
Stochastic Solvency Testing
in Life Insurance

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Statement of Originality

This thesis contains no material that has been accepted for the award of any other degree or diploma in any University, and to the best of my knowledge and belief, contains no material published or written by another person, except where due reference is made in the thesis.

Genevieve Hayes

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Abstract

Stochastic solvency testing methods have existed for more than 20 years, yet there has been little research conducted in this area, particularly in Australia. This is for a number of reasons, the most pertinent of which being the lack of computing capabilities available in the past to implement more sophisticated techniques. However, recent advances in computing have made stochastic solvency testing possible in practice and have resulted in a trend towards this being done in advanced studies.

The purpose of this thesis is to develop a realistic solvency testing model in a form that can be implemented by Australian Life Insurers, in anticipation that the Australian insurance regulator, APRA, will ultimately follow the world trend and require stochastic solvency testing to be carried out in Australia. The model is constructed from three interconnected stochastic sub-models used to describe the economic environment and the mortality and lapsation experience of the portfolio of policies under consideration. Australian economic and Life Insurance data is used to fit a number of possible sub-models, such as generalised linear models, over-dispersion models and asset models, and the “best” model is selected in each case. The selected models are a modified CAS/SOA economic sub-model; either a Poisson or negative binomial (NB1) distribution (depending on the policy type considered) as the mortality sub-model; and a normal-Poisson lapsation sub-model.

Based on tests carried out using this model, it is demonstrated that, for portfolios of level and yearly-renewable term insurance business, the current deterministic solvency capital requirements provide little protection against insolvency. In fact, for the test portfolios of term insurance policies considered, the deterministic capital requirements have levels of sufficiency of less than 2% (on a Value at Risk basis) when compared to the change in capital distribution over a three year time horizon. This is of concern, as yearly-renewable term insurance comprises a significant volume of Life Insurance business in Australia, with there being over 426,000 yearly-renewable

term insurance policies on the books of Australian Life Insurers in 1999 and more business expected since then.

A sensitivity analysis shows that the results of the stochastic asset requirement calculations are sensitive to the choice of sub-model used to forecast economic variables and to the choice of formulae used to describe the mean mortality and lapsation rates. The implication of this is that, if APRA were to require Life Insurers to calculate their solvency capital requirements on a stochastic basis, some guidance would need to be provided regarding the components of the solvency testing model used. The model is not, however, sensitive to whether an allowance is made for mortality or lapsation rate over-dispersion, nor to whether dependency relationships between mortality rates, lapsation rates and the economy are allowed for. Thus, over-dispersion and dependency relationships between the sub-models can be ignored in a stochastic solvency testing model without significantly impacting the calculated solvency requirements.

Contents

1	Introduction	1
2	Life Insurance Solvency Testing: A Review	5
2.1	Introduction	5
2.2	Reserves and Policy Liabilities	6
2.3	An Overview of Policy Valuation Techniques	8
2.4	Solvency	10
2.5	Stochastic Claims Reserving and Solvency Testing	12
2.6	Risk Measures	13
2.7	Reserving and Solvency Legislation	15
2.8	Conclusion	21
3	A Framework for Stochastic Solvency Testing	23
3.1	Introduction	23
3.2	The Model Framework	23
3.3	Research Questions	29
4	Statistical Models for Mortality and Economic Data	31
4.1	Introduction	31
4.2	Generalised Linear Models	31
4.2.1	Generalised Linear Models - An Overview	31
4.2.2	A Review of Existing Generalised Linear Models for Mortality Data	32
4.2.3	Graduation by Reference to a Standard Table	34

4.2.4	Generalised Linear Models for Mortality Data Using a Standard Table	36
4.2.5	Mortality Reduction Factors	37
4.3	Over-Dispersion Models and Testing	38
4.3.1	Testing for Over-Dispersion	38
4.3.2	Models for Over-Dispersion	40
4.4	Time Series Statistical Models	45
4.5	Software	47
5	The Data	49
5.1	Introduction	49
5.2	The IAAust Data Set	49
5.3	The Single Insurer Data Set	54
5.3.1	The Single Insurer Mortality Data	54
5.3.2	The Single Insurer Lapsation Data	58
5.4	The Economic Data	60
6	Dependency Relationships	69
6.1	Introduction	69
6.2	Withdrawal Rates and Mortality	70
6.2.1	Selective Lapsation	70
6.2.2	Existing Tests for Selective Lapsation	71
6.2.3	A GLM-Based Test for Selective Lapsation	73
6.2.4	Inferring Policy Discontinuances from the Single Insurer Data	74
6.2.5	Results	76
6.2.6	Conclusion	78
6.3	Mortality and the Economy	79
6.3.1	Population Mortality and the Economy	79
6.3.2	Existing Tests for a Relationship Between Economic Fluctuations and Mortality	81
6.3.3	A GLM-Based Test for a Relationship Between Economic Fluctuations and Mortality	84
6.3.4	The Data	88
6.3.5	GLM Modelling Results	92
6.3.6	Conclusion	95
6.4	Withdrawal Rates and the Economy	95
6.4.1	The Interest Rate and Emergency Fund Hypotheses	95
6.4.2	Existing Tests for a Relationship Between Economic Fluctuations and Lapse Rates	96

6.4.3	A GLM-Based Test for a Relationship Between Economic Fluctuations and Lapsation	98
6.4.4	The Data	99
6.4.5	Results	101
6.4.6	Conclusion	104
7	Stochastic Sub-Models	105
7.1	Introduction	105
7.2	Stochastic Economic Models	105
7.2.1	Introduction	105
7.2.2	A Review of Existing Stochastic Economic Models	106
7.2.3	Fitting the Models	115
7.2.4	A Discussion of the Appropriateness of the Stochastic Economic Models	115
7.2.5	Tests for Comparing Stochastic Asset Models	117
7.2.6	The Data	118
7.2.7	Results	119
7.2.8	Conclusion	130
7.3	Stochastic Mortality Models	130
7.3.1	Poisson and Binomial Models	130
7.3.2	Testing for Over-Dispersion in the Insured Life Mortality Data	132
7.3.3	Results	133
7.4	Stochastic Lapsation Models	138
7.4.1	Introduction	138
7.4.2	Testing for Over-Dispersion in the Lapsation Data	138
7.4.3	Results	139
7.5	Summary	145
8	Solvency Testing Methodology	147
8.1	Introduction	147
8.2	Solvency Testing Methodology	147
8.3	Simulation	150
8.3.1	Simulation Techniques	150
8.3.2	Convergence and Accuracy	151
8.4	The Spreadsheet Models	153
8.4.1	Introduction	153
8.4.2	The Policy Model	153
8.4.3	The Payment Model	155
8.4.4	The Asset Model	159
8.4.5	The Capital Model	160

8.5	Model Assumptions	160
8.5.1	Introduction	160
8.5.2	Portfolio Specifications	160
8.5.3	Expenses	162
8.5.4	Asset Mix	162
8.5.5	Surrender Values	164
8.5.6	Premiums	165
8.5.7	Starting Values	171
8.6	Solvency and Capital Adequacy Assumptions	171
8.6.1	Introduction	171
8.6.2	Solvency Assumptions	172
8.6.3	Capital Adequacy Assumptions	176
8.7	Cost of Capital Risk Margin Calculations	178
8.8	Sensitivity Analysis	179
9	Results	183
9.1	Introduction	183
9.2	Base Case Simulation Results	183
9.2.1	Capital Adequacy	183
9.2.2	Asset Requirements	189
9.3	Sensitivity Analysis	192
9.3.1	Economic Sub-Model Sensitivities	192
9.3.2	Mortality Sub-Model Sensitivities	201
9.3.3	Lapsation Sub-Model Sensitivities	201
9.3.4	Mortality and Lapsation Sub-Model Sensitivities	204
9.3.5	Dependency Sensitivities	205
9.3.6	Distributional Mean Sensitivities	209
9.3.7	General Comments Relating to the Sensitivity Analyses	216
10	Conclusion	219
10.1	Responses to the Research Questions	219
10.2	Implications	221
10.3	Limitations and Further Research	223
	Appendices	225
A	Standard Tables	225
A.1	Mortality	225
A.2	Lapsation	229

B Australian Solvency and Capital Adequacy Requirements	231
B.1 Introduction	231
B.2 The Solvency Requirement	231
B.3 The Capital Adequacy Requirement	233
C Proofs of Mathematical Results	235
C.1 Change in Capital $\Delta C(1)$	235
C.2 VaR of the $-\Delta C(1)$ Distribution	236
C.3 Change in Capital $\Delta C(t)$	237
C.4 VaR of the $-\Delta C_{min}(0,3)$ Distribution	238
D Detailed Sensitivity Analysis Outputs	239
D.1 Economic Sub-Model Sensitivity Outputs	240
D.2 Mortality Sub-Model Sensitivity Outputs	243
D.3 Lapsation Sub-Model Sensitivity Outputs	244
D.4 Mortality and Lapsation Sub-Model Sensitivity Outputs	246
D.5 Dependencies Sensitivity Outputs	247
D.6 Distributional Mean Sensitivity Outputs	253
Glossary of Acronyms and Abbreviations	259
Glossary of Insurance Terms	261
Bibliography	265

List of Tables

4.1	A Comparison of Reduction Factors at Selected Ages	38
5.1	Exposures by Sex, Policy Type and Year	50
5.2	Deaths by Sex, Policy Type and Year	51
5.3	Exposures by Age Band, Sex and Duration Band for Type 1 Policies	51
5.4	Exposures by Age Band, Sex and Duration Band for Type 2 Policies	52
5.5	Exposures by Age Band, Sex and Duration Band for Type 3 Policies	52
5.6	Exposures by Age Band, Sex and Duration Band for Type 4 Policies	53
5.7	Exposures by Sex, Policy Type and Year	55
5.8	Deaths by Sex, Policy Type and Year	55
5.9	Non-Death Terminations by Sex, Policy Type and Year	56
5.10	Exposures by Age Band, Sex and Duration Band for Type 1 Policies	56
5.11	Exposures by Age Band, Sex and Duration Band for Type 4 Policies	57
5.12	Exposures as a % of the Total for each Policy Type/Sex/Data Set Combination by Age Band	57
5.13	Exposures as a % of the Total for each Policy Type/Sex/Data Set Combination by Duration Band	58
5.14	Exposures and Withdrawals by Policy Type and Year	59
5.15	Exposures by Duration Band and Policy Type	60
5.16	Exposures as a % of the Total for each Policy Type by Duration Band	60
5.17	A Summary of the Economic Data	62
5.18	Summary Statistics for the Economic Data	63
5.19	Correlations Between the Economic Variables with p -values Shown in Brackets	67

6.1	Correlations Between Mortality Ratios and Lapse Ratios by Lag, Sex and Policy Type (All Ages) with p -values Shown in Brackets	76
6.2	Correlations Between Mortality Ratios and Lapse Ratios by Lag, Sex and Policy Type (Ages 15–64 Only) with p -values Shown in Brackets	76
6.3	Drop in Deviance Between the Two Selective Lapsation GLMs by Maximum Lag	77
6.4	Correlations Between Insured Life Mortality Ratios and Economic Variables (1995–1999) with p -values Shown in Brackets	89
6.5	Correlations Between Population Mortality Ratios and Economic Variables with p -values Shown in Brackets	91
6.6	Drop in Deviance between the Two Economic Fluctuation versus Mortality GLMs	93
6.7	Analysis of Deviance Table for Model 2 for the IAAust Data	93
6.8	Fitted Coefficients for the Economic Fluctuation versus Mortality GLMs with Standard Errors Shown in Brackets	94
6.9	% Change in the Mortality Ratio Associated with a 1% Change in the Short-Term Interest Rate	95
6.10	Correlations Between (All-Duration) Lapse Ratios and Economic Variables (Single Insurer Mortality Data) with p -values Shown in Brackets	101
6.11	Correlations Between (All-Duration) Lapse Ratios and Economic Variables (Single Insurer Lapsation Data) with p -values Shown in Brackets	101
6.12	Decrease in Deviance between the Two Economic Fluctuation versus Mortality GLMs	102
6.13	Fitted Coefficients for the Economic Fluctuation versus Mortality GLMs (Model 2) with Standard Errors Shown in Brackets	103
6.14	% Change in the Lapsation Ratio Associated with a 1% Change in Each of the Economic Variables for the Single Insurer Mortality Data	103
7.1	Fitted Parameter Values for the Kemp Model with Standard Errors Shown in Brackets	121
7.2	Correlation Matrix for the Kemp Model with p -values Shown in Brackets	121
7.3	Goodness of Fit Statistics for the Kemp Random Walk Model	122
7.4	A Summary of the Wilkie Model Sub-Models Used in this Thesis	124
7.5	Fitted Parameter Values for the Wilkie Model	124
7.6	Goodness of Fit Statistics for the Wilkie Model Time Series Processes	124
7.7	Fitted Parameter Values for the CAS/SOA Model	126
7.8	Goodness of Fit Statistics for the CAS/SOA Model Time Series Processes	126

7.9	A Comparison of the AIC for the Kemp, Wilkie and CAS/SOA Models	127
7.10	Fitted Parameter Values for the Modified CAS/SOA Model	129
7.11	Goodness of Fit Statistics for the Modified CAS/SOA Model	129
7.12	A Summary of the Over-Dispersion Models Fitted to the Data	133
7.13	Over-Dispersion Statistics for the Poisson GLMs with p -values Shown in Brackets	134
7.14	Over-Dispersion Parameters for the Over-Dispersion Models with p -values Shown in Brackets	135
7.15	AIC for the Models Fitted to the Type 1 Males and Females Data	137
7.16	Fitted Coefficients for the Over-Dispersion Models with Standard Errors Shown in Brackets	137
7.17	Over-Dispersion Statistics for the Lapsation Data Poisson GLMs with p -values Shown in Brackets	140
7.18	Over-Dispersion Parameters for the Lapsation Over-Dispersion Models with p -values Shown in Brackets	141
7.19	AIC for the Lapsation Over-Dispersion Models	142
7.20	Fitted Coefficients for the Normal-Poisson Random Coefficient Model with Standard Errors Shown in Brackets	144
7.21	Fitted Variances and Covariances for the Normal-Poisson Random Coefficient Model with Standard Errors Shown in Brackets	144
8.1	Model Portfolio Compositions	161
8.2	Duration Assumptions (Years) by Age Band, Sex and Policy Type	162
8.3	Expense Assumptions per Policy	163
8.4	Composition of each of the Model Asset Portfolios	164
8.5	Regular Annual Premiums Assumed for Type 1 and 3 Policies by Age Band	167
8.6	Single Premiums Assumed for the Type 2 (M and F) Policies Under Each Investment Option by Age Band	168
8.7	Implicit Profit Margins (% of After-Tax Investment Earnings) for Type 2 Policies Under Each Investment Option	168
8.8	Premium Rate Scale for Type 4 Policies (\$)	170
8.9	Implicit Profit Margins (% of Claims) for Type 4 Policies	170
8.10	Economic Variable Starting Values	171
8.11	Proportion of New Policies in Each Age Band by Policy Type and Sex	174
8.12	Regular Annual Premiums Assumed for New Type 1 and 3 Policies	174
8.13	Single Premiums Assumed for New Type 2 (M and F) Policies Under Each Investment Option	174

8.14	Selected Capital Adequacy Margins	177
8.15	Fitted Values of α_d and α_w by Policy Type and Sex	181
9.1	VaR and TVaR Per Policy for the Base Case Scenarios (\$)	185
9.2	Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Base Case Scenarios (\$)	186
9.3	Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Requirements for the Base Case Scenarios	187
9.4	Summary Statistics for the Base Case Simulations	187
9.5	Number of Iterations Performed, the Time Required and the Number of Iterations Needed for Convergence for the Base Case Scenarios	188
9.6	p -values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Base Case Scenarios	189
9.7	Stochastic Minimum Asset Requirements for the Base Case Scenarios (\$)	190
9.8	Ratios of the LPS2.04 and LPS3.04 Solvency and Capital Adequacy Requirements to the Stochastic Minimum Asset Requirements for the Base Case Scenarios	191
9.9	p -values for the Two-Sample Kolmogorov-Smirnov Tests for the Economic Sub-Model Sensitivity Scenarios	193
9.10	VaR and TVaR Per Policy for the Economic Sub-Model Sensitivity Analysis Scenarios (\$) with Ratios of these Amounts to the Base Case Capital Requirements Shown in Brackets	195
9.11	Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Economic Sub-Model Sensitivity Analysis Scenarios (\$)	196
9.12	Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Amounts for the Economic Sub-Model Sensitivity Analysis Type 3 and 4 Policy Scenarios	197
9.13	Ratios of the Economic Sub-Model Sensitivity Analysis Stochastic Minimum Asset Requirements to the Base Case Requirements for the Same Portfolio	198
9.14	Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Economic Sub-Model Sensitivity Analysis Scenarios	199
9.15	p -values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Economic Sub-Model Sensitivity Analysis Scenarios	200
9.16	p -values for the Two-Sample Kolmogorov-Smirnov Tests for the Mortality Sub-Model Sensitivity Scenarios	201

9.17	<i>p</i> -values for the Two-Sample Kolmogorov-Smirnov Tests for the Lapsation Sub-Model Sensitivity Scenarios	202
9.18	VaR and TVaR Per Policy for the Lapsation Sub-Model Sensitivity Analysis Scenarios (\$) with Ratios of these Amounts to the Economic Base Case Capital Requirements Shown in Brackets	203
9.19	<i>p</i> -values for the Two-Sample Kolmogorov-Smirnov Tests for the Mortality and Lapsation Sub-Model Sensitivity Scenarios	204
9.20	<i>p</i> -values for the Two-Sample Kolmogorov-Smirnov Tests for the Dependencies Sensitivity Scenarios	206
9.21	VaR and TVaR Per Policy for the Dependencies Sensitivity Analysis Scenarios (\$) with Ratios of these Amounts to the Economic Base Case Capital Requirements Shown in Brackets	208
9.22	<i>p</i> -values for the Two-Sample Kolmogorov-Smirnov Tests for the Distributional Mean Sensitivity Analysis Scenarios	210
9.23	VaR and TVaR Per Policy for the Distributional Mean Sensitivity Analysis Scenarios (\$) with Ratios of these Amounts to the Economic Base Case Capital Requirements Shown in Brackets	212
9.24	Ratios of the Distributional Mean Sensitivity Analysis Stochastic Minimum Asset Requirements to the Economic Base Case Requirements for the Same Portfolio	213
9.25	Ratios of the Distributional Mean Sensitivity Analysis Stochastic Minimum Asset Requirements to the (Original) Base Case Requirements for the Same Portfolio	215
9.26	Average Numbers of Iterations Required for Simulation Convergence and Accuracy	217
A.1	IA95-97 M and F	226
A.2	ALT95-97 M and F	227
A.3	Mortality Reduction Factors.	228
A.4	Expected Lapse Rates by Curtate Duration (Years) and Policy Type.	229
D.1	Stochastic Minimum Asset Requirements for the Economic Sub-Model Sensitivity Analysis Scenarios (\$)	240
D.2	Ratios of the LPS2.04 Solvency Requirements to the Stochastic Minimum Asset Requirements for the Economic Sub-Model Sensitivity Analysis Scenarios	241
D.3	Ratios of the LPS3.04 Capital Adequacy Requirements to the Stochastic Minimum Asset Requirements for the Economic Sub-Model Sensitivity Analysis Scenarios	242

D.4	Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Mortality Sub-Model Sensitivity Analysis Scenarios	243
D.5	Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Lapsation Sub-Model Sensitivity Analysis Scenarios	244
D.6	Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Amounts for the Lapsation Sub-Model Sensitivity Analysis Scenarios	245
D.7	Stochastic Minimum Asset Requirements for the Lapsation Sub-Model Sensitivity Analysis Scenarios (\$)	245
D.8	p -values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Lapsation Sub-Model Sensitivity Analysis Scenarios	245
D.9	Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Mortality and Lapsation Sub-Model Sensitivity Analysis Scenarios	246
D.10	Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Dependencies Sensitivity Analysis Scenarios	247
D.11	Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Dependencies Sensitivity Analysis Scenarios (\$)	248
D.12	Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Amounts for the Dependencies Sensitivity Analysis Scenarios	249
D.13	Stochastic Minimum Asset Requirements for the Dependencies Sensitivity Analysis Scenarios (\$)	250
D.14	Ratios of the LPS2.04 and LPS3.04 Requirements to the Stochastic Minimum Asset Requirements for the Dependencies Sensitivity Analysis Scenarios	251
D.15	p -values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Dependencies Sensitivity Analysis Scenarios	252
D.16	Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Distributional Mean Sensitivity Analysis Scenarios	253
D.17	Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Distributional Mean Sub-Model Sensitivity Analysis Scenarios (\$)	254
D.18	Stochastic Minimum Asset Requirements for the Distributional Mean Sensitivity Analysis Scenarios (\$)	255
D.19	Ratios of the LPS2.04 Solvency Requirements to the Stochastic Minimum Asset Requirements for the Distributional Mean Sensitivity Analysis Scenarios	256

D.20 Ratios of the LPS3.04 Capital Adequacy Requirements to the Stochastic Minimum Asset Requirements for the Distributional Mean Sensitivity Analysis Scenarios	257
D.21 p -values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Distributional Mean Sensitivity Analysis Scenarios	258

List of Figures

3.1	A Simplified Pictorial View of the Solvency Testing Model Framework	28
5.1	CPI Inflation ($\Delta \ln Q(t)$)	64
5.2	Share Price Index Growth ($\Delta \ln P(t)$)	64
5.3	Dividend Yield ($\ln Y(t)$)	65
5.4	Interest Rates ($\ln(1 + R(t))$ and $\ln(1 + F(t))$)	65
5.5	Property Yield ($\Delta \ln Z(t)$)	66
5.6	Unemployment Rate ($U(t)$)	66
6.1	A Scree Plot for the Economic Data.	87
6.2	All-Age All-Duration Mortality Ratios for the IAAust Data Set . . .	88
6.3	All-Age Mortality Ratios for the Australian Population Data Set . .	90
6.4	All-Age Mortality Ratios versus Short-Term Interest Rate (%) for the Australian Population Data Set	91
6.5	All-Age Mortality Ratios versus Unemployment Rate (%) for the Aus- tralian Population Data Set	92
6.6	All-Age, All-Duration Insured Life Lapse Ratios (Single Insurer Mor- tality Data)	100
6.7	All-Duration Insured Life Lapse Ratios (Single Insurer Lapse Data) .	100
7.1	Relationships Between the Variables in the Wilkie Model	110
7.2	Relationships Between the Variables in the CAS/SOA Model	112
7.3	Property Yield Correlogram ($\Delta \ln Z(t)$)	120
7.4	A Simplified Pictorial View of the Stochastic Sub-Model Cascade Structure	145

9.1	Average Numbers of Iterations Required For Simulation Convergence by Liability Portfolio and Type of Analysis	217
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CHAPTER 1

Introduction

The most important goal of any business is to remain solvent, as it can no longer continue its operations if it becomes insolvent, except in special circumstances where the government bails out the company by injecting capital. In the case of financial services institutions, such as banks and insurance companies, continued solvency is of importance, not just to the institution, but also to account/policyholders who could potentially face economic hardship if such an institution were to collapse. The 2001 collapse of Australian General Insurer HIH Insurance illustrates the adverse consequences to the public of such an event. As a result, the financial services industry is one of the most highly regulated industries, and all advanced economies have in place legislation designed to minimise the risk of a financial services institution going bankrupt. The legislation generally requires such institutions to hold capital greater than a specified minimum amount, often referred to as the *solvency capital requirement*, at all points in time. This thesis focuses on the calculation of this quantity in the context of the Australian Life Insurance industry.

Currently, Australian Life Insurers are required to calculate their solvency capital requirements on a deterministic basis using formulae set out in Life Insurance Prudential Standards LPS2.04 and LPS3.04¹. However, recently there has been a trend in advanced economies, such as Switzerland, through the Swiss Solvency Test, and the European Union countries, through Solvency II, towards calculating insurer

¹LPS2.04 and LPS3.04 specify methodologies the insurer must follow in order to determine its solvency and capital adequacy requirements, respectively. The insurer must hold assets greater than both of these requirements at all times. If the insurer's assets fall below the solvency requirement, the insurer is considered to be insolvent for statutory purposes, while if its assets fall below the capital adequacy requirement, the Australian insurance regulator, the Australian Prudential Regulatory Authority (APRA), will intervene in the hope of preventing the insurer from becoming insolvent.

solvency capital requirements using stochastic techniques, thereby requiring insurers to hold a capital amount that satisfies a probability-based criterion. For example, insurers might be required to hold an amount of capital sufficiently large so that there is a 99.5% chance that, in one year's time, the insurer's assets will exceed its liabilities. In order to satisfy such a criterion, the insurer must attempt to determine the probability distributions of the values of its assets and liabilities, or sometimes just of its capital holdings, at future points in time. These distributions usually need to be determined using computer-intensive simulation techniques. It was due to the unavailability of inexpensive, high-speed computers in the past that deterministic solvency testing techniques were used almost exclusively in all countries, and it is because of the easier access to such computers in recent years that stochastic solvency testing techniques have suddenly come to prominence.

It is anticipated that the Australian insurance regulator, the Australian Prudential Regulatory Authority (APRA), will ultimately require Australian Life Insurers to calculate their solvency capital requirements using stochastic methods. Such being the case, there is a need to develop a realistic asset-liability model that can be used for this purpose. In this thesis, such a model is constructed and the model is then used to assess whether the current Australian deterministic solvency capital criteria are appropriate, based on four commonly used stochastic solvency criteria: the 99.5% Value at Risk (VaR) and Tail Value at Risk (TVaR) of the change in capital distribution over a one year time horizon, and the 95% VaR and TVaR of the change in capital distribution over a three year time horizon. The developed model is a simulation model comprising three interconnected stochastic sub-models used to describe the economic environment and the mortality and lapsation experience. It is demonstrated, using Australian economic and Life Insurance data, that the "best" sub-model in each case (out of the range of models under consideration) is a modified CAS/SOA² economic sub-model, a Poisson or negative binomial (depending on the policy type considered) mortality sub-model, and a normal-Poisson lapsation sub-model.

Tests conducted in this thesis demonstrate that, although the current deterministic requirements are sufficiently high for portfolios of investment-linked or "traditional" (endowment insurance) policies, they provide very little protection against insolvency for portfolios of "traditional" term insurance or for portfolios of "modern" yearly-renewable term insurance under some of the solvency criteria. Sensitivity tests conducted in association with these investigations show that the (stochastic) total asset requirements calculated using the solvency testing model are virtually unaffected by ignoring the over-dispersion that was found to be present in the mortality and lapsation data used in this thesis, or dependency relationships that were found

²Casualty Actuarial Society/Society of Actuaries.

to exist between the economy and mortality rates, and the economy and lapsation rates. However, for some policy types, the requirements are significantly affected by changing the sub-model used to forecast the economic variables, or simplifying the formulae used to determine the mean mortality and lapsation rates in the sub-models used to forecast future mortality and lapsation experience. The implication of this latter result is that, if APRA is to require Life Insurers to calculate their solvency capital requirements using stochastic methods, then, in order to ensure consistency between insurers, some guidance should be provided with regard to the nature of the solvency testing model used.

The structure of this thesis is as follows: Chapter 2 provides a review of the existing stochastic valuation and solvency testing literature, as well as providing an overview of the current solvency legislation in place in Australia and in several other countries throughout the world. In Chapter 3, a framework for the stochastic solvency testing model built in this thesis is developed and a number of research questions are posed. Chapter 4 gives background details on many of the statistical models (including generalised linear models and time series models) and tests used in this thesis. The main data sets used in this thesis are described in Chapter 5, and in Chapters 6 to 8, a realistic stochastic solvency testing model, intended for use by Australian Life Insurers, is developed based on this data, and the method of implementation of the model is set out. Chapter 9 summarises the results of comparing the solvency capital requirements calculated using the stochastic solvency model developed in the previous chapters with those calculated under LPS2.04 and LPS3.04, and provides a sensitivity analysis of these results; and Chapter 10 concludes this thesis by discussing its limitations and suggesting possibilities for future research.