Applications of Continuous Spatial Models in Multiple Antenna Signal Processing

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Declaration

The contents of this thesis are the results of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work in this thesis has been published or submitted for publication as journal papers or conference proceedings. These papers are:

- G. Dickins and L. W. Hanlen, "Fast calculation of singular values for MIMO wireless systems," in *Proceedings of the 5th Australian Communications Theory Workshop*, (Newcastle, Australia), pp. 185–190, 2004.
- G. Dickins and L. W. Hanlen, "On finite dimensional approximation in MIMO," in *Proceedings of the 11th Asia-Pacific Conference on Communications APCC2005*, (Perth, Australia), pp. 710–714, 2005.
- M. I. Y. Williams, G. Dickins, R. A. Kennedy and T. D. Abhayapala, "Spatial Limits on the Performance of Direction of Arrival Estimation", in *Proceedings of the 6th Australian Communications Theory Workshop*, (Brisbane, Australia), pp. 189–194, 2005.
- G. Dickins, M. I. Y. Williams and L. W. Hanlen, "On the dimensionality of spatial fields with restricted angle of arrival," in *Proceedings of the IEEE International Symposium on Information Theory*, (Adelaide, Australia), pp. 1033–1037, 2005.
- M. I. Y. Williams, G. Dickins, R. A. Kennedy, T. S. Pollock and T. D. Abhayapala, "A Novel Scheme for Spatial Extrapolation of Multipath," in *Proceedings of the 2005 Asia-Pacific Conference on Communications*, (Perth, Australia), pp. 784–787, 2005.
- G. Dickins, T. Betlehem and L. W. Hanlen, "A Stochastic MIMO Model Utilising Spatial Dimensionality and Modes," in *Proceedings of the IEEE Vehicular Technology Conference*, (Melbourne, Australia), 2006.

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- G. Dickins, M. I. Y. Williams and R. A. Kennedy, "Spatial limits to direction of arrival estimation," *Submitted to IEEE Signal Processing Letters*, 2006.

The research represented in this thesis has been performed jointly with Professor Rodney A. Kennedy, Dr. Leif W. Hanlen, Dr. Terence Betlehem and M. I. Y. Williams. The substantial majority of this work is my own.

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Abstract

This thesis covers the investigation and application of continuous spatial models for multiple antenna signal processing. The use of antenna arrays for advanced sensing and communications systems has been facilitated by the rapid increase in the capabilities of digital signal processing systems. The wireless communications channel will vary across space as different signal paths from the same source combine and interfere. This creates a level of spatial diversity that can be exploited to improve the robustness and overall capacity of the wireless channel. Conventional approaches to using spatial diversity have centered on smart, adaptive antennas and spatial beam forming. Recently, the more general theory of multiple input, multiple output (MIMO) systems has been developed to utilise the independent spatial communication modes offered in a scattering environment.

Underlying any multiple antenna system is the basic physics of electromagnetic wave propagation. Whilst a MIMO system may present a set of discrete inputs and outputs, each antenna element must interact with the underlying continuous spatial field. Since an electromagnetic disturbance will propagate through space, the field at different positions in the space will be interrelated. In this way, each position in the field cannot assume an arbitrary independent value and the nature of wave propagation places a constraint on the allowable complexity of a wave-field over space. To take advantage of this underlying physical constraint, it is necessary to have a model that incorporates the continuous nature of the spatial wave-field.

This thesis investigates continuous spatial models for the wave-field. The wave equation constraint is introduced by considering a natural basis expansion for the space of physically valid wave-fields. This approach demonstrates that a wave-field over a finite spatial region has an effective finite dimensionality. The optimal basis for representing such a field is dependent on the shape of the region of interest and the angular power distribution of the incident field. By applying the continuous spatial model to the problem of direction of arrival estimation, it is shown that the spatial region occupied by the receiver places a fundamental limit on the number and accuracy with which sources can be resolved. Continuous spatial models also provide a parsimonious representation for modelling the spatial communications channel independent of specific antenna array configurations. The continuous spatial model is also applied to consider limits to the problem of wireless source direction and range localisation.

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Notation, Symbols and Acronyms

Symbol	Definition
a_p	source amplitude
$\boldsymbol{a}(\theta)$	array response vector for direction θ
$\mathbf{A}$ $\mathbf{A}(\boldsymbol{\theta})$	array response matrix
$\mathbb{B}_R^2$	two-dimensional ball (disc) of radius R
$\mathbb{B}_R^3$	three-dimensional ball (sphere) of radius R
C_m	array response scaling constant for mode m
$ds(\widehat{\boldsymbol{\theta}})$	surface area element of $\mathbb{S}^2$
D	essential dimensionality
$\mathbf{D}(\boldsymbol{\theta})$	derivative of array response matrix
$e^{\cdots}$	exponent function
$E\{\cdots\}$	expectation of a random process
$\mathcal{F}$	space of far-field angular distributions
$g(\widehat{\boldsymbol{\theta}})$ $g(\theta)$	angular domain representation function
$g_n(\widehat{\boldsymbol{\theta}})$ $g_n(\theta)$	angular domain basis function
$g_N(\widehat{\boldsymbol{\theta}})$ $g_N(\theta)$	truncated finite dimensional angular domain representation
$\mathbf{H}$	MIMO channel matrix
$\mathbf{H}_S$	modal channel matrix
$H_n(\cdot)$	Hankel function of order n
$\mathbf{I}$	identity matrix
$j = \sqrt{-1}$	imaginary number
$j_n(\cdot)$	spherical Bessel function of order n
$J_n(\cdot)$	Bessel function of order n
$\mathbf{J}_R$	receiver modal configuration matrix
$\mathbf{J}_T$	transmitter modal configuration matrix
$k = 2\pi/\lambda$	wave number
$\log(\cdot)$	logarithm

Symbol	**Definition**
m	mode or other integer index
M	maximum mode or truncation order
M_R	mode truncation for receiver region
M_T	mode truncation for transmitter region
n	time sample or other integer index
N	number of time samples or max integer index
N_R	number of receive antennas
N_T	number of transmit antennas
p	source number integer index
P	number of sources
$P(\widehat{\boldsymbol{\theta}})$	angular power spectrum
q	sensor number integer index
Q	number of sensors
r	radius
R	maximum radius of region
$\mathbf{R}$	covariance matrix
$\mathbb{R}$	real numbers
$\mathbb{R}^2$	two-dimensional space
$\mathbb{R}^3$	three-dimensional space
$\mathrm{sinc}(\cdot)$	sinc function $= \sin(z)/z$
$s_p(n)$	source signal sample
$\boldsymbol{s}(n)$	vector of source signal samples
S	minimum radius of sources
$\mathbb{S}^1$	unit sphere in $\mathbb{R}^2$, equivalent to $[0, 2\pi]$
$\mathbb{S}^2$	unit sphere in $\mathbb{R}^3$
$\mathcal{S}$	space of spatial fields
t	continuous time
T	maximum time
$u(\boldsymbol{x})$	spatial field
$u(\boldsymbol{x}, n)$	spatial field at time sample n
$\mathcal{U}$	general functional space
$w_q(n)$	sensor noise sample
$\mathbf{W}$	modal coupling weight matrix
$\boldsymbol{w}(n)$	sensor noise vector
$\boldsymbol{x}$ $\boldsymbol{x}'$	position vector

Symbol	Definition
$y \;\; y_q$	output signal
$\boldsymbol{y} \;\; \boldsymbol{y}(n)$	vector of output signals
Y_n	Neumann function of order n
z	arbitrary argument
$z_m(n)$	modal output signal
$\boldsymbol{z}(n)$	vector of modal output signals
α_n	coefficient of expansion
β_n	basis function
$\Gamma(\cdot)$	Gamma function
$\delta(\cdot)$	Dirac delta function
δ_{mn}	Kronecker delta function
ε	error or small perturbation
σ^2	noise variance
λ	wavelength
λ_n	eigenvalue
Λ	spatial region of interest
Ω	range of angular domain $P(\widehat{\boldsymbol{\theta}}) \neq 0$
$\rho(\boldsymbol{x}, \boldsymbol{y})$	spatial correlation function
θ	angle in $[0, 2\pi]$
$\theta_{\boldsymbol{x}}$	polar co-ordinate angle for point $\boldsymbol{x}$
θ_p	direction of source p
$\widehat{\boldsymbol{\theta}}$	unit vector on $\mathbb{S}_2$
$\widehat{\boldsymbol{\theta}}_{\boldsymbol{x}}$	unit vector in direction of point $\boldsymbol{x}$
ϕ	angle in $[0, 2\pi]$
$\widehat{\boldsymbol{\phi}}$	unit vector on $\mathbb{S}_2$
$\triangle$	Laplacian operator
$\lVert\cdot\rVert_{\mathcal{U}}$	norm of element in indicated space
$\lvert \cdot \rvert$	absolute value of argument
$\langle \cdot, \cdot \rangle_{\mathcal{U}}$	inner product of elements in indicated space
$[a, b] \;\; (a, b)$	closed / open interval from a to b
$\odot$	Schur-Hadamard elementwise matrix product
$\otimes$	Kronecker matrix product
$.^T$	transpose of matrix or vector
$.^H$	Hermetian (conjugate) transpose of matrix
$.^{-1}$	matrix inverse
$\overrightarrow{\cdot}$	vectorise operation – stack matrix columns

Acronym	**Meaning**
APS	Angular Power Spectrum
AWGN	Additive White Gaussian Noise
BLAST	Bell Labs Layered Space Time communications system
CRB	Cramér-Rao Bound for variance of parameter estimation
DOA	Direction of Arrival
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Techniques
MIMO	Multiple-Input, Multiple-Output communications system
MMSE	Minimum Mean Squared Error
MUSIC	Multiple Signal Classification algorithm
RMS	Root Mean Squared average
SNR	Signal to Noise Ratio
UCA	Uniform Circular Array
ULA	Uniform Linear Array
WSSUS	Wide Sense Stationary Uncorrelated Scatter statistical model

Chapter 1

Introduction

Any sufficiently advanced technology is indistinguishable from magic.
Arthur C. Clarke, 1961.

1.1 History and Background

For most of history, the ability of people to communicate without any physical connection was nothing but a magical fantasy. In 1865, James Clerk Maxwell published a seminal work showing that "*an electromagnetic disturbance in the form of waves*" could propagate through space [1]. This inspired work by Hertz, Marconi and Tesla that lead to the demonstration of wireless communication over significant distances at the dawn of the twentieth century.

The concept of the mobile telephone emerged in 1947, with commercial systems becoming available in the early 1980s and rapid consumer uptake in the 1990s [2]. Now mobile phones are ubiquitous and an accepted part of our culture. The demand for wireless communications continues to increase, driven by the high data rate connectivity requirements of mobile computing and multimedia devices.

A wireless device must be designed to meet the regulatory emission and bandwidth constraints whilst also maximising battery life through low power usage. Such constraints motivate the search for ways to improve the efficiency of wireless communications systems – to send more with less. Understanding the wireless communications channel and how to fully and efficiently exploit it is an important area of research and development.

In 1948, Claude Shannon [3] introduced a mathematical theory for understanding communications and the field of Information Theory was born. Among other things, this work established the notion of capacity for a continuous communications channel in the presence of noise. For a channel with additive white Gaussian noise, the capacity is related to the logarithm of the signal to noise ratio η. For a channel of bandwidth B, the capacity is given by

$$C = B \log(1 + \eta) \tag{1.1}$$

in bits per second using a logarithm of base 2. This represents an upper bound on the information that can be passed through the channel without error and is known as the "Shannon Limit".

When multiple transmitters use the same frequency spectrum, the signal detected by a receiver will be a combination of all the transmissions. For this reason, conventional systems were developed with each independent broadcaster occupying a unique spectral band or spreading code[1] within the range of radio coverage. Cellular systems were designed to achieve some level of spectral reuse over large distances. With this approach, the Shannon Limit implies that the only way to increase capacity is to increase the signal to noise ratio, or increase the signal bandwidth. The noise floor is not easily reduced and increasing the transmitted power results only in a logarithmic growth in capacity. Increasing the spectrum usage is generally not possible due to practical or regulatory constrains. For much of the twentieth century, this was thought to fundamentally limit the capacity of the wireless communication channel.

For mobile wireless communications, the variation of the channel characteristics over time and space presents many challenges [4]. There has been much research into ways of mitigating or dealing with the effects of the fading wireless channel. The variation of the wireless channel over space is known as spatial diversity Recently there has been a significant shift in the research community toward the idea of spatial diversity as an advantage rather than a problem for wireless communications. The basic principle centres around taking advantage of this spatial diversity in the communications channel by using multiple receiver and transmitter antennas.

Early work by Winters [5] hinted at the possibility of sending multiple streams of data simultaneously using multiple antennas. Further research cemented the theoretical results [6] and practical architectures for achieving them [7]. Experiments at Bell Labs demonstrated these techniques in practice [8, 9], creating great excitement by effectively shattering the single channel Shannon Limit for communications spectral efficiency. The theory and practice

[1] Spread spectrum systems or code division multiple access systems utilise different spreading codes to create signal diversity over the same spectrum.

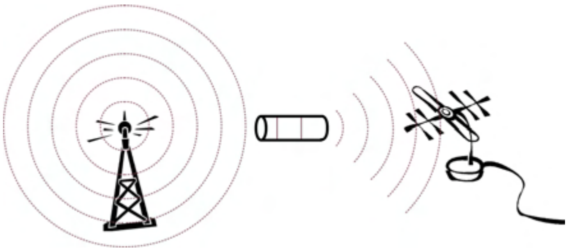
suggested a capacity limit of the wireless channel that would increase linearly with the number of antenna elements used. These events spawned the area of research and development known as MIMO (multiple input, multiple output) communications.

MIMO is now becoming accepted in practice with the recent IEEE standards 802.11n and 802.16e both providing for higher data rates using spatial multiplexing. Despite the extensive research and practical implementations of MIMO systems, there are some important questions that do not yet have satisfactory answers. The development of MIMO communications theory, reviewed in the following section, stems from strong mathematical results for a general system with multiple inputs and outputs. Whilst the mathematical results are well established, there remains open questions regarding the applicability of such results to practical systems of multiple antennas. A critique of much of the research in this area is that the assumptions follow mathematical convenience rather than arising from a study of the physical MIMO communications system.

The underlying physical process responsible for wireless communications is the propagation of electromagnetic waves. A suitable model of this must be able to represent the associated physical value of the electric and magnetic fields continuously across a region of space. However, by construction, the central ideas in MIMO theory rest on the assumption that there is only a discrete set of input and output signals. The work of this thesis seeks to develop the ideas central to multiple antenna signal processing from the underlying perspective of a continuous spatial field. The development of the continuous spatial models to represent a wave-field is proposed as a way forward to improve the theoretical understanding and development of signal processing algorithms.

The use of a continuous spatial model permits the constraints inherent in electromagnetic radiation to be implicitly embodied in the signal processing frameworks developed. Research in this area will help to illuminate the physical processes and fundamental limitations critical to the performance of MIMO communications systems. The development of a continuous spatial framework will facilitate the effective representation, detection and signal processing for the physical electromagnetic fields that carry information. The goal is to extend the theory of MIMO communications systems beyond that of a discrete set of inputs and outputs, and to elegantly incorporate relevant aspects of spatial wave propagation.

This thesis develops a framework for continuous spatial models and considers their application to several problems in multiple antenna signal processing. The work will consider optimal finite dimensional approximations, intrinsic limits and efficient statistical signal models for the continuous spatial field associated with wireless communications. In covering a fairly broad range of areas, the results vary in depth from observations and conjectures through to



(a) Conventional view of wireless communications. Space is filled by a broadcast as if it were a single dimensional pipe for information.

(b) MIMO wireless communications. Different spatial paths create spatial diversity at receiver and transmitter and allow re-use of the spectrum.

Figure 1.1: Conceptual comparison of conventional and MIMO systems. To the extent that each received signal is a linearly independent combination of the transmitted signals, it is possible to exploit the channel as if it were multiple independent communications channels. Spectral reuse is facilitated by the spatial diversity of the transmitter and receiver antennas, along with the multiple propagation paths introduced by the scattering environment.

well developed frameworks, theorems and proofs. It provides a contribution to communications theory to better reflect the medium over which the signal is being transmitted – in this case the spatial dimension.

1.2 Multiple Antenna Communications

The fundamental premise of multiple antenna (MIMO) systems is that the physical environment in which the wireless signal is transmitted provides a degree of diversity through the existence of independent signal paths. With such spatial diversity, and through appropriate signal processing and detection, it is possible to achieve the transmission of multiple symbols using the same time and spectrum resource within a single wireless communications cell. To the extent that the received signal combinations are linearly independent, the channel can be utilised as if there were multiple independent channels. A conceptual comparison between the conventional view, and that adopted in MIMO systems, is shown in Figure 1.1.

1.2.1 Multiple Antenna Channel Framework

This section presents the conventional framework for modelling and representation of the MIMO communications channel Consider a system with n_T transmitter antennas and n_R

receiver antennas. We define $\boldsymbol{s}(t) = [s_1(t) \cdots s_{n_T}(t)]^T$ as the vector of signals transmitted at time t. Assuming a linear system, the received signal $\boldsymbol{y}(t) = [y_1(t) \cdots y_{n_R}(t)]^T$ is constructed by the convolution of the input signal with a set of channel impulse responses,

$$y_m(t) = \sum_{n=1}^{n_T} \int_{-\infty}^{\infty} h_{mn}(t,\tau) s(t-\tau) d\tau + w_m(t) \quad m = 1, \ldots, n_R \tag{1.2}$$

$$\boldsymbol{y}(t) = \int_{-\infty}^{\infty} \mathbf{H}(t,\tau) \boldsymbol{s}(t-\tau) d\tau + \boldsymbol{w}(t), \tag{1.3}$$

where $\mathbf{H}$ is a matrix of channel impulse responses $h_{mn}(t,\tau)$ representing the contribution at time t of the signal at receive element n from transmit element m at time $t-\tau$. The vector $\boldsymbol{w}(t) = [w_1(t) \cdots w_{n_R}(t)]^T$ represents an additive noise process.

Depending on the signalling bandwidth, we need only consider samples of the baseband signals at an appropriate interval, T, such that $\boldsymbol{y}[n] = \boldsymbol{y}(nT)$. The other signal vectors $\boldsymbol{s}[n] = \boldsymbol{s}(nT)$ and $\boldsymbol{w}[n] = \boldsymbol{w}(nT)$ and sampled channel matrix $\mathbf{H}[n,k] = \mathbf{H}(nT, kT)$. Assuming the channel is causal, we obtain a discrete time representation of the channel

$$\boldsymbol{y}[n] = \sum_{k=0}^{\infty} \mathbf{H}[n,k] \boldsymbol{s}[n-k] + \boldsymbol{w}[n]. \tag{1.4}$$

In the case of frequency flat fading, or where appropriate equalisation has been performed to eliminate inter-symbol interference, we can simplify the model to consider the transmission of a single symbol,

$$\boldsymbol{y} = \mathbf{H}\boldsymbol{s} + \boldsymbol{w}, \tag{1.5}$$

where $\boldsymbol{s}$ is the transmitted symbol on the n_T antenna, $\boldsymbol{y}$ is the received symbol on the n_R antenna, $\mathbf{H}$ is the instantaneous $n_R \times n_T$ channel transfer matrix and $\boldsymbol{w}$ is the noise vector. This equation represents the effect of each “channel use” and is the general signal framework adopted in works investigating the multiple antenna communication link such as [10].

For a given channel realisation $\mathbf{H}$ we can calculate the theoretical channel capacity by considering the number and strength of independent single dimensional channels supported by $\mathbf{H}$. This is dependent on the rank and the eigenvalues of $\mathbf{H}$ with a value related to the logarithmic determinant of the system matrix [11]. The capacity will be

$$C = B \log \det \left[\mathbf{I}_{n_R} + \frac{\eta}{n_T} \mathbf{H}\mathbf{H}^H \right] \tag{1.6}$$

bits per second for a base 2 logarithm, where $\mathbf{I}_{n_R}$ is the $n_R \times n_R$ identity matrix, and $\mathbf{H}^H$ is the Hermitian or complex transpose of $\mathbf{H}$. The signal to noise ratio η is interpreted in the context of the components of $\mathbf{H}$ having unity expected power. Provided there is sufficient transmitter diversity, the capacity can scale linearly with the number of antenna n_R. This can be compared to the the single antenna case, (1.1), which would only allow a logarithmic increase in capacity as the addition of receiver antennas increased the effective signal to noise ratio.

1.2.2 Statistical Model of Channel Matrix

At typical radio frequencies, the presence of multiple signal paths and their subtle time variations will cause random fluctuations in the individual antenna coupling parameters of $\mathbf{H}$ [4]. For such situations, it is expected that the value and statistics of the channel capacity will be of interest in a system design context.

Significant interest in the use of multiple antennas to achieve higher spectral efficiency in the wireless channel commenced around 1995. The mathematical results of Telatar and Foschini were key to demonstrating the potential for capacity gains when the channel $\mathbf{H}$ was considered as a statistical process [6, 12–14]. Some practical demonstrations soon followed that demonstrated such potential in laboratory environments [7–9]. These activities catalysed an explosion of research investigating the potential and realisable capacities for various classes of random matrix $\mathbf{H}$. With a relatively simple channel model, (1.5), and armed with decades of statistical, matrix, and information theory many capacity results were presented as being informative of the practical MIMO communications problem [15].

Prior to the increased interest in MIMO, the statistics of a single antenna wireless channel were well studied. However, the statistics of the channel ensemble between two antenna arrays was a challenging and open problem. The application of a complete physical and electromagnetic propagation model had been considered for somewhat similar problems in optics [16] and introduced to communications [17]. In the case of a complex scattering environment such an approach becomes unwieldy and is best suited to specific geometrical investigations [18].

The characteristic behaviour and statistics of the channel model $\mathbf{H}$ depends on an array of physical properties and environmental characteristics: the antenna properties, radiation patterns, array geometry, orientation, scattering environment, movement and the overriding laws of electromagnetic radiation. As depicted in Figure 1.2, the matrix equation conceals the complexity and often abstracts the spatial aspects of the multiple antenna channel.

At the outset of the MIMO developments, it was realised that as antenna separation decreased, signals would become correlated, impacting system performance [19]. This

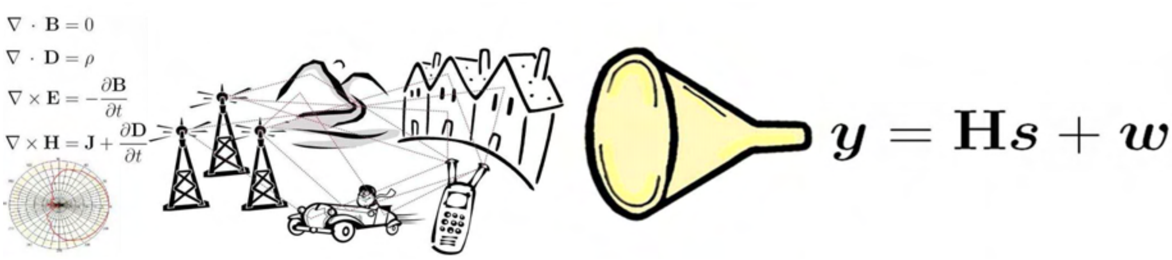


Figure 1.2: The compact form of the MIMO matrix equation. The discrete MIMO matrix equation represents the effects of a broad range of complex physical properties and processes.

prompted work to introduce additional models for correlation between the channel components of $\mathbf{H}$ [19–25]. There has also been significant interest in conducting measurement campaigns to fit empirical distributions to observed data [26–29]. Other efforts have sought to adopt convenient statistical distributions for analytic purposes [30–32]. A further review of MIMO channel models is presented in Section 1.2.5.

Such models provide a numerical framework to characterise antenna correlation, without reference to the physical processes that cause it [25, 33–36]. Since these models are not directly related to the physical propagation, they can be misleading. For example, the framework permits degenerate "keyhole" channels [22, 37, 38], however in practice these are rare [39] and even difficult to reproduce in artificial situations [40]. The development of MIMO theory around statistical channel distributions became an independent research field, and arguably some results were of little practical significance.

1.2.3 Introducing Space into MIMO Channel Models

Around 2003, there was movement toward incorporating the spatial constraint of the MIMO arrays into the channel modelling. Some results suggested a finite dimensionality of a multipath field over a region of space [41–43] and discuss the impact of this on channel modelling [44]. It was recognised that discrete statistical channel models ignored the fundamental aspects of wave propagation inherent in the problem [39, 45, 46].

The performance of a MIMO system will be directly related to the degree of spatial diversity available. However, for much of the MIMO literature, the spatial diversity and correlation of antenna channels was assumed or approximated. Ironically, to address this, the concept of "space" needed to be introduced in to the MIMO framework [47, 48].

This work is a continuation of the development of a spatial theory intended to model, analyse and design optimal signal processing for multiple antenna systems. Rather than being specific to a particular antenna configuration, the use of a continuous spatial model moves closer to understanding the underlying dimensionality and appropriate representation of the spatial field.

1.2.4 Suggested MIMO Review Articles

Since the explosion in the level of research interest in MIMO systems, there has been numerous publications on the subject. This section presents briefly some of the more useful review and summary articles available.

One particular work [49] developed a wider interest in the field early on. A review by Gesbert et al. addresses theoretical and practical aspects of MIMO systems [50] with explanations and useful interpretations. Paulraj et al. present an overview of MIMO as the solution to meet the needs of high data rate links [51].

Special issues of the Journal of Wireless Communications and Mobile Computing [52, 53], EURASIP Journal on Applied Signal Processing [54], IEEE Transactions on Signal Processing [55] and IEEE Journal on Selected areas in Communications [56, 57] contain a collection of relevant articles. Some key books on the subject have been compiled by Durgin [58], Jankiraman [59], Paulraj et al. [60], Gershman and Sidiropoulos [61] and Tsoulos [62].

1.2.5 Review of MIMO Channel Models

A fairly central theme of this work is the representation and modelling of the MIMO channel using the continuous spatial fields. Whilst there is some work in this area, the majority of MIMO channel models present a statistical model for the the discrete channel matrix specific to a given antenna configuration. This section presents a review of the literature in this area.

The purpose of a channel model is to provide a way of capturing and simulating the behaviour of the channel matrix $\mathbf{H}$. A good channel model should allow for the development and testing of systems to work in real practical situations. The quality and utility of a model depends on the intended application of the model and how well the model captures the parameters of the channel critical to the application [63]. A comprehensive review of the various MIMO channel models developed can be found in the work by Yu and Ottersen [64] and Jensen and Wallace [65].

The models that have been developed can be grouped into two main categories. Statistical or non-physical models directly model the statistics of the entries in the channel matrix $\mathbf{H}$

with statistics based on experimental measurements or convenient probability distributions [22, 35, 36]. Given a system with n_T transmitters and n_R receivers, characterising the correlations between the elements of $\mathbf{H}$ requires $(n_T n_R)^2$ parameters. Various models reduce this by assuming certain structures of the correlations. For example, the Kronecker model [23] assumes the overall correlation is separable as a product of receive side and transmitter side correlation. The virtual channel model [66] assumes a Fourier structure and the Weichselberger model [67] assumes a Kronecker style eigenbasis. Simple statistical models, such as the Kronecker, can provide satisfactory results for small numbers of antenna elements but will fail with more complex configurations [27, 68, 69]. Statistical models are easy to implement and can provide adequate modelling for some purposes. The effects of the propagation channel and the transmit and receive arrays are coupled together in the resultant model.

Geometrical or physical models characterise the spatial propagation aspects of the channel in terms of the directions of arrival and directions of departure [70]. Developed from early work on the nature of the time response of radio channels [71], the models incorporate the idea of distributed scatterers and clusters of scatterers interacting with the wireless signal. Models for the distribution and effect of scatterers can be based on geometric models, such as the one-ring and two-ring and other arrangements [72]. Alternately, the angular characteristics can be modelled as statistical processes [73]. Distributions such as the Laplacian [74] and Von-Mises [31] are used to characterise the angular spread of a scattering cluster. Such models can be fitted to experimental data by identifying scattering paths in array measurement data. This is typically achieved using subspace techniques for estimating direction of arrival.

For specific physical scenarios, it is possible to use point wise ray tracing methods to model the channel [75]. With sufficient model detail, these have been shown to provide a good match to the physical measurements [76]. The experimental validation of channel models is an important area of research [29]. Complex models have been developed that incorporate many of the attributes discussed above and play a role in the development of future wireless standards [77].

An alternative to direct modelling of $\mathbf{H}$ or an angular representation is provided by considering a modal spatial decomposition of the channel [41, 42, 44, 48, 78–81]. The coupling between the receive and transmit volume is described in terms of modes related to the essential dimensionality and degrees of freedom of the spatial field. It is these classes of models that are further developed and investigated in Chapter 2, Chapter 4 and Chapter 6. The estimation of direction of arrival is also an important topic for the development and validation of MIMO channel models. This is investigated in Chapter 5.

1.3 Motivation and Scope of Thesis

There is an extensive amount of existing research on antennas and electromagnetic propagation. The direct application of such results to the field of multiple antenna signal processing can create an onerous and often unnecessary level of complexity. The statistical models for MIMO analysis can provide an over simplification and be guided by mathematical elegance rather than practical correspondence. The motivation of this work is to develop the idea of continuous spatial model in a signal processing context in order to introduce a more appropriate level of complexity and physical correspondence to the MIMO problem. It is anticipated that this will be advantageous in the pursuit of understanding fundamental limits and achieving optimal system design.

In many practical applications, system design will be based on approximation or heuristics. While conventional designs may adopt a half wavelength antenna spacing, it is important to understand if this is efficient and optimal, or if there is room for improvement. Furthermore, as the antenna array is extended in three-dimensional space, a single antenna cannot completely characterise the array geometry.

The motivation of this thesis is to understand spatial fields and multipath diversity to better inform system design, antenna geometries and signal processing used for multiple antenna communications systems.

Pioneering work in this area [41, 42, 44, 47, 78, 79, 82–84] has considered the limits of dimensionality of a multipath field. The electromagnetic wave equation imposes a structure and constraint on the permissable wave-fields over a region of space. This work further develops the proposal of continuous spatial models to naturally incorporate this constraint into the problem formulation. The scope of the topics vary across optimal representations, parameter estimation and statistical modelling in the area of multiple antenna systems. Since the work is largely exploratory, the contributions of the thesis vary in strength from reviews and observations through to detailed frameworks and theorems.

The structure and main ideas of the thesis are arranged as follows:-

- The remainder of this chapter provides some further background material related to electromagnetic fields and multiple antenna communications.
- Chapter 2 provides a review of the key results regarding the spatial dimensionality and the impact it has on the multiple antenna systems. Some developments and conjectures are provided towards improving the bounds and limits in this area.

- Chapter 3 considers the specific problem of modelling a field with restricted direction of arrival. Formal proof of the relationship between dimensionality and angular spread is provided along with a constructive approximation for the optimal representation.

- Chapter 4 contains a significant technical contribution of the thesis in the formal development of the framework required to determine the optimal representation of a spatial field. It is shown clearly how the optimal basis depends on the angular power spectrum and the shape of the region of interest. Several examples are solved and investigated numerically.

- Chapter 5 presents a detailed derivation of a fundamental bound for system performance of direction of arrival estimation. This is a contribution in that the bound is independent of the specific sensor geometry and has been derived for multiple sources. It is shown that the number of sources that can be resolved is directly related to the essential dimensionality of the spatial field independent of the algorithm employed.

- Chapter 6 presents a new continuous space statistical channel model. This model is validated against experimental and simulated data and is shown to provide a more efficient representation of experimental data than existing models. By using the spatial model, this approach facilitates the prediction and optimisation of alternate antenna array geometries from measurement data.

- Chapter 7 presents an exploratory investigation of the implications of the continuous spatial model in the resolution of source location. Some new approaches are developed leading to some useful bounds for the problem defined.

- Chapter 8 offers concluding remarks and provides a set of open areas of research and conjectures that have been identified through this research work.

Understanding the wave equation and how it constrains the signal subspace and thus performance of an antenna array is not a simple matter. It bears a strong resemblance to the issues of sampling and understanding the dimensionality of bandlimited functions [85], an issue which was prevalent for several decades in the middle of last century. Similar developments in relation to multiple antennas and spatial fields will lead to a body of research to guide engineering developments in the area.

1.4 Space, Waves and Intrinsic Limits

Electromagnetic wireless communication requires the creation and detection of an electromagnetic field. By controlling a current distribution across a region of space, the transmitter is able to generate or excite the field. The strength and direction of the electromagnetic field is a physical quantity that varies over space and time, extending beyond the region occupied by the transmitter. The continuous electromagnetic field, defined over the constrained region of the receiver, carries information about the transmitted signal. The interaction of the electromagnetic field with antenna elements at the receiver will generate current and voltage signals.

Complete electromagnetic modelling of a MIMO system is generally prohibitive due to the scope of the propagation environment. A review by Jensen and Wallace [86] lists the physical parameters that are relevant to system performance:

- antenna sensitivity and impedance matching,
- array size and configuration,
- element radiation patterns,
- polarisation,
- mutual coupling, and
- multipath propagation.

Modelling such parameters will increase the accuracy and applicability of the MIMO channel representation. This will provide a benefit when the increase in complexity is justified by a valuable improvement to matching and prediction of the model.

The first three of these items relate to the configuration of the sensor array. In practice, arrays should be designed to maximise their ability to transmit or receive information from the region of the electromagnetic field with which they interact. Jensen and Wallace suggest that the "average capacity is relatively insensitive to array configuration" [86], which leads to the concept of considering the intrinsic capacity of a region of space.

This section reviews literature covering the aspects of electromagnetic radiation relevant to MIMO systems. Some recent ideas and results relating to the essential dimensionality of a spatial field and resultant intrinsic limits are also reviewed. These works represent a foundation and motivation for much of the work in this thesis regarding the study of continuous spatial models for multiple antenna communication and signal processing.

1.4.1 Wave Equation

The physics and associated mathematics of wave propagation and wave motion is an area that has received a significant amount of attention [87, 88] and is accepted as a general engineering principle [89]. A similar theory can be applied across a wide range of physical waves, such as acoustic waves and electromagnetic radiation [90]. A central relationship is known as the reduced wave equation, or Helmholtz equation [91],

$$\triangle u(\boldsymbol{x}) + k^2 u(\boldsymbol{x}) = 0, \tag{1.7}$$

where $u(\boldsymbol{x})$ is a scalar valued field representing some spatial property of the medium, $k = 2\pi/\lambda$ is the wave-number related to the wavelength, λ, of waves in that medium and $\triangle$ is the Laplacian operator equal to the sum of second order partial derivatives of $u(\boldsymbol{x})$ on a unitary orthogonal co-ordinate system. For three-dimensional cartesian coordinates

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \tag{1.8}$$

The second order differential equation (1.7) characterises the spatial distribution of a narrow-band wave-field across a region free of any sources. The time varying physical parameter is obtained from considering

$$U(\boldsymbol{x}, t) = \mathrm{Re}\left\{u(\boldsymbol{x})e^{-j\omega t}\right\} \tag{1.9}$$

where $\mathrm{Re}\left\{\cdot\right\}$ is the real component, $j = \sqrt{-1}$ and $\omega = 2\pi f$ is the angular frequency of the waves.

This equation is widely studied in acoustics where it is derived from a linearisation of Eulers's equation and the equation of continuity for a compressible medium [91, 92]. The scalar field, $u(\boldsymbol{x})$, is related to the velocity potential or localised pressure of the medium.

In considering electromagnetic radiation, we have the additional complexity of considering a vector field. The field at a point is fully characterised by six components – the electric field vector $\boldsymbol{E}(\boldsymbol{x})$ and the magnetic field vector $\boldsymbol{H}(\boldsymbol{x})$. These fields must satisfy the vector Helmholtz equations,

$$\triangle \boldsymbol{E}(\boldsymbol{x}) + k^2 \boldsymbol{E}(\boldsymbol{x}) = 0 \qquad \triangle \boldsymbol{H}(\boldsymbol{x}) + k^2 \boldsymbol{H}(\boldsymbol{x}) = 0. \tag{1.10}$$

Where the region is free of sources, the fields will also be divergence free [91]. The magnetic field and electric field are not independent; each field can be derived from the other. The complete constraint on the field can be expressed

$$\triangle \boldsymbol{E}(\boldsymbol{x}) + k^2 \boldsymbol{E}(\boldsymbol{x}) = 0 \qquad \nabla \cdot \boldsymbol{E}(\boldsymbol{x}) = 0 \qquad \boldsymbol{H}(\boldsymbol{x}) = \frac{\nabla \times \boldsymbol{E}(\boldsymbol{x})}{ik} \quad \text{or} \tag{1.11}$$

$$\triangle \boldsymbol{H}(\boldsymbol{x}) + k^2 \boldsymbol{H}(\boldsymbol{x}) = 0 \qquad \nabla \cdot \boldsymbol{H}(\boldsymbol{x}) = 0 \qquad \boldsymbol{E}(\boldsymbol{x}) = \frac{-\nabla \times \boldsymbol{H}(\boldsymbol{x})}{ik} \tag{1.12}$$

where ∇ is the vector differential operator

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \tag{1.13}$$

for three-dimensional space with orthogonal unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ and respective cartesian coordinates (x, y, z). The divergence and curl operations on the vector field $\boldsymbol{E}(\boldsymbol{x})$ are then defined by the scalar or dot product and the cross product as $\nabla \cdot \boldsymbol{E}(\boldsymbol{x})$ and $\nabla \times \boldsymbol{E}(\boldsymbol{x})$.

The divergence constraint implies that the electric or magnetic field has only two degrees of freedom. From this it is apparent that the complete electromagnetic field can be characterised by a two-dimensional scalar field satisfying the wave equation. A similar case for the importance of the wave equation was made in [93] where it was shown that the Green's function for radiating waves satisfying Maxwell's equations has two degrees of freedom.

This brief analysis demonstrates why the properties of scalar fields satisfying the wave equation (1.7) are central to understanding the limits of wireless communications. To facilitate the analysis, we will investigate the single dimensional scalar field. This approach matches physical implementations that make use of unpolarised antennas to interact with the field. The issue of polarisation will be discussed further in the next section.

1.4.2 Polarisation

Early work in the field demonstrated that different polarisation modes of the radio channel could exhibit uncorrelated amplitudes [94]. The complete electromagnetic field has six components, suggesting that six communication modes are theoretically available [95, 96], however simple antenna designs will generally only excite or detect three modes [97]. Where the polarisation modes are independent, the use of polarisation will offer improved system performance in the form of a diversity gain [98].

For scatterers in the far-field, the electric and magnetic fields are not independent. The rank of the far-field array response matrix is only two [84]. In practice, compact trimode antenna have been proposed [99] and performance approaching three [100] or four [101] independent Rayleigh fading channels have been observed. Whilst such antenna offer multiple signals from one antenna location, the antenna itself must have some spatial extent to couple with the component modes of the electromagnetic field. It is likely that such results arising from the array may also affect the pattern or directional diversity [102].

In this work we consider scatterers to be a reasonable distance from the array and thus in the far-field. It is the far-field excitation and response of the transmitter and receiver array which are of interest. In addition to satisfying the wave equation, these response matrices will have two degrees of freedom. The use of polarisation could increase the available degrees of freedom by a factor of 2. In this way, limits of capacity or system performance utilising polarisation would be increased by a factor between 1 and 2 depending on the amount of cross polarisation diversity. This approach has also been followed by others to develop a MIMO spatial channel model incorporating polarisation [103].

1.4.3 Mutual Coupling

Practical antennas will exhibit coupling between the elements as they are brought close together. This effect is known as mutual coupling. Initial studies of this effect [104–107] suggested a small improvement in system performance since mutual coupling would introduce antenna pattern diversity, decorrelating the antenna signals. Other works suggested the coupling would be detrimental [108] with a loss in signal to noise ratio degrading capacity [109]. Practical measurements showed that degradation in radiation efficiency would outweigh any increase in pattern diversity leading to a loss in performance [110].

Conflicting views in the existing research literature on this topic are largely due to different scopes and underlying assumptions [111]. Careful analysis shows a tradeoff between any diversity enhancement and the directional characteristics of the channel [112]. It is not possible to make definite predictions without considering the complete impedance network model of the antennas [113] and resultant changes in response and efficiency [114]. A rigourous approach and framework for investigating the effects of mutual coupling was proposed in [115]. With appropriate matching networks it has been shown that it is possible to decrease correlation without loss in gain [116], however the system bandwidth is significantly reduced.

Most approaches to mutual coupling consider the main source of noise to be that generated in the receiver amplifiers. When this is combined with power constraints based on the radiated power rather than any internal element currents, it is possible to benefit from “super directivity” with multiple antennas [117]. However, it is known that when circuit elements are coupled, the thermal noise components generated within them generate correlated noise at the network outputs [118]. This should be considered when analysing the effects of mutual coupling [119, 120].

Whichever approach to mutual coupling is considered, the underlying field incident on the antenna array must satisfy the wave equation constraint. The mutual coupling effects and antenna impedance matching network can be considered to perform a processing operation on the wave-field. This can be well modelled by a linear transformation and consequently cannot increase the information content of the underlying spatial field [121]. Thus mutual coupling is a factor related to the efficiency of a particular antenna configuration, rather than having an impact on the fundamental limits for spatial communication.

1.4.4 Dimensionality

We define a continuous spatial field, $u(\boldsymbol{x})$, to be a scalar function varying over three-dimensional space $\boldsymbol{x} = (x, y, z)$. We are interested in modelling the field over some domain of interest $\Lambda \subset \mathbb{R}^3$ which we require be bounded in extent such that $\boldsymbol{x}, \boldsymbol{y} \in \Lambda$ implies that $\|\boldsymbol{x} - \boldsymbol{y}\| < \infty$. We also require that Λ is not a set of measure zero, and thus contains at least some open interval. We assume the field, $u(\boldsymbol{x})$, is continuous, bounded and integrable over this domain. With these assumptions we can define an inner product and induced norm

$$\langle u, v \rangle = \int_\Lambda u(\boldsymbol{x})\overline{v(\boldsymbol{x})} d\boldsymbol{x} \qquad\qquad \|u\|_\Lambda = \int_\Lambda |u(\boldsymbol{x})|^2 d\boldsymbol{x}. \tag{1.14}$$

Define $\mathcal{S}$ as the space of fields $u(\boldsymbol{x})$ created from this inner product and norm. The space $\mathcal{S}$ is isomorphic to a separable Hilbert space with countable basis. For example, a Fourier basis of spatial complex sinusoids can be easily constructed for an arbitrary region. Since the fields are continuous, the dimensionality of the space of fields $\mathcal{S}$ over the bounded region Λ will be countably infinite.

If the field $u(\boldsymbol{x})$ is required to satisfy the narrow-band wave equation, (1.7), this implies an additional second order differential constraint. Define $\mathcal{S}'$ as the space of functions satisfying the wave equation (1.7) on the bounded region Λ. The space $\mathcal{S}'$ is a strict subspace of the space $\mathcal{S}$ and is again isomorphic to a countably infinite Hilbert space.

Consider a finite region $\Lambda' \subset \Lambda$ whose closure lies in the interior of Λ. A similar norm can be defined on Λ' as in (1.14). Any member of $\mathcal{S}'$ with unit norm $\|u\|_\Lambda$ can be approximated on the region Λ' with arbitrary precision with a fixed basis $\beta_m(\boldsymbol{x})$ for $m = 1, \ldots, M$ for some $M < \infty$. That is, given an arbitrary ϵ, there exists a number M and set of basis functions β_m such that

$$\min_{\alpha_m} \left\| u - \sum_{m=1}^{M} \alpha_m \beta_m \right\|_{\Lambda'} < \epsilon \qquad \forall \quad u(\boldsymbol{x}) : \|u\|_\Lambda = 1. \tag{1.15}$$

This result implies that provided a spatial field satisfies the wave equation over some larger region Λ, an arbitrary field over a bounded finite volume $\Lambda' \subset \Lambda$ is essentially finite dimensional. The combination of the wave equation constraint, a bounded domain of interest, and a finite precision representation leads to a fixed number of degrees of freedom. This is investigated further in Chapter 2 and forms an underlying theme for this thesis. The notion that a field is essentially finite dimensional leads to results regarding the efficient representation of fields and fundamental limits to system performance.

The idea of dimensionality for the multipath spatial field in wireless communications was developed recently [41], leading to a string of results regarding capacity limits [82, 122–124], modelling [44, 48, 78, 125], extrapolation [126, 127] and direction of arrival estimation [128, 129]. Similar ideas were developed by considering a suitable basis representation for the signals observed by a spherical antenna array [43, 84, 130].

The idea of dimensionality and degrees of freedom has been investigated for a scattered field resulting from objects in a finite volume [131–133]. This problem can be thought of as the dual of that considered in this work, where we are interested in the dimensionality of the electromagnetic field itself in a finite volume.

1.4.5 Intrinsic Limits

In wireless communications systems, transmission is achieved by means of a modulated narrow-band radio frequency transmission sent from a finite transmitter region and received in a finite receiver region. It follows then that the concept of the essential dimensionality of a wave-field developed in Section 1.4.4 will be related to the intrinsic ability to send information between the two regions. In the field of Wireless Communications and Information Theory there have been several results presented towards understanding these limits. This section presents a brief literature review of that area.

The assumption of independently fading channel coefficients must be examined in the context of the wave equation [46]. The intrinsic limit can be related to the properties of a continuous operator describing the electromagnetic coupling between the two spatial regions [134, 135]. The laws of electromagnetism will have an effect on the maximum achievable spatial capacity [136, 137].

The interaction with the electromagnetic field through a continuous or distributed sensor, across the receiver and transmitter spatial region, suggests an intrinsic upper bound on the capacity of a wireless channel [138, 139]. A similar result is obtained by taking the limit of a finite element approximation of the spatial channel [140]. The essentially finite dimensionality of the spatial field can be used to derive bounds for the scaling of the capacity of a constrained antenna array [79, 82]. An extensive numerical investigation has been presented with similar conclusions [121].

A recent detailed work by Jensen and Wallace reviewed the capacity saturation that results from considering the laws of electromagnetism [141]. A more mathematical approach based on the dimensionality of the spatial field is presented in [81].

Whilst this thesis will consider the application of continuous spatial models to several specific problems, it does not extend to incorporate the capacity limits established above. The review in this section has presented the works that have taken the notion of the field dimensionality and applied it to the communications capacity problem. However, since some of the elements and aspects of the continuous spatial model remain poorly established, most of these results sit on tenuous foundations. The motivation of this research and thesis has been to provide a more systematic development of some of the aspects and applications of continuous spatial models.

The following chapter leads into this work by a more thorough review of the dimensionality results and analysis of their application to two-dimensional multipath fields.