel knowledge \( C_{hh} \) is not required in (5.19), it needs channel taps’ relative delays \( \Delta_r[l] \) for \( \hat{V} \) calculation. In this study, we suppose the longest relative tap delay is within the CP length, and for all \( 1 \leq l \leq \hat{L}_h = N_g \), we let \( \Delta_r[l] = (l - 1) \). This is equivalent to assuming that the true channel impulse response can be represented by a \( N_g \times 1 \) vector, which is generally true. And, because the possible values of \( N_g \) are known at the receiver, one can compute \( D_B \) in advance to avoid the matrix inverse as required by the Method A. The simplified joint timing, CFO and channel estimator can then be written as

\[ \left\{ \hat{\epsilon}, \hat{\tau} \right\} = \arg \max_{\epsilon, \tau} \left( G(\tau) \odot \hat{H}(\epsilon) \right)^H \hat{V} \left( \hat{V}^H \hat{V} \right)^{-1} \hat{V}^H \left( G(\tau) \odot \hat{H}(\epsilon) \right). \] (5.20)

If there is no timing offset, the cost function of the Method B reduces to that of the maximum likelihood estimator (MLE) in [14]. However, in our case, the fractional CFO has already been corrected and the grid search is only needed for
the integer part. This simplifies the algorithm and improves the performance due to the reduced ICI.

Later in Section 5.4.7, we show the timing ambiguity resulted from the difference between the true channel length and its estimate \( \hat{L}_h \). A solution to that problem is presented in Section 5.6 using the proposed successive timing estimator.

5.4.3 Method C

In practical OFDM systems, the unused subcarriers at two ends of the spectrum usually only occupy a small proportion of the whole spectrum. For instance in IEEE 802.16 [4] (WiMAX) systems, the guard band occupies about 16% of the total bandwidth. Considering the small proportion, we neglect the guard band and approximate \( \hat{V}^H \hat{V} \approx \frac{N_p[1]}{N} I \). Using (5.19), we can simplify the cost function to

\[
\Lambda_C(\hat{\epsilon}, \hat{\tau}) \triangleq \frac{N}{N_p[1]} \left( G(\hat{\tau}) \odot \hat{H}(\hat{\epsilon}) \right)^H \hat{V}^H \hat{V} \left( \hat{H}(\hat{\epsilon}) \odot G(\hat{\tau}) \right) = \frac{1}{\sqrt{N_p[1]}} \sum_{l=1}^{\hat{L}_h} \sum_{m=1}^{N_p[1]} |\hat{H}(\hat{\epsilon})[m] e^{j2\pi(a_1[1]+(m-1)L_1)(l-1+\hat{\tau})}|^2. \quad (5.21)
\]

Using (5.21) to replace the cost function in (5.16), we obtain another simplified algorithm for joint timing, CFO and channel estimation. It only takes one IFFT to compute the cost function for all \( \hat{\tau} \) under the given \( \hat{\epsilon} \), so the overall complexity for this method is \( O(M_fN \log_2 N) \), much lower than \( O(N^2) \) as required by the Method A and B.

Actually, according to [62], for any given \( \hat{\tau} \) and \( \hat{\epsilon} \), the IFFT of \( \hat{H}(\hat{\epsilon}) \) gives the least square (LS) estimate of the channel, so the cost function \( \Lambda_C(\hat{\epsilon}, \hat{\tau}) \) can be interpreted as the energy of the truncated channel impulse response. When the CFO estimate is correct, the channel estimate resembles the real channel, whose energy is expected to concentrate in a number of taps following the true timing offset. When the timing estimate is also correct, the channel taps’ energy is summed up to yield a large value. As for incorrect CFO hypotheses, the channel estimates look like random noise with less energy in the channel truncation window. Figure 5.1 visualizes the phenomenon described above with a 10-path channel that has an exponentially decaying power delay profile.
5.4.4 Method D

The ML estimator assumes the channel to be as long as the CP. However, using $N_p$ subcarriers in the training symbol actually allows the ML estimation of a channel up to $N_p$ samples long. Hence, we can assume ($\hat{L}_h = N_p$), which is equivalent to not taking advantage of the knowledge that the multipath delay of the channel is expected to be shorter than the CP. As a result, $\hat{V}$ becomes a square matrix and $\hat{V} \left( \hat{V}^H \hat{V} \right)^{-1} \hat{V}^H = I$. This reduces the cost function to

$$\Lambda_D(\hat{\epsilon}, \hat{\tau}) \triangleq \frac{N}{N_p \Gamma}(\hat{H}(\hat{\epsilon}) \otimes \text{diag}(G(\hat{\tau})))^H I (\hat{H}(\hat{\epsilon}) \otimes \text{diag}(G(\hat{\tau})))^H = \|Y(\hat{\epsilon})\|^2. \quad (5.22)$$

Using (5.22) to replace the cost function in (5.16), we obtain another simplified algorithm for joint timing, CFO and channel estimation. Because the right hand side of (5.22) is a function only about $\hat{\epsilon}$, we can tell that the joint estimator will not be able to give any timing estimate if the channel is really that long. Similar observation was also reported for other ML based CFO estimators [6,15,56]. In the sequel, we omit the argument $\hat{\tau}$ and write the cost function as $\Lambda_D(\hat{\epsilon})$ for conciseness. It is worth noting that for the special case where the training symbol consists of two identical halves, the Method D reduces to the integer CFO estimator in [52].

As we mentioned earlier, it only takes one FFT to calculate $Y(\hat{\epsilon})$ for all $\hat{\epsilon}$, and then $2N$ real multiplications for the norm operation. Overall, the complexity is
5.4 Simplified Joint Estimation Algorithms

\[ O(N \log_2 N). \]

5.4.5 Complexity Analysis

In Table 5.1, we list the complexity of the simplified algorithms in terms of real multiplications, and every complex multiplication is counted as four real ones [14]. The average complexity for \( D \) matrix calculation is denoted by \( C_D \), which depends on the estimated channel length \( \hat{L}_h \) and the channel knowledge update rate. Assume Gaussian Elimination Algorithm [123] is used for matrix inverse, \( C_D \) is given by

\[
C_D = \frac{4}{M_D} \left( 3N_p[1]\hat{L}_h^2 + \hat{L}_h(\hat{L}_h - 1)(3\hat{L}_h + 1) + N_p[1]^2\hat{L}_h \right),
\]

(5.23)

where \( M_D \) is the number of OFDM packets between every two updates of the channel statistical information.

<table>
<thead>
<tr>
<th>Table 5.1: Complexity of the simplified algorithms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

Later, in Section 5.5.5, the complexity of different methods are plotted against their performance in Figure 5.11 for a typical IEEE 802.16 OFDM (WiMAX) system.

5.4.6 Comparison between the Method A and B

We compare the Method A and B under the assumption that they have the same information about the channel length and tap delays. This implies that both methods have the same estimated DFT matrix \( \hat{\mathbf{V}} \). At high SNR, it is clear that

\[
\lim_{\sigma_w^2 \to 0} D = \mathbf{V} (\hat{\mathbf{V}}^H \hat{\mathbf{V}})^{-1} \hat{\mathbf{V}}^H = D_B,
\]

(5.24)

so the performance of the Method A will be the same as that of the Method B.

When the SNR is low, from (5.11), (5.14) and (5.15), we have

\[
\Lambda_A(\hat{\epsilon}, \hat{\tau}) = \hat{h}(\hat{\epsilon}, \hat{\tau})^H \left( N \hat{\mathbf{V}}^H \hat{\mathbf{V}} + 2 \frac{N_p[1]\sigma_w^2}{N} C_{hh}^{-1} \right) \hat{h}(\hat{\epsilon}, \hat{\tau}).
\]

(5.25)
Since \( \left( N \hat{\mathbf{V}}^H \hat{\mathbf{V}} + 2 \frac{N_p}{N} \sigma_w^2 \mathbf{C}_{hh}^{-1} \right) \) is a positive-definite Hermitian matrix, its singular values \( \{ \lambda_1, \lambda_2, \ldots, \lambda_{L_h} \} \) are all real and positive, we can formulate (5.25) into a quadratic form as

\[
\Lambda_A(\epsilon, \hat{\tau}) = \left\| \text{diag} \left( \sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{L_h}} \right) \mathbf{U}^H \hat{\mathbf{h}}(\epsilon, \hat{\tau}) \right\|^2,
\]

(5.26)

where \( \mathbf{U} \) is a unitary matrix of rank \( \hat{L}_h \).

**Figure 5.2:** Implementation of the function \( \Lambda_A(\epsilon, \hat{\tau}) \).

Figure 5.2 shows an implementation of the function \( \Lambda_A(\epsilon, \hat{\tau}) \) according to the right hand side of (5.26). It is revealed that the likelihood of CFO and timing estimates is actually measured by the channel estimates computed from them. The channel estimates are de-correlated by the matrix \( \mathbf{U} \), then weighted by the vector \( \{ \sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{L_h}} \} \) before the energy is summed up and becomes the likelihood measure of the given pair of timing and CFO estimates.

In the case of the Method B, the singular values do not take into account the channel statistical information, so the weighting is not adaptive to channel conditions. Also, the Method B is optimized for high SNR scenarios, which results in a performance loss at low SNR.

### 5.4.7 Timing Ambiguity

In practical wireless communication systems where the exact channel length is unknown at the receiver, there can be a mismatch between the estimated channel length \( \hat{L}_h \) and the true one \( L_h \). In this case, we show the ambiguity in the timing estimates cannot be solved by the joint estimators based on the cost function given by (5.14).

Assume there is no noise and \( \hat{L}_h = L_h + 1 \). Suppose the channel estimates are
perfect except for the padded zeros, i.e., \( \hat{h}(\epsilon_0, \tau_0) = [h, 0]^T \), \( \hat{h}(\epsilon_0, \tau_0 - 1) = [0, h]^T \).

Ignore the noise term in (5.25), we have
\[
\Lambda(\epsilon_0, \tau_0) = N \left( \hat{h}(\epsilon_0, \tau_0 - 1) \right)^H \left( \hat{V}^H \hat{V} \right) \hat{h}(\epsilon_0, \tau_0 - 1)
\]
\[
= N \left\| \hat{V} \hat{h}(\epsilon_0, \tau_0 - 1) \right\|^2
\]
\[
= \sum_{m=1}^{N_p[1]} \left| \hat{h}(\epsilon_0, \tau_0 - 1)[l] e^{-j \frac{2\pi}{N} (o_1[1] + (m-1)L_1)} \right|^2
\]
\[
= \sum_{m=1}^{N_p[1]} \left| e^{-j \frac{2\pi}{N} (o_1[1] + (m-1)L_1)} \sum_{l'=1}^{\hat{L}_h - 1} \hat{h}(\epsilon_0, \tau_0)[l'] e^{-j \frac{2\pi}{N} (o_1[1] + (m-1)L_1)l'} \right|^2
\]
\[
= \Lambda(\epsilon_0, \tau_0).
\]

This means that one will not be able to tell whether the correct timing is \((\tau_0 - 1)\) or \(\tau_0\) from the cost function, the timing ambiguity is unsolvable.

We give a straightforward example of the timing ambiguity problem in Figure 5.3. Assume the Method C is used for the joint timing and CFO estimation in a single-path noiseless channel. The assumed channel length is \( \hat{L}_h = 4 \), and we suppose the channel estimate is perfect. As shown in the figure, the cost function will have a plateau near the true timing position which causes the timing ambiguity. A solution to this problem is presented in Section 5.6 using the proposed successive timing estimator.
5.4.8 Performance Analysis of the Method D

The Method D is based on the autocorrelation of the frequency domain signal. It reduces to the ML integer CFO estimator of [52] for the $L_1 = 2$ case. We analyze the performance of the Method D and shed new light on the existing autocorrelation based method [52].

Define the difference between the outputs of the cost functions for two integer CFO hypotheses that are $(m - l)L_1$ and $mL_1$ away from the true residual CFO as

$$z_m \triangleq \Lambda_D(\epsilon_0 + (m - 1)L_1) - \Lambda_D(\epsilon_0 + mL_1)$$

$$= \|Y(\epsilon_0 + (m - 1)L_1)\|^2 - \|Y(\epsilon_0 + mL_1)\|^2$$

$$= |H[m] X_1[m] + \hat{\omega}[m']|^2 - |\hat{\omega}[m' + L_1N_p[1]]|^2,$$  \hspace{1cm} (5.28)

where $H[m]$ is the true frequency domain channel for subcarrier $m$, $m' = \epsilon_0 + o_1[m]$ is the physical subcarrier index of the $m^{th}$ used subcarrier, $\hat{\omega}[m']$ is the frequency domain AWGN sample which has the same statistical properties as the time domain sample $w[n]$. If $z_1 < 0$, which means that the cost function gives a higher value for $\epsilon_0 + L_1$ than that of the true residual CFO, an integer CFO estimation error event occurs.

Denote $f_z(\cdot)$ as the probability density function (pdf) of the random variable $Z$. The pdf of $z_m$ can be written as

$$f_{z_m}(u) = \int_0^{+\infty} f_{|H[m] X_1[m] + \hat{\omega}[m']|^2}(v_1) f_{|\hat{\omega}[m' + L_1N_p[1]]|^2}(v_1 - u) dv_1$$

$$= \int_0^{+\infty} f_{|H[m] X_1[m] + \hat{\omega}[m']|^2}(v_2) \frac{f_{|\hat{\omega}[m' + L_1N_p[1]]|^2}(\sqrt{v_2^2 - u})}{2\sqrt{v_2^2 - u}} dv_2,$$  \hspace{1cm} (5.29)

where the second equality follows the fact that $f_{|Z|^2}(|u|^2) = f_{Z}(|u|)/(|2u|)$. We know $|\hat{\omega}[m' + L_1N_p[1]]|$ follows Rayleigh distribution whose pdf is

$$f_{|\hat{\omega}[m' + L_1N_p[1]]|^2}(v) = f_{(ra)}(v|0.5 \sigma_w^2),$$  \hspace{1cm} (5.30)

where $f_{(ra)}(|\cdot|\sigma^2)$ denotes the pdf of Rayleigh distribution which can be written as

$$f_{(ra)}(v|\sigma^2) \triangleq \frac{v}{\sigma^2} e^{-\frac{v^2}{2\sigma^2}}.$$  \hspace{1cm} (5.31)

And, for a given $H[m]$, $|H[m] X_1[m] + \hat{\omega}[m']|$ is the magnitude of a Gaussian random
variable with mean $|H[m]|\sqrt{N\sigma_1^2/N_p[1]}$, so it follows Rice distribution:

$$f_{H[m]X_1[m]+\hat{w}[m']}(v) = f_{(ri)} \left( v | H[m] | \sqrt{N\sigma_1^2/N_p[1]}, 0.5\sigma_w^2 \right),$$  \hspace{1cm} (5.32)

where $f_{(ri)}(\cdot|u, \sigma^2)$ represents the pdf of Rice distribution that can be written as

$$f_{(ri)}(v|u, \sigma^2) \triangleq \frac{v}{\sigma^2} e^{-\frac{v^2+2u^2}{2\sigma^2}} I_0\left(\frac{vu}{\sigma^2}\right).$$  \hspace{1cm} (5.33)

In (5.33), $I_0(\cdot)$ represents the modified Bessel function of the first kind with order zero.

The channel gain $|H[m]|$ is a random variable whose pdf depends on the channel condition. In AWGN and single-path fading channels, the pdf of $|H[m]|$ is given by

$$f_{|H[m]|}(a) = \begin{cases} 
\delta(a-1), & \text{AWGN;} \\
(f_{(ri)}(a|0.5), & \text{Fading.} 
\end{cases}$$  \hspace{1cm} (5.34)

Using (5.34), we can marginalize the channel gain in (5.32) and obtain the pdf of $|\hat{H}(e_0)[m]X_1[m]+\hat{w}[m']|$ as

$$f_{H[m]X_1[m]+\hat{w}[m']}(v) = \int_{0}^{+\infty} f_{|H[m]|}(a) f_{(ri)}(v|a^2N\sigma_1^2/N_p[1], 0.5\sigma_w^2) da.$$  \hspace{1cm} (5.35)

Combining (5.29), (5.35) and (5.34), we can compute the pdf of $z_m$ as

$$f_{z_m}(u) = \int_{0}^{+\infty} \int_{0}^{+\infty} f_{|H[m]|}(a) f_{(ri)}(v|a^2N\sigma_1^2/N_p[1], 0.5\sigma_w^2) da \\ \times f_{(ri)}(\sqrt{v^2-u|0.5\sigma_w^2}) \frac{2\sqrt{v^2-u}}{2\sqrt{v^2-u}} dv.$$

A lower bound on the error probability of the Method D can be obtained by only considering the events of $z_1 < 0$ and $z_{-1} < 0$. Because these two events have the same probability to happen, we can calculate the lower bound as

$$P_{e}^{(D)} \geq 2 \int_{-\infty}^{0} f_{z_1}(u) du.$$  \hspace{1cm} (5.37)

Using (5.37), we numerically evaluate the error probability for the Method D in AWGN and single-path rayleigh fading channels respectively, and plot the results in Figure 5.4. The coarse timing estimate is modeled by an integer number of samples uniformly distributed in the range of $[-N_g+1, 0]$ samples from the time when the
first sample of the OFDM symbol following the CP is received. The true CFO is modeled by a multiple of the subcarrier spacing uniformly distributed in the range of \([-20, 20]\) subcarrier spacing. Every point in the figure is an average of at least \(10^5\) independent experiments. We can see that the analysis matches the simulation results very well. The figure indicates a significant performance degradation of the Method D in the fading channel.

5.4.9 Numerical Results

A typical IEEE 802.16 system with 512 subcarriers and 5.6MHz bandwidth is modeled in the simulations to evaluate the performance of the proposed algorithms. There are 42 null subcarriers on each side of the spectrum, so the true CFO is modeled by a uniformly distributed random variable in \([-42, 42]\) subcarrier spacing. The CP is \(1/8\) of one symbol duration, and the coarse timing is provided by the peak of the timing metric proposed in Chapter 3 within the timing window of \([-N_g, N_g]\) around the ideal timing position. Nevertheless, the search window for residual timing offset is set to \(\left[\frac{-N_p[1]}{2}, \frac{N_p[1]}{2}\right]\), the maximum distinguishable range for residual timing offsets. The fractional CFO estimate is provided by the universal estimator introduced in Chapter 4.
The channel models used in the simulations are described in Chapter 2. CH-A is a stationary wireless channel model with deep and long fading, while CH-B is a mobile wireless channel model with up to 388.9Hz Doppler frequency.

For the Method A, we assume the perfect SNR is available to the receiver, but the statistical channel knowledge is imperfect. We plot the receiver’s statistical channel power delay profiles in Figure 5.5 to compare with the true mean power delay profiles of the channels. For the Method B and C, the channel length is assumed to be $\hat{L}_h = N_g$ and the tap delays are consecutive integers from 0 to $N_g - 1$.

![Figure 5.5: Imperfect channel knowledge used by the Method A.](image)

The simulation results are plotted in Figure 5.6. Every point in the figure is an average of at least 100 estimation errors or 1000 independent experiments. As predicted by the performance analysis, the Method D does not work very well in the multipath fading channels. At error rate $10^{-2}$, a performance gap as large as 20dB is observed between the Method D and the other methods. The Method A takes advantage of the channel knowledge, which gives 2dB gain over the Method B. The complexity of the Method B is much higher than that of the Method C, however, it does not give any performance gain in the low SNR region. This can be explained by the noise enhancement effect of the matrix inverse operation in the Method B.
As the SNR increases, the Method B eventually outperforms the Method C and approaches the performance of the Method A.

The steepness of the curves of the Method A, B and C indicates the performance of those estimators are highly sensitive to SNR. As we show in Chapter 2, CH-A is more likely to see a long and deep fading, which leads to the higher error rates than those in CH-B. This result also indicates the estimators’ robustness to the channel’s time-selectivity.

Later, we discuss the cost-effectiveness of the methods in Section 5.5.5. More specifically, we compare the complexity of the methods and the required SNRs to achieve the given performance.

### 5.5 A Hybrid CFO Estimation Scheme

The simulation results presented in Section 5.4.9 show that the simplified joint estimation algorithms have either very high complexity or poor performance in multipath channels. In this section, we propose a hybrid scheme to provide flexible performance and complexity tradeoffs and bridge the performance gap between the Method D and the other methods.
5.5 A Hybrid CFO Estimation Scheme

5.5.1 The Method

The hybrid integer CFO estimation scheme consists of two steps, namely **Focus** and **Zoom**. In the Focus step, we compute a set of $\tilde{M}_f$ integer CFO hypotheses $\mathcal{F}$ such that for all $\tilde{\epsilon}_1 \in \mathcal{F}$ and $\tilde{\epsilon}_2 \notin \mathcal{F}$, $\Lambda_D(\tilde{\epsilon}_1) \geq \Lambda_D(\tilde{\epsilon}_2)$. In other words, the set $\mathcal{F}$ contains $\tilde{M}_f$ most likely integer CFO hypotheses according to the cost function of the Method D.

Then, in the Zoom step, we use the cost function of the Method C to determine the most likely integer CFO hypotheses from the set $\mathcal{F}$. More precisely, the integer CFO estimate is given by

$$\hat{\epsilon} = \arg\max_{\epsilon \in \mathcal{F}} \max_{\tau} \Lambda_C(\epsilon, \tau).$$

(5.38)

The complexity of the hybrid method is proportional to the design parameter $\tilde{M}_f$ rather than the number of integers in the whole estimation range $M_f$.

5.5.2 Complexity Analysis

The complexity of the hybrid method is between that of the Method C and D depending on the $\tilde{M}_f$ setting. If $\tilde{M}_f = 1$, only one hypothesis is in the set $\mathcal{F}$, the Zoom step can be skipped and the algorithm reduces to the Method D. Another extreme case is to set $\tilde{M}_f = M_f$, all possible integer CFOs are passed to the Zoom step, the hybrid method becomes the same as the Method C.

For all $\tilde{M}_f \in [2, M_f]$, the complexity of the hybrid method in terms of real multiplications is given by

$$4N \log_2 N + 2N + \tilde{M}_f(4N \log_2 N + 2N_p[1]).$$

(5.39)

Therefore, the complexity of the hybrid method is at the same order of magnitude as that of the Method D when $\tilde{M}_f$ is small. Nevertheless, as shown by both our analytical and simulation results presented in the following sections, the hybrid method with $\tilde{M}_f = 4$ can improve the performance of the Method D by up to 7dB in Rayleigh fading channels.

5.5.3 Implementation

The proposed hybrid algorithm can be efficiently implemented by digital logics using the architecture shown in Figure 5.7. In the Focus step, (5.22) is implemented...
with a series of summation and norm operations; while in the Zoom step, (5.21) is realized by IFFT, norm, and summation operations. For both steps, the building blocks are the circuits of FFT/IFFT, norm operation, and the integrate-and-dump algorithm. Using a time-multiplexing scheme, only one instance of those modules is needed to implement the whole hybrid joint timing and integer CFO estimator, which is quite efficient. When a short processing delay is more desirable, the Zoom step can be performed in parallel using multiple instances of the core modules. The tradeoff between the speed and circuit area is flexible and straightforward.

Figure 5.7: Implementation of the hybrid algorithm.

5.5.4 Performance Analysis

We analyze the performance of the proposed hybrid estimator. When an estimation error occurs, it can be either in the Focus step that \( \epsilon_0 \notin \mathcal{F} \), or in the Zoom step that there exists \( \epsilon_w \in \mathcal{F} \) and \( \Lambda_C(\epsilon_w) > \Lambda_C(\epsilon_0) \). Because the latter is much less likely to happen, we assume the Zoom step is ideal and concentrate on the error probability of the Focus step. We also assume that for all \( \epsilon \in \mathcal{F} \), there exists \( m \in [-\tilde{M}_f, \tilde{M}_f] \) such that \( \epsilon = \epsilon_0 + m L_1 \), and for all \( k \) between 0 and \( m \), \( \Lambda_D(\epsilon_0 + k L_1) > \Lambda_D(\epsilon_0) \). This means that only the hypotheses around the true integer CFO can be included in the set \( \mathcal{F} \), and the distance between the hypotheses and the true integer CFO must be a multiple of \( L_1 \). In Figure 5.8, we illustrate the considered error patterns as well as those not taken into account for the \( (\tilde{M}_f = 3) \) and \( (L_1 = 1) \) case. The above two assumptions allow us to lower bound the error probability of the hybrid method by the most likely estimation error patterns.

Define the difference between the correlation outputs of the true integer CFO and the hypothesis that is \( mL_1 \) subcarrier spacing away from the true integer CFO.
Figure 5.8: Most likely error patterns considered for the ($\tilde{M}_f = 3$) and ($L_1 = 1$) case.

as

$$Z_m \triangleq \Lambda_D(\epsilon_0) - \Lambda_D(\epsilon_0 + mL_1) = \sum_{k=1}^{m} z_k. \quad (5.40)$$

As shown in (5.29), the pdf of $z_k$ depends on the channel condition. We consider two kinds of channels, AWGN and single path Rayleigh fading. In both cases, the pdf of $z_k$ under the given channel gain $a$ can be written as

$$f_{z_k}(u|a) = \int_{0}^{+\infty} f_{(ri)}(v|\sqrt{a^2\sigma_t^2}, 0.5 \sigma_w^2) f_{(ra)}(\sqrt{v^2 - u}|0.5 \sigma_w^2) \frac{\sqrt{v^2 - u}}{2\sqrt{v^2 - u}} \, dv. \quad (5.41)$$

where $a$ equals to 1 for the AWGN channel, and follows Rayleigh distribution with unit variance in the fading channel.

When $m = 1$, $Z_1 = z_1$, the pdf of $Z_1$ has been given by (5.41). For $m > 1$, under the condition that for all $k \in [1, m-1]$, $Z_k < 0$, the pdf of $Z_m$ can be recursively evaluated by

$$f_{Z_m}(u|a) = \int_{-\infty}^{0} f_{Z_{m-1}}(v|a) f_{z_m}(u - v|a) \, dv. \quad (5.42)$$

With the pdf of $Z_m$, we can further compute the probability for certain error pattern to occur. For instance, the probability for $m$ consecutive integer CFO hypotheses to have correlation outputs higher than that of the true integer CFO is given by

$$p_Z(m|a) \triangleq \int_{-\infty}^{0} f_{Z_m}(v|a) \, dv \quad (5.43)$$

and $p_Z(0|a) = 1$. For a given number of elements in the set $\mathcal{F}$, there can be $m \in [0, \tilde{M}_f]$ consecutive integer CFO hypotheses having correlation outputs higher
than that of the true integer CFO on one side, and \((\tilde{M}_f - m)\) such hypotheses on the other side, so the error probability for a given channel gain \(a\) can be lower bounded by

\[
P_e^{(H)}(\tilde{M}_f | a) \geq \sum_{m=0}^{M_f} p_Z(m | a) p_Z(\tilde{M}_f - m | a).
\]

The impact of the channel gain \(a\) can be marginalized under the given probability distribution as

\[
P_e^{(H)}(\tilde{M}_f) = \int_{0}^{+\infty} f_{|H(\cdot)|}(a) P_e^{(H)}(\tilde{M}_f | a) da \\
\geq \int_{0}^{+\infty} f_{|H(\cdot)|}(a) \sum_{m=0}^{M_f} p_Z(m | a) p_Z(\tilde{M}_f - m | a) da,
\]

where the pdf of the channel magnitude \(f_{|H(\cdot)|}(a)\) is given by (5.34).

![Figure 5.9: Performance analysis of the hybrid method.](image)

We numerically compute (5.45) for a 512-subcarrier OFDM system and compare it to the simulation results in Figure 5.9. The coarse timing estimate is modeled by an integer number of samples uniformly distributed in the range of \([-N_g + 1, 0]\) samples from the ideal timing position, where \(N_g = 64\) in this case. The true CFO is modeled by a multiple of the subcarrier spacing uniformly distributed in
the range of \([-20, 20]\) subcarrier spacing. Every point in the figure is an average of at least \(10^5\) independent experiments. It shows that the analysis agrees with the simulations reasonably well. For large \(\tilde{M}_f\) values, a larger number of error patterns are neglected by the lower bounds, so the gaps between the analytical and simulated curves are slightly widened. The \(\tilde{M}_f = 1\) curve gives the performance of the original Method D, which is improved by the hybrid method even with a small \(\tilde{M}_f\). Larger performance improvement by the hybrid method is observed in the fading channel rather than the AWGN. For instance, at error rate \(10^{-2}\) and with \(\tilde{M}_f = 4\), the hybrid method has 5dB gain over the Method D in AWGN, and 7dB in the fading channel.

### 5.5.5 Numerical Results

We simulate the proposed hybrid integer CFO estimator in an IEEE 802.16 system as described in Section 5.4.9. The system parameters and channel models used in the simulations are exactly the same for easy comparison. We also compare the performance of the proposed hybrid scheme to an upper bound where the Zoom step is assumed to be ideal such that no estimation error occurs as long as the correct integer CFO estimate is in the output of the Focus step \(\mathcal{F}\).

![Figure 5.10: Performance of the hybrid integer CFO estimators.](image-url)
It is shown in Figure 5.10 that at error rate $10^{-2}$, the hybrid method with $\tilde{M}_f = 4$ achieves 7 dB gain over the Method D in the multipath fading channels. This coincides with our prediction in Section 5.5.4. For the same reason as we explained in Section 5.4.9, the hybrid method has slightly better performance in CH-B than that in CH-A, which indicates its robustness to mobile wireless channel’s time selectivity. The figure also shows that the performance of the hybrid method is very close to that of the Ideal-Zoom bound, which means that it is the Focus step rather than the Zoom step that limits the hybrid method’s overall performance. Actually, because the error probability of the Method C is negligible compared to that of the Method D at high SNR, the Zoom step is already very close to the ideal case, there is no need for more complex algorithms like the Method A or B in the Zoom step.

![Figure 5.11: Complexity vs performance of joint CFO and timing estimators.](image)

To compare the cost-effectiveness of the joint CFO and timing estimators, we plot their complexity versus the performance in Figure 5.11. The complexity is measured by the number of real multiplications, which is listed in Table 5.1 for the simplified joint estimators and (5.39) for the hybrid method respectively. The performance is measured by the required SNR to achieve the target error probability $10^{-2}$ in CH-B, which can be obtained from Figure 5.6 for the joint estimators and Figure 5.10 for the hybrid method respectively. Since the estimators have quite
similar behaviors in CH-A, the curves will not change much in that channel.

We can see that the Method A, B and C are all at the top-left corner of the figure, which represents very high complexity and excellent performance; whereas the Method D is at the bottom-right corner meaning low complexity and poor performance. Both of them are extreme choices in favor of either the performance or complexity. The hybrid methods are in the middle of the figure bridging the gap between them. For the SNR range between 0 and 15dB, which is the operating region of most practical communication systems, the hybrid methods achieve satisfactory performance with relatively low complexity.

5.6 Successive Joint Channel and Timing Estimation

As explained in Section 5.4.7, when the exact number of channel taps is unknown to the receiver, the approximated ML joint frequency and timing estimator and its simplified versions cannot solve the timing ambiguity. In this section, we propose a successive joint channel and timing estimator to solve that problem. The new estimator successively detects the valid channel paths, then remove their components in the observed signal until no more valid channel path is found or the power of the residual signal is below a preset threshold. This way, the channel delay profile can be accurately estimated and the delay of the first channel tap gives the residual timing offset.

5.6.1 Overview

A diagram of the proposed successive joint channel and timing estimator is shown in Figure 5.12. We assume the integer CFO estimate $\hat{\epsilon}$ is available, and the input to the joint channel and timing estimator is the least-square frequency domain channel estimate $\hat{H}(\hat{\epsilon})$. In every iteration, the Path Detector provides the delay of the most likely undetected channel tap, which, together with the delays of other taps detected in previous iterations, is utilized by the Channel Re-estimator for an updated channel estimate. The difference between the estimated channel and the original least-square frequency domain channel estimate is given to the Path Detector again for the search of undetected path in the next iteration. This iterative process repeats until one of the following conditions is satisfied:

- The re-estimated channel $\tilde{H}_i$ is very close to the original channel estimate
Refined Timing and Frequency Estimation

The minimum delay of detected paths

Figure 5.12: Diagram of the successive joint channel and timing estimator.

\[ \hat{H}(\epsilon) : \| \hat{H}(\epsilon) - \tilde{H}_i \|^2 < \mathcal{E}_t, \] where \( \mathcal{E}_t \) is a pre-defined threshold.

- The likelihood measure of the newly detected channel path does not reach the pre-defined threshold \( \lambda_t \).

- The maximum number of iterations \( \mathcal{I}_{max} \) is reached.

When the iteration finishes, the minimum delay of all detected paths is determined to be the residual timing offset. We omit the path validity check at the first iteration because there must be at least one channel path in the input signal. Later in Section 5.6.6 we provide the guidelines for threshold settings based on the performance analysis.

In the following subsections, the details about the Path Detector and Channel Re-estimator are given, then we shed new light on the proposed successive algorithm by interpreting it as a series of vector operations. To avoid the matrix inverse operation, we propose a simplified successive estimator by replacing the ML channel estimator with a least square (LS) one. Performance analysis shows that the proposed methods work very well even in deep fading channels.
5.6.2 Path Detector

Define $\tilde{H}_i$ as the re-estimated channel at iteration $i$, and at beginning, $\tilde{H}_0 = 0$. The input signal to the Path Detector can be expressed by

$$\tilde{H}_i \triangleq \hat{H}(\dot{\epsilon}) - \tilde{H}_{i-1}, \quad i \geq 1. \quad (5.46)$$

Without knowing the exact number of channel taps and their relative delays, we assume there is only one channel path and reduce $\dot{\epsilon}$ to a column vector with all one elements scaled by a constant $\frac{1}{\sqrt{N}}$. From (5.11), we can write the channel estimate for the given timing offset $\tau$ at iteration $i$ as

$$\hat{h}_i(\tau) = \frac{\sqrt{N}}{N_p[1]} \tilde{V}^H(\tilde{H}_i \odot G(\tau)) = \frac{1}{N_p[1]} \sum_{m=1}^{N_p[1]} \tilde{H}_i[m] e^{j \frac{2\pi}{N} (o_1[1] + (m-1)L_1) \tau}, \quad (5.47)$$

where $\tilde{H}_i[m]$ is the $m$th element of the vector $\tilde{H}_i$. It only takes one IFFT to evaluate (5.47) for all $\tau$.

We measure the likelihood of the existence of the channel tap at the given delay $\tau$ by the normalized magnitude of the channel estimate defined as

$$\dot{r}_i(\tau) \triangleq \frac{N_p[1] |\hat{h}_i(\tau)|}{\sqrt{\sum_{m=1}^{N_p[1]} |\tilde{H}_i[m]|^2}}. \quad (5.48)$$

Since a valid channel path is expected to have higher magnitude than that of invalid paths, the delay of the most likely valid channel path is

$$\hat{\tau}_i \triangleq \arg \max_{\tau \in [-N_g, N_g]} \dot{r}_i(\tau). \quad (5.49)$$

If $\dot{r}_i(\hat{\tau}_i)$ is above the threshold $\lambda_t$, which means $\hat{\tau}_i$ is quite likely to be the delay of a valid channel tap, we append $\hat{\tau}_i$ to the estimated channel delay profile, and update the channel estimate using the algorithm explained in the next subsection.

5.6.3 Channel Re-estimator

At iteration $i$, denote the delays of the estimated channel taps as $\{\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\tau}_i\}$. According to [122], the ML frequency domain channel estimate is given by

$$\tilde{H}_i = \tilde{V}_i(\tilde{V}_i^H \tilde{V}_i)^{-1} \tilde{V}_i^H \hat{H}(\dot{\epsilon}), \quad (5.50)$$
where $\tilde{V}_i$ expands the matrix $\tilde{V}_{i-1}$ with vector $\sqrt{\frac{1}{N}} G(-\hat{\tau}_i)$ as $\tilde{V}_i = (\tilde{V}_{i-1}, \sqrt{\frac{1}{N}} G(-\hat{\tau}_i))$, and $\tilde{V}_1 = \sqrt{\frac{1}{N}} G(-\hat{\tau}_1)$.

We know direct implementation of $\tilde{V}_i (\tilde{V}_i^H \tilde{V}_i)^{-1} \tilde{V}_i^H$ requires $\mathcal{O}(N_p[1]^3)$ complexity, which is very high. To reduce the complexity, we provide a recursive matrix update algorithm based on the following theorem:

**Theorem 5.1.** Define $B_i = (B_{i-1}, b_i)$, and $D_i = B_i \left( B_i^H B_i \right)^{-1} B_i^H$. Assume $D_i$ and $b_{i+1}$ are known, then $D_{i+1}$ can be obtained recursively as

$$D_{i+1} = D_i + \frac{(D_i b_{i+1} - b_{i+1})(D_i b_{i+1} - b_{i+1})^H}{b_{i+1} b_{i+1} - b_{i+1}^H D_i b_{i+1}}. \quad (5.51)$$

**Proof.** Decompose the matrices about $B_{i+1}$ as

$$B_{i+1}^H B_{i+1} = \begin{pmatrix} B_i^H B_i & B_{i+1}^H b_{i+1} \\ b_{i+1}^H B_i & b_{i+1}^H b_{i+1} \end{pmatrix}, \quad \left( B_{i+1}^H B_{i+1} \right)^{-1} = \begin{pmatrix} Q_i & q_i \\ q_i^H & z_i \end{pmatrix}. \quad (5.52)$$

Solving the equation that $(B_{i+1}^H B_{i+1}) (B_{i+1}^H B_{i+1})^{-1} = I$, we obtain

$$z_i = \frac{1}{b_{i+1}^H b_{i+1} - b_{i+1}^H B_i (B_i^H B_i)^{-1} B_i b_{i+1}}, \quad (5.53)$$

$$q_i = -\frac{(B_i^H B_i)^{-1} b_{i+1}}{b_{i+1}^H b_{i+1} - b_{i+1}^H B_i (B_i^H B_i)^{-1} B_i b_{i+1}}, \quad (5.54)$$

$$Q_i = (B_i^H B_i)^{-1} \left( I + \frac{B_i^H b_{i+1} B_{i+1}^H B_i (B_i^H B_i)^{-1}}{b_{i+1}^H b_{i+1} - b_{i+1}^H B_i (B_i^H B_i)^{-1} B_i b_{i+1}} \right). \quad (5.55)$$

These equations establish (5.51) according to the definition of $D_i$. \qed

When $i = 1$, $\tilde{V}_1$ is a vector, so the calculation of $\tilde{V}_1 (\tilde{V}_1^H \tilde{V}_1)^{-1} \tilde{V}_1$ is trivial. Afterwards, one can use Theorem 5.1 to update the matrices recursively at $\mathcal{O}(N_p[1]^2)$ complexity, which is much lower than that of the direct implementation.

### 5.6.4 An Interpretation

The proposed successive joint channel and timing estimation algorithm can be interpreted as a series of vector operations. For all possible residual timing offsets $\tau \in \left[ -\frac{N_p[1]}{2}, \frac{N_p[1]}{2} \right]$, the vectors $\{G(\tau)\}$ are pseudo-orthogonal. Neglect the noise, the frequency domain channel $\tilde{H}(\hat{\epsilon})$ should be a vector in the space spanned by the vectors $\{G(\tau)\}$. To find the minimum set of elements in $\{G(\tau)\}$ to approximate $\tilde{H}(\hat{\epsilon})$, following vector operations are performed:
1. Correlate $\tilde{H}_i = \hat{H}(\varepsilon) - \tilde{H}_{i-1}$ with each of the pseudo-orthogonal vectors $G(\tau)$ using (5.47) and (5.48).

2. Find the $G(\hat{\tau}_i)$ with maximum component of $\tilde{H}_i$ using (5.49).

3. If the component is above the threshold, continue; otherwise, stop.

4. Project $\hat{H}(\varepsilon)$ onto the null space of all detected vectors $\{G(\hat{\tau}_k)\}$, $1 \leq k \leq i$, using (5.46) and (5.50), which are equivalent to

$$\tilde{H}_{i+1} = \hat{H}(\varepsilon) - \tilde{H}_i = \left(I - \tilde{\mathbf{V}}_i \left(\tilde{\mathbf{V}}_i^H \tilde{\mathbf{V}}_i\right)^{-1} \tilde{\mathbf{V}}_i^H\right) \hat{H}(\varepsilon).$$

(5.56)

Because $G(\tau)$ is a $N_p[1] \times 1$ vector, (5.56) reveals the fact that the maximum number of distinguishable channel taps is equal to $N_p[1]$, the number of used subcarriers in the preamble symbol. Therefore, it is safe to set the maximum number of iterations $I_{\text{max}}$ to $N_p[1]$ without any performance loss.

### 5.6.5 The Simplified Algorithm

The joint timing and channel estimator proposed above has two major blocks that perform path detection and channel estimation tasks respectively. In the following, we introduce a method to simplify the channel estimator so that the overall complexity of the algorithm can be significantly reduced.

In practical OFDM systems, the unused subcarriers at two ends of the spectrum usually only occupy a small proportion of the total spectrum. For instance in IEEE 802.16 [4] (WiMAX) systems, the guard band occupies about 16% of the total bandwidth. Considering the small proportion, we neglect the guard band and approximate $\tilde{\mathbf{V}}_i^H \tilde{\mathbf{V}}_i = \frac{N[I]}{N} \mathbf{I}$. This greatly simplifies the Channel Re-estimator described in Section 5.6.3 because the channel update algorithm in each iteration becomes

$$\tilde{H}_{i+1} = \hat{H}(\varepsilon) - \tilde{H}_i = \hat{H}(\varepsilon) - \frac{N}{N_p[1]} \tilde{\mathbf{V}}_i \tilde{\mathbf{V}}_i^H \hat{H}(\varepsilon) = \hat{H}(\varepsilon) - \frac{N}{N_p[1]} \left( \tilde{\mathbf{V}}_{i-1} \tilde{\mathbf{V}}_{i-1}^H + \frac{1}{N} G(-\hat{\tau}_i)G(-\hat{\tau}_i)^H \right) \hat{H}(\varepsilon) = \hat{H}_i - G(-\hat{\tau}_i) \hat{h}_i(\hat{\tau}_i).$$

(5.57)

Since $\hat{h}_i(\hat{\tau}_i)$ is an intermediate variable already computed by the Path Detector in
each iteration, the channel re-estimation can be implemented at $O(N_p[1])$ complexity rather than $O(N_p[1]^2)$ as required by the original algorithm.

5.6.6 Performance Analysis

We define the correct timing offset as the residual delay of the first channel tap $\hat{\tau}_0[1]$. Ideally, in every iteration, the successive timing estimator will detect one valid channel path and remove its component in the observed signal for next iteration. The most likely error event occurs when the first channel tap is in deep fading and has no chance to be detected until all the other valid channel paths are detected. At iteration $L_h$, we compute the correct detection probability for the first channel tap assuming the other valid channel taps have already been detected and their components are perfectly removed in the input to the Path Detector. This genie-aided approach gives the performance upper bound for the scenario when the first channel tap is in deep fading.

For the given channel gain $|h[1]|^2$, at the correct timing position $\hat{\tau}_0[1]$, the term in the square root of the denominator of $\hat{r}_i(\hat{\tau}_0[1])$ is the power of the residual component of the channel, and its variation is only caused by the noise. When $N_p[1]$ is large, the mean of the term in the square root of the denominator is much larger than its standard deviation. We approximate that term in the denominator of $\hat{r}_i(\hat{\tau}_0[1])$ with its mean,

$$
\hat{r}_i(\hat{\tau}_0[1]) \approx \frac{N_p[1] |\hat{h}_i(\tau)|}{\sqrt{E \left\{ \sum_{m=1}^{N_p[1]} |\hat{H}_i[m]|^2 \right\}}} = \frac{|\hat{h}_i(\tau)|}{\sqrt{\frac{N}{N_p[1]} |h[1]|^2 + \frac{\sigma_w^2}{\sigma_i^2}}}.
$$

The numerator of (5.58) is the magnitude of a Gaussian random variable with non-zero mean, so we can approximate $\hat{r}_i(\hat{\tau}_0[1])$ by a random variable following the pdf of Rice distribution:

$$
f_{\hat{r}_i(\hat{\tau}_0[1])}(u||h[1]|^2) \approx f_{(ri)} \left( u \left| \sqrt{\frac{N}{N_p[1]} \frac{\sigma_i^2|h[1]|^2 + \sigma_w^2}{\sigma_i^2}} \right| \right)
\approx f_{(ri)} \left( u \left| \sqrt{\frac{1 + N_p[1]}{1 + \zeta}} \right| \frac{\sqrt{\zeta}}{\sqrt{2(1 + \zeta)}} \right),
$$

where $\zeta \triangleq N|h[1]|^2\sigma_i^2/(N_p[1]\sigma_w^2)$ represents the average signal to noise ratio on each subcarriers.
At an incorrect timing position \( \bar{\tau} \) where \( \bar{\tau} \neq \tau_0^{[1]} \), the numerator of \( r_i(\bar{\tau}) \) is the magnitude of the summation of a large number of Gaussian random variables, so it approximately follows Rayleigh distribution. We use the same approximation in (5.58) to the term in the denominator of \( r_i(\bar{\tau}) \), then the pdf of \( r_i(\bar{\tau}) \) approximately follows that of the Rayleigh distribution:

\[
f_{r_i(\bar{\tau})}(u) \approx f_{(ra)}(u|0.5).
\] (5.60)

The correct detection of the first channel tap requires \( r_i(\tau_0^{[1]}) \) to be higher than all the \( r_i(\bar{\tau}) \), the probability of which is given by

\[
P_{c}^{(\tau_0^{[1]})} \approx \int_{0}^{\infty} f_{\sqrt{\zeta}(a)} \int_{0}^{\infty} f_{(ri)} \left( u \left| \frac{1 + N_p^{[1]} a^2}{1 + a^2}, \frac{a}{\sqrt{2(1 + a^2)}} \right. \right) \times \left( \int_{0}^{u} f_{(ra)}(v|0.5)dv \right)^{M_t - 1} du \, da,
\] (5.61)

where \( M_t \) is the number of possible timing offsets, and the distribution of \( \sqrt{\zeta} \) depends on the channel condition and is given by

\[
f_{\sqrt{\zeta}(a)} = \begin{cases} 
\delta \left( a - \sqrt{\frac{N_p^{[1]} \sigma_w^2}{N_p^{[1]} \sigma_v^2}} \right), & \text{Fixed gain;} \\
\frac{N_p^{[1]} \sigma_w^2}{2N_p^{[1]} \sigma_v^2}, & \text{Rayleigh fading.}
\end{cases}
\] (5.62)

We numerically compute (5.61) for a 512-subcarrier OFDM system and compare that with the simulation results in Figure 5.13. The channels used in the simulations have only one tap, whose gain either is fixed or follows Rayleigh distribution. Every simulated point in the figure is an average of at least 10,000 independent experiments. It shows that the analysis matches the simulations reasonably well. When the tap’s gain is fixed, the curve exhibits a threshold around -15dB, and few detection errors occur above that SNR level. This means that for a signal 10dB above the noise floor, the timing is quite likely to be correctly estimated even if the first channel tap is in a fading as deep as -20dB. When the first channel tap follows Rayleigh fading, we can see that the curve becomes flatter and more estimation errors occur at low SNR. However, at 0dB and above, more than 97% OFDM bursts are expected to have ideal timing estimates.

Next, we give design guidelines for the threshold settings based on the analysis on their impacts on the estimator’s performance.

When deriving the joint channel and timing estimator, we implicitly assume
$\hat{H}(\epsilon_0)$ is the true frequency-domain channel in the space spanned by $\{G(\tau)\}$. However, due to the noise and other channel impairments, that assumption may not hold, especially when the signal strength is low. To avoid overfitting the assumed signal model, a threshold $\epsilon_t$ is designed to terminate the iterative process when the estimate is close enough to the observation. We propose to set the threshold according to noise variance as

$$\epsilon_t = \frac{1}{2} N_p[1] \sigma_w^2,$$

where $\sigma_w^2$ can be estimated from the variance of the observed signal on the unused subcarriers of a training symbol.

Below we study the impact of the other threshold $\lambda_t$ on the performance of the proposed timing estimator. For all $\bar{\tau}$ not being the delay of a valid channel tap, we assume that $\{\hat{r}_i(\bar{\tau})\}$ can be approximated by independent identically distributed Rayleigh random variables. In each iteration, the probability for the maximum of those random variables to reach the threshold $\lambda_t$ can be evaluated as

$$P \left( \max_{\bar{\tau}} \hat{r}_i(\bar{\tau}) > \lambda_t \right) \approx 1 - \left( \int_{0}^{\lambda_t} f_{r_i}(u|0.5) \, du \right)^{M_t-L_h} \approx M_t e^{-\lambda_t^2},$$

where $\sigma_w^2$ can be estimated from the variance of the observed signal on the unused subcarriers of a training symbol.
5.6 Successive Joint Channel and Timing Estimation

where the first approximation is due to the Rayleigh approximation of \( r_i(\tau) \), and the second one follows the fact that \( M_t \) is much larger than \( L_h \), and the threshold \( \lambda_t \) needs to be relatively large to limit the error probability in practical systems. Because (5.64) can be evaluated without the knowledge of the channel, we propose to set the threshold \( \lambda_t \) according to the target error probability \( \hat{P}_t \) as

\[
\lambda_t = \sqrt{\log M_t - \log \hat{P}_t}.
\]  (5.65)

For instance, in a 512-subcarrier OFDM system with \( M_t = N_p[1] = 143 \), if the desirable error probability is \( \hat{P}_t = 10^{-3} \), then one can translate that into a threshold setting \( \lambda_t = 3.45 \) using (5.65).

When \( \tau \) is the delay of a valid channel tap, the distribution of \( r_i(\tau) \) largely depends on the channel condition. We consider a single-path fading channel to simplify the analysis. For the given SNR on each subcarrier \( \zeta \), the possibility of \( (r_i(\hat{\tau}_0[1]) < \lambda_t) \) can be computed as

\[
P(r_i(\hat{\tau}_0) < \lambda_t|\zeta) \approx \int_0^{\lambda_t} f_{(r)}(u) \left( u \left[ \frac{1}{1 + N_p[1] \zeta} \right]^{1/2}, \frac{\sqrt{\zeta}}{\sqrt{2(1 + \zeta)}} \right) du.
\]  (5.66)

We numerically evaluate (5.66) for a 512-subcarrier OFDM system with \( N_p[1] = 143 \), and find the error probability is about \( 10^{-3} \) for \( \lambda_t = 3.45 \) and \( \zeta = -7 \text{dB} \). This indicates the proposed threshold setting for \( \lambda_t \) given by (5.65) has left a sufficiently large margin for channel fading in most practical OFDM systems.

In the next chapter, we extend the successive path detection method to the ranging channel of uplink OFDMA systems where multiple users with different timing offsets sharing the same set of subcarriers need to be detected and their transmission parameters need to be estimated.

5.6.7 Numerical Results

We simulate the proposed successive joint channel and timing estimator in an IEEE 802.16 system as described in Section 5.4.9. The same system parameters and channel models are used in the simulations of this section. The coarse timing is given by the peak of the timing metric proposed in Chapter 3 within the timing window of \([-N_g, N_g]\) samples relative to the ideal timing position, and the coarse fractional CFO estimate is provided by the best universal CFO estimator described in Chapter 4. The true integer CFO is assumed to be perfectly known at the
receiver. The maximum number of iterations for the full complexity algorithm is set to $N_p[1]$, and for the simplified method, $I_{max} = 8$. The threshold $\lambda_t$ is set to 3.45, which gives an error probability around $10^{-3}$ for the channel taps whose energy is 7dB below the noise floor according to our analysis. The noise variance $\sigma_w^2$ is estimated from the unused subcarriers in each training symbol, and we set $\mathcal{E}_t$ according to (5.63).

Two conventional methods designed for single-path channels are also simulated for performance comparison. One method assumes the first arrived channel path is stronger than the others, and determines the timing offset of the strongest path from the time-domain channel estimate. This method is equivalent to the first iteration of the proposed successive algorithm. The other conventional method is originally suggested in [85] for ranging channel detection. The method assumes the channel multipath spread is not very long so that the frequency domain channel does not change too much on adjacent used subcarriers except for a phase rotation resulted from the residual timing offset. This assumption can be written as

$$\hat{H}(\hat{\epsilon})[m+1] \approx \hat{H}(\hat{\epsilon})[m]e^{-j \frac{2\pi}{N} \hat{\epsilon} L_1}. \quad (5.67)$$

Hence, the timing offset can be estimated by

$$\hat{\tau} = \frac{N}{2\pi L} \arg \left( \sum_{m=1}^{N_p[1]-1} \hat{H}(\hat{\epsilon})[m] \hat{H}(\hat{\epsilon})[m+1]^* \right). \quad (5.68)$$

We further round $\hat{\tau}$ to the nearest integer to give a timing estimate as an integer number of OFDM samples.

Figure 5.14(a) compares the performance of the proposed algorithms to that of the conventional methods we described above. Every point in the figure is an average of at least $10^3$ independent experiments. It is shown that the conventional methods fail to decrease the timing estimation errors as the SNR increases in the multipath channels. In the contrast, the proposed successive methods achieve very good performance in both CH-A and CH-B channel models. The error probability is within 5% at 0dB SNR, and further reduces to 1% for SNRs above 8dB. A slight performance degradation in CH-B is observed for the proposed methods, however, that becomes marginal for the SNR at 5dB or above. Another interesting observation is that the simplified method has exactly the same performance as the full-complexity successive estimator. This is because the guard band in IEEE 802.16 system only occupies a small proportion of the spectrum, the approx-
Figure 5.14: Simulation results for the successive timing estimators.
To better understand the behaviors of the proposed successive estimators, we compare the average number of iterations in Figure 5.14(b). Ideally, the number of iterations should equate to the number of channel taps, which is 3 and 6 for CH-A and CH-B respectively. At high SNR, we can see the full-complexity algorithm almost always detects all the channel taps and terminates the iteration process right after the detection of all valid paths. However, the simplified algorithm does not follow this behavior. We find the simplified channel estimator cannot effectively remove the residual signal of detected channel taps, which results in a repeated detection of the same channel tap in multiple iterations. Although this does not necessarily affect the timing estimation performance, we suggest to use a relative small $I_{max}$ value to force the simplified algorithm to stop properly. The value of $I_{max}$ represents a performance and complexity tradeoff. If $I_{max} = 1$, the proposed successive methods reduce to the strongest path timing estimator.

5.7 Summary

In the previous two chapters, we have discussed coarse timing and frequency estimation in the time domain which allow us to further process the received signal in the frequency domain without significant ISI and ICI. In this chapter we have derived and analyzed a number of refined timing and frequency estimation algorithms to solve the remaining ambiguities in the coarse timing and frequency estimates.

We derived an approximated maximum likelihood joint channel, frequency and timing estimator, and then simplified that by different assumptions and approximations. To provide improved tradeoffs between the complexity and performance, we proposed a hybrid scheme that could achieve a satisfactory integer CFO estimation error probability with the complexity feasible for hardware real-time implementation. When there was a mismatch between the true and estimated channel lengths, the timing ambiguity needs to be solved after the integer frequency offset is estimated. We proposed a successive algorithm and its simplified version to solve the problem.

The performance of the proposed methods were theoretically analyzed, and the results agreed with the simulations very well in both stationary and mobile wireless communication channels.
Chapter 6

Successive Ranging Detection

6.1 Introduction

The uplink synchronization in practical OFDMA systems like the IEEE 802.16 [4] is realized by a mechanism called ranging. A subscriber station (SS) obtains initial timing and frequency offsets using the preamble-based synchronization method in the downlink before any uplink transmission. A specially designed signal is then transmitted on a randomly selected ranging opportunity, which is a set of subcarriers in certain OFDM symbols allocated by the base station (BS). The timing and frequency offsets are estimated by the BS, and then sent to the SSs for adjustments until they are within an acceptable range. These uplink synchronization procedures force all SSs to be synchronized before any transmission on data sub-channels, which minimizes inter-carrier interference (ICI) and inter-symbol interference (ISI).

Contention based multiple access schemes are usually employed in the ranging channel for higher efficiency. For instance, the ranging channel specified in the IEEE 802.16 standard is a multi-carrier code-division multiple access (MC-CDMA) channel whose performance is limited by the multiple access interference (MAI). The ranging detection methods of [80–84] are based on single user detection algorithms so the performance degrades rapidly as the number of ranging subscriber stations (RSSs) increases. It is also worth mentioning that the existing ranging channel detection algorithms assume the maximum propagation delay is within the length of the cyclic prefix (CP), which is not necessarily the case for outdoor mobile wireless communication systems.

Other ranging detection approaches [85–87, 124–127] were proposed to mitigate the MAI by specially designed ranging codes and modulation schemes. Because
this study focuses on the detection algorithm, we confine ourselves to the more general contention based OFDMA ranging channel model, which is compliant to the IEEE 802.16 standard. Recently, the space-alternating generalized expectation-maximization (SAGE) algorithm [76,77] and its variation (MSAGE) [78] were proposed for OFDMA uplink synchronization. Those algorithms were developed for preamble based synchronization schemes and not applicable to the ranging channel where the BS has no knowledge of active SSs in the cell. Detailed comparison between the SAGE based algorithm and the proposed method is given in Section 6.4.2.

In this chapter, we propose a novel successive multiuser detector (SMUD) for the OFDMA uplink ranging channel. The SMUD detects the most likely channel path of the active RSSs in each iteration, then jointly estimates the channels for all the detected paths and removes their interference. This way, near single-user performance can be achieved. The iterative process of the ranging detector stops when the power of the residual ranging signal becomes lower than the preset threshold, or none of the channel paths of all possible ranging codes reach the detection threshold. This mechanism is quite effective to ensure the proposed algorithm works properly in the presence of severe interference and channel impairments.

When a ranging signal has a timing offset larger than the CP length, the samples in the first Fast Fourier Transform (FFT) window do not make up a whole OFDM symbol, leading to the interference to other SSs sharing the same time slot. The proposed SMUD effectively mitigates the interference in two ways: first, since the RSSs are more likely to be detected by the SMUD at lower power levels, their interference to the data subscriber stations (DSSs) is expected to be weaker; second, the SMUD estimates all the transmission parameters of the RSSs to reconstruct the ranging signals so that their interference can be accounted for in DSS decoding.

A reduced-complexity SMUD (RC-SMUD) which can be implemented using a number of IFFT functions is also proposed in this chapter to further improve the efficiency of the successive detector. Simulation results show that significant complexity reduction can be achieved by the RC-SMUD at the cost of slight performance degradation.

### 6.2 Successive Multiuser Detection and Interference Cancellation

In this section, we first derive the proposed SMUD and introduce the interference cancellation scheme for DSSs. The methods to efficiently implement the SMUD are
explained, followed by the complexity analysis and comparison to the MSAGE [78] algorithm, which has lower complexity than other SAGE algorithms.

### 6.2.1 Derivation of the SMUD

We consider the ranging signal model given in Section 2.3.3. For one channel path of an active RSS, assume the RSS’s BPSK modulated ranging code is $C_n$, the timing offset of the path is $\tau$, then the observed ranging signal without any channel impairment can be written as

$$ b(n, \tau) \triangleq C_n \odot v(\tau), \quad (6.1) $$

where $\odot$ denotes vector element-wise product that reflects the phase rotation caused by the timing offset on different subcarriers, and $v(\tau)$ is a $N_r \times 1$ vector whose elements are given by

$$ v(\tau)[m] \equiv e^{-j \frac{2\pi}{N_o} (\tau - (N_g - \tau_{\text{max}}))}. \quad (6.2) $$

Let the maximum propagation delay be $d_{\text{max}}$, the maximum channel length be $L_{\text{max}}$, and $M_t \triangleq \tau_{\text{max}} + d_{\text{max}} + L_{\text{max}} + 1$ is the number of possible timing offsets. We know for all possible $N_c$ ranging codes, there are $N_c \times M_t$ such possible paths and the observed ranging signal can be expressed by a linear combination of them as

$$ \tilde{Y}_2 = \sum_{n=1}^{N_c} \sum_{\tau=-\tau_{\text{max}}}^{d_{\text{max}}+L_{\text{max}}} \tilde{h}(n, \tau) b(n, \tau) + \tilde{w}_2, \quad (6.3) $$

where $\tilde{w}_2$ is the noise and interference terms combined, and $\tilde{h}(n, \tau)$ is given by

$$ \tilde{h}(n, \tau) = \sum_{k=1}^{K_t} \sum_{l=1}^{L_k} h_k[l] \delta_{n, n_k} \delta_{\tau, \tau_k[l]}, \quad (6.4) $$

In (6.4), $n_k$ represents the ranging code used by RSS $k$, $\delta_{i_1, i_2}$ is the Kronecker delta function. Also, for (6.4) and the remainder of this section, the subscript $k$ indicates that the corresponding variables are referring to RSS $k$. If $\tilde{h}(n, \tau) \neq 0$, we refer to $b(n, \tau)$ as a valid path and $\tilde{h}(n, \tau)$ is its complex channel gain.

At the receiver, without knowing which paths are valid, it is not feasible to estimate the channels for all possible paths. Consider a typical IEEE 802.16 OFDMA system with $N_c = 32$ ranging codes and $M_t = 100$ samples, the number of subcarriers in the ranging opportunity is $N_r = 144$, which is only a fraction of the required
\( N_c \times M_t = 3200 \) independent observations for the estimation of all possible channel paths. Therefore, conventional channel estimation methods including those of the SAGE algorithms \([76–78]\) do not work for the ranging channel due to the limited number of subcarriers in the ranging channel.

We propose an algorithm to successively identify the most likely valid paths and jointly estimate the channels for all detected paths. At iteration \( i \), assume \( \tilde{n}_1, \ldots, \tilde{n}_{i-1} \) are the detected codes and the corresponding delays are \( \tilde{\tau}_1, \ldots, \tilde{\tau}_{i-1} \). Define \( \tilde{b}_i \triangleq b(\tilde{n}_i, \tilde{\tau}_i) \) and \( \tilde{B}_i \triangleq (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_i) \). The successive detection algorithm for one iteration can be described as follows:

1. Project \( \tilde{Y}_2 \) onto the null space of the detected paths as
   \[
   \tilde{Y}_{2,i} \triangleq \left( I - B_{i-1} \left( B_{i-1}^H B_{i-1} \right)^{-1} B_{i-1}^H \right) \tilde{Y}_2. \tag{6.5}
   \]

2. If the residual signal in the null space is smaller than the preset threshold, \( \left( \tilde{Y}_{2,i}^H \tilde{Y}_{2,i} \leq E_t \right) \), the detection finishes; Otherwise, go to the next step.

3. Compute vector correlation function
   \[
   \mathcal{R}(n, \tau | \tilde{Y}_{2,i}) \triangleq \frac{|b(n, \tau)^H \tilde{Y}_{2,i}|}{\sqrt{\tilde{Y}_{2,i}^H \tilde{Y}_{2,i}}} \tag{6.6}
   \]

4. The most likely valid path is determined by
   \[
   \{\tilde{n}_i, \tilde{\tau}_i\} = \arg \max_{n \in [1, N_c], \tau \in [-\tau_{max}, d_{max}+L_{max}]} \mathcal{R}(n, \tau | \tilde{Y}_{2,i}). \tag{6.7}
   \]

Define \( r_i \triangleq \mathcal{R}(\tilde{n}_i, \tilde{\tau}_i | \tilde{Y}_{2,i}) \). If \( r_i < \lambda_t \), where \( \lambda_t \) is a pre-defined threshold, we think it is unlikely to be a valid path, the iteration stops; Otherwise, increment the iteration index \( i \) and start next iteration from Step 1).

A diagram of the proposed SMUD is shown in Figure 6.1. It is worth noting that for all detected channel paths, the maximum likelihood channel estimate is given by \([122]\)

\[
\tilde{h}_i = (B_i^H B_i)^{-1} B_i^H \tilde{Y}_2, \tag{6.8}
\]

so (6.5) actually combines the MAI mitigation and channel estimation steps as

\[
\tilde{Y}_{2,i} = \tilde{Y}_2 - B_{i-1} \tilde{h}_i = \left( I - B_{i-1} \left( B_{i-1}^H B_{i-1} \right)^{-1} B_{i-1}^H \right) \tilde{Y}_2. \tag{6.9}
\]
When the iteration finishes, one obtains $\mathcal{I}$ sets of transmission parameters, each set corresponds to an estimated valid path and comprises the ranging code index $\tilde{n}_i$, the timing offset $\tilde{\tau}_i$, and the channel estimate $\hat{h}_{i\mathcal{I}}$. As some of the detected paths belong to the same RSS, it is more convenient to post-process the transmission parameter sets grouped by users. The following operations are able to group the detected paths into a link data structure for each individual RSS:

1. Initialize $\hat{K}_r = 0$.

2. For $i = 1 : \mathcal{I}$,
   
   (a) Find $k \in [1 : \hat{K}_r]$ such that $\tilde{n}_i = \hat{n}_k$.

   (b) If $k$ is not found, initialize the new RSS as
      
      i. $k = \hat{K}_r + 1$;
      ii. $\hat{K}_r = k$; $\hat{n}_k = \tilde{n}_i$; $\hat{L}_k = 0$.

   (c) Update the parameters for RSS $k$ as
      
      i. $\hat{L}_k = \hat{L}_k + 1$;
The detected RSSs’ CFOs can be estimated from the phase offset between the channel estimates obtained from the two consecutive ranging symbols. With the estimated valid paths, we estimate the channel from the first ranging symbol as

$$\hat{h} = (B^H I B)^{-1} B^H \left( v(-\tau_{max}) \odot \tilde{Y}_1 \right),$$  \hspace{1cm} (6.10)

where $\tilde{Y}_1$ is the observed ranging signal for the first ranging symbol, and the vector element-wise product aligns the timing reference of the channel estimate to that of the second symbol $\hat{h}_I$. For the $k^{th}$ estimated active RSS in the ranging opportunity, the CFO estimate is given by

$$\hat{\epsilon}_k = \frac{N}{2\pi(N + N_g)} \arg \left( \sum_{l=1}^{\hat{L}_k} \hat{h}[l]^* \hat{h}_I[l] \delta_{\hat{n}_k, \hat{n}_l} \right),$$  \hspace{1cm} (6.11)

where $(\cdot)^*$ denotes the complex conjugate.

Besides CFOs, the BS also needs to send timing and power adjustment instructions to the RSSs based on the timing offset and power estimates given by $(\min_{1 \leq l \leq \hat{L}_k} \hat{\tau}_k[l])$ and $(\sum_{l=1}^{\hat{L}_k} |\hat{h}_k[l]|^2)$ respectively.

### 6.2.2 Interference Cancellation for DSS

When a ranging signal has a timing offset larger than the CP length, the samples in the first FFT window do not make up a whole OFDM symbol and incur considerable ICI. This problem has not yet been addressed in the existing literature. The novel SMUD algorithm estimates the transmission parameters for all the detected RSSs, which allows the reconstruction of the time-domain samples of the ranging signal in the first FFT window so that their interference can be accounted for in the decoding of DSSs’ data.

With the estimated RSS transmission parameters, the time-domain ranging signal can be re-constructed as

$$\hat{y}[n] = \sum_{k=1}^{K_r} e^{j2\pi \hat{\epsilon}_k(n-(N+2N_g-\tau_{max}))} \left( \sum_{l=1}^{\hat{L}_k} \hat{h}_k[l] c_{n_l} [n - \hat{\tau}_k[l]] \right),$$  \hspace{1cm} (6.12)
where
\[ c_n[n] \triangleq (s(n) - s(n - 2(N + N_g))) \times \frac{1}{\sqrt{N}} \sum_{m=1}^{N_r} C_n[m] e^{j \frac{2\pi}{N} (n - N_g) a(m)} \] (6.13)

The interference suppressed DSS signal is obtained by subtracting \( \hat{y}[n] \) from \( y[n] \).

### 6.2.3 Implementation

The proposed SMUD needs to compute (6.5) and (6.6) in each iteration, whose direct implementation is highly complex. The method to simplify (6.6) has been proposed in [128] which shows that for the given ranging code \( n \), it only takes one IFFT to compute \( b(n, \tau) \) for all possible \( \tau \) at complexity \( O(N \log_2 N) \). Usually the number of subcarriers in one ranging opportunity \( N_r \) is much smaller than \( N \), it is possible to further reduce the complexity by using the FFT pruning technique proposed in [129].

The complexity of (6.5) mainly comes from the calculation of the matrix
\[
\tilde{B}_i \triangleq B_i (B_i^H B_i)^{-1} B_i^H.
\] (6.14)

When \( i = 1 \), \( B_1 \) is a vector, and the calculation of \( \tilde{B}_1 \) is trivial. For \( i > 1 \), assume \( \tilde{B}_{i-1} \) and \( \tilde{b}_i \) are available, according to Theorem 5.1, we have
\[
\tilde{B}_i = \tilde{B}_{i-1} + \frac{\left( \tilde{B}_{i-1} \tilde{b}_i - \tilde{b}_i \right) \left( \tilde{B}_{i-1} \tilde{b}_i - \tilde{b}_i \right)^H}{N_r - \tilde{b}_i^H \tilde{B}_{i-1} \tilde{b}_i}.
\] (6.15)

Therefore, \( \tilde{B}_i \) can be computed recursively with much lower complexity than that of the direct implementation.

The MSAGE algorithm [78] also requires matrix inverse, and to avoid the prohibitive complexity, it was proposed to pre-calculate and store the matrix inverse results. For instance, if there are \( N_c = 32 \) possible ranging codes, \( N_s = 201 \) search grids, \( N_r = 144 \) subcarriers for each SS, assume the CP length is \( N_g = 64 \), the MSAGE needs to pre-calculate and store at least \( N_c N_s (N_g^2 + N_g N_r) \approx 8.56 \times 10^7 \) complex values for each possible ranging channel allocation, which requires a memory space beyond the capacity of many practical systems.
6.3 Reduced-Complexity SMUD

We propose a reduced-complexity successive multiuser detection (RC-SMUD) algorithm to provide flexible performance and complexity tradeoffs. The RC-SMUD has the same successive detection and interference cancellation structure as the original SMUD algorithm. However, instead of searching only the most likely path in each iteration, the RC-SMUD looks for all the paths that reach the detection thresholds; instead of estimating the channel with ML algorithm as the SMUD does, the RC-SMUD uses LS estimates that are already computed as intermediate results of the path search; instead of allowing maximum up to $N_r$ iterations as the SMUD does, the RC-SMUD limits the number of iterations within a smaller number $I_{\text{max}}$.

Below we give the details of the proposed RC-SMUD algorithm. The calculation procedures of the RC-SMUD in each iteration follow those of the SMUD given in Section 6.2.1, and we only modify the first and last steps for higher efficiency. For the ease of understanding, we start from the path detection and then explain the channel estimation and MAI mitigation steps.

At iteration $i$, for every code $n$, define

$$\hat{\tau}_i(n) \triangleq \arg \max_{\tau \in [-\tau_{\text{max}}, d_{\text{max}} + L_{\text{max}}]} R(n, \tau | \tilde{Y}_{2,i}),$$

and

$$\gamma_i(n) \triangleq R(n, \hat{\tau}_i(n) | Y_{2,i}).$$

For all the codes, we define

$$\tilde{n}_i \triangleq \arg \max_{n \in [1, N_c]} \gamma_i(n),$$

and it follows that $r_i = \gamma_i(\tilde{n}_i)$. In the SMUD case, only the path $b(\tilde{n}_i, \hat{\tau}_i(\tilde{n}_i))$ is detected in each iteration, while the RC-SMUD will pick up all the paths that are likely to be valid. The most likely valid paths make up a set $S_i$ that can be computed by

$$S_i = \left\{ \begin{array}{ll} \emptyset, & r_i < \lambda_t; \\ \{n | \gamma_i(n) > \lambda_t\}, & r_i \geq \lambda_t \text{ and } i = I_{\text{max}}; \\ \{n | \gamma_i(n) > \lambda_s\}, & r_i \geq \lambda_s \text{ and } i < I_{\text{max}}; \\ \{\tilde{n}_i\}, & \lambda_s \geq r_i \geq \lambda_t \text{ and } i < I_{\text{max}}. \end{array} \right.$$
In (6.19), $I_{\text{max}}$ is the pre-defined maximum number of iterations, $\lambda_s$ and $\lambda_t$ are two preset thresholds satisfying $\lambda_s > \lambda_t$. We will discuss their impacts on the performance and give design guidelines in Section 6.4.4. Figure 6.2 visualizes $S_i$ calculation for different scenarios listed in (6.19). The iteration terminates if $S_i$ is an empty set, or the maximum number of iterations $I_{\text{max}}$ is reached.

\[ \gamma_i(n) \]

\begin{align*}
\text{Figure 6.2: Examples of } S_i \text{ in different cases.}
\end{align*}

Denote $\hat{n}$ as an element of the set $S_i$, the least square channel estimate for that path is given by

\[ \hat{h}_i[\hat{n}] = \frac{1}{N_r} b(\hat{n}, \hat{\tau}_i(\hat{n}))^H \hat{Y}_{2,i}. \]  

(6.20)

Because $\hat{h}_i[\hat{n}]$ has been calculated in Step 3) as the argument of the absolute function in the numerator of $\mathcal{R}(\hat{n}, \hat{\tau}_i(\hat{n}))[\hat{Y}_{2,i}]$, there is no extra computational complexity required for channel estimation. At iteration $i$, with the estimated transmission parameters of the detected paths, one can update the transmission parameters of the RSS whose ranging code is $\hat{n} \in S_i$ as follows:

1. Find $k \in [1 : \hat{K}_r]$ such that $\hat{n}_k = \hat{n}$.

2. If $k$ is not found, initialize the newly detected RSS as
   
   (a) $k = \hat{K}_r + 1$;
   
   (b) $\hat{K}_r = k; \hat{n}_k = \hat{n}; \hat{L}_k = 0$.

3. Update the parameters for RSS $k$ as
   
   (a) Find $l \in [1 : \hat{L}_k]$ such that $\hat{\tau}_k[l] = \hat{\tau}_i(\hat{n})$.

   (b) If $l$ is not found, initialize the new path as

   i. $l = \hat{L}_k + 1$;

   ii. $\hat{L}_k = l; \hat{\tau}_k[l] = \hat{\tau}_i(\hat{n}); \hat{h}_k[l] = \hat{h}_i[\hat{n}]$. 

(c) If \( l \) is found, update the parameters for RSS \( k \) and path \( l \) as

\[
\hat{h}_k[l] = \hat{h}_k[l] + \hat{h}_i[\tilde{n}].
\]

Before the first iteration, the number of detected RSSs \( \hat{K}_r \) is initialized to 0. And, when the iteration stops, the transmission parameters are already organized for every RSS, the mapping from the estimated valid paths to detected RSSs described in Section 6.2.1 is not needed by the RC-SMUD.

The null space projection of the SMUD can be decomposed into two operations, channel estimation and MAI mitigation. For the RC-SMUD, the channel estimates are updated incrementally, so the interference cancellation can be simplified to

\[
\hat{Y}_{2,i+1} = \hat{Y}_2 - \sum_{k=1}^{\hat{K}_r} \sum_{l=1}^{\hat{L}_k} \hat{h}_k[l] b(\hat{n}_k, \hat{\tau}_k[l])
\]

\[
= \hat{Y}_{2,i} - \sum_{\tilde{n} \in S_i} \hat{h}_i[\tilde{n}] b(\tilde{n}, \hat{\tau}_i(\tilde{n})).
\] (6.21)

The CFO estimation for the RC-SMUD is based on the phase offset between the channel estimates obtained from the two consecutive ranging symbols. For RSS \( k \) and channel tap \( l \in [1, \hat{L}_k] \), the channel estimate based on the observation \( \hat{Y}_1 \) is given by

\[
\hat{h}_k[l] = \frac{1}{N_r} b(\hat{n}_k, \hat{\tau}_k[l])^H \left( v(-\tau_{\text{max}}) \odot \hat{Y}_1 \right).
\] (6.22)

And, the CFO estimate for that RSS is

\[
\hat{\epsilon}_k = \frac{N}{2\pi(N + N_g)} \arg \left( \sum_{l=1}^{\hat{L}_k} \hat{h}_k[l] \hat{h}_k[l] \right).
\] (6.23)

### 6.4 Performance Analysis

In this section, we provide more insights into the proposed SMUD by analyzing the impacts of the interference on the behavior of the SMUD. We also compare the SMUD to the SAGE [76–78] and correlation based algorithms [83, 84]. Design guidelines for the threshold settings are presented at the end of this section. These guidelines are based on performance analysis rather than empirical simulation results as suggested in [83, 84].
6.4.1 Interference Analysis

Denote \( L_0 = \sum_{k=1}^{K_r} \sum_{l=1}^{L_k} L_k \) as the total number of valid paths. For all \( k \in [1, L_0] \), we represent the path by \( \tilde{b}_k \) and the corresponding complex channel gain by \( \tilde{h}_k \), respectively. The ranging signal after interference cancellation at iteration \( i \) can be expressed by

\[
\tilde{Y}_{2,i} = \sum_{k=1}^{L_0} \vartheta_{i,k} \tilde{h}_k \tilde{b}_k + \varpi_i,
\]

where \( \vartheta_{i,k} \) is the ratio of the residual channel estimation error to the original channel, \( \varpi_i \) represents the combined effect of AWGN, ICI, and wrongly detected invalid paths. The effect of interference cancellation over iterations is reflected by the values of \( \{ \vartheta_{i,k} \} \). At the first iteration, all of them equal 1, which means that no interference has been cancelled; then, if a valid path \( \tilde{b}_k \) is detected, \( |\vartheta_{i,k}|^2 \) is expected to be reduced – in the case of ideal interference cancellation, \( |\vartheta_{i,k}|^2 = 0 \).

Without loss of generality, at iteration \( i \),

\[
\mathcal{R}(n_i, n_i[1]|\tilde{Y}_{2,i}) = \frac{N_r \tilde{h}_l + \sum_{k \neq l} \vartheta_{i,k} \tilde{h}_k \tilde{b}_k^H \tilde{b}_k + \tilde{b}_k^H \varpi_i}{\sqrt{N_r \sum_{k=1}^{L_0} |\vartheta_{i,k} \tilde{h}_k|^2 + 2\Re(\Upsilon_i) + \varpi_i^H \varpi_i}},
\]

where \( \Re(\cdot) \) denotes the function that takes the real part of its argument, and

\[
\Upsilon_i \triangleq \sum_{k_1=1}^{L_0} \sum_{k_2=1}^{k_1-1} \vartheta_{i,k_1}^* \vartheta_{i,k_2} \tilde{h}_{k_1}^* \tilde{h}_{k_2} \tilde{b}_{k_1}^H \tilde{b}_{k_2} + \sum_{k=1}^{L_0} \vartheta_{i,k}^* \tilde{h}_k b_k^H \varpi_i.
\]

It is revealed by (6.25) that

\begin{itemize}
  \item A path with higher power has a better chance to be detected earlier. This means that the user detection order of the SMUD agrees with that of the optimum multiuser detector in an unequal-power CDMA system [130].
  
  \item The correct detection of valid paths reduces the corresponding \( \vartheta_{i,k} \) values, which results in a reduced variation of the numerator and smaller value of the denominator in (6.25), both giving a better chance of correct detection for the remaining valid paths.
  
  \item When there is severe interference in the ranging channel, the denominator of (6.25) becomes larger, and the correlation output becomes less likely to reach the detection threshold. This effect applies the back pressure to the iterative
\end{itemize}
Successive Ranging Detection

process such that the process terminates itself in the presence of excessive interference and estimation errors.

6.4.2 Comparison with the SAGE

The SAGE algorithm [76, 77] and its variation (MSAGE) [78] are proposed for OFDMA uplink synchronization. Those algorithms also have successive interference cancellation structures, however, they are designed for training-symbol based synchronization schemes and do not work for ranging channel detection due to the following reasons:

- The SAGE represents the SSs’ channels by zero-padded vectors as long as the CP, and requires the number of independent observations to be as large as the product of the number of SSs and the CP length. In the ranging channel there are not sufficient subcarriers to provide the required number of independent observations to distinguish all RSSs’ channels. Moreover, the timing offsets of RSSs vary in a large range from a negative integer to several times the CP length, it is not feasible to represent their channels by zero-padded vectors. This justifies the path-by-path approach of the SMUD which avoids estimating the channel for any invalid virtual paths.

- The SAGE has a pre-determined iteration order, which is not feasible for the ranging channel where the active RSSs are unknown. The SMUD, however, adaptively schedules the iterations by prioritizing the paths by their strengths, which agrees with the behavior of the optimum multi-user detector in an unequal power CDMA system [130].

- The SAGE mitigates the ICI caused by the relative CFOs of different subcarrier sets rather than the MAI which is caused by the multiple access scheme itself as in the ranging channel. This means that the SAGE’s single-user channel and CFO estimators are expected to have a dramatic performance loss in the ranging channel due to the increased interference and correlation among the paths. The SMUD, instead, jointly estimate the channels for all detected paths to avoid this problem.

- The SAGE stops the iterative process by detecting the convergence of the CFO estimates, which is difficult to achieve in the presence of significant ICI caused by the receiver and channel impairments other than the CFO, e.g., large timing offsets, channel time-selectivity, IQ-imbalance, etc. The SMUD,
instead, uses two thresholds to control the iterative process so that it is more robust to severe interference including, but is not limited to, that caused by CFO.

### 6.4.3 Comparison with the Single-User Detector

The time-domain correlation based algorithms [83, 84] were proposed for ranging channel detection. Because of the equivalence of the time-domain and frequency-domain correlations, those methods are equivalent to the single-user detector (SUD) described as follows:

1. Initialize $n = 1$.

2. Compute $\tilde{\tau}_1(n)$ and $\gamma_1(n)$ using (6.16) and (6.17) respectively.

3. If $\gamma_1(n) > \lambda_t$, the ranging code $n$ is determined to be active with timing offset $\tilde{\tau}_1(n)$, and power $\frac{1}{N_p} |b(n, \tilde{\tau}_1(n))^H \tilde{Y}_2|^2$.

4. If $n < N_c$, increment $n$ by one and go to Step 1); otherwise, finish.

We can see that the SUD is equivalent to one iteration of the proposed successive algorithms, i.e., if $I_{\text{max}} = 1$, the RC-SMUD reduces to the SUD. The performance of the SUD is compromised by the MAI among the active RSSs and the correlation among the channel paths due to the following facts.

First, the SUD only estimates the strongest channel path for each RSS, which may be different from the first path and does not represent the timing offset of the RSS. This also results in the performance loss in power estimation because only the strongest path is taken into account.

Second, the correlation among the possible paths increases the chance of false alarm. Assume $b(n_1, \tau_1)$ and $b(n_2, \tau_2)$ are highly correlated. If $b(n_1, \tau_1)$ is a valid path, at the receiver the SUD sees a high correlation at $b(n_2, \tau_2)$ as well, which may cause a false alarm. The SMUD mitigates this problem by detecting the most likely valid path at each iteration and projecting the observation $\tilde{Y}_2$ to the null space of the detected paths. This way, once $b(n_1, \tau_1)$ is detected, its component in the ranging signal will be suppressed in the following iterations, which leads to a much smaller chance for $b(n_2, \tau_2)$ to reach the threshold and become a false alarm.

Third, the SUD is not likely to detect the low-power RSSs who are immersed in the MAI of the high-power users. The SMUD also tends to detect the high-power RSSs in the first several iterations, but the low-power RSSs will have better
and better chances of correct detection as the residual interference factors $|\vartheta_{i,k}|^2$ decrease over the iterations. Comparing to the SUD case where $|\vartheta_{i,k}|^2$ is fixed to 1, we expect the SMUD to have a much higher probability to detect those low-power RSSs.

### 6.4.4 Determining the Thresholds

The iterative process of the SMUD and RC-SMUD is controlled by three thresholds $\mathcal{E}_t$, $\lambda_t$, and $\lambda_s$, where $\mathcal{E}_t$ is the minimal residual ranging signal strength, $\lambda_t$ and $\lambda_s$ determine whether a path is likely to be valid or not. Higher thresholds result in a better chance for the iteration to stop before all the valid paths are detected, while lower thresholds may lead to the detection of invalid paths (false alarms). Therefore, the threshold settings represent the tradeoffs between the missed detection and false alarm probabilities. Different from [83, 84] where the thresholds are determined empirically by simulations, we propose to set the thresholds based on the analysis of their impacts on the ranging detection performance.

We first consider the scenario when no RSS is active in the ranging opportunity. In this case, $\mathbf{Y}_{2,i}$ is a vector comprising only noise and interference terms. For large $N_r$, according to the Central Limit theorem, $\mathbf{Y}_{2,i}^H \mathbf{Y}_{2,i}$ can be well approximated by a Gaussian random variable with mean $N_r \hat{\sigma}_w^2$ and variance $2N_r \hat{\sigma}_w^2$, where $\hat{\sigma}_w^2$ is the average power of noise plus interference on each subcarrier, which can be estimated from the unused subcarriers in each OFDM symbol. The probability for $\mathbf{Y}_{2,i}^H \mathbf{Y}_{2,i}$ to reach $\mathcal{E}_t$ is given by

$$P_{fa}(\mathcal{E}_t) = 0.5 \text{erfc} \left( \frac{\mathcal{E}_t - N_r \hat{\sigma}_w^2}{2 \hat{\sigma}_w^2 \sqrt{N_r}} \right), \quad (6.27)$$

where $\text{erfc}(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2} dt$. If there is an undetected path whose received signal strength is $|\hat{h}_1|^2$, the variance of $\mathbf{Y}_{2,i}^H \mathbf{Y}_{2,i}$ remains the same but the mean is increased by $N_r |\hat{h}_1|^2$. The possibility for the power of that ranging signal to be lower than the threshold is given by

$$P_{md}(\mathcal{E}_t) = 1 - 0.5 \text{erfc} \left( \frac{\mathcal{E}_t - N_r |\hat{h}_1|^2 - N_r \hat{\sigma}_w^2}{2 \hat{\sigma}_w^2 \sqrt{N_r}} \right). \quad (6.28)$$

As a tradeoff between the missed detection and false alarm probabilities, we propose to set

$$\mathcal{E}_t = N_r \hat{\sigma}_w^2 \left( 1 + 0.5 \text{SINR} \right), \quad (6.29)$$
where $\hat{\text{SINR}}$ is the target signal to noise and interference ratio (SINR) for the paths that are expected to reach the threshold.

Now we consider the scenario when all the valid paths are detected, but due to the high interference and residual channel estimation errors, the threshold $E_t$ fails to terminate the ranging detection. The other threshold $\lambda_t$ should be designed to cope with this scenario. Assume $\tilde{\vec{b}} = \vec{b}(\bar{n}, \bar{\tau})$ is an invalid path, using (6.24), we can write

$$\left| R(\bar{n}, \bar{\tau}| \tilde{Y}_2, i) \right|^2 = 1 + \frac{2 \Re \left( \sum_{m_1=1}^{N_r} \sum_{m_2=1}^{m_1-1} \tilde{b}^*[m_1] \tilde{b}[m_2] \tilde{Y}_{i,2}[m_1] \tilde{Y}_{i,2}^*[m_2] \right)}{\tilde{Y}_{i,2}^H \tilde{Y}_{i,2}}. \tag{6.30}$$

Because the numerator of the second term is the summation of a large number of phase rotating terms with weak correlation, the expectation is 0. This means that $R(\bar{n}, \bar{\tau}| \tilde{Y}_2, i)$ can be well approximated by a Rayleigh distributed random variable whose probability distribution function (pdf) is

$$f_{R(\bar{n}, \bar{\tau}| \tilde{Y}_2, i)}(u) \approx f_{(\text{ra})}(u|0.5) = 2ue^{-u^2}. \tag{6.31}$$

We assume the correlation outputs of invalid paths are not correlated. When all valid paths are detected, $r_i$ is the maximum of $(N_c \times M_t)$ such weakly correlated Rayleigh distributed random variables, the probability for $r_i$ to reach $\lambda_t$ is given by

$$P_{fa}(\lambda_t) \approx 1 - \left( \int_0^{\lambda_t} 2ue^{-u^2} \, du \right)^{N_c M_t} = 1 - (1 - e^{-\lambda_t^2})^{N_c M_t} \approx N_c M_t e^{-\lambda_t^2}, \tag{6.32}$$

where the first approximation neglects the correlation among the invalid paths, and the second ignores the higher order exponential terms which decay quickly for large $\lambda_t$ values.

For any given threshold $\lambda_t$, the missed detection probability can vary dramatically in different channels. Therefore, we propose to set the threshold according to the preset false alarm probability $\hat{P}_{fa}$ as

$$\lambda_t = \sqrt{\log (N_c M) - \log \left( \hat{P}_{fa} \right)}. \tag{6.33}$$

It is worth noting that the threshold setting given by (6.33) is also applicable to the SUD proposed in [83,84] where originally the detection thresholds were determined empirically by Monte-Carlo simulations.
The analysis of the threshold $\lambda_s$ is similar to that of $\lambda_t$, however, because all the codes that reach $\lambda_s$ will be determined to be active, it should be set considerably higher than $\lambda_t$ so that it is very unlikely to be reached by inactive codes. Therefore, we propose to use (6.33) to set $\lambda_s$ as well but with a smaller target false alarm probability $\tilde{P}_{fa}$. For two extreme cases where $\lambda_s = \lambda_t$ and $\lambda_s = +\infty$, the false alarm probability of the RC-SMUD will be the same as that of the SUD and SMUD respectively.

### 6.5 Complexity Analysis

We analyze the complexity of the algorithms by the number of complex multiplications. First, we compare the ranging channel detectors and the results are listed in Table 6.1, where $|S_i|$ denotes the number of elements in the set $S_i$, and we do not take advantage of the FFT pruning algorithm [129]. It is shown that the RC-SMUD reduces the complexity of the SMUD by using fewer iterations and a simpler channel estimator. Because the MAI cancellation has much lower complexity than that of the path detection, the overall complexity of the RC-SMUD is $I_{max}$ times that of the SUD. In Section 6.6, the simulation results show that the extra complexity of the proposed successive methods is well paid off by the much better performance than that of the SUD.

<table>
<thead>
<tr>
<th></th>
<th>SUD [83], [84]</th>
<th>SMUD</th>
<th>RC-SMUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Iterations</td>
<td>1</td>
<td>$[1, N_r]$</td>
<td>$[1, I_{max}]$</td>
</tr>
<tr>
<td>Path Detection</td>
<td>$N_cN \log_2 N$</td>
<td>$N_cN \log_2 N$</td>
<td>$N_cN \log_2 N$</td>
</tr>
<tr>
<td>Channel Estimation</td>
<td>0</td>
<td>$3N_r^2 + 2N_r$</td>
<td>0</td>
</tr>
<tr>
<td>MAI Cancellation</td>
<td>0</td>
<td>$N_r$</td>
<td>$</td>
</tr>
</tbody>
</table>

Below we compare the complexity of the SMUD to that of the MSAGE algorithm [78]. Because both algorithms have iterative structures, we compare the complexity of one iteration of the SMUD to that of one expectation-maximization (EM) operation of the MSAGE for one SS who is allocated $N_r$ subcarriers in the training symbol. According to [78], the E-step and M-step of the MSAGE need $N(N_g + 1)$ and $N_sN_g(2BN_r + 1)$ multiplications respectively, where $N_s$ and $B$ are design parameters for grid search and subcarrier masking operations.

In a 1024-subcarrier OFDMA system where $N_r = 1024$, $N_g = N/16 = 64$, $N_c = 32$, we numerically evaluate the complexity of the SMUD and MSAGE. As recommended by [78], we suppose $N_s = 201$ and $B = 3$. Using these parameters,
we obtain the numbers of complex multiplications for the SMUD and MSAGE are 390176 and 11193920 respectively. This means that even with the help of the huge amount of pre-calculated data, the MSAGE is still more than 28 times as complex as the SMUD in each iteration. Similarly we can compute that the complexity of the RC-SMUD in each iteration for the \(|S| = 16\) case is 329984, which is only 2.9% of that of the MSAGE. Because the SMUD and RC-SMUD have similar complexity in each iteration, the complexity saving of the RC-SMUD is mainly attributed to a reduced number of iterations.

It is worth mentioning that the simplified algorithm in [78] is only applicable to the training symbols with a repetition structure in the time domain, so it cannot be used for ranging detection and is not suitable for the complexity comparison with the proposed algorithm.

6.6 Numerical Results

A 1024-subcarrier IEEE 802.16 [4] OFDMA system is modeled in the simulations. The system has 10MHz bandwidth, 3.5GHz carrier, and 5-kilometer cell radius. The length of the CP is set to 1/16 of one OFDMA symbol, or 64 OFDM samples. The maximum propagation delay is \(d_{\text{max}} = 334\) samples, and following the work in [80–87] the timing offsets for RSSs are modeled by independent random variables uniformly distributed in \([-8, 334]\) samples. Each ranging opportunity lasts for two OFDM symbols and consists of \(N_r = 144\) subcarriers. The subcarrier permutation pattern and the ranging code generation process are both specified by the IEEE 802.16 standard. We allocate all the subcarriers not belonging to the ranging opportunity to one single DSS, who is perfectly synchronized to the BS. To simplify the simulations, we assume the DSS has a single-path channel with fixed gain and the data on each subcarrier is QPSK modulated without forward error control coding.

Two channel models described in Chapter 2 are used in the simulations. CH-A is a stationary wireless channel model with deep and long fading, CH-B is a mobile wireless channel model with up to 388.9Hz Doppler frequency resulted from a vehicle speed at 120km/h. The relative tap delays are all rounded to the nearest sample-spaced positions, and the length of CH-A and CH-B in terms of samples are 10 and 26 respectively. The maximum channel length \(L_{\text{max}}\) is set to half of the CP length, which equals 32 OFDM samples, regardless of the channel models. The channels for different RSSs are independently generated following the same
channel model, either CH-A or CH-B. Each RSS's channel gain is normalized over a long period of time, however, in each ranging opportunity, the received power levels can differ dramatically.

If not explicitly stated, the long-term average signal to noise ratio (SNR) for each RSS and DSS is 10dB, and the CFOs are modeled by independent Gaussian random variables with standard deviation 1% subcarrier spacing. The preset false alarm rate $\hat{P}_{fa}$ is 2%, and the threshold $\lambda_t$ is calculated from (6.33). For the RC-SMUD, $\lambda_s$ is calculated from the same equation using a much lower preset false alarm rate 0.1% as a tradeoff between the performance and efficiency. We estimate noise and interference variance $\hat{\sigma}_w^2$ from the unused subcarriers at two ends of the spectrum, and set the threshold $E_t$ according to (6.29) with $\text{SINR} = 3$dB. Every point shown in the figures is an average of at least $10^4$ independent experiments.

### 6.6.1 User Detection Performance

In one ranging opportunity, we count one occurrence of the false alarm event if at least one of the detected ranging codes is actually inactive. The false alarm probability per ranging opportunity is shown in Figure 6.3. We can see that for both channels the proposed threshold setting algorithm can effectively limit the false alarm probabilities within the preset desirable level.

In one ranging opportunity, we count one occurrence of the missed detection event if at least one of the active ranging codes is not detected. The missed detection probability per ranging opportunity is shown in Figure 6.4. The results suggest that much lower missed-detection probabilities are achieved by the SMUD and RC-SMUD than that of the SUD in both channels. Compared to the SMUD, the RC-SMUD’s slightly degraded performance is a tradeoff for its much lower complexity. The performance degradation of both successive algorithms in CH-B can be explained by the channel’s time selectivity which increases ICI and channel estimation error. However, even in that challenging channel condition, the SMUD and RC-SMUD still can detect double RSSs as the SUD does at the same missed detection rate.

The missed detection probability is defined from the BS’s point of view, however, for each individual RSS, a more sensible performance measure is the expected number of ranging attempts needed for a successful ranging detection at the given received signal strength. Figure 6.5 shows the results for the RSSs whose power levels are 6dB and 3dB lower than the long-term average. The figure shows that the low-power RSSs have a much better chance to be detected if the BS is using the
6.6 Numerical Results

Figure 6.3: False alarm probability.

Figure 6.4: Missed detection probability.
successive detector rather than the SUD. When the ranging opportunity is shared by six or more RSSs, the RSS whose transmit power is 6dB lower than the average has virtually no chance of detection by the SUD without boosting the transmit power. In the contrast, the SMUD and RC-SMUD almost surely detect those low-power RSSs even when there are up to 7 other RSSs. This allows the low-power RSSs to be detected and synchronized before boosting the power, which reduces their interference to the DSSs sharing the ranging time slots.

For the correctly detected RSSs, we plot the standard deviation of their timing estimation errors in Figure 6.6. Due to the SMUD and RC-SMUD’s much higher sensitivity to low-power RSSs, we expect the RSSs detected by the successive methods to have considerably lower signal strengths than those of the SUD. Nevertheless, the results show that the SMUD and RC-SMUD still achieve better performance than that of the SUD in both channels. For the same reason as we explained earlier, the time selectivity of CH-B slightly degrades the performance of the successive algorithms, and the SMUD outperforms the RC-SMUD at the cost of higher complexity.

Also for the correctly detected RSSs, we evaluate the normalized power estima-
6.6 Numerical Results

Figure 6.6: Timing estimation performance.

Figure 6.7: Power estimation performance.
tion error by [83]

\[
\frac{|\sum_{l=1}^{L_k} |h_k[l]|^2 - \sum_{l=1}^{L_k} |\hat{h}_k[l]|^2|}{\sum_{l=1}^{L_k} |h_k[l]|^2}.
\] (6.34)

and plot the results in Figure 6.7. The figure agrees with our performance analysis that the SMUD has much better performance than that of the SUD despite of the lower power levels of its detected RSSs. The RC-SMUD outperforms the SUD only when the interference is not too high, i.e., in CH-A or fewer than six RSSs in CH-B, otherwise, the performance degrades to that of the SUD. The superior performance of the SMUD and RC-SMUD can be explained by the fact that they are designed to find all the valid paths with significant strengths, while the SUD only takes into account the strongest path for each code.

### 6.6.2 Complexity

Figure 6.8 shows the average number of iterations as a function of the number of RSSs in both channel models. Theoretically, if the interference is perfectly canceled, we expect the number of iterations to increase linearly with the number of RSSs. However, the simulation results follow that theoretical prediction only when the number of RSSs is small. As the number of RSSs increases, ideal interference cancellation is far from the reality, more and more low-power RSSs or their channel taps are not able to reach the threshold \(\lambda_t\) due to the residual interference, which leads to an early termination of the iterative detection process. This behavior of the SMUD and RC-SMUD agrees with our analysis that in the presence of severe interference and channel impairments the proposed ranging detectors tend to terminate the detection earlier rather than wasting computational efforts on unnecessary iterations. The figure also shows that on average the RC-SMUD runs considerably fewer iterations than that of the SMUD, which explains its much lower overall complexity.

For the same OFDMA system, the MSAGE [78] needs at least \(N_c = 32\) EM operations to complete one scan of all the possible ranging codes, so the number of iterations is much higher than that of the SMUD. The MSAGE terminates the iterative process by detecting the convergence of the estimates, which may be difficult to achieve in challenging channel conditions like CH-B where the ICI is not only caused by CFOs but also the time-selective channel. As shown in [78], the residual interference reduces the speed of convergence and makes the complexity of the MSAGE even higher.
Figure 6.8: Complexity as a function of the number of active RSSs.

Figure 6.9: CFO estimation performance.


6.6.3 CFO Estimation Performance

We model the RSSs’ initial CFOs as independent Gaussian random variables, and let the standard deviation to increase from 1% to 20% of the subcarrier spacing. We plot the standard deviation of CFO estimation error as a function of the input CFO standard deviation in Figure 6.9. The diagonal line in the figure represents the case when the deviations are equal. The figure shows that the minimum error standard deviations the SMUD and RC-SMUD can achieve are about 2% and 3.5% of the subcarrier spacing respectively. If all RSSs are expected to have small CFOs within 2% of the subcarrier spacing, it is a good strategy to skip the estimation and directly neglect the CFOs. Nevertheless, if the RSSs have large CFOs, as shown in the figure, the SMUD and RC-SMUD can effectively keep the output standard deviation within 4% and 5% of the subcarrier spacing respectively even when that of the input is as high as 15%.

6.6.4 Data Subscriber Station’s Bit Error Rate

We allocate all the subcarriers not belonging to the ranging channel to one single DSS who uses QPSK to modulate the data on the subcarriers without forward error control coding. Because the ICI in the first OFDM symbol of the ranging opportunity is not only caused by CFOs but also the RSSs’ large timing offsets, we expected much more severe interference in the first OFDM symbol than that in the second. In Figure 6.10, this fact is reflected by the error floors of the SUD, which are as high as 1% and 3% respectively for the 8-RSS and 12-RSS cases. The proposed SMUD can effectively suppress the interference from as many as 8 RSSs and lower the error floors to $10^{-5}$ and $4 \times 10^{-4}$ in CH-A and CH-B respectively. When there are 12 RSSs many of whom are not detected by the SMUD, our interference cancellation scheme still manages to lower the error floors to a fraction of their original levels in both channels. Compared to the SMUD, the RC-SMUD has only 1dB performance loss in all the cases, which means that it is a quite cost-effective alternative if the complexity of the SMUD is too high for some practical OFDMA systems.

6.7 Summary

The contention based ranging channel is allocated for joint user detection and synchronization in the uplink of practical OFDMA communication systems. Because
Figure 6.10: Data subscriber station bit error rate.
multiple users share the same set of time slots and subcarriers for uplink transmission, the multiple access interference (MAI) limits the performance of ranging channel.

In this chapter we have proposed a successive ranging channel detector to mitigate the MAI and improve the user detection performance. In every iteration, the proposed method detects the most likely channel path of active RSSs, then jointly estimates the channels for all the detected paths and removes their interference before next iteration. Using this approach, near single-user performance was achieved. It was shown that the user-by-user iteration approach of the SAGE based algorithms could not work for the ranging channel due to the insufficient number of subcarriers to distinguish the multipath channels of all possible ranging codes. Moreover, the complexity of the SAGE methods is much higher than that of the SMUD. Compared to the conventional single-user detection methods, the proposed algorithm is able to detect low-power users at much higher success rate. Not only the ranging users, but also the data subscriber stations benefit from the reduced ICI on the data subcarriers. This was realized by removing the interference of the reconstructed ranging signal, and detecting the ranging users before they boost the power. To further improve the efficiency of the proposed successive multiuser detector, we have proposed a reduced complexity version of the original algorithm that can be implemented simply by a number of IFFT functions. Simulation results showed a slight performance degradation resulted from the reduced complexity.
Chapter 7

System Performance of Ranging Detectors

7.1 Introduction

Ranging is the process of establishing an initial link in the uplink and initiating closed loop timing control such that ISI and ICI can be minimized. In the IEEE 802.16 [4] standard, two ranging methods are specified, initial ranging and periodic ranging. The initial ranging is needed when a SS enters a network or performs a handover. Once the SS receives an acknowledgement from the BS with the allocated connection identification number (CID), the SS enters the network and uses periodic ranging to request bandwidth and keep track of its transmission parameters.

Consider a wireless communication system where a BS serves a number of SSs with a given number of ranging opportunities. In the previous chapter, we have obtained the missed detection probabilities for the ranging channel detectors via simulations, however, it is more interesting from a system point of view to know the maximum number of SSs a BS can serve, or, for a given number of SSs what is the minimum number of ranging opportunities required. This information is useful for the BS to optimize the bandwidth allocation for ranging and avoid under-supplying or over-supplying the ranging opportunities, both of which incur a loss in system data throughput.

In this chapter, we quantify the relationship between the number of ranging opportunities and the maximum number of SSs a BS can serve with a given ranging channel detector whose missed detection probabilities have been obtained from the simulations. For periodic ranging, we assume the SSs are stationary within a cell,
and analyze the maximum number of users that can be served by the BS with a given number of ranging opportunities. For initial ranging, we assume the SSs are entering from neighboring cells and need to perform the handover operations. We analyze the handover success rate and the average time needed by the handover process.

For illustration purpose, three ranging channel detection algorithms are considered in this study: the differential decoding based method (Diff) [131], correlation based method (SUD) [83, 84], and the reduced complexity successive multiuser detection method (RC-SMUD) proposed in Chapter 6 of this thesis. The complexity of these ranging channel detectors is summarized in Table 7.1, where \( N_c \) is the number of available ranging codes, \( N_r \) is the number of subcarriers allocated to ranging channel, \( N \) is the number of subcarriers, \( I_{\text{max}} \) is the pre-defined maximum number of iterations for the RC-SMUD. Nevertheless, the methods developed in this chapter for system-level performance analysis are also applicable to other ranging channel detectors, e.g., the full complexity SMUD algorithm described in Chapter 6 of this thesis.

![Table 7.1: Complexity of the ranging channel detectors.](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diff [131]</td>
<td>( O(N_c N_r) )</td>
</tr>
<tr>
<td>SUD [83]</td>
<td>( O(N_c N \log_2 N) )</td>
</tr>
<tr>
<td>RC-SMUD</td>
<td>( O(I_{\text{max}} N_c N \log_2 N) )</td>
</tr>
</tbody>
</table>

### 7.2 Assumptions And Definitions

A simplified diagram of the ranging procedures is shown in Figure 7.1. Every time a RSS starts the ranging process, it randomly selects a ranging opportunity in a predefined backoff window \([T_{bks}, T_{bkw}]\) with equal probability. Then, the RSS transmits the ranging signal in the selected ranging opportunity, and waits for the response from the BS. If the response is not received within the time duration \( T_{\text{exp}} \), the RSS assumes the previous ranging attempt has failed and restarts the ranging process. In this study we assume the downlink transmission and reception is perfect, which means that once a RSS is detected by the BS, it surely receives the response before the expiry time.

When we analyze the periodic ranging performance, we assume:

- Once the RSS has successfully completed a periodic ranging, it waits for a period of \( T_{\text{rng}} \) and then ranges again.
7.2 Assumptions and Definitions

- The RSS keeps ranging until success.
- All RSSs in the cell have equal long-term averaged power.

When we analyze the initial/handover ranging performance, we assume:

- The density of RSSs is identical throughout the cell of radius $R_c$.
- All RSSs move in straight lines at speed $\nu$ and the directions are uniformly distributed in $[0, 2\pi)$.
- The same number of RSSs move out of and into the cell within arbitrarily small time interval. This implies that the number of RSSs in the cell remains constant.
- A successful handover requires $N_{\text{succ}}$ times successful initial ranging for any RSS entering the cell.
- A handover is assumed to fail if the RSS does not complete $N_{\text{succ}}$ times successful initial ranging with the hand-over BS before its distance to the serving BS is larger than the cell radius.
The system parameters we use in the simulations and performance analysis are listed in Table 7.2.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers</td>
<td>$N$</td>
<td>-</td>
<td>1024</td>
</tr>
<tr>
<td>Number of ranging subcarriers</td>
<td>$N_r$</td>
<td>-</td>
<td>144</td>
</tr>
<tr>
<td>Length of cyclic prefix</td>
<td>$N_g$</td>
<td>-</td>
<td>128</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>$BW$</td>
<td>MHz</td>
<td>10</td>
</tr>
<tr>
<td>Ranging response expiry time</td>
<td>$T_{exp}$</td>
<td>ms</td>
<td>200</td>
</tr>
<tr>
<td>Start of backoff window</td>
<td>$T_{bks}$</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>End of backoff window</td>
<td>$T_{bkw}$</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>Nominal periodic ranging interval</td>
<td>$T_{rng}$</td>
<td>second</td>
<td>2</td>
</tr>
<tr>
<td>Cell radius</td>
<td>$R_c$</td>
<td>km</td>
<td>5</td>
</tr>
<tr>
<td>Required number of successful ranging</td>
<td>$N_{succ}$</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td>Ranging opport. interval (Periodic)</td>
<td>$T_{op}$</td>
<td>ms</td>
<td>5, 20</td>
</tr>
<tr>
<td>Ranging opport. interval (Initial)</td>
<td>$T_{op}$</td>
<td>ms</td>
<td>20, 40</td>
</tr>
<tr>
<td>Terminal Speed</td>
<td>$\nu$</td>
<td>km/h</td>
<td>120</td>
</tr>
</tbody>
</table>

### 7.3 Performance Analysis

Consider a cell with a given number of ranging opportunities. If the number of SSs in a cell keeps increasing, each ranging opportunity is shared by more and more RSSs, the chance of their correct detection becomes lower and lower. When the correct detection probability decreases to a certain point where the number of undetected RSSs starts to accumulate, the chance of correct detection will quickly diminishes to 0 and all the SSs in the cell will eventually be trapped in the ranging state without being detected. Once this situation occurs, the system will not be able to recover by itself, and the communication system is effectively jammed. In this section, we investigate the condition for that to happen, and translate the ranging channel detectors’ missed detection probabilities into the maximum number of SSs can be served by the BS with a given number of ranging opportunities.

#### 7.3.1 Maximum Number of New Ranging Users

The RSSs in one ranging opportunity can be divided into two categories: new ones and remaining ones. The new RSSs have just started a new ranging process, while the remaining ones were undetected by the BS in their previous ranging attempts.
In the ranging opportunity \( m \), we denote the number of new and remaining RSSs as \( K_{\text{new}}^{(m)} \) and \( K_{\text{rem}}^{(m)} \) respectively, then the total number of RSSs is given by

\[
K_r^{(m)} = K_{\text{new}}^{(m)} + K_{\text{rem}}^{(m)}. \tag{7.1}
\]

For the ranging opportunity \((m + 1)\), the expected number of RSSs can be evaluated as

\[
E \left\{ K_r^{(m+1)} \right\} = K_{\text{new}}^{(m+1)} + E \left\{ K_{\text{rem}}^{(m+1)} \right\} = K_{\text{new}}^{(m+1)} + P_{\text{md}}(K_r^{(m)}) K_r^{(m)}, \tag{7.2}
\]

where \( P_{\text{md}}(K_r) \) is the probability for a RSS being miss detected when there are \( K_r \) RSSs in the ranging opportunity. The values of \( P_{\text{md}}(K_r) \) can be obtained from simulations for different ranging detectors, and for a given \( K_r \), a smaller \( P_{\text{md}}(K_r) \) indicates superior performance of the ranging detector. To ensure the convergence of the sequence \( E \left\{ K_r^{(m)} \right\} \), it requires that

\[
E \left\{ K_r^{(m+1)} \right\} = K_{\text{new}}^{(m+1)} + P_{\text{md}}(K_r^{(m)}) \leq 1, \tag{7.3}
\]

which is equivalent to

\[
K_{\text{new}}^{(m+1)} \leq \left\lfloor (1 - P_{\text{md}}(K_r^{(m)})) K_r^{(m)} \right\rfloor = K_t, \tag{7.4}
\]

where \( \lfloor \cdot \rfloor \) denotes the flooring function that outputs the largest integer not larger than the argument, and we define \( K_t \) as the maximum average number of new RSSs that can enter the system in each ranging opportunity without increasing the expected number of RSSs in the next ranging opportunity.

For the three ranging channel detectors, Figure 7.2 plots \( K_t \) as a function of \( K_r \) and \( P_{\text{md}}(K_r) \) for different ranging detectors. In Table 7.3, we list the maximum numbers of new RSSs obtained from the figure.

### 7.3.2 Maximum Number of Users in a Cell

In the previous subsection, the connection between the missed detection probabilities and the maximum number of new RSSs has been established. Next, we derive the relationship between the number of new RSSs and the total number of SSs in the cell, which is denoted by \( K_c \).
For periodic ranging, assume the number of RSSs in every ranging opportunity is uniformly distributed, then we have

$$E \{ K_{new} \} = K_c \frac{T_{op}}{T_{rng}} \leq K_t,$$

(7.5)

where $K_t$ is defined in (7.4), and $T_{op}$, $T_{rng}$ are defined in Table 7.2. It immediately follows that

$$K_c \leq K_t \frac{T_{rng}}{T_{op}}.$$

(7.6)

For initial ranging, the position of a RSS entering the cell is shown in Figure 7.3. For the RSSs whose distance to the edge of the cell is $x$, assume the directions of their trajectories are uniformly distributed within $[0, 2\pi)$, then the chance for one
7.4 Numerical Results

In this section, we verify our analytical results by the numerical data obtained from the simulations. The system parameters used in both analysis and simulation are listed in Table 7.2.
7.4.1 Periodic Ranging

The nominal periodic ranging interval $T_{rng}$ is fixed to 2 seconds. Using (7.6), we calculate the maximum number of SSs in a cell and list the results in Table 7.4. Because the maximum number of SSs in a cell is proportional to the average number of new ranging users, the ratios between the maximum SSs supported by the detection algorithms are the same as those listed in Table 7.3.

<table>
<thead>
<tr>
<th>$T_{op}$</th>
<th>Diff</th>
<th>SUD</th>
<th>RC-SMUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ms</td>
<td>400</td>
<td>1600</td>
<td>3600</td>
</tr>
<tr>
<td>20 ms</td>
<td>100</td>
<td>400</td>
<td>900</td>
</tr>
</tbody>
</table>

In Figure 7.4, we plot the average time needed by one successful ranging process as a function of the number of SSs in the cell. The vertical dot-dash lines correspond to the maximum numbers of users we calculate from (7.6), which are also listed in Table 7.4 as well. We can see that the analytical results match the simulations reasonably well. When the number of SSs in a cell is less than the threshold given by (7.6), the average time needed for a successful ranging process is quite small, however, it quickly becomes impractically large as the number of users exceeds the threshold.

7.4.2 Initial/Handover Ranging

The RSS speed $\nu$ is set to 120km/h, and we list the maximum number of SSs in the cell calculated from (7.8) in Table 7.5 for different ranging opportunity time intervals.

<table>
<thead>
<tr>
<th>$T_{op}$</th>
<th>Diff</th>
<th>SUD</th>
<th>RC-SMUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 ms</td>
<td>1413</td>
<td>4241</td>
<td>8482</td>
</tr>
<tr>
<td>40 ms</td>
<td>706</td>
<td>2120</td>
<td>4241</td>
</tr>
</tbody>
</table>

In Figure 7.5 and 7.6, we plot the average time needed by a successful handover process and the success rate as functions of the number of SSs in the cell. The vertical dot-dash lines correspond to the maximum numbers of SSs computed from (7.8), which are also listed in Table 7.5. We can see that when the number of SSs in a cell is less than the threshold given by (7.8), the average time needed for a successful handover is quite small, and the success rate is close to 100%. However,
7.4 Numerical Results

![Graph of periodic ranging performance](image)

(a) Periodic ranging opportunity interval $T_{op} = 5$ms.

(b) Periodic ranging opportunity interval $T_{op} = 20$ms.

Figure 7.4: Periodic ranging performance.
Figure 7.5: Initial/handover ranging ($\nu = 120$ km/h, $T_{op} = 20$ ms).
7.4 Numerical Results

![Graph showing probability of successful handover and average time of successful handover.

(a) Probability of successful handover

(b) Average time

Figure 7.6: Initial/handover ranging ($\nu = 120\text{km/h}, T_{op} = 40\text{ms}$).
the handover time quickly becomes impractically large when the number of users exceeds the threshold, and the success rate diminishes quickly to 0 as well. This suggests that our analysis is quite accurate.

7.5 Summary

The missed detection probabilities of the ranging channel detectors can be obtained from simulations, however, from a system point of view, a more sensible measure of the performance is the number of users a BS can serve with a given number of ranging opportunities. We have derived the algorithms for the conversion for periodic and initial/handover ranging methods respectively. Our analysis agreed with the simulation results reasonably well. The results showed the proposed RC-SMUD could double the number of users supported by the SUD, and allow up to 5 times more users in a cell than the differential method.
Chapter 8

Bounds for Frequency-Domain Equalization

8.1 Introduction

Even with the most advanced CFO and channel estimation algorithms, there can still be residual ICI caused by the channel’s time selectivity or other impairments. Because the ICI in the frequency domain has a similar effect to the ISI in the time domain, it is feasible to use equalization techniques [91, 92] to mitigate ICI. We consider the idea of equalizing neighboring subcarriers so that the residual interference mainly comes from the subcarriers outside of the equalization window. The longer an equalization window is, the lower residual interference can be. The relationship between the equalization window length and residual interference is quantified in this chapter, and the results show that the power of residual ICI is inversely proportional to the number of equalizer taps. This provides useful insights into the frequency-domain equalization techniques.

8.2 OFDM over Time-varying Channel

In the time domain, an OFDM signal can be represented by

\[ x_i(t) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N_{p[i]}} X_i[m] e^{j \frac{2\pi}{N} (t/t_s - N_{p})(\alpha[i] + (m-1))}. \]  

(8.1)
We assume the frequency domain data symbols have unit power and are independent to each other,
\[ E \{ X_i[m_1]X_i^*[m_2] \} = \delta_{m_1,m_2}. \]  
\hspace{1cm} (8.2)

As shown in [90], there is no loss of generality to consider a flat fading channel for ICI evaluation. This is because for a multipath channel with uncorrelated taps, the total ICI power equals the summation of that on each channel tap. As long as the total gain of the channel is fixed, no matter whether it is single-path or not, the ICI power remains the same. For simplicity, we denote the impulse response of the single-path channel as \( h(t,\tau) = \gamma(t)\delta(\tau), \) where \( \gamma(t) \) is a wide-sense stationary stochastic process with zero mean and unit variance. Define \( P(f) \) as the spectral density function that satisfies
\[ E \{ \gamma(t_0)\gamma^*(t_0 + t) \} = \int_{-\infty}^{+\infty} P(f) e^{j2\pi ft} \, df. \]  
\hspace{1cm} (8.3)

We consider three channel models in this chapter, namely classical, uniform and two-path. The time-varying properties of the channels are characterized by their spectral density functions:
\[ P(f) = \begin{cases} 
  s(f_d - |f|) \left( \pi \sqrt{f_d^2 - f^2} \right)^{-1}, & \text{Classical;} \\
  s(f_d - |f|)(2f_d)^{-1}, & \text{Uniform;} \\
  \frac{1}{2}(\delta(f + f_d) + \delta(f - f_d)), & \text{Two-path.}
\end{cases} \]  
\hspace{1cm} (8.4)

In (8.4), \( f_d \) is the maximum Doppler frequency, and \( s(t) \) is the unit step function, i.e., for \( t < 0, \) \( s(t) = 0, \) \( t \geq 0, \) \( s(t) = 1. \) The two-path channel model corresponds to the scenario where the receiver has a fixed CFO equal to \( f_d. \)

Hence, if the additive white noise is ignored, the signal received through the channel can be expressed as
\[ \tilde{x}_i(t) = \int h(t,\tau)x_i(t - \tau)d\tau = \gamma(t)x_i(t) \]  
\hspace{1cm} (8.5)

and the demodulated signal becomes
\[ \dot{X}_i[m] = \frac{1}{T_s} \int_0^{T_s} \gamma(t) \left( \sum_{k=1}^{N_o[k]} X_i[k]e^{j\frac{2\pi}{T_s}k\omega_i[k]} \right) e^{-j\frac{2\pi}{T_s}\omega_i[m](t/t_s)} \, dt 
  = \sum_{|k| \leq M_q} a_k X_i[m - k] + \sum_{|k| > M_q} a_k X_i[m - k], \]  
\hspace{1cm} (8.6)
where $M_q$ is the number of neighboring subcarriers on each sides of the interested subcarrier, and the second term represents ICI. In (8.6), we define

$$a_k = \frac{1}{T_s} \int_0^{T_s} \gamma(t) e^{j2\pi(k-m)t/T_s} \, dt,$$

(8.7)

which is the component of the signal on subcarrier $m - k$ that is interfering the interested subcarrier. By employing a frequency-domain equalizer, the interference from the neighboring subcarriers within the equalization window can be suppressed, and the residual ICI mainly comes from the subcarriers outside of the equalization window. We assume that on each side of the interested subcarrier the equalizer has $M_q$ taps, so the total number of taps is given by $2M_q + 1$. It is worth noting that the ICI problem studied in [88–90] corresponds to the special case of $(M_q = 0)$ here.

### 8.3 ICI Analysis

We define the residual ICI power as

$$P_{\text{ICI}}(M_q) \triangleq E \left\{ \left| \sum_{|k| > M_q} a_k X_i[m-k] \right|^2 \right\} = \sum_{|k| > M_q} E \left\{ |a_k|^2 \right\},$$

(8.8)

where the second equality follows the linear property of expectation and the assumption in (8.2).

From equation (A.2) in the appendix of [90], we have

$$E \left\{ |a_k|^2 \right\} = \int_{-1}^{1} \phi(T_s x)(1 - |x|) e^{-j2\pi k x} \, dx$$

$$= \int_{-1}^{1} \left( \int_{-\infty}^{+\infty} P(f) e^{j2\pi f T_s x} \, df \right) (1 - |x|) e^{-j2\pi k x} \, dx$$

$$= 4 \int_{0}^{f_s} P(f) \int_{0}^{T_s} \cos(2\pi f T_s x) \cos(2\pi k x)(1 - x) \, dx \, df$$

$$= \frac{1}{\pi^2} \int_{0}^{f_s} P(f) \sin^2(\pi T_s f) \left( \frac{1}{(T_s f + k)^2} + \frac{1}{(T_s f - k)^2} \right) \, df,$$

(8.9)

where the second equality uses (8.3), and the last equality follows the integral with
respect to $x$. Combining (8.9) and (8.8), one has

$$P_{\text{ICI}}(M_q) = \frac{1}{\pi^2} \int_0^{f_d} P(f) \sin^2(\pi T_s f) \sum_{|k| > M_q} \left( \frac{1}{(T_s f + k)^2} + \frac{1}{(T_s f - k)^2} \right) df$$

$$= \frac{2}{\pi^2} \int_0^{f_d} P(f) \sin^2(\pi T_s f) \Phi(T_s f, M_q) df,$$

(8.10)

where we defined $\Phi(\varepsilon, M_q) = \sum_{k > M_q} (\varepsilon + k)^{-2} + (\varepsilon - k)^{-2}$. This result is equivalent to the Equation (20) of [91], but our derivation is much more concise.

### 8.4 Closed-Form Bounds

In this section we derive tight closed-form bounds for $P_{\text{ICI}}(M_q)$. Since for fixed $M_q$ and $T_s$, $\Phi(T_s f, M_q) > 0$ and monotonously increases with $f$, from (8.10), we have

$$\Phi(0, M_q) \Psi(T_s, f_d) < P_{\text{ICI}}(M_q) < \Phi(T_s f_d, M_q) \Psi(T_s, f_d),$$

(8.11)

where $\Psi(T_s, f_d) \triangleq \frac{2}{\pi^2} \int_0^{f_d} P(f) \sin^2(\pi T_s f) df$, whose closed-form expression can be computed from (8.4) as

$$\Psi(T_s, f_d) = \begin{cases} 
(2\pi^2)^{-1} (1 - I_0(2\pi T_s f_d)), & \text{Classical;} \\
(2\pi^2)^{-1} (1 - \text{sinc}(2\pi T_s f_d)), & \text{Uniform;} \\
\pi^{-2} \sin^2(\pi T_s f_d), & \text{Two-path.} 
\end{cases}$$

(8.12)

In (8.12), $I_0(\cdot)$ represents the modified Bessel function of the first kind with order zero, and we define $\text{sinc}(x) \triangleq \sin(\pi x)/(\pi x)$. As for the other term $\Phi(\varepsilon, M_q)$ in (8.11), its bounds are given by the following theorem.

**Theorem 8.1.** Define

$$\beta_0 \triangleq \frac{16(1 + M_q)}{8M_q^2 + 12M_q + 5} - \frac{4}{N - 1},$$

(8.13)

$$\beta_1 \triangleq \frac{4}{2M_q + 1},$$

(8.14)

$$\beta_2 \triangleq \frac{96}{7 + 54M_q + 96M_q^2 + 48M_q^3}.$$

(8.15)

For all $|\varepsilon| \leq 0.5$,

$$\beta_0 \leq \Phi(\varepsilon, M_q) \leq \beta_1 + \beta_2 \varepsilon^2.$$  

(8.16)
Proof. Solving the equation
\[
\int_{k-\frac{1}{2}}^{k+\frac{1}{2}} (x - \varrho_k(\varepsilon))^{-2} \, dx = \frac{1}{2} \left( \frac{1}{(k-\varepsilon)^2} + \frac{1}{(k+\varepsilon)^2} \right)
\]  
(8.17)
with the constrain that $|\varrho_k(\varepsilon)| < 1$, we have
\[
\varrho_k(\varepsilon) = k - \sqrt{\frac{(k^2 - \varepsilon^2)^2}{k^2 + \varepsilon^2} + \frac{1}{4}}.
\]  
(8.18)
It can be shown that, for all $k \geq M_q + 1$ and $|\varepsilon| < \frac{1}{2}$,
\[
\varrho_k(0) = k - \sqrt{k^2 + \frac{1}{4}} > k - \left( k + \frac{1}{8k} \right) \geq -\frac{1}{8(M_q + 1)}
\]  
(8.19)
and
\[
\varrho_k(\varepsilon) \leq k - \sqrt{\frac{(k^2 - \varepsilon^2)^2}{k^2 + \varepsilon^2} + \varepsilon^2} \leq \frac{k(k + \varepsilon^2)/(2k) - (k^2 - \varepsilon^2/2)}{\sqrt{k^2 + \varepsilon^2}} = \frac{\varepsilon^2}{\sqrt{(M_q + 1)^2 + \varepsilon^2}}.
\]  
(8.20)
Hence, according to the definition of $\Phi(\varepsilon, M_q)$,
\[
\Phi(0, M_q) > 2 \int_{M_q + \frac{1}{2}}^{N-\frac{1}{2}} \left( x + \frac{1}{8(M_q + 1)} \right)^{-2} \, dx > \frac{16(M_q + 1)}{8M_q^2 + 12M_q + 5} - \frac{4}{N-1} = \beta_0,
\]  
(8.21)
\[
\Phi(\varepsilon, M_q) < 2 \int_{M_q + \frac{1}{2}}^{+\infty} \left( x - \frac{\varepsilon^2}{\sqrt{(M + 1)^2 + \varepsilon^2}} \right)^{-2} \, dx = 2 \left( \frac{M_q + 1}{2} - \frac{\varepsilon^2}{\sqrt{(M_q + 1)^2 + \varepsilon^2}} \right)^{-1} < \frac{96\varepsilon^2}{2M_q + 1} + \frac{4}{7} + 54M_q + 96M_q^2 + 48M_q^3 = \beta_1 + \beta_2\varepsilon^2.
\]  
(8.22)
Combining (8.11) and (8.16), the closed-form upper and lower bounds are given
by

$$\beta_0 \Psi(T_s, f_d) < P_{ICI}(M_q) < (\beta_1 + \beta_2(T_s f_d)^2) \Psi(T_s, f_d).$$  (8.23)

Another method to compute the upper bound is to substitute \(\Phi(T_s f_d, M_q)\) in (8.10) with (8.22) and integrate directly. This would yield a more complex bound that gives less insight into the problem. Our numerical results also suggest that the marginal performance improvement of the direct integral approach over (8.23) may not be able to justify the much higher complexity.

Cai and Giannakis [91] simplified the ICI evaluation by upper bounding \(\Phi(T_s f_d, M_q)\) with \(\Phi(1, M_q)\) so that the term \(\Phi(T_s f_d, M_q)\) can be extracted from the integral. As \(\Phi(T_s f_d, M_q) < \Phi(1, M_q)\), our upper bounds must be tighter. Moreover, the results of [91] are not explicit functions of \(M_q\), which provide less insight than ours. For instance, from (8.23), one can tell that for large \(M_q\), \(\beta_0 \approx \beta_1\) and \(\beta_2 \approx 0\), so \(P_{ICI}(M_q) \propto (2M_q + 1)^{-1}\), which is hidden in the results of [91].

### 8.5 Polynomial Bounds

Polynomial bounds are easier to evaluate, and often provide more insights into the problem. Using the inequality that for all \(x > 0\),

$$x^2 - \frac{x^4}{3} < \sin^2(x) < x^2$$  (8.24)

and substituting \(\Phi(\varepsilon, M)\) in (8.10) with its bounds, we have

$$P_{ICI}(M_q) > 2\beta_0 \int_0^{fd} P(f) \left( (T_s f)^2 - \frac{\pi^2}{3} (T_s f)^4 \right) df$$

$$= \alpha_1 \beta_0 (T_s f_d)^2 - \frac{\pi^2}{3} \alpha_2 \beta_0 (T_s f_d)^4,$$  (8.25)

$$P_{ICI}(M_q) < 2 \int_0^{fd} P(f) \left( \beta_1(T_s f)^2 + \beta_2(T_s f)^4 \right) df$$

$$= \alpha_1 \beta_1 (T_s f_d)^2 + \alpha_2 \beta_2 (T_s f_d)^4,$$  (8.26)

where \(\alpha_1\) and \(\alpha_2\) are constants listed in Table 8.1, which were also given in [90].

<table>
<thead>
<tr>
<th></th>
<th>Classical</th>
<th>Uniform</th>
<th>Two-path</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td>1/2</td>
<td>1/3</td>
<td>1</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>3/8</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>
8.5 Polynomial Bounds

Figure 8.1: Bounds for classical channel model.

Figure 8.2: Bounds for uniform channel model.
8.6 Comparison of the Bounds

In this section, we numerically compare the bounds derived in the previous sections to those in the literature [91]. A 256-subcarrier OFDM is modeled in the calculations. The exact ICI power given by (8.10) is also plotted for reference. The proposed closed-form upper and lower bounds are calculated from (8.23), while the polynomial upper and lower bounds are given by (8.25) and (8.26) respectively. Figure 8.1, 8.2, and 8.3 plot the exact curves along with the various bounds for the three channel models respectively. It is shown that our closed-form bounds are quite tight for all the models and different $T_s f_d$ values. The polynomial bounds are tight only for small $T_s f_d$ values, and become loose as $T_s f_d$ increases. This is because the approximation in (8.24) is accurate only for small arguments. The bounds of [91] is looser than our polynomial bounds for most cases except for the two-path channel at $T_s f_d = 0.5$. The figure also shows that residual ICI diminishes as the length of the equalization window increases. Unfortunately, the implementation of a maximum likelihood equalizer with $(2M_q + 1)$ taps for BPSK modulated symbols requires $O(2^{2M_q+1})$ complexity. This means that when $M_q$ is large, using two additional equalizer taps brings marginal performance gain $\frac{2M_q+3}{2M_q+1}$ at the
cost of quadruple increase in the complexity.

8.7 Summary

In this chapter we have derived closed-form and polynomial bounds for the ICI power that could not be mitigated by a frequency-domain equalizer with a given number of taps under three different channel models. The closed-form bounds were tight, and the polynomial bounds were straightforward to evaluate. The results facilitated the evaluation of ICI power in OFDM systems and revealed the fact that residual ICI power is inversely proportional to the number of equalizer taps and the gain of employing each additional equalization tap diminishes as the equalization window extends.
Chapter 9

Turbo Equalization Using Grouped Particle Filter

9.1 Introduction

In the previous chapter, we have derived the bounds for the ICI power out of the equalization window; in this chapter, we discuss a method to mitigate the ICI within the equalization window. Because the ICI in the frequency domain is the dual problem of ISI in the time domain, the equalization techniques developed for single-carrier time-domain signals are generally applicable to OFDM systems without significant modification.

The idea of Turbo equalization [93–95, 97] is to concatenate the equalizer and error control decoder in an iterative manner so that they can take advantage of the \textit{a priori} information obtained from the previous iterations and achieve better performance. However, to make the Turbo receiver work, the outputs of the equalizer and decoder need to be probabilities (soft values) rather than hard decisions on the data. Particle filtering is such a technique, which is capable of tracking the dynamics of unknown parameters in the system and estimating their probability distributions [98, 99]. So, the particle filter should perform as well as the soft-input-soft-out (SISO) blocks used for Turbo equalization.

However, the theory agrees with the practice only when the number of particles is large enough so that the probabilities output by the equalizer are close to their true values. The complexity of conventional particle filters is proportional to the number of particles, hence a large number of particles require prohibitive computational complexity to implement.

In this chapter, we propose a new particle filter with grouping to improve the
efficiency of conventional methods. The idea of grouping the trajectories of particles was mentioned in [102,103], and we extend that idea with deterministic sample drawing and group merging techniques, which allow the system to work with a large number of particles without dramatically increasing the computational complexity. The proposed grouped particle filter (GPF) can be interpreted as a reduced state MAP algorithm [132, 133], while instead of using a fixed threshold, it adaptively adjusts the threshold for state pruning as an implicit function of the probability distribution of previous and current symbols. Simulation results confirm its “adaptive effort” similar to that of the T-algorithm [132,133]. Two simple early-stopping criteria are developed to further reduce the computational complexity and processing delay of the proposed algorithm with little performance loss.

9.2 Turbo Equalization

In this section we introduce the system model of Turbo equalization for single-carrier time-domain signals. The extension to the ICI in the frequency domain is straightforward. Figure 9.1 shows a simplified diagram of a communication system that uses Turbo equalization. The information bits are encoded by a convolutional encoder, permuted by an interleaver, and then modulated into a constellation alphabet \( \mathcal{A} \triangleq \{a_1, \ldots, a_J\} \), where \( J \) is the size of the alphabet. Assume the channel has \( L_h \) taps continuously placed with delay \( \tau[l] = l - 1 \), then the discrete samples of the received signal can be represented by

\[
y[n] = \sum_{l=1}^{L_h} h[l] x[n - l + 1] + w[n],
\]

where \( \{w[n]\} \) are the AWGN samples with variance \( \sigma_w^2 \). At the receiver, the iterative equalization and decoding process, namely “Turbo equalization”, is performed between the Demodulator & Equalizer and Convolutional Decoder, whose inputs and outputs are probabilities rather than hard-decisions as in conventional receivers. For the equalizer, the outputs are extrinsic probabilities for the modulated symbols \( \{p(x[n] = a_m|y), a_m \in \mathcal{A}\} \) based on the observation \( y \) and a priori information of the symbols given by the convolutional decoder from the previous iteration.

In this study, we assume perfect knowledge about the channel and noise variance is available to the receiver, and focus on the SISO equalization algorithm using the proposed GPF method.
9.3 Brief Review of Particle Filtering

We briefly review the conventional particle filtering technique in this section. Particle filtering is based on sequential importance sampling and Bayesian theory. The random samples that are drawn from the probability distribution of the variable space are called particles, and the strings of particles that are drawn sequentially in the temporal order are called trajectories. The underlying principle of particle filtering is to approximate the probability distributions with random measures composed of particles and their associated weights [99]. In the case of equalization, this approximation can be written as

\[
p(x[n - L_d + 1] = a_m | y_{1:n}) \approx \sum_{k=1}^{R} \nu_n^{(k)} \delta_{x_{n-L_d+1}^{(k)}, a_m},
\]

where \(L_d\) is the decision delay, \(k\) is the index of the trajectories, \(R\) is the number of particle trajectories, \(x_n^{(k)}\) is the particle drawn for the trajectory \(x_{1:n-1}^{(k)}\) at time instance \(n\) from a trial distribution \(\psi(x_{1:n-1}^{(k)}|y_{1:n})\). The weight \(\nu_n^{(k)} = \frac{p(x_n^{(k)}|y_{1:n})}{\psi(x_{1:n-1}^{(k)}|y_{1:n})}\) is the Radon-Nikodym derivative which is the ratio between the true distribution \(p(x_n^{(k)}|y_{1:n})\) and the trial distribution \(\psi(x_{1:n-1}^{(k)}|y_{1:n})\). The optimal trial distribution in terms of minimal variance of \(\nu_n^{(k)}\) conditionally on the particle trajectory \(x_{1:n-1}^{(k)}\) and the observation \(y_{1:n}\) is given by [134]

\[
\psi(x_{1:n}^{(k)}|y_{1:n}) = p(x[n]|x_{1:n-1}^{(k)}, y_{1:n})\psi(x_{1:n-1}^{(k)}|y_{1:n-1}) \\
\propto p(y[n]|x[n], x_{1:n-1}^{(k)}) p(x[n]).
\]

Let

\[
\mu_{n,m}^{(k)} \triangleq p(y[n]|x[n] = a_m, x_{1:n-1}^{(k)}) p(x[n] = a_m),
\]

Figure 9.1: Diagram of communication systems using Turbo equalization.
the particles can be drawn from an equivalent probability distribution

\[
q^{(k)}(x[n] = a_m) = \frac{\mu^{(k)}_{n,m}}{\sum_{m' = 1}^{J} \mu^{(k)}_{n,m'}}. \tag{9.5}
\]

Meanwhile, using (9.3), we can derive the formula for iterative updates of the weights \(v^{(k)}_n\) as

\[
v^{(k)}_n = \frac{p(x[n]|x^{(k)}_{1:n-1}, y_{1:n})p(y[n]|x^{(k)}_{1:n-1})p(x^{(k)}_{1:n-1}|y_{1:n-1})}{p(y[n]|y_{1:n-1})p(x[n]|x^{(k)}_{1:n-1}, y_{1:n})\psi(x^{(k)}_{1:n-1}|y_{1:n-1})} \propto p(y[n]|x^{(k)}_{1:n-1}) v^{(k)}_{n-1} = v^{(k)}_{n-1} \sum_{m=1}^{J} \mu^{(k)}_{n,m}. \tag{9.6}
\]

We summarize the calculation procedures of conventional particle filters as follows.

- For trajectory index \(k=1:K\),
  - Calculate \(\mu^{(k)}_{n,m}\) for the signal alphabet \(A\) with (9.4).
  - Extend the trajectory \(x^{(k)}_{1:n-1}\) with the new particle \(x^{(k)}_n\) from the distribution given by (9.5).
  - Update \(v^{(k)}_n\) with (9.6).
- Normalize trajectory weights \(\{v^{(k)}_n\}_{k=1}^{q}\).
- Compute the probabilities with (9.2).
- If the variance of weights \(\{v^{(k)}_n\}_{k=1}^{q}\) is over certain threshold, do re-sampling.

(a) Conventional particle filter

(b) Exact LMMSE [97]

Figure 9.2: Equalizer output histogram in Turbo schemes (SNR=7dB).
Figure 9.2 compares the output histogram of the conventional particle filter to that of the linear minimum mean square error (LMMSE) equalizer [97]. The particle filter equalizer employs maximum 50 trajectories of particles for the 5-tap channel, so the complexity has already exceeded that of the MAP equalizer. It shows that the histogram of the LMMSE starts to exhibit two peaks from the fifth iteration, while the conventional particle filter’s output are very close to hard-decisions even in the first few iterations. This significantly degrades the performance of the Turbo equalizer, and explains why the conventional particle filter does not work for Turbo receivers.

9.4 Grouped Particle Filter Equalizer

We define trajectory groups as follows. At time instance \( n \), for a given trajectory length \( L_g \geq L_h \), two trajectories \( x^{(k)} \) and \( x^{(l)} \) are in one trajectory group \( G \) if and only if they have the equal weights \( v_n^{(k)} = v_n^{(l)} \) and particles \( x^{(k)} = x^{(l)} \) for all \( \tau \in [n - L_g + 1, n - 1] \). This definition implies

- The number of non-empty groups must be smaller than the number of trajectories.
- Every trajectory group only has one weight, and all the particles in one group have no difference.

A diagram of the proposed GPF equalizer is shown in Figure 9.3 with an example we will explain later.

9.4.1 Sample Drawing

Taking advantage of the groups, we propose to assign \( N_m^{(G)} \) trajectories to pick up \( a_m \in A \) as their new particles rather than randomly drawing them individually for each trajectory. We can show that it not only saves considerable computational complexity, but also minimizes the bias from the true distribution if \( N_m^{(G)} \) is given by

\[
N_m^{(G)} = \left[ E \{ N_m^{(G)} \} \right] = \left[ N^{(G)} \frac{\mu_{n,m}^{(G)}}{\sum_{l=1}^{J} \mu_{n,l}^{(G)}} \right], \tag{9.7}
\]

where \([\cdot]\) denotes integer rounding function, and \( N^{(G)} \) denotes the number of trajectories in the group \( G \).
9.4.2 Group Merging

We propose to use group merging instead of re-sampling to maintain the efficiency of the particles. Similar to the merging of trellis in the MAP algorithm, the group merging operation is performed on multiple groups which have identical recent \((L_g - 1)\) particles. We use the merging of two groups as an example while merging of more groups is straightforward by recursively performing the procedure. The size and associated weight for the new group after merging \(H\) and \(G\) are given by

\[
\hat{N}^{(G)} = N^{(G)} + N^{(H)}
\]

\[
\hat{\nu}_n^{(G)} = \frac{\nu_n^{(G)} N^{(G)} + \nu_n^{(H)} N^{(H)}}{\hat{N}^{(G)} + N^{(H)}}.
\]

The group merging operation bounds the number of groups by \(J^{L_g - 1}\) after merging and \(J^{L_g}\) after sample drawing, where \(J\) is the size of the transmit signal alphabet. The merging operation also helps to suppress the sampling bias by increasing the number of trajectories in each group.

![Diagram of GPF with an example \((L_g = 4, A = \{-1, +1\})\).

Figure 9.3 gives an illustrative example of the sample drawing and group merging operations. Group 1 has 5 trajectories with weight 0.1, at time instance \(n\), we assign \(N_1^{(1)} = 5 \times q^{(1)}(-1) = 3\) trajectories to extend themselves with -1, and \(N_2^{(1)} = 5 \times q^{(1)}(1) = 2\) trajectories to extend with 1. Similarly, we assign two out of three trajectories in Group 2 with -1, and the other one with 1. At time
instance \((n+1)\), the new groups that have the same new particles \(x_n^{(i)}\) and originate from Group 1 and 2 respectively are merged together to become a new group whose weight is determined by (9.9), and the number of trajectories in that group is given by (9.8).

### 9.4.3 Computation Procedures

At time instance \(n\), suppose there are \(G\) groups, we summarize the procedures of the proposed GPF as follows.

- Merge all the groups that have identical recent \((L_g - 1)\) trajectories with (9.8) and (9.9), and update the number of groups \(G\).

- For group index \(G=1:G\),
  - Calculate \(\mu_{m,m}^{(G)}\) for the signal alphabet \(A\) with (9.4).
  - Update \(\nu_n^{(G)}\) as in (9.6).
  - Calculate \(N_m^{(G)}\) for the signal alphabet \(A\), and if \(N_m^{(G)}\) is not zero, generate a new group \(H\) as \(x_n^{(H)} = a_m, \nu_n^{(H)} = \nu_n^{(G)}, x_{n-L_g+1:n-1} = x_{n-L_g+1:n-1}^{(G)}\).

- Normalize group weights \(\{\nu_n^{(G)}\}_{G=1}^{G}\).

- Calculate the probability for the symbol \((n - L + 1)\) as

\[
p(x[n - L + 1] = a_m|y_{1:n}) \approx \sum_{G=1}^{G} \nu_n^{(G)} \delta_{x_{n-L+1:n-1}^{(G)}, a_m}.
\]

Comparing the procedures of the GPF to those of conventional particle filters, we can see that the basic idea of grouping is to avoid redundant computation on identical trajectories. The merging of groups can be seen as the re-sampling operation among identical trajectories, which averages the weights and sums up the numbers of trajectories to form a union of the groups.

Figure 9.4 compares the output histograms of the GPFs with different numbers of trajectories. When there are only 100 trajectories, the equalizer starts to output extreme values in the first few iterations, which is very similar to the behavior of conventional particle filters. It means that the GPF does not fundamentally alter the behavior of conventional particle filters, but improves their efficiency so that it is affordable to employ a huge number of trajectories to have truly “soft” estimates required by the Turbo receiver.
The T-Algorithm prunes the states whose normalized metrics are below a preset threshold \([132, 133]\), which is similar to the group pruning in the GPF that occurs when the number of trajectories in one group equals 0. However, in the GPF case, the threshold for group-pruning is not a fixed constant, but implicitly determined by \(N(G)\) and \(q(G)(x[n])\), which are calculated from the trial distributions for current and previous symbols. Therefore, the pruning criterion becomes more adaptive to the probability distributions of the trajectories, and may vary for different groups.

One extreme situation occurs when there are infinite number of trajectories and all new groups will be preserved. In this case, the number of groups will always reach the upper bound \(J^L_g\) after new particle drawing, and the complexity will be the same as that of the full-complexity MAP algorithm if \((L_g = L_h)\).

### 9.4.5 Early-Stopping Criteria

The efficiency of Turbo equalization can be improved by terminating the iterative process when further performance improvement is unlikely to be attained. A good indicator of the stopping time is the reliability of the decoder output, e.g., the mean of output log-likelihood ratio \([135]\). And, according to \([136]\), good early-stopping criteria should detect both early-convergence and non-convergence cases, because it is a waste of time and computational complexity to continue the iterative process in both cases.

For the GPF, the average number of groups can indicate the convergence of the iterative process quite well in most cases. Generally speaking, with more and more reliable \textit{a priori} information, the average number of groups should keep decreasing.
over iterations, and eventually becomes 1 because the highly reliable a priori information will enable one of the trajectories to dominate all the others. Denote the average number of groups at iteration $i$ as $G_i$, we propose to terminate the iteration if $(G_i < 1.1)$ or $(G_i - 1 < \eta_i)$, where $\eta_i$ is a preset threshold. These two criteria are designed to detect the convergence and non-convergence cases respectively.

9.5 EXIT Chart Analysis

We analyze the proposed GPF using the extrinsic information transfer (EXIT) chart [137] plotted in Figure 9.5. The curves of well-known full-complexity MAP and exact LMMSE [97] equalizers are also included for comparison. The signal to noise ratio (SNR) is set to 7dB, and the channel is taken from [138, p.631], namely “Proakis C”: $h = [0.227 0.460 0.688 0.460 0.227]^T$. As shown in [139], the Proakis C channel has severe frequency selectivity and simple interference cancellation schemes do not work very well in that channel. To validate the EXIT curves, we also plot mutual information trajectories obtained from Monte Carlo simulations in the figure. For the conciseness of notation, in the sequel we use $n$-GPF to denote the GPF that employs $n$ trajectories.

![Figure 9.5: EXIT chart analysis.](image-url)
It is shown that the $10^2$-GPF fails to work in such a severe ISI channel, and its mutual information trajectory does not agree with the EXIT curve very well. When the number of trajectories rises to $10^5$, the simulated mutual information trajectory matches the EXIT curve reasonably well, and we can predict from the curves that the performance of the $10^5$-GPF is somewhere between the full-complexity MAP detector and the exact LMMSE equalizer.

9.6 Numerical Results

We use a system model similar to that of [97] in the simulations for straightforward comparison. The coded bits are BPSK modulated and passed through the Proakis C channel. We set both the decision delay $L_d$ and group trajectory depth $L_g$ to the channel length $L_h$. The interleaver size is 32768 bits, and the (5,7) convolutional code is used for all the simulations.

Figure 9.6(a) compares the bit error rate (BER) performance of the proposed GPF to that of the full MAP and exact LMMSE equalizers. As predicted by the EXIT chart analysis, the $10^2$-GPF fails to work in this severe ISI channel. The $10^3$-GPF outperforms the LMMSE for SNRs up to 6dB, however, an error floor at $10^{-5}$ occurs for high SNRs. This is because with $10^3$ trajectories, the minimum representable probability by the particles is $10^{-3}$, which is not accurate enough to represent the particle drawing probabilities at high SNRs. When the number of trajectories rises to $10^5$, the performance of the GPF is significantly better than that of the LMMSE, and just slightly inferior to that of the MAP equalizer.

The computational complexity of conventional particle filters is proportional to the number of trajectories, while that of the GPF is proportional to the number of groups. Figure 9.6(b) plots the average number of groups over iterations at different SNRs. The maximum number of groups for this 5-tap channel is 32, however, the figure shows that at high SNR the actual complexity is much lower, which demonstrates the GPF’s adaptive computational effort. The figure also suggests that more trajectories do not necessarily lead to higher computational complexity after the first few iterations.

Employing the early-stopping criteria with $\eta_r = 0.002$ and limiting the maximum number of iterations to be within 15, we plot the BER performance of the GPF in Figure 9.7(a). All the curves of the GPFs with the early-stopping criteria are quite close to their counterparts whose numbers of iterations are fixed to 15. Figure 9.7(b) plots the average number of iterations as a function of SNR. It can
9.6 Numerical Results

(a) BER performance.

(b) Computational complexity.

Figure 9.6: Simulation results for GPF in Proakis C channel.
Figure 9.7: Simulation results for GPF using the early-stopping criteria.
be seen, by comparison with Figure 9.6(b), that in most cases the number of iterations is significantly reduced, so are the processing delay and overall complexity. When the iterative process starts to converge, which occurs at 4dB SNR for the GPFs, the early-stopping criteria do not terminate the iterative process because it is converging slowly and every iteration can improve the performance. All these behaviors match our expectation of an adaptive-effort equalizer.

Figure 9.8: Computational complexity of GPF using the early-stopping criteria.

We measure the computational complexity by the average number of groups processed for each block of data normalized by the complexity of the MAP algorithm. The results are shown in Figure 9.8 as a function of SNR. Because the average number of groups changes over iterations, the curves of Figure 9.8 do not proportionally follow that in Figure 9.7(b). It is also shown that the proposed early-stopping criteria reduce the complexity of the GPF in low SNR region where the numbers of groups usually remain high throughout the iterations. For high SNR, as the number of groups decreases to 1 rapidly, more iterations do not increase much complexity. Nevertheless, comparing to the MAP algorithm, the proposed method saves about 80% computational complexity in low and high SNR regions, and at least 35% in the medium SNR region where the convergence starts to occur.
9.7 Summary

The ICI in the frequency domain is the dual problem of the ISI in the time domain, so the equalization techniques developed in this chapter for single-carrier time-domain systems are generally applicable to OFDM systems without significant modification. We proposed a grouped particle filter for Turbo equalization schemes where the outputs of the equalizer are required to be probabilities rather than hard decisions. The novel particle filter improves the efficiency of the conventional methods by grouping the trajectories, and uses deterministic sample drawing and group merging techniques to allow a large number of particles without dramatically increasing the computational complexity. Two early-stopping criteria were developed to terminate the iterative process when further performance improvement is unlikely to be attained. In this way, both the processing delay and the overall complexity of the equalizer are significantly reduced. The simulation results agreed with the EXIT-chart analysis very well and confirmed the performance of the proposed methods.
Chapter 10

Conclusions and Suggestions for Future Work

10.1 Summary

This thesis has primarily focused on the synchronization algorithms for practical OFDM systems. The algorithms developed in this thesis are applicable to many existing and future OFDM systems, robust to challenging mobile wireless channel conditions, and feasible for hardware real-time implementation.

In Chapter 2 we described the mathematical modeling of the wireless channel and the OFDM system of interest. The classic wireless propagation and shadowing models were briefly reviewed, and the details about the channel models used in the simulations were provided. We generalized the definition of training symbols in practical OFDM standards such that they only comprise highly correlated but not necessarily identical segments in the time domain. The OFDMA uplink ranging channel was also introduced, and the timing positions of FFT windows were suggested to minimize ICI and ISI.

In Chapter 3 we proposed a universal coarse timing estimator that was robust to various channel conditions. The proposed method could take advantage of multiple training symbols composed of highly correlated but not necessarily identical segments. The new algorithm worked for all the preamble structures specified in the IEEE 802.11 [3] and IEEE 802.16 [4] standards, including the downlink of IEEE 802.16 OFDMA (WiMAX) systems where the training symbol consisted of three non-identical segments. Our analysis provided more insights into the proposed method and shed new light on existing works.

In Chapter 4 we suggested an method to accurately estimate the fractional part
of CFO from the generalized training symbols. The proposed estimator did not require the training symbol to have identical segments, and could take advantage of multiple training symbols to achieve a linear decrease in mean square error. The flexibility of the proposed method was illustrated by three schemes with different performance and complexity tradeoffs. For some training symbol structures, the proposed estimator outperformed existing methods with lower complexity.

In Chapter 5 we derived an approximated maximum likelihood joint channel, frequency and timing estimator, and then simplified that by different assumptions and approximations. To provide improved complexity and performance tradeoffs, we proposed a hybrid scheme that achieved satisfactory performance with relatively low computational complexity. When the actual channel length was different from the assumption, we proposed a successive timing estimator and its simplified version to solve the timing ambiguity problem. The performance analysis of the proposed methods agreed with the simulations reasonably well.

Chapter 6 gave detailed information about the proposed successive multiuser detector (SMUD) for contention based ranging channels. The new method took a path-by-path iterative approach to successively estimate the ranging users and their channels. The suppressed interference from the detected ranging users and their paths gave the low-power users a much better chance of correct detection. A simplified algorithm was also proposed to reduce the complexity and improve the efficiency of the SMUD. Besides the extraordinary user detection performance, the proposed algorithms were also able to mitigate the interference from ranging users to data subscriber stations, and thus considerably improve system data throughput.

Chapter 7 investigated the performance of the ranging detectors from a system point of view. More specifically, we translated the ranging detectors’ missed detection probabilities into the number of users a base station could serve with a given number of ranging opportunities, which was a more sensible performance measure for ranging bandwidth allocation. Our analytical results matched the simulations reasonably well and revealed that the proposed reduced-complexity SMUD could allow more than double the number of users in a cell compared to the conventional methods.

We analyzed the ICI caused by the channel impairments in Chapter 8 and derived the bounds of the power of residual ICI that could not be mitigated by the frequency-domain equalizer with a given number of taps. The results gave useful guidelines for equalizer design and simplified performance analysis. It was indicated that the power of ICI was inversely proportional to the number of equalizer taps,
which meant less and less performance gain could be achieved by increasing the number of taps.

Chapter 9 studied a new soft-input-soft-output particle filter that could be used in Turbo equalization schemes. The method was developed for single-carrier time-domain signals but also applicable to OFDM systems without significant modification. The proposed equalization method approached the performance of the Maximum A Posteriori (MAP) equalizer with lower complexity. The efficiency of the proposed method was further improved by an early-stopping algorithm.

10.2 Suggestions for Future Work

Constrained by the time frame of the PhD program, there are still many remaining and emerging synchronization problems in practical OFDM systems that have not been addressed in this thesis. A short but far from complete list of the problems that need to be investigated by future research are as follows.

10.2.1 Automatic Gain Control and Synchronization

Automatic gain control (AGC) in single-carrier systems usually is integrated in the RF circuit and realized by analog components. This approach can be used in OFDM systems [140], however, without the timing information provided by the digital circuits, a purely analog AGC can introduce unnecessary ICI by adjusting the gain in the middle of an OFDM symbol, which has a similarly adverse impact on the system as that of a time-varying channel. References [141] and [142] investigated the problem for the IEEE 802.11 wireless local-area networks and proposed to use state machines to align the AGC adjustments to the boundaries of OFDM symbols. This method works quite well in indoor environments where the terminal mobility is low and the channel fluctuation is relatively small, however, the joint AGC and synchronization design for outdoor OFDM systems with high-mobility is largely an open question.

Recently, we have developed a three-state joint AGC and synchronization algorithm and implemented it in the hardware of an IEEE 802.16 receiver. Initial field trial results are quite positive, however, more theoretical and experimental evidence is needed to justify the effectiveness of our approach and identify the possibility of further improvement.
10.2.2 Stochastic Filters for Synchronization

The synchronization algorithms discussed in this thesis are all based on a single OFDM burst, and as shown by our simulation results, the performance is limited by the time-varying nature of wireless communication channels. Due to the channel’s occasional deep fading, it is almost inevitable to see a number of preambles undetected by the receiver, which could interrupt the data reception for a considerably long period and require the receiver to establish the synchronization again.

A method to improve the robustness of communication links is to employ stochastic filters for synchronization. Stochastic filtering is a very general Bayesian framework for sequential estimation in a model-based setting, and for linear and Gaussian models, the densities being propagated have a closed-form solution and the result is simply the well-known Kalman filter. To analyze the synchronization problem in a stochastic framework, one can define the states of the receiver by the data that is being processed, i.e., preamble, normal data, and purely noise; then, the timing synchronization problem can be formulated into a classic state estimation problem for which the stochastic filters are designed. The well-established theories of stochastic filtering not only allow us to smooth the past estimates, but also to predict the timing positions of future bursts. Hence, even if the preambles of several OFDM bursts fail to reach the threshold, the receiver still can process the data in those bursts with predicted timing positions.

10.2.3 Training Symbols in Emerging Standards

There are emerging standards adopting OFDM modulation in their physical layers, e.g., the IEEE 802.15.3c [143], the 3GPP LTE [144], etc. At the time when this thesis is written, those standards have not been finalized and only the recommendations from various industrial players and research institutes are available to the general public. Because every standard is made for specific channel conditions under various constraints and practical considerations, there is a chance that the training symbol structure and random access channel design in existing standards are not suitable for direct adoption. Then, new research efforts are needed to provide novel solutions to the emerging standards and applications. As always, it is the new challenges that drive the advance of new technologies.
Acronyms

ADC analog-to-digital converter 27
AGC automatic gain control 27
AWGN additive white Gaussian noise 9, 33, 172

BER bit error rate 180
BLUE best linear unbiased estimator 10, 74
BPSK binary phase shift keying 9, 35, 125, 168, 180

BS base station 13, 26, 123, 149

CDMA code-division multiple access 17
CFO carrier-frequency offset 5, 89, 161

CH-A the channel model type A used in simulations, which follows the SUI-3 channel model 28, 67, 85

CH-B the channel model type B used in simulations, which follows the Vehicle-A power delay profile recommended by ITU.1225 and uses Jakes’ model to emulate the time-selectivity at 120km/h speed in a 3.5GHz-carrier OFDM system 28, 67, 85

CID connection identification number 149
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>cyclic prefix</td>
<td>2, 102, 123</td>
</tr>
<tr>
<td>CRB</td>
<td>Cramer-Rao bound</td>
<td>19, 81</td>
</tr>
<tr>
<td>DAB</td>
<td>digital audio broadcasting</td>
<td>1</td>
</tr>
<tr>
<td>DFT</td>
<td>discrete Fourier transform</td>
<td>90</td>
</tr>
<tr>
<td>Diff</td>
<td>differential ranging channel detector</td>
<td>150</td>
</tr>
<tr>
<td>DSS</td>
<td>data subscriber station</td>
<td>34, 124</td>
</tr>
<tr>
<td>DVB-T</td>
<td>terrestrial digital video broadcasting</td>
<td>1</td>
</tr>
<tr>
<td>EM</td>
<td>expectation maximization</td>
<td>11, 138</td>
</tr>
<tr>
<td>ESCA</td>
<td>extended Schmidl and Cox algorithm</td>
<td>10, 73</td>
</tr>
<tr>
<td>etc</td>
<td>et cetera</td>
<td>1</td>
</tr>
<tr>
<td>EXIT</td>
<td>extrinsic information transfer</td>
<td>179</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
<td>1, 89, 124</td>
</tr>
<tr>
<td>GAIC</td>
<td>generalized Akaike information criterion</td>
<td>12, 90</td>
</tr>
<tr>
<td>GHz</td>
<td>Gigahertz</td>
<td>4, 29</td>
</tr>
<tr>
<td>GPF</td>
<td>grouped Particle filter</td>
<td>23, 172</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz</td>
<td>29</td>
</tr>
<tr>
<td>ICI</td>
<td>inter-carrier interference</td>
<td>2, 123, 161, 171</td>
</tr>
<tr>
<td>IFFT</td>
<td>inverse fast Fourier transform</td>
<td>21, 90, 124</td>
</tr>
<tr>
<td>ISI</td>
<td>inter-symbol interference</td>
<td>1, 89, 123, 171, 180</td>
</tr>
<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
<td>28</td>
</tr>
<tr>
<td>kHz</td>
<td>kilohertz</td>
<td>5</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
<td>Page(s)</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>km/h</td>
<td>kilometers per hour</td>
<td>29</td>
</tr>
<tr>
<td>LS</td>
<td>least square</td>
<td>95, 112</td>
</tr>
<tr>
<td>MAI</td>
<td>multiple access interference</td>
<td>14, 123</td>
</tr>
<tr>
<td>MAP</td>
<td>Maximum A Posteriori</td>
<td>17, 172, 187</td>
</tr>
<tr>
<td>MC-CDMA</td>
<td>multi-carrier code-division multiple access</td>
<td>14, 123</td>
</tr>
<tr>
<td>MHz</td>
<td>Megahertz</td>
<td>4, 152</td>
</tr>
<tr>
<td>MMO</td>
<td>multiple-input-multiple-output</td>
<td>16</td>
</tr>
<tr>
<td>ML</td>
<td>maximum-likelihood</td>
<td>9, 112</td>
</tr>
<tr>
<td>MLE</td>
<td>maximum-likelihood estimator</td>
<td>94</td>
</tr>
<tr>
<td>MMSE</td>
<td>minimum mean square error</td>
<td>92</td>
</tr>
<tr>
<td>ms</td>
<td>millisecond</td>
<td>29, 152</td>
</tr>
<tr>
<td>MSAGE</td>
<td>modified space-alternating generalized expectation-maximization</td>
<td>124</td>
</tr>
<tr>
<td>MSE</td>
<td>mean square error</td>
<td>81</td>
</tr>
<tr>
<td>NL-RLS</td>
<td>non-linear recursive least squares algorithm</td>
<td>11</td>
</tr>
<tr>
<td>OFDM</td>
<td>orthogonal frequency-division multiplexing</td>
<td>1</td>
</tr>
<tr>
<td>OFDMA</td>
<td>orthogonal frequency-division multiple access</td>
<td>7, 123</td>
</tr>
<tr>
<td>PAPR</td>
<td>peak to average power ratio</td>
<td>4</td>
</tr>
<tr>
<td>pdf</td>
<td>probability density function</td>
<td>47, 100</td>
</tr>
<tr>
<td>ppb</td>
<td>parts per billion</td>
<td>6</td>
</tr>
<tr>
<td>ppm</td>
<td>parts per million</td>
<td>5</td>
</tr>
<tr>
<td>PSD</td>
<td>power spectral density</td>
<td>28</td>
</tr>
<tr>
<td>PSK</td>
<td>phase shift keying</td>
<td>42</td>
</tr>
<tr>
<td>PUSC</td>
<td>partial usage of subchannels</td>
<td>16</td>
</tr>
<tr>
<td>QAM</td>
<td>quadrature amplitude modulation</td>
<td>1, 60</td>
</tr>
<tr>
<td>QPSK</td>
<td>quadrature phase shift keying</td>
<td>60</td>
</tr>
<tr>
<td>RC-SMUD</td>
<td>reduced complexity successive multiuser detector</td>
<td>124, 150</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
<td>Page Numbers</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td>--------------</td>
</tr>
<tr>
<td>RSS</td>
<td>ranging subscriber station</td>
<td>14, 25, 33, 123, 150</td>
</tr>
<tr>
<td>SAGE</td>
<td>space-alternating generalized expectation-maximization</td>
<td>13, 124</td>
</tr>
<tr>
<td>SINR</td>
<td>signal to interference and noise ratio</td>
<td>137</td>
</tr>
<tr>
<td>SISO</td>
<td>soft-input-soft-output</td>
<td>16, 171</td>
</tr>
<tr>
<td>SMUD</td>
<td>successive multiuser detector</td>
<td>19, 124, 150, 186</td>
</tr>
<tr>
<td>SNR</td>
<td>signal to noise ratio</td>
<td>10, 27, 74, 179</td>
</tr>
<tr>
<td>SS</td>
<td>subscriber station</td>
<td>13, 26, 123, 149</td>
</tr>
<tr>
<td>SUD</td>
<td>single-user detector</td>
<td>135, 150</td>
</tr>
<tr>
<td>SUI</td>
<td>Stanford University Interim</td>
<td>28</td>
</tr>
<tr>
<td>TDD</td>
<td>Time Division Duplex</td>
<td>33</td>
</tr>
<tr>
<td>WiFi</td>
<td>wireless local area network</td>
<td>1</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide Inter-operability for Microwave Access</td>
<td>1, 26</td>
</tr>
</tbody>
</table>
## Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>⌈·⌉</td>
<td>the ceiling function that outputs the smallest integer not smaller than the argument</td>
<td>80</td>
</tr>
<tr>
<td>⌊·⌋</td>
<td>the floor function that outputs the largest integer not larger than the argument</td>
<td>79, 153</td>
</tr>
<tr>
<td>⌊·⌋</td>
<td>the rounding function that outputs the integer nearest to the argument</td>
<td>79, 175</td>
</tr>
<tr>
<td>⌊·⌋_1</td>
<td>the function that takes the fractional part of the argument</td>
<td>76</td>
</tr>
<tr>
<td>(·)*</td>
<td>complex conjugate function</td>
<td>128</td>
</tr>
<tr>
<td>(·)^H</td>
<td>the matrix conjugate transpose function</td>
<td>90</td>
</tr>
<tr>
<td>⊙</td>
<td>vector element-wise product</td>
<td>92, 125</td>
</tr>
<tr>
<td>α_1</td>
<td>the constant coefficient for polynomial approximation of the ICI bounds</td>
<td>166</td>
</tr>
<tr>
<td>α_2</td>
<td>the constant coefficient for polynomial approximation of the ICI bounds</td>
<td>166</td>
</tr>
<tr>
<td>β_0</td>
<td>a variable defined for the lower bound on ICI power as a function of the length of the equalization window</td>
<td>164</td>
</tr>
<tr>
<td>β_1</td>
<td>a variable defined for the upper bound on ICI power as a function of the length of the equalization window</td>
<td>164</td>
</tr>
<tr>
<td>β_2</td>
<td>a variable defined for the upper bound on ICI power as a function of the length of the equalization window</td>
<td>164</td>
</tr>
<tr>
<td>β</td>
<td>the coefficient vector that combines the component CFO estimates according to the BLUE principle</td>
<td>77</td>
</tr>
<tr>
<td>δ(·)</td>
<td>Dirac delta function</td>
<td>32</td>
</tr>
</tbody>
</table>
\( \delta_{i_1, i_2} \)  
Kronecker delta function that equals 1 when \( i_1 = i_2 \), 0 otherwise 79, 125

\( \Delta_\tau[l] \)  
the estimated channel tap relative delays 91

\( \Delta_f \)  
subcarrier spacing 30

\( \Delta_t[l] \)  
the relative delay of the \( i \)th channel path 27

\( \epsilon_0 \)  
the true carrier frequency offset normalized by the subcarrier spacing 33

\( \hat{\epsilon}_c(\hat{\tau}_i, d_{i,k}) \)  
the component fractional CFO estimate based on the \( i \)th training symbol and correlation interval \( d_{i,k} \) 76

\( \epsilon_k \)  
the \( k \)th subscriber station’s carrier frequency offset normalized by the subcarrier spacing 36

\( \hat{\epsilon}_k \)  
CFO estimate for RSS \( k \) 128

\( \hat{\epsilon}_0 \)  
the true residual CFO 90

\( \eta_i(d_{i,k}) \)  
the function that gives the performance loss of the component CFO estimators due to imperfect sampling 81

\( \eta_t \)  
the threshold for GPF early-stopping criteria 179

\( \gamma_i(n) \)  
the value of the correlation function for the most likely path at iteration \( i \) for the given ranging code \( n \) 130

\( \gamma(t) \)  
the wide-sense stationary stochastic process with zero mean and unit variance that describes the time-varying property of the channel 162

\( \mu_{n,m}^{(k)} \)  
the product of the \textit{a priori} probability of \( x[n] \) and probability of \( y[n] \) for the given \( x[n] \) and the trajectory \( x_{1:n-1}^{(k)} \) 173

\( \lambda_s \)  
the higher threshold used by the RC-SMUD algorithm for path validity check 131

\( \lambda_t \)  
the threshold to indicate whether the detected path is valid 112, 126, 131

\( \Lambda_A(\epsilon, \hat{\tau}) \)  
the cost function of the simplified joint channel and residual CFO and timing estimation method A 93
\(\Lambda_C(\epsilon, \hat{\tau})\) the cost function of the simplified joint channel and residual CFO and timing estimation method C

\(\Lambda_D(\epsilon)\) the cost function of the simplified integer CFO estimation method D

\(\Lambda_D(\epsilon, \hat{\tau})\) the cost function of the simplified integer CFO estimation method D, which actually is a function only about \(\epsilon\)

\(\Lambda(\epsilon, \hat{\tau})\) the cost function of the approximated maximum likelihood joint channel, residual CFO and timing estimation

\(\nu\) the speed of initial/handover RSSs

\(\{\omega_{i,k}\}\) the weighting coefficient for the \(k^{th}\) component timing metric in training symbol \(i\)

\(\Phi(\epsilon, M_q)\) defined to be \(\sum_{k>M_q}(\epsilon + k)^{-2} + (\epsilon - k)^{-2}\) and is a term in the expression of \(P_{ICI}(M_q)\)

\(\phi_i(d)\) the phase rotation of the product of signal terms in the \(R[\hat{\tau}_i, d]\)

\(\psi(x^{(k)}_{1:n}|y_{1:n})\) the trial distribution for the \(k^{th}\) trajectory based on the observation from time instance 1 to \(n\)

\(\psi_i(d)\) the difference in the phase of the correlator and that resulted from the true CFO

\(\Psi(T_s, f_d)\) a function defined for the derivation of ICI power bounds which reflects the impact of channel power spectrum

\(\Psi\) the component CFO estimation error correlation matrix

\(\Psi_{i}\) the component CFO estimation error correlation matrix of the \(i^{th}\) training symbol

\(\rho_i(d)\) the magnitude of the product of signal terms in the \(R[\hat{\tau}_i, d]\)

\(\bar{\sigma}_{i,k}^2\) the variance of the component timing metric \(T_c[\tau, d_{i,k}]\) at an incorrect timing position

\(\sigma_i^2\) the average received power of the samples in training symbol \(i\) excluding the cyclic prefix
\( \tilde{\sigma}_w^2 \) the average noise plus interference variance on each subcarrier allocated to the ranging opportunity

\( \sigma_w^2 \) the variance of the sampled additive white Gaussian noise

\( \bar{\tau} \) an incorrect timing position at the \( L_h \) iteration of the successive joint channel and timing estimator

\( \bar{\tau}_i \) a timing position that is far away from the ideal timing position of training symbol \( i \)

\( \hat{\tau}_i \) the estimated starting position for the \( i^{th} \) training symbol

\( \check{\tau}_i \) the ideal timing for the \( i^{th} \) training symbol

\( \check{\hat{\tau}}_i \) the estimated channel tap delay at the \( i^{th} \) iteration for the successive joint channel and timing estimator

\( \hat{\tau}_{k}[l] \) the estimated timing offset for the \( l^{th} \) tap of RSS \( k \)

\( \tau_{k}[l] \) the delay of the \( l^{th} \) channel tap of subscriber station \( k \) in terms of OFDM samples

\( \tau[l] \) the delay of the \( l^{th} \) channel tap in terms of OFDM samples

\( \tau_{\text{max}} \) the absolute value of the maximum timing offset for synchronized subscriber stations

\( \hat{\tau}_0[l] \) the delay of the \( l^{th} \) channel tap relative to the residual timing offset

\( \hat{\tau}_i \) the timing offset of the most likely valid path detected at iteration \( i \)

\( \check{\hat{\tau}}_i(n) \) the timing offset of the most likely path at iteration \( i \) for the given ranging code \( n \)

\( \hat{\theta}_i \) the variance of the BLUE based on the training symbol \( i \)

\( \Theta(\tau,d) \) the product of noise and signal terms in the correlator \( R[\tau,d] \)

\( \check{\hat{\tau}}_{n}^{(G)} \) the weight of the trajectories in group \( G \) after merging
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varpi )</td>
<td>the summation of the variance of weighted component timing metrics, which determines the exponential decay rate of the false alarm probability</td>
</tr>
<tr>
<td>( \varpi_i )</td>
<td>the combined effect of AWGN, ICI and wrongly detected invalid paths at iteration ( i )</td>
</tr>
<tr>
<td>( \vartheta_{i,k} )</td>
<td>the ratio of the residual channel estimation error to the original channel</td>
</tr>
<tr>
<td>( \nu^{(k)}_{m} )</td>
<td>the Radon-Nikodym derivative, which is the ratio between the true distribution ( p(x^{(k)}_{1:n}</td>
</tr>
<tr>
<td>( \chi )</td>
<td>channel shadowing factor</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>the signal to noise ratio per subcarrier at the ( L^{th} ) iteration of the successive joint channel and timing estimator</td>
</tr>
<tr>
<td>( a_k )</td>
<td>the power level of RSS ( k )</td>
</tr>
<tr>
<td>( a_m )</td>
<td>the ( m^{th} ) constellation point in the alphabet</td>
</tr>
<tr>
<td>( a_k )</td>
<td>the component from the signal on subcarrier ( m - k ) to the signal on subcarrier ( m )</td>
</tr>
<tr>
<td>( \mathcal{A} )</td>
<td>the constellation alphabet of the modulation scheme</td>
</tr>
<tr>
<td>( \bar{b} )</td>
<td>an invalid path in the ranging opportunity</td>
</tr>
<tr>
<td>( \bar{b}_k )</td>
<td>the linearly indexed valid ranging signal path</td>
</tr>
<tr>
<td>( b(n, \tau) )</td>
<td>the observed ranging signal of code ( n ) and timing offset ( \tau ) without any channel impairment</td>
</tr>
<tr>
<td>( \tilde{b}_i )</td>
<td>the most likely valid path detected at iteration ( i )</td>
</tr>
<tr>
<td>( B_i )</td>
<td>the matrix of all detected paths at iteration ( i )</td>
</tr>
<tr>
<td>( c_n[n] )</td>
<td>the time domain samples of the ranging code indexed ( n )</td>
</tr>
<tr>
<td>( C_F )</td>
<td>path loss correction factor for urban and suburban areas</td>
</tr>
<tr>
<td>( C_{f}(h_m) )</td>
<td>mobile station antenna correction factor</td>
</tr>
<tr>
<td>( C_{hh} )</td>
<td>the statistical channel correlation matrix</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$C_n$</td>
<td>the $n^{th}$ pseudo-random ranging code</td>
</tr>
<tr>
<td>$C_D$</td>
<td>the average complexity of the $D$ matrix computation for the joint estimation method A</td>
</tr>
<tr>
<td>$d$</td>
<td>the correlation interval that defines a correlator</td>
</tr>
<tr>
<td>$d_m$</td>
<td>the distance between the mobile terminal and base station</td>
</tr>
<tr>
<td>$d_{max}$</td>
<td>the maximum delay of a RSS in one cell</td>
</tr>
<tr>
<td>$\hat{d}_i$</td>
<td>the correlation interval for the best component CFO estimator based on training symbol $i$</td>
</tr>
<tr>
<td>$d_{i,k}$</td>
<td>the correlation interval for the $k^{th}$ component estimator based on training symbol $i$</td>
</tr>
<tr>
<td>$D$</td>
<td>the matrix defined for the joint channel and residual CFO and timing estimation</td>
</tr>
<tr>
<td>$D_B$</td>
<td>the matrix defined for the joint channel and residual CFO and timing estimation method B</td>
</tr>
<tr>
<td>$\text{erfc}(x)$</td>
<td>the complementary error function $\text{erfc}(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2} dt$</td>
</tr>
<tr>
<td>$E_t$</td>
<td>the threshold for the successive joint channel and timing estimator and ranging channel detector that gives the minimal input signal strength for current iteration</td>
</tr>
<tr>
<td>$f_c$</td>
<td>carrier frequency</td>
</tr>
<tr>
<td>$f_d$</td>
<td>maximum Doppler frequency of the channel</td>
</tr>
<tr>
<td>$f_{(ra)}(\cdot</td>
<td>\sigma^2)$</td>
</tr>
<tr>
<td>$f_{(ri)}(\cdot</td>
<td>u,\sigma^2)$</td>
</tr>
<tr>
<td>$f_Z(\cdot)$</td>
<td>the probability density function of the given random variable $Z$</td>
</tr>
</tbody>
</table>
**Notation**

\[ F(\epsilon) \]
the vector of time-domain sample phase rotations as a result of the residual frequency offset

\[ \mathcal{F} \]
the set of most likely integer CFO hypotheses outputted by the Focus step in the hybrid joint CFO and timing estimator

\[ G(\tau) \]
the vector of subcarrier phase rotations as a result of the residual timing offset

\[ G \]
the trajectory group \( G \)

\[ \mathcal{G} \]
the number of trajectory groups

\[ \mathcal{G}_i \]
the average number of groups at iteration \( i \)

\( h_b \)
base station antenna height

\( \hat{h}_k \)
the channel of the linearly indexed valid ranging signal path

\( \hat{h}_k[l] \)
the estimated channel gain for RSS \( k \) and path \( l \) using the first ranging OFDM symbol in the ranging opportunity

\( h_k[l](t) \)
the time-varying complex gain for the \( l^{th} \) channel tap at time instance \( t \) for subscriber station \( k \)

\( h[l](t) \)
the time-varying complex gain for the \( l^{th} \) channel tap at time instance \( t \)

\( h_m \)
mobile terminal antenna height

\( \hat{h}(n, \tau) \)
the channel for the path of active ranging code \( n \) whose timing offset is \( \tau \)

\( \hat{h}_i(\tau) \)
the single path channel estimate for the given \( \tau \) at the \( i^{th} \) iteration of the successive joint channel and timing estimator

\( \hat{h}_k[l] \)
the estimated complex gain for the \( l^{th} \) channel tap of RSS \( k \)

\( \mathbf{h} \)
the vector of the true channel complex gains

\( \hat{\mathbf{h}} \)
the estimated channel from the first ranging symbol based on the paths detected from the second symbol
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{h}_i )</td>
<td>the estimated channel at iteration ( i )</td>
</tr>
<tr>
<td>( \hat{h}(\hat{\epsilon}, \hat{\tau}) )</td>
<td>the maximum likelihood channel estimate under the given residual CFO estimate ( \hat{\epsilon} ) and timing estimate ( \hat{\tau} )</td>
</tr>
<tr>
<td>( \hat{H}(\hat{\epsilon}) )</td>
<td>the least square channel estimate under the given residual CFO estimate ( \hat{\epsilon} )</td>
</tr>
<tr>
<td>( \tilde{H}_i )</td>
<td>the input to the Path Detector of the successive joint channel and timing estimator at iteration ( i )</td>
</tr>
<tr>
<td>( I_p )</td>
<td>the number of training symbols in the preamble</td>
</tr>
<tr>
<td>( I_r )</td>
<td>the number of OFDM symbols in one ranging opportunity</td>
</tr>
<tr>
<td>( I_0(\cdot) )</td>
<td>the modified Bessel function of the first kind with order zero</td>
</tr>
<tr>
<td>( \mathcal{I} )</td>
<td>the number of iterations that have valid paths detected</td>
</tr>
<tr>
<td>( \Im(\cdot) )</td>
<td>the function that takes the imaginary part of its complex argument</td>
</tr>
<tr>
<td>( \mathcal{I}_{max} )</td>
<td>the maximum number of iterations for the successive estimators and detectors</td>
</tr>
<tr>
<td>( J )</td>
<td>the size of constellation alphabet</td>
</tr>
<tr>
<td>( \hat{K}_r )</td>
<td>the estimated number of RSSs</td>
</tr>
<tr>
<td>( K_c )</td>
<td>the total number of SSs in one cell</td>
</tr>
<tr>
<td>( K_d )</td>
<td>the number of DSSs sharing the OFDM symbols of the ranging opportunity</td>
</tr>
<tr>
<td>( K_i )</td>
<td>the number of component timing metrics for training symbol ( i )</td>
</tr>
<tr>
<td>( K_{new}^{(m)} )</td>
<td>the number of new RSSs in ranging opportunity ( m )</td>
</tr>
<tr>
<td>( K_r )</td>
<td>the number of active RSSs in the ranging opportunity</td>
</tr>
</tbody>
</table>
### Notation

- $K_{rem}^{(m)}$: the number of remaining RSSs in ranging opportunity $m$
- $K_r^{(m)}$: the total number of RSSs in ranging opportunity $m$
- $K_t$: the maximum average number of new RSSs in one ranging opportunity
- $\hat{\mathcal{R}}$: the number of particle trajectories for conventional particle filters
- $L_0$: the total number of valid channel paths of the ranging signals in one ranging opportunity
- $\hat{L}_k$: the estimated number of channel taps for RSS $k$
- $L_d$: the decision delay of the equalizer
- $L_g$: the length of the trajectories
- $L_h$: the number of channel taps
- $L_h[k]$: the number of channel taps for subscriber station $k$
- $L_i$: the number of highly correlated time-domain segments in the $i^{th}$ training symbol
- $L_{max}$: the maximum relative delay of a RSS’s channel taps
- $\max(\cdot)$: the function of taking the maximum of the arguments
- $M_f$: the number of possible integer CFOs in the estimation range
- $\tilde{M}_f$: the number of most likely integer CFO hypotheses given by the Focus step in the hybrid estimation method
- $M_t$: the number of possible residual timing offsets
- $M_D$: the number of OFDM packets between every two updates of the channel statistical information
\( M_q \) the number of neighboring subcarriers on each side of the equalization window 163

\( \hat{n}_k \) the estimated ranging code index for RSS \( k \) 127

\( \bar{n}_i \) the index of the pseudo-random ranging code of the most likely valid path detected at iteration \( i \) 126

\( n_k \) the index of the pseudo-random ranging code selected by RSS \( k \) 35

\( N \) the number of subcarriers in each OFDM symbol 11, 30, 150

\( N_c \) the number of ranging codes 35, 125, 150

\( N_g \) the length of the cyclic prefix in terms of OFDM samples 31, 152

\( N^G \) the number of trajectories in group \( G \) 175

\( N^G_m \) the number of trajectories in group \( G \) whose new particles are \( a_m \) 175

\( \tilde{N}^G \) the number of trajectories in group \( G \) after merging 176

\( N_{p[i]} \) the number of used subcarriers in OFDM symbol \( i \) 30

\( N_{p[i, k]} \) the number of used subcarriers allocated to subscriber station \( k \) in the \( i^{th} \) OFDM symbol 35

\( N_r \) the number of OFDM subcarriers in one ranging opportunity 33, 125, 150

\( N_{succ} \) the number of successful initial ranging required for every handover 151

\( \mathcal{N}(u, \sigma^2) \) a Gaussian distribution with mean \( u \) and variance \( \sigma^2 \) 26

\( o_{i,k} \) the mapping from the logical to physical subcarriers allocated to DSS \( k \) in the \( i^{th} \) OFDM symbol 35

\( o^{(r)} \) the mapping from the logical to physical subcarriers allocated to the ranging opportunity 35
\( o_i \) the mapping from the logical to physical sub-carriers in the \( i^{th} \) OFDM symbol

\( p_Z(m|a) \) the possibility for all \( Z_k (k \leq m) \) to be less than 0 conditionally on the given channel gain \( a^2 \)

\( P_c^{(\hat{\tau}_0 | 1)} \) the probability to detect the first channel tap at the \( L_h^{th} \) iteration for the successive joint channel and timing estimator

\( P_c^{(D)} \) the analytical integer CFO estimation error probability for the Method D

\( \hat{P}_t \) the target probability for a detected path to be invalid at one iteration for the successive joint channel and timing estimator

\( P(f) \) the power spectrum density function characterizing the time-varying property of the channel

\( \hat{P}_{fa} \) the target false alarm probability for threshold settings

\( P_{ICI}(M_q) \) the ICI power out of the \((2M_q + 1)\)-long equalization window

\( q^{(k)}(x[n] = a_m) \) the probability distribution from which the \( n^{th} \) particle of trajectory \( k \) is drawn

\( r_i \) the value of the correlation function for the most likely path at iteration \( i \)

\( \hat{r}_i(\tau) \) the normalized channel gain for the given \( \tau \) at the \( i^{th} \) iteration of the successive joint channel and timing estimator

\( R(\tau, d) \) the \( N \)-sample long correlator starting at sample \( \tau \) with interval \( d \)

\( R_{B,i}[\hat{\tau}_i] \) the intermediate variable defined for the derivation of the coarse timing missed detection probability for the Case B

\( R_c \) the cell radius
\( \mathcal{R}(\cdot) \) the function that takes the real part of its complex argument 58, 76, 133

\( \mathcal{R}(n, \tau | \tilde{Y}_{2,i}) \) the correlation function defined for the path \( b(n, \tau) \) and observed ranging signal \( \tilde{Y}_{2,i} \) 126

\( s(t) \) the unit step function 30, 162

\( \text{sinc} (\cdot) \) the sinc function: \( \text{sinc}(x) \triangleq \frac{\sin(\pi x)}{(\pi x)} \) 164

\( \text{SNR}_i \) the signal to noise ratio for training symbol \( i \) 42

\( \text{SINR} \) the target SINR for threshold setting 137

\( |S_i| \) the number of elements in the set \( S_i \) 138

\( S_i \) the set of ranging codes that are likely to be active based on the MAI mitigated ranging signal at iteration \( i \) 130

\( t_i \) the start time of the \( i^{th} \) OFDM symbol 30

\( t_s \) the duration of one OFDM sample 30

\( T_{bks} \) the start time of ranging backup window 150

\( T_{bkw} \) the end time of ranging backup window 150

\( T_c[\tau, d_{i,k}] \) the component timing metric defined by the correlation interval \( d_{i,k} \) 44

\( T_{exp} \) the expiry time for the response to a ranging attempt 150

\( T_{rng} \) the nominal periodical ranging interval 150

\( T[\tau] \) the proposed timing metric that linearly combines the component timing metrics to minimize the false alarm probability 50

\( T_{op} \) the time interval between every two ranging opportunities 152

\( T_s \) the duration of one OFDM symbol 30, 67, 85

\( v(\tau) \) the vector that depicts the phase rotating effect on the subcarriers in the ranging opportunity resulted from the timing offset \( \tau \) 125
\( \tilde{V}_i \) the truncated discrete Fourier transform matrix using the estimated channel tap delays at the \( i^{th} \) iteration for the successive joint channel and timing estimator

\( V \) the truncated discrete Fourier transform matrix with true channel tap relative delays

\( \hat{V} \) the truncated discrete Fourier transform matrix with estimated channel tap relative delays

\( w[n] \) the \( n^{th} \) sampled additive white Gaussian noise

\( \hat{w}[n] \) the AWGN noise samples following the same statistical properties as \( w[n] \)

\( \tilde{w}_2 \) the noise and interference terms combined in the ranging signal

\( w \) the AWGN noise samples falling in an FFT window

\( W \) the truncated discrete Fourier transform matrix

\( x_i(t) \) the waveform of the transmit signal for the \( i^{th} \) OFDM symbol

\( x[n] \) the transmit signal sampled at time interval \( t_s \)

\( \hat{x}[n] \) the sampled transmit signal after the multi-path channel

\( x_n^{(k)} \) the particle of trajectory \( k \) drawn at time instance \( n \)

\( x(t) \) the waveform of an OFDM transmit signal

\( x_{1:n-1}^{(k)} \) the \( k^{th} \) trajectory of the particles drawn from time instance 1 to \( n - 1 \)

\( X_i \) the data modulated onto the subcarriers of OFDM symbol \( i \)

\( X_{i,k} \) the vector of data symbols modulated onto the subcarriers of the \( i^{th} \) OFDM symbol by DSS \( k \)
\( X_k \) the vector of a ranging code modulated onto the subcarriers of the ranging opportunity by RSS \( k \)

\( \hat{y}[n] \) reconstructed time-domain ranging signal with the estimated transmission parameters

\( y \) the received signal samples falling in an FFT window

\( y_{1:n} \) the observation from time instance 1 to \( n \)

\( \tilde{Y}_2 \) the observed signal on the subcarriers of the second OFDM symbol allowed to the ranging opportunity

\( Y_i \) the observation of all the subcarriers in the \( i^{th} \) OFDM symbol of the ranging opportunity

\( Y(\hat{\epsilon}) \) the vector of the received signal in the frequency domain under the given residual CFO estimate \( \hat{\epsilon} \)

\( \tilde{Y}_{2,i} \) the observed ranging signal projected to the null space of all detected paths at iteration \( i \)

\( z_m \) the difference of the Method D’s cost function between two adjacent integer CFO hypotheses \( mL_1 \) away from the true CFO

\( Z_m \) the difference of the Method D’s cost function between the true CFO and the hypothesis \( mL_1 \) subcarriers away from it

\( Z_A[\hat{\tau}_1] \) the intermediate variable defined for the derivation of the coarse timing missed detection probability for the Case A

\( Z_B[\hat{\tau}_1] \) the intermediate variable defined for the derivation of the coarse timing missed detection probability for the Case B

\( Z_C[\hat{\tau}_1] \) the intermediate variable defined for the derivation of the coarse timing missed detection probability for the Case C
References


[57] Darryl Dexu Lin, Ryan A. Pacheco, Teng Joon Lim, and Dimitrios Hatzinakos, “Joint estimation of channel response, frequency offset, and phase


[66] H. Minn and V.K. Bhargava, “A joint time and frequency synchronization and channel estimation for OFDM systems using one training symbol,” in
References


[84] Yue Zhou, Zhaoyang Zhang, and Xiangwei Zhou, “OFDMA initial ranging for IEEE 802.16e based on time-domain and frequency-domain approaches,”


