Stochastic Solvency Testing
in Life Insurance

Genevieve Katherine Hayes

A thesis submitted for the degree of Doctor of Philosophy of
The Australian National University

September 2008
This thesis contains no material that has been accepted for the award of any other degree or diploma in any University, and to the best of my knowledge and belief, contains no material published or written by another person, except where due reference is made in the thesis.

Genevieve Hayes

September 2008
Acknowledgements

Over the course of my PhD, I have received advice and direction from a number of people, first among them being Prof. Ross Maller. Ross taught me the importance of statistical evidence in justifying opinions, no matter how wise they may appear. I have never met anyone with higher standards than Ross, both of himself and of his students, and although, at times, I wished that he would lower these standards, I am now glad that he never did. I could not imagine a better supervisor than Ross.

I would also like to thank Dr. Steven Stern, David Service, Aaron Bruhn, Adam Butt, Tim Higgins, Jeff Dukes, Sue Howes, Winnie Luo and Prof. David Wilkie, for their assistance and feedback.

Thank you to the Australian National University School of Finance and Applied Statistics for providing me with the resources necessary to complete my research, and to the following individuals for assisting me in gaining access to the data used in this thesis: Dr. Len Smith of the Australian Centre for Population Research at the Australian National University, who provided access to the Australian Population Data Set; Anthony Brien and Sue Clark of the Institute of Actuaries of Australia Life Risk Insurance Committee, who provided access to the IAAust Data Set; and the Australian Life Insurer (which must remain anonymous for confidentiality reasons) that supplied the Single Insurer Data Set.

Finally, I would like to thank my parents, who lived through this PhD with me. They never stopped believing in me, even on those occasions when I thought that I would never make it to the end of it all.
Stochastic solvency testing methods have existed for more than 20 years, yet there has been little research conducted in this area, particularly in Australia. This is for a number of reasons, the most pertinent of which being the lack of computing capabilities available in the past to implement more sophisticated techniques. However, recent advances in computing have made stochastic solvency testing possible in practice and have resulted in a trend towards this being done in advanced studies.

The purpose of this thesis is to develop a realistic solvency testing model in a form that can be implemented by Australian Life Insurers, in anticipation that the Australian insurance regulator, APRA, will ultimately follow the world trend and require stochastic solvency testing to be carried out in Australia. The model is constructed from three interconnected stochastic sub-models used to describe the economic environment and the mortality and lapsation experience of the portfolio of policies under consideration. Australian economic and Life Insurance data is used to fit a number of possible sub-models, such as generalised linear models, over-dispersion models and asset models, and the “best” model is selected in each case. The selected models are a modified CAS/SOA economic sub-model; either a Poisson or negative binomial (NB1) distribution (depending on the policy type considered) as the mortality sub-model; and a normal-Poisson lapsation sub-model.

Based on tests carried out using this model, it is demonstrated that, for portfolios of level and yearly-renewable term insurance business, the current deterministic solvency capital requirements provide little protection against insolvency. In fact, for the test portfolios of term insurance policies considered, the deterministic capital requirements have levels of sufficiency of less than 2% (on a Value at Risk basis) when compared to the change in capital distribution over a three year time horizon. This is of concern, as yearly-renewable term insurance comprises a significant volume of Life Insurance business in Australia, with there being over 426,000 yearly-renewable
term insurance policies on the books of Australian Life Insurers in 1999 and more business expected since then.

A sensitivity analysis shows that the results of the stochastic asset requirement calculations are sensitive to the choice of sub-model used to forecast economic variables and to the choice of formulae used to describe the mean mortality and lapsation rates. The implication of this is that, if APRA were to require Life Insurers to calculate their solvency capital requirements on a stochastic basis, some guidance would need to be provided regarding the components of the solvency testing model used. The model is not, however, sensitive to whether an allowance is made for mortality or lapsation rate over-dispersion, nor to whether dependency relationships between mortality rates, lapsation rates and the economy are allowed for. Thus, over-dispersion and dependency relationships between the sub-models can be ignored in a stochastic solvency testing model without significantly impacting the calculated solvency requirements.
# Contents

1 Introduction

2 Life Insurance Solvency Testing: A Review
   2.1 Introduction ........................................... 5
   2.2 Reserves and Policy Liabilities ........................ 6
   2.3 An Overview of Policy Valuation Techniques .......... 8
   2.4 Solvency ................................................. 10
   2.5 Stochastic Claims Reserving and Solvency Testing .... 12
   2.6 Risk Measures ........................................... 13
   2.7 Reserving and Solvency Legislation ..................... 15
   2.8 Conclusion .............................................. 21

3 A Framework for Stochastic Solvency Testing ............... 23
   3.1 Introduction ........................................... 23
   3.2 The Model Framework .................................... 23
   3.3 Research Questions ...................................... 29

4 Statistical Models for Mortality and Economic Data ......... 31
   4.1 Introduction ........................................... 31
   4.2 Generalised Linear Models ............................... 31
      4.2.1 Generalised Linear Models - An Overview .......... 31
   4.2.2 A Review of Existing Generalised Linear Models for Mortality Data .................................. 32
   4.2.3 Graduation by Reference to a Standard Table ....... 34
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.4</td>
<td>Generalised Linear Models for Mortality Data Using a Standard Table</td>
<td>36</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Mortality Reduction Factors</td>
<td>37</td>
</tr>
<tr>
<td>4.3</td>
<td>Over-Dispersion Models and Testing</td>
<td>38</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Testing for Over-Dispersion</td>
<td>38</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Models for Over-Dispersion</td>
<td>40</td>
</tr>
<tr>
<td>4.4</td>
<td>Time Series Statistical Models</td>
<td>45</td>
</tr>
<tr>
<td>4.5</td>
<td>Software</td>
<td>47</td>
</tr>
<tr>
<td>5</td>
<td>The Data</td>
<td>49</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>49</td>
</tr>
<tr>
<td>5.2</td>
<td>The IAAust Data Set</td>
<td>49</td>
</tr>
<tr>
<td>5.3</td>
<td>The Single Insurer Data Set</td>
<td>54</td>
</tr>
<tr>
<td>5.3.1</td>
<td>The Single Insurer Mortality Data</td>
<td>54</td>
</tr>
<tr>
<td>5.3.2</td>
<td>The Single Insurer Lapsation Data</td>
<td>58</td>
</tr>
<tr>
<td>5.4</td>
<td>The Economic Data</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>Dependency Relationships</td>
<td>69</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>69</td>
</tr>
<tr>
<td>6.2</td>
<td>Withdrawal Rates and Mortality</td>
<td>70</td>
</tr>
<tr>
<td>6.2.1</td>
<td>Selective Lapsation</td>
<td>70</td>
</tr>
<tr>
<td>6.2.2</td>
<td>Existing Tests for Selective Lapsation</td>
<td>71</td>
</tr>
<tr>
<td>6.2.3</td>
<td>A GLM-Based Test for Selective Lapsation</td>
<td>73</td>
</tr>
<tr>
<td>6.2.4</td>
<td>Inferring Policy Discontinuances from the Single Insurer Data</td>
<td>74</td>
</tr>
<tr>
<td>6.2.5</td>
<td>Results</td>
<td>76</td>
</tr>
<tr>
<td>6.2.6</td>
<td>Conclusion</td>
<td>78</td>
</tr>
<tr>
<td>6.3</td>
<td>Mortality and the Economy</td>
<td>79</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Population Mortality and the Economy</td>
<td>79</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Existing Tests for a Relationship Between Economic Fluctuations and Mortality</td>
<td>81</td>
</tr>
<tr>
<td>6.3.3</td>
<td>A GLM-Based Test for a Relationship Between Economic Fluctuations and Mortality</td>
<td>84</td>
</tr>
<tr>
<td>6.3.4</td>
<td>The Data</td>
<td>88</td>
</tr>
<tr>
<td>6.3.5</td>
<td>GLM Modelling Results</td>
<td>92</td>
</tr>
<tr>
<td>6.3.6</td>
<td>Conclusion</td>
<td>95</td>
</tr>
<tr>
<td>6.4</td>
<td>Withdrawal Rates and the Economy</td>
<td>95</td>
</tr>
<tr>
<td>6.4.1</td>
<td>The Interest Rate and Emergency Fund Hypotheses</td>
<td>95</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Existing Tests for a Relationship Between Economic Fluctuations and Lapse Rates</td>
<td>96</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>6.4.3 A GLM-Based Test for a Relationship Between Economic Fluctuations and Lapsation</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>6.4.4 The Data</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>6.4.5 Results</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>6.4.6 Conclusion</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>7 Stochastic Sub-Models</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>7.2 Stochastic Economic Models</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>7.2.1 Introduction</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>7.2.2 A Review of Existing Stochastic Economic Models</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>7.2.3 Fitting the Models</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>7.2.4 A Discussion of the Appropriateness of the Stochastic Economic Models</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>7.2.5 Tests for Comparing Stochastic Asset Models</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>7.2.6 The Data</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>7.2.7 Results</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>7.2.8 Conclusion</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>7.3 Stochastic Mortality Models</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>7.3.1 Poisson and Binomial Models</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>7.3.2 Testing for Over-Dispersion in the Insured Life Mortality Data</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>7.3.3 Results</td>
<td>133</td>
<td></td>
</tr>
<tr>
<td>7.4 Stochastic Lapsation Models</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>7.4.1 Introduction</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>7.4.2 Testing for Over-Dispersion in the Lapsation Data</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>7.4.3 Results</td>
<td>139</td>
<td></td>
</tr>
<tr>
<td>7.5 Summary</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>8 Solvency Testing Methodology</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>8.1 Introduction</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>8.2 Solvency Testing Methodology</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>8.3 Simulation</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>8.3.1 Simulation Techniques</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>8.3.2 Convergence and Accuracy</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>8.4 The Spreadsheet Models</td>
<td>153</td>
<td></td>
</tr>
<tr>
<td>8.4.1 Introduction</td>
<td>153</td>
<td></td>
</tr>
<tr>
<td>8.4.2 The Policy Model</td>
<td>153</td>
<td></td>
</tr>
<tr>
<td>8.4.3 The Payment Model</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>8.4.4 The Asset Model</td>
<td>159</td>
<td></td>
</tr>
<tr>
<td>8.4.5 The Capital Model</td>
<td>160</td>
<td></td>
</tr>
</tbody>
</table>
8.5 Model Assumptions ......................................................... 160
  8.5.1 Introduction .......................................................... 160
  8.5.2 Portfolio Specifications ............................................ 160
  8.5.3 Expenses ............................................................. 162
  8.5.4 Asset Mix ............................................................ 162
  8.5.5 Surrender Values ................................................... 164
  8.5.6 Premiums ............................................................. 165
  8.5.7 Starting Values ....................................................... 171
8.6 Solvency and Capital Adequacy Assumptions .............................. 171
  8.6.1 Introduction .......................................................... 171
  8.6.2 Solvency Assumptions .............................................. 172
  8.6.3 Capital Adequacy Assumptions ................................... 176
8.7 Cost of Capital Risk Margin Calculations ................................ 178
8.8 Sensitivity Analysis ....................................................... 179

9 Results ........................................................................... 183
  9.1 Introduction ............................................................... 183
  9.2 Base Case Simulation Results ......................................... 183
    9.2.1 Capital Adequacy .................................................. 183
    9.2.2 Asset Requirements ............................................... 189
  9.3 Sensitivity Analysis ..................................................... 192
    9.3.1 Economic Sub-Model Sensitivities ............................. 192
    9.3.2 Mortality Sub-Model Sensitivities .............................. 201
    9.3.3 Lapsation Sub-Model Sensitivities .............................. 201
    9.3.4 Mortality and Lapsation Sub-Model Sensitivities .......... 204
    9.3.5 Dependency Sensitivities ........................................ 205
    9.3.6 Distributional Mean Sensitivities ............................... 209
    9.3.7 General Comments Relating to the Sensitivity Analyses .... 216

10 Conclusion ..................................................................... 219
  10.1 Responses to the Research Questions ............................ 219
  10.2 Implications ............................................................. 221
  10.3 Limitations and Further Research ................................ 223

Appendices ....................................................................... 225
A Standard Tables .................................................................. 225
  A.1 Mortality ................................................................. 225
  A.2 Lapsation ................................................................. 229
B Australian Solvency and Capital Adequacy Requirements 231
  B.1 Introduction ............................................. 231
  B.2 The Solvency Requirement ................................ 231
  B.3 The Capital Adequacy Requirement ....................... 233

C Proofs of Mathematical Results 235
  C.1 Change in Capital $\Delta C(1)$ .......................... 235
  C.2 VaR of the $-\Delta C(1)$ Distribution .................. 236
  C.3 Change in Capital $\Delta C(t)$ .......................... 237
  C.4 VaR of the $-\Delta C_{min}(0,3)$ Distribution .......... 238

D Detailed Sensitivity Analysis Outputs 239
  D.1 Economic Sub-Model Sensitivity Outputs ................. 240
  D.2 Mortality Sub-Model Sensitivity Outputs ............... 243
  D.3 Lapsation Sub-Model Sensitivity Outputs ............... 244
  D.4 Mortality and Lapsation Sub-Model Sensitivity Outputs 246
  D.5 Dependencies Sensitivity Outputs ....................... 247
  D.6 Distributional Mean Sensitivity Outputs ............... 253

Glossary of Acronyms and Abbreviations 259

Glossary of Insurance Terms 261

Bibliography 265
List of Tables

4.1 A Comparison of Reduction Factors at Selected Ages ............. 38

5.1 Exposures by Sex, Policy Type and Year ......................... 50
5.2 Deaths by Sex, Policy Type and Year ............................ 51
5.3 Exposures by Age Band, Sex and Duration Band for Type 1 Policies 51
5.4 Exposures by Age Band, Sex and Duration Band for Type 2 Policies 52
5.5 Exposures by Age Band, Sex and Duration Band for Type 3 Policies 52
5.6 Exposures by Age Band, Sex and Duration Band for Type 4 Policies 53
5.7 Exposures by Sex, Policy Type and Year ......................... 55
5.8 Deaths by Sex, Policy Type and Year ............................ 55
5.9 Non-Death Terminations by Sex, Policy Type and Year ............ 56
5.10 Exposures by Age Band, Sex and Duration Band for Type 1 Policies 56
5.11 Exposures by Age Band, Sex and Duration Band for Type 4 Policies 57
5.12 Exposures as a % of the Total for each Policy Type/Sex/Data Set Combination by Age Band ................................. 57
5.13 Exposures as a % of the Total for each Policy Type/Sex/Data Set Combination by Duration Band ............................. 58
5.14 Exposures and Withdrawals by Policy Type and Year ............. 59
5.15 Exposures by Duration Band and Policy Type .................... 60
5.16 Exposures as a % of the Total for each Policy Type by Duration Band 60
5.17 A Summary of the Economic Data ................................. 62
5.18 Summary Statistics for the Economic Data ....................... 63
5.19 Correlations Between the Economic Variables with $p$-values Shown in Brackets ..................................................... 67
6.1 Correlations Between Mortality Ratios and Lapse Ratios by Lag, Sex and Policy Type (All Ages) with p-values Shown in Brackets 76
6.2 Correlations Between Mortality Ratios and Lapse Ratios by Lag, Sex and Policy Type (Ages 15–64 Only) with p-values Shown in Brackets 76
6.3 Drop in Deviance Between the Two Selective Lapsation GLMs by Maximum Lag 77
6.4 Correlations Between Insured Life Mortality Ratios and Economic Variables (1995–1999) with p-values Shown in Brackets 89
6.5 Correlations Between Population Mortality Ratios and Economic Variables with p-values Shown in Brackets 91
6.6 Drop in Deviance between the Two Economic Fluctuation versus Mortality GLMs 93
6.7 Analysis of Deviance Table for Model 2 for the IAAust Data 93
6.8 Fitted Coefficients for the Economic Fluctuation versus Mortality GLMs with Standard Errors Shown in Brackets 94
6.9 % Change in the Mortality Ratio Associated with a 1% Change in the Short-Term Interest Rate 95
6.10 Correlations Between (All-Duration) Lapse Ratios and Economic Variables (Single Insurer Mortality Data) with p-values Shown in Brackets 101
6.11 Correlations Between (All-Duration) Lapse Ratios and Economic Variables (Single Insurer Lapsation Data) with p-values Shown in Brackets 101
6.12 Decrease in Deviance between the Two Economic Fluctuation versus Mortality GLMs 102
6.13 Fitted Coefficients for the Economic Fluctuation versus Mortality GLMs (Model 2) with Standard Errors Shown in Brackets 103
6.14 % Change in the Lapsation Ratio Associated with a 1% Change in Each of the Economic Variables for the Single Insurer Mortality Data 103
7.1 Fitted Parameter Values for the Kemp Model with Standard Errors Shown in Brackets 121
7.2 Correlation Matrix for the Kemp Model with p-values Shown in Brackets 121
7.3 Goodness of Fit Statistics for the Kemp Random Walk Model 122
7.4 A Summary of the Wilkie Model Sub-Models Used in this Thesis 124
7.5 Fitted Parameter Values for the Wilkie Model 124
7.6 Goodness of Fit Statistics for the Wilkie Model Time Series Processes 124
7.7 Fitted Parameter Values for the CAS/SOA Model 126
7.8 Goodness of Fit Statistics for the CAS/SOA Model Time Series Processes 126
7.9 A Comparison of the AIC for the Kemp, Wilkie and CAS/SOA Models .................................. 127
7.10 Fitted Parameter Values for the Modified CAS/SOA Model ............................... 129
7.11 Goodness of Fit Statistics for the Modified CAS/SOA Model .............................. 129
7.12 A Summary of the Over-Dispersion Models Fitted to the Data ...................... 133
7.13 Over-Dispersion Statistics for the Poisson GLMs with p-values Shown in Brackets ...................................................... 134
7.14 Over-Dispersion Parameters for the Over-Dispersion Models with p-values Shown in Brackets ...................................................... 135
7.15 AIC for the Models Fitted to the Type 1 Males and Females Data ............... 137
7.16 Fitted Coefficients for the Over-Dispersion Models with Standard Errors Shown in Brackets ...................................................... 137
7.17 Over-Dispersion Statistics for the Lapsation Data Poisson GLMs with p-values Shown in Brackets ...................................................... 140
7.18 Over-Dispersion Parameters for the Lapsation Over-Dispersion Models with p-values Shown in Brackets ...................................................... 141
7.19 AIC for the Lapsation Over-Dispersion Models ................................................. 142
7.20 Fitted Coefficients for the Normal-Poisson Random Coefficient Model with Standard Errors Shown in Brackets ...................................................... 144
7.21 Fitted Variances and Covariances for the Normal-Poisson Random Coefficient Model with Standard Errors Shown in Brackets ...................................................... 144

8.1 Model Portfolio Compositions ................................................................................. 161
8.2 Duration Assumptions (Years) by Age Band, Sex and Policy Type ............... 162
8.3 Expense Assumptions per Policy ........................................................................... 163
8.4 Composition of each of the Model Asset Portfolios ........................................ 164
8.5 Regular Annual Premiums Assumed for Type 1 and 3 Policies by Age Band ...................................................... 167
8.6 Single Premiums Assumed for the Type 2 (M and F) Policies Under Each Investment Option by Age Band ...................................................... 168
8.7 Implicit Profit Margins (% of After-Tax Investment Earnings) for Type 2 Policies Under Each Investment Option ...................................................... 168
8.8 Premium Rate Scale for Type 4 Policies ($) ....................................................... 170
8.9 Implicit Profit Margins (% of Claims) for Type 4 Policies .................................. 170
8.10 Economic Variable Starting Values ..................................................................... 171
8.11 Proportion of New Policies in Each Age Band by Policy Type and Sex ......... 174
8.12 Regular Annual Premiums Assumed for New Type 1 and 3 Policies .......... 174
8.14 Selected Capital Adequacy Margins ........................................... 177
8.15 Fitted Values of $\alpha_d$ and $\alpha_w$ by Policy Type and Sex .............. 181

9.1 VaR and TVaR Per Policy for the Base Case Scenarios ($) .................. 185
9.2 Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Base Case Scenarios ($) ............... 186
9.3 Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Requirements for the Base Case Scenarios ......................... 187
9.4 Summary Statistics for the Base Case Simulations .......................... 187
9.5 Number of Iterations Performed, the Time Required and the Number of Iterations Needed for Convergence for the Base Case Scenarios ... 188
9.6 $p$-values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Base Case Scenarios ........................................... 189
9.7 Stochastic Minimum Asset Requirements for the Base Case Scenarios ($) ................................................................. 190
9.8 Ratios of the LPS2.04 and LPS3.04 Solvency and Capital Adequacy Requirements to the Stochastic Minimum Asset Requirements for the Base Case Scenarios ........................................... 191
9.9 $p$-values for the Two-Sample Kolmogorov-Smirnov Tests for the Economic Sub-Model Sensitivity Scenarios ..................................... 193
9.10 VaR and TVaR Per Policy for the Economic Sub-Model Sensitivity Analysis Scenarios ($) with Ratios of these Amounts to the Base Case Capital Requirements Shown in Brackets .............................. 195
9.11 Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Economic Sub-Model Sensitivity Analysis Scenarios ($) ........................................... 196
9.12 Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Amounts for the Economic Sub-Model Sensitivity Analysis Type 3 and 4 Policy Scenarios ........................................... 197
9.13 Ratios of the Economic Sub-Model Sensitivity Analysis Stochastic Minimum Asset Requirements to the Base Case Requirements for the Same Portfolio ........................................... 198
9.14 Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{min}(0,3)$ Simulations for the Economic Sub-Model Sensitivity Analysis Scenarios ............. 199
9.15 $p$-values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Economic Sub-Model Sensitivity Analysis Scenarios ............. 200
9.16 $p$-values for the Two-Sample Kolmogorov-Smirnov Tests for the Mortality Sub-Model Sensitivity Scenarios ..................................... 201
9.17  p-values for the Two-Sample Kolmogorov-Smirnov Tests for the Lapsation Sub-Model Sensitivity Scenarios ........................................ 202
9.18  VaR and TVaR Per Policy for the Lapsation Sub-Model Sensitivity Analysis Scenarios ($) with Ratios of these Amounts to the Economic Base Case Capital Requirements Shown in Brackets ....................... 203
9.19  p-values for the Two-Sample Kolmogorov-Smirnov Tests for the Mortality and Lapsation Sub-Model Sensitivity Scenarios ..................................... 204
9.20  p-values for the Two-Sample Kolmogorov-Smirnov Tests for the Dependencies Sensitivity Scenarios ..................................................... 206
9.21  VaR and TVaR Per Policy for the Dependencies Sensitivity Analysis Scenarios ($) with Ratios of these Amounts to the Economic Base Case Capital Requirements Shown in Brackets ....................... 208
9.22  p-values for the Two-Sample Kolmogorov-Smirnov Tests for the Distributional Mean Sensitivity Analysis Scenarios ......................................... 210
9.23  VaR and TVaR Per Policy for the Distributional Mean Sensitivity Analysis Scenarios ($) with Ratios of these Amounts to the Economic Base Case Capital Requirements Shown in Brackets ....................... 212
9.24  Ratios of the Distributional Mean Sensitivity Analysis Stochastic Minimum Asset Requirements to the Economic Base Case Requirements for the Same Portfolio ..................................................... 213
9.25  Ratios of the Distributional Mean Sensitivity Analysis Stochastic Minimum Asset Requirements to the (Original) Base Case Requirements for the Same Portfolio ..................................................... 215
9.26  Average Numbers of Iterations Required for Simulation Convergence and Accuracy ...................................................................................... 217

A.1  IA95-97 M and F ................................................................. 226
A.2  ALT95-97 M and F ............................................................... 227
A.3  Mortality Reduction Factors .................................................... 228
A.4  Expected Lapse Rates by Curtate Duration (Years) and Policy Type. 229

D.1  Stochastic Minimum Asset Requirements for the Economic Sub-Model Sensitivity Analysis Scenarios ($) .................................................. 240
D.2  Ratios of the LPS2.04 Solvency Requirements to the Stochastic Minimum Asset Requirements for the Economic Sub-Model Sensitivity Analysis Scenarios .................................................. 241
D.3  Ratios of the LPS3.04 Capital Adequacy Requirements to the Stochastic Minimum Asset Requirements for the Economic Sub-Model Sensitivity Analysis Scenarios .................................................. 242
D.4 Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{\text{min}}(0, 3)$ Simulations for the Mortality Sub-Model Sensitivity Analysis Scenarios ....... 243
D.5 Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{\text{min}}(0, 3)$ Simulations for the Lapsation Sub-Model Sensitivity Analysis Scenarios ....... 244
D.6 Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Amounts for the Lapsation Sub-Model Sensitivity Analysis Scenarios ................................. 245
D.7 Stochastic Minimum Asset Requirements for the Lapsation Sub-Model Sensitivity Analysis Scenarios ($) ................................................ 245
D.8 p-values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Lapsation Sub-Model Sensitivity Analysis Scenarios ................. 245
D.9 Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{\text{min}}(0, 3)$ Simulations for the Mortality and Lapsation Sub-Model Sensitivity Analysis Scenarios ............................................. 246
D.10 Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{\text{min}}(0, 3)$ Simulations for the Dependencies Sensitivity Analysis Scenarios ................. 247
D.11 Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Dependencies Sensitivity Analysis Scenarios ($) ....................................... 248
D.12 Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Amounts for the Dependencies Sensitivity Analysis Scenarios 249
D.13 Stochastic Minimum Asset Requirements for the Dependencies Sensitivity Analysis Scenarios ($) ................................................ 250
D.14 Ratios of the LPS2.04 and LPS3.04 Requirements to the Stochastic Minimum Asset Requirements for the Dependencies Sensitivity Analysis Scenarios ............................................. 251
D.15 p-values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Dependencies Sensitivity Analysis Scenarios ................................. 252
D.16 Summary Statistics for the $-\Delta C(1)$ and $-\Delta C_{\text{min}}(0, 3)$ Simulations for the Distributional Mean Sensitivity Analysis Scenarios ................. 253
D.17 Best Estimate Liabilities, and Solvency and Capital Adequacy Capital Requirements Per Policy for the Distributional Mean Sub-Model Sensitivity Analysis Scenarios ($) .......................................... 254
D.18 Stochastic Minimum Asset Requirements for the Distributional Mean Sensitivity Analysis Scenarios ($) ................................................ 255
D.19 Ratios of the LPS2.04 Solvency Requirements to the Stochastic Minimum Asset Requirements for the Distributional Mean Sensitivity Analysis Scenarios ............................................. 256
D.20 Ratios of the LPS3.04 Capital Adequacy Requirements to the Stochastic Minimum Asset Requirements for the Distributional Mean Sensitivity Analysis Scenarios ........................................ 257
D.21 \( p \)-values for the Kolmogorov-Smirnov and Anderson-Darling Tests for the Distributional Mean Sensitivity Analysis Scenarios ........................................ 258
List of Figures

3.1 A Simplified Pictorial View of the Solvency Testing Model Framework 28

5.1 CPI Inflation ($\Delta \ln Q(t)$) ........................................... 64
5.2 Share Price Index Growth ($\Delta \ln P(t)$) ........................................... 64
5.3 Dividend Yield ($\ln Y(t)$) ............................................................. 65
5.4 Interest Rates ($\ln (1 + R(t))$ and $\ln (1 + F(t))$) ......................... 65
5.5 Property Yield ($\Delta \ln Z(t)$) ...................................................... 66
5.6 Unemployment Rate ($U(t)$) .......................................................... 66

6.1 A Scree Plot for the Economic Data ............................................. 87
6.2 All-Age All-Duration Mortality Ratios for the IAAust Data Set ........ 88
6.3 All-Age Mortality Ratios for the Australian Population Data Set ...... 90
6.4 All-Age Mortality Ratios versus Short-Term Interest Rate (%) for the Australian Population Data Set ........................................... 91
6.5 All-Age Mortality Ratios versus Unemployment Rate (%) for the Australian Population Data Set ........................................... 92
6.6 All-Age, All-Duration Insured Life Lapse Ratios (Single Insurer Mor
tality Data) ................................................................. 100
6.7 All-Duration Insured Life Lapse Ratios (Single Insurer Lapse Data) . 100

7.1 Relationships Between the Variables in the Wilkie Model ............. 110
7.2 Relationships Between the Variables in the CAS/SOA Model .......... 112
7.3 Property Yield Correlogram ($\Delta \ln Z(t)$) ........................................... 120
7.4 A Simplified Pictorial View of the Stochastic Sub-Model Cascade Structure ................................................................. 145
9.1 Average Numbers of Iterations Required For Simulation Convergence  
by Liability Portfolio and Type of Analysis  . . . . . . . . . . . . . . 217
The most important goal of any business is to remain solvent, as it can no longer continue its operations if it becomes insolvent, except in special circumstances where the government bails out the company by injecting capital. In the case of financial services institutions, such as banks and insurance companies, continued solvency is of importance, not just to the institution, but also to account/policyholders who could potentially face economic hardship if such an institution were to collapse. The 2001 collapse of Australian General Insurer HIH Insurance illustrates the adverse consequences to the public of such an event. As a result, the financial services industry is one of the most highly regulated industries, and all advanced economies have in place legislation designed to minimise the risk of a financial services institution going bankrupt. The legislation generally requires such institutions to hold capital greater than a specified minimum amount, often referred to as the solvency capital requirement, at all points in time. This thesis focuses on the calculation of this quantity in the context of the Australian Life Insurance industry.

Currently, Australian Life Insurers are required to calculate their solvency capital requirements on a deterministic basis using formulae set out in Life Insurance Prudential Standards LPS2.04 and LPS3.04\(^1\). However, recently there has been a trend in advanced economies, such as Switzerland, through the Swiss Solvency Test, and the European Union countries, through Solvency II, towards calculating insurer

\(^1\)LPS2.04 and LPS3.04 specify methodologies the insurer must follow in order to determine its solvency and capital adequacy requirements, respectively. The insurer must hold assets greater than both of these requirements at all times. If the insurer’s assets fall below the solvency requirement, the insurer is considered to be insolvent for statutory purposes, while if its assets fall below the capital adequacy requirement, the Australian insurance regulator, the Australian Prudential Regulatory Authority (APRA), will intervene in the hope of preventing the insurer from becoming insolvent.
solvency capital requirements using stochastic techniques, thereby requiring insurers to hold a capital amount that satisfies a probability-based criterion. For example, insurers might be required to hold an amount of capital sufficiently large so that there is a 99.5% chance that, in one year’s time, the insurer’s assets will exceed its liabilities. In order to satisfy such a criterion, the insurer must attempt to determine the probability distributions of the values of its assets and liabilities, or sometimes just of its capital holdings, at future points in time. These distributions usually need to be determined using computer-intensive simulation techniques. It was due to the unavailability of inexpensive, high-speed computers in the past that deterministic solvency testing techniques were used almost exclusively in all countries, and it is because of the easier access to such computers in recent years that stochastic solvency testing techniques have suddenly come to prominence.

It is anticipated that the Australian insurance regulator, the Australian Prudential Regulatory Authority (APRA), will ultimately require Australian Life Insurers to calculate their solvency capital requirements using stochastic methods. Such being the case, there is a need to develop a realistic asset-liability model that can be used for this purpose. In this thesis, such a model is constructed and the model is then used to assess whether the current Australian deterministic solvency capital criteria are appropriate, based on four commonly used stochastic solvency criteria: the 99.5% Value at Risk (VaR) and Tail Value at Risk (TVaR) of the change in capital distribution over a one year time horizon, and the 95% VaR and TVaR of the change in capital distribution over a three year time horizon. The developed model is a simulation model comprising three interconnected stochastic sub-models used to describe the economic environment and the mortality and lapsation experience. It is demonstrated, using Australian economic and Life Insurance data, that the “best” sub-model in each case (out of the range of models under consideration) is a modified CAS/SOA economic sub-model, a Poisson or negative binomial (depending on the policy type considered) mortality sub-model, and a normal-Poisson lapsation sub-model.

Tests conducted in this thesis demonstrate that, although the current deterministic requirements are sufficiently high for portfolios of investment-linked or “traditional” (endowment insurance) policies, they provide very little protection against insolvency for portfolios of “traditional” term insurance or for portfolios of “modern” yearly-renewable term insurance under some of the solvency criteria. Sensitivity tests conducted in association with these investigations show that the (stochastic) total asset requirements calculated using the solvency testing model are virtually unaffected by ignoring the over-dispersion that was found to be present in the mortality and lapsation data used in this thesis, or dependency relationships that were found.

---

2Casualty Actuarial Society/Society of Actuaries.
to exist between the economy and mortality rates, and the economy and lapsation rates. However, for some policy types, the requirements are significantly affected by changing the sub-model used to forecast the economic variables, or simplifying the formulae used to determine the mean mortality and lapsation rates in the sub-models used to forecast future mortality and lapsation experience. The implication of this latter result is that, if APRA is to require Life Insurers to calculate their solvency capital requirements using stochastic methods, then, in order to ensure consistency between insurers, some guidance should be provided with regard to the nature of the solvency testing model used.

The structure of this thesis is as follows: Chapter 2 provides a review of the existing stochastic valuation and solvency testing literature, as well as providing an overview of the current solvency legislation in place in Australia and in several other countries throughout the world. In Chapter 3, a framework for the stochastic solvency testing model built in this thesis is developed and a number of research questions are posed. Chapter 4 gives background details on many of the statistical models (including generalised linear models and time series models) and tests used in this thesis. The main data sets used in this thesis are described in Chapter 5, and in Chapters 6 to 8, a realistic stochastic solvency testing model, intended for use by Australian Life Insurers, is developed based on this data, and the method of implementation of the model is set out. Chapter 9 summarises the results of comparing the solvency capital requirements calculated using the stochastic solvency model developed in the previous chapters with those calculated under LPS2.04 and LPS3.04, and provides a sensitivity analysis of these results; and Chapter 10 concludes this thesis by discussing its limitations and suggesting possibilities for future research.