The Liar Paradox and its Relatives

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Declaration

This dissertation is my own work, except where otherwise acknowledged.

[Signature]

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The Contradictions.

(1) The oldest contradiction of the kind in question is the *Epimenides*. Epimenides the Cretan said that all Cretans were liars, and all other statements made by Cretans were certainly lies. Was this a lie? The simplest form of this contradiction is afforded by the man who says "I am lying;" if he is lying, he is speaking the truth, and vice versa.

(2) Let $w$ be the class of all those classes which are not members of themselves. Then, whatever class $x$ may be, "$x$ is a $w$" is equivalent to "$x$ is not an $x$." Hence, giving to $x$ the value $w$, "$w$ is a $w$" is equivalent to "$w$ is not a $w$.”

(3) Let $T$ be the relation which subsists between two relations $R$ and $S$ whenever $R$ does not have the relation $R$ to $S$. Then, whatever relations $R$ and $S$ may be, "$R$ has the relation $T$ to $S$" is equivalent to "$R$ does not have the relation $R$ to $S." Hence, giving the value $T$ to both $R$ and $S$, "$T$ has the relation $T$ to $T$" is equivalent to "$T$ does not have the relation $T$ to $T$.”

Bertrand Russell, 1908.

“Mathematical Logic as based on the Theory of Types”

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- ‘Relatives of the Liar Paradox including a New One’, my mid-term presentation June 2003. This incorporated immature elements of the next two papers, and sketched relationships with epistemic paradoxes that I hope to return to after this thesis.
- ‘Relatives of the Liar Paradox including a New One’, 2004 Australasian Association of Philosophy conference (AAP2004), which is the basis of Chapters 2 and 3.
- ‘Paradoxes of Truth, Satisfaction and membership’, AAP2005 conference, which is incorporated in Chapter 4.
- ‘Paradoxes and Hypodoxes of Time Travel’, AAP2006 conference, an extended version of the above paper.

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Abstract

My thesis aims at contributing to classifying the Liar-like paradoxes (and related Truth-teller-like expressions) by clarifying distinctions and relationships between these expressions and arguments. Such a classification is worthwhile, firstly, because it makes some progress towards reducing a potential infinity of versions into a finite classification; secondly, because it identifies a number of new paradoxes, and thirdly and most significantly, because it corrects the historically misplaced distinction between semantic and set-theoretic paradoxes. I emphasize the third result because the distinction made by Peano [1906] and supported by Ramsey [1925] has been used to warrant different responses to the semantic and set-theoretic paradoxes. I find two types among the paradoxes of truth, satisfaction and membership, but the division is shifted from where it has historically been drawn. This new distinction is, I believe, more fundamental than the Peano-Ramsey distinction between semantic and set-theoretic paradoxes. The distinction I investigate is ultimately exemplified in a difference between the logical principles necessary to prove the Liar and those necessary to prove Grelling’s and Russell’s paradoxes. The difference relates to proofs of the inconsistency of naive truth and satisfaction; in the end, we will have two associated ways of proving each result.

Another principled division is intuitively anticipated. I coin the term hypodox (adj.: hypodoxical) for a generalization of Truth-tellers across paradoxes of truth, satisfaction, membership, reference, and where else it may find applicability. I make and investigate a conjecture about paradox and hypodox duality: that each paradox (at least those in the scope of the classification) has a dual hypodox.

In my investigation, I focus on paradoxes that might intuitively be thought to be relatives of the Liar paradox, including Grelling’s (which I present as a paradox of satisfaction) and, by analogy with Grelling’s paradox, Russell’s paradox. I extend these into truth-functional and some non-truth-functional variations, beginning with the Epimenides, Curry’s paradox, and similar variations. There are circular and infinite variations, which I relate via lists. In short, I focus on paradoxes of truth, satisfaction and some paradoxes of membership.

Among the new paradoxes, three are notable in advance. The first is a non-truth functional variation on the Epimenides. This helps put the Epimenides on a par with Curry’s as a paradox in its own right and not just a lesser version of the Liar. I
find the second paradox by working through truth-functional variants of the paradoxes. This new paradox, call it ‘the ESP’, can be either true or false, but can still be used to prove some other arbitrary statement. The third new paradox is another paradox of satisfaction, distinctively different from Grelling’s paradox. On this basis, I make and investigate the new distinction between two different types of paradox of satisfaction, and map one type back by direct analogy to the Liar, and the other by direct analogy to Russell’s paradox.
The Liar Paradox and its Relatives

Audience

As a thesis, I am writing for a number of readers; but I am also trying to provide a general resource for ongoing research into the logical paradoxes. To this end, I assume that the reader has a mastery of first-order classical logic with identity and naive set-theory, and is familiar with the concepts of second-order logic, substitutional and objective quantification, the principle of bivalence, the (so-called) law of non-contradiction (also known as “the law of contradiction”), and has some general knowledge of philosophical theories of truth.

Structure

My introduction and Chapter 1 are deliberately discursive. I want to set the scene. The introduction includes a list of conundrums and paradoxes with the intent to motivate my suggestion that the time has come to begin a taxonomy or classification of paradoxes. Chapter 1 gives some historical intellectual background with particular focus on the Epimenides. The Epimenides has been viewed as just a version of the Liar, whereas Curry’s paradox is treated as a distinct paradox. Yet the variations of the Epimenides and Curry’s paradoxes can be put into one-to-one correspondence.

Chapter 2 is about truth, particularly in relation to the Liar paradox. It contains a discussion of the T-schema and the concept of paradox, focusing attention on the ongoing theme of how to individuate paradoxes.

Chapter 3 gives my basic classification of the nuclear family, as it were, of Liar-like paradoxes.

The Peano-Ramsey distinction between semantic and set-theoretic paradoxes is introduced in chapter 4, as this is when my classification is extended to paradoxes of satisfaction and membership. Chapter 4 introduces the extended family, as it were, relating paradoxes of truth to paradoxes of satisfaction and set-theory. The relationship is not straight-forward, a distinction is drawn between two types of paradox of satisfaction. This distinction is then extended among the paradoxes of membership. The primary distinction that I draw between paradoxes, instead of the Peano-Ramsey distinction remains only semi-formal in this chapter.

Chapter 5 focuses on formally grounding the distinction between two types of paradox in the logical principles necessary for each type of paradox. It may not be
very surprising that there are logical principles used to derive Liar-like paradoxes that are not necessary to derive Grelling’s or Russell’s. I think it is more interesting that there is a logical principle necessary for deriving Grelling’s and Russell’s that is not necessary for the Liar. Furthermore, I will show that when the use of truth in the Liar is mapped to a paradox of satisfaction, the result is not Grelling’s paradox; and, when the use of satisfaction in deriving Grelling’s is mapped to a derivation using the truth predicate, it is not the Liar paradox that results. I also explore the implications for proofs of the inconsistency of naive truth and satisfaction in Chapter 5.

In concluding, I summarise the results of my investigation and the elements that combine to individuate variations among the types of paradoxes.
Introduction

In times past, and today, people constructed and still construct lists of intuitively related conundrums, sophisms and paradoxes such as the list following. Most collections have been roughly grouped or simply listed, such as Michael Clark’s [2002] Paradoxes A-Z. Even efforts focused on classification, such as Ramsey [1925], Prior [1961], and Sorensen [1998] make only a few principled divisions. Indeed, there is a challenge to provide a formal basis for any division [Priest 1994]. My chief concern will be to provide a more systematic means of classifying paradoxes of truth, satisfaction and membership. I begin then with an extensive, discursive list intended to work on two levels; firstly, to motivate a more systematic classification of Liar-like paradoxes, and secondly, to illustrate and introduce some of the material that such a classification would aspire to encompass.

Here then are some teasers, all of them variations on a theme:

80% of statistics are false.

If 80% of statistics are false, other things being equal, there is an 80% chance that this sentence is false.

By analogous reasoning the following sentence is false:

100% of statistics are false.

As a statistic itself, this sentence would seem to say something about itself, to its own demerit.

Yet reasoning about whether the next two sentences are true or false will prove I am rich:

(1) Both this and the next sentence are false.
(2) There are a million dollars in my bank account.

If (1) is true, then it is false. So (1) is not true and is therefore false. A conjunction like (1) is false only if at least one of its conjuncts is false. Given that (1) is false, then that is what its first conjunct says about (1), so its first conjunct is true. Therefore (1) must be wrong about the next sentence being false, so (2) must be true. (A genuine consolation of philosophy to run past one’s bank manager – after all, it is pure logic, and therefore only reasonable!)

Lucian (125-192 CE) is an early source of a ‘Liar sentence’. In the
Introduction to *True History*, he writes:

I see no reason for resigning my right to that inventive freedom which others enjoy; and, as I have no truth to put on record, having lived a humdrum life, I fall back on falsehood – but falsehood of a more consistent variety; for I now make the only true statement you are to expect – that I am a liar. This confession is, I consider, a full defence against all imputations. My subject is, then, what I have neither seen, experienced, nor been told, what neither exists nor could conceivably do so. I humbly solicit my readers’ incredulity.¹

[Lucian, 1905 trans., p. 137.]

Lucian’s sentence borders on the worrisome Liar claim to be presently lying in saying he is lying, which would be self-devaluing like the earlier examples except then, on reflection, it would seem to warrant its own claim. Lucian seems to tie lying to telling falsehoods. If a liar always lies, then his claim to be lying is self-devaluing; so, logic dictates he must make some other true statement in the book. If, in fact, he does not, then logic meets contingency in paradox.

Compare this with broad statements by Bandler and Grinder, the originators of Neuro-Linguistic Programming, in their self-help classic *Frogs into Princes*:

Everything we’re going to tell you is a lie. All generalizations are lies. Since we have no claim on truth or accuracy, we will be lying to you consistently throughout this seminar.

[Bandler and Grinder 1979, p. 18]

If they claim to lie throughout the book, which is a record of a seminar, and this statement is itself made in the book, then is this statement true or false? It would seem that provided there is some true and sincere statement in the book, then this claim is simply false; and as we have seen such claims must be false, yet it follows from the necessary falsity of this claim that there must be some other truth in the book. Notice also that since ‘All generalizations are lies’ is itself a generalization, it entails that it itself is false, or that it is being asserted insincerely or with intent to mislead.

The reasoning behind these sorts of teasers (not necessarily what their various

authors intended) is similar to a type of conundrum or paradox known as the
Epimenides, a relative of the Liar Paradox. Statements like these examples
somewhat surprisingly are provably false in the circumstances. In certain additional
circumstances they may be paradoxical.²

Let me briefly introduce the Liar with reference to my favourite sentence. My
favourite sentence just happens to be ‘My favourite sentence is false’. If my
favourite sentence is false, it is true; but if it is true, it is false.

Trading in intuitions will keep us in business for a very long time; but if we
are serious about clarifying and distinguishing the issue or issues involved in these
exhibits, we need a more comprehensive classification of Liar-like paradoxes.
Nevertheless, let us savour a little more variety in this genre before settling down to
the analytical task at hand. I knowingly admit:

My thesis contains at least one overstatement.

(Treat what this sentence says in my thesis as a statement.) Even if my thesis
contained no other overstatement, this statement would seem to be true. Assume for
the moment that the rest of my thesis contains no overstatement; then, what this
statement says seems overstated; for if it were not, then the thesis contains less
overstatements than this statement says it does; but if it is overstated, in virtue of
being overstated it must be somewhat in error or have said something more than the
truth, and is therefore false. As a contradiction results from the assumption that the
rest of my thesis contains no other overstatement, my thesis must contain an
overstatement. So the statement has been proven.

Perhaps this does not follow if there is some defect in meaning, as the
following statement seems hard to interpret.

This statement is overstated.

Perhaps then also, the following sentence is not false, but meaningless; yet, that is
what it says:

This sentence is meaningless.

At least, the little that its dual, as given below, says seems intelligible, and
presumably true.

² Cf. Kripke [1975/1984, p. 55]. Kripke’s point about the inter-relationship between empirical
circumstances and paradox is discussed in Chapter 3.
This sentence is meaningful.

How is one to determine the truth of the next sentence?

This very sentence is true.

If it is true, it is true; if it is false, it is false; but there is a lack of some basis for determining which it is. This is a ‘Truth-teller’. A similar phenomenon can be found for other expressions and even in naive set theory. Consider the set of all sets that are members of themselves. Does it belong to itself or not? I will introduce the general concept of hypodox for such expressions.

Contrast the above Truth-teller sentence with another Liar sentence:

This sentence is false.

If this sentence is true, it is false; but if this sentence is false, it is true. This is the paradox of the Liar. In contrast to the Truth-teller, there is a problem consistently allocating this sentence a truth value.

I will conjecture in Chapter 3 that Liar-like sentences and Truth-teller-like sentences come in pairs, and that the members of each pair stand in a dual relationship to each other. Indeed, this duality extends to paradoxical expressions of satisfaction and at least some set-theoretic paradoxes. I make such a conjecture with reference to the naive concepts of truth, satisfaction and membership. However, in doing so, naively, I have not distinguished between ‘is false’ and ‘is not true’; otherwise, perhaps I should conjecture that each Truth-teller comes with a pair of paradoxes. My intent is to produce a classification of paradoxes as they occur in natural language, in their natural state, as it were.

Let me make some brief comments on the truth predicate and its negation, and on truth values; and then put these matters aside until Chapter 2.

In some sophisticated logics, ‘is false’ and ‘is not true’ are associated with distinguishable paradoxes, the Liar and the Strengthened Liar. Sophisticated logics may represent the ‘is not true’ predicate to the exclusion of the ‘is false’ predicate and yet distinguish between two valuations of being not true and being false; something could be not true without necessarily being false. Julius Caesar is not the sort of thing that is true, but he is not therefore false. Perhaps some sentences, like the Liar, may be neither true nor false. Naively though, we may reason as follows:

If the Liar sentence given above were true, it would be false; so it is necessarily false, but its being false is sufficient for its being true.
One sophisticated response is to restrict the extension of ‘is false’ to a subset of what is not evaluated as true. There is a truth value gap. So that, with reference to ‘This sentence is false’, one might argue that:

If the Liar sentence were true, it would be false, so it is necessarily not true; but its not being true does not necessarily mean it is false, and so is not sufficient for its being true. Therefore, it is not true (and not false).

This argument will not succeed (on its own) in avoiding paradox when used with the Strengthened Liar, ‘This sentence is not true’; for proving that this sentence cannot be true and is therefore not true will once again suffice for its being true. In the way truth value gaps are motivated above, the negation of the ‘is true’ predicate would be interpreted as ‘is false’. That is, from a naive point of view, the ‘is not true’ ought to be interpreted as meaning being false, but nothing represents everything that is not true, as naively understood. This is not Kripke’s motivation and not the case for his truth predicate.

Kripke [1975] implements a yet more sophisticated account of truth value gaps, and uses a meta-language to say that the Liar sentence is not true. Kripke [1975] provides a different motivation for gaps, for which I will provide an appropriate metaphor for truth-value gaps in Chapter 2. Kripke’s implementation of gaps has just the truth predicate and its negation in the object-language; however, the interpretation of ‘is not true’ in the meta-language is different to that of the object-language. Kripke appeals to a different conception of truth, based on Herzberger’s [1970] notion of groundedness. I will introduce groundedness in Chapter 2; and Kripke’s account in a section of Chapter 3. For present purposes, this is a subtle example of how theories affect the representation of the paradoxes.

I will for the most part assume being not true is being false; and devote a brief section to strengthened versions in Chapter 2. Before we seek to remedy the paradoxes, Strengthened versions are treated as a relatively fine-grained distinction. Their significance increases depending on the approach one takes to resolving the paradoxes; but I am engaged in classifying them as they are found, prior to any resolution. I will compare my classification with Kripke’s theory in Chapter 3.

Liar paradoxes are intuitive. Having explained the Liar to my elder children some years ago, I asked them to come up with versions of their own. My son, Leif, then twelve, suggested:
A policeman asks a suspect whether he is lying, and the criminal just says “Yes”.

One can see how this would work. Yet one can easily overlook a paradox too, or perhaps look too hard for one. Kripke [1975 /1984, p. 55] relates the story of Russell asking Moore whether he always told the truth, to which Moore replied in the negative, and yet Russell thought this the only lie that Moore ever told. Kripke evidently thinks Russell’s thought was paradoxical, and not just false. Indeed, as Kripke suggests, if Russell is right that all other statements made by Moore are true, then Moore’s reply to Russell is paradoxical.

After some time, my elder daughter, Veronique, then ten, came up with the Pinocchio paradox pictured on the front cover of my thesis. Pinocchio’s nose grows if and only if (iff) what he is stating is false. Pinocchio says “My nose is growing.” So Pinocchio’s nose is growing iff it is not growing. It has an obvious Truth-teller variant; but its main feature is that it moves away from using a synonym for truth in the Liar statement. Having one’s nose grow is not a semantic feature. Pinocchio’s nose grows just when he is telling an untruth, but the relationship is supposed to be causal not semantic – if Pinocchio’s nose is growing it is because he is saying something false; otherwise, it is not growing. In any case, the scenario is logically possible, whether or not it is physically possible. It is in a way a counter-example to theories that define semantic predicates in a meta-language, because ‘is growing’ is not a semantic predicate. Intuitively, predicates like ‘is growing’ are typical of just the sorts of predicates one wants in a useful object-language. Tarski’s analysis of the Liar led him to conclude it arose from semantic predicates being included in the object-language. If empirical predicates like ‘is growing’ may need to be defined in the meta-language, the intuitive bounds on which predicates may need to be defined in the meta-language to avoid Liar-like paradoxes have been breached.

I will discuss the bearers of truth in chapter 2. I wish to remain as ecumenical about these as I can. I talk about ‘sentences’ and ‘statements’, using the latter, generally speaking, for sentences involving indexicality.

Philosophers who espouse various theories of truth are very serious about what is wrong with ‘Liar sentences’; after two millennia there are a plethora of theories about the Liar – too much would have to be wrong with these sentences for all these theories to be correct. Nevertheless, paradoxes are sometimes fertile areas for research. The Pythagoreans seem to have been confounded by a paradox, which
was eventually resolved by distinguishing and admitting irrational numbers. In some sense, avoiding the Liar has already helped conceive modern semantics.

Extensive lists of examples, like the above, have been produced at various stages in the history of logic. I suggest such lists exhibit the need for principled classification. There is still debate about whether even a primary distinction is justified between the semantic and set-theoretic paradoxes. It seems to me that classifying variants of the Liar and Liar-like paradoxes would help to clarify the boundaries and some of the structure of the problem — in short, such a classification would help to map out the problem space surrounding the Liar.

My investigation is intentionally dealing with the paradoxes prior to any attempt at resolution. This does not mean it is pre-theoretic. There have been significant advances in modern times in the representation of the paradoxes, in formally representing the intuitions involved in the reasoning to a contradiction, and even in investigating weakened forms of those intuitions. In this respect, my investigation is similar in spirit to Herzberger’s [1982] Notes on Naive Semantics. Nevertheless, my work is perhaps distinguished by trying to find paradoxes rather than avoid or resolve them. I am primarily concerned to individuate, formalize and classify the paradoxes as found in the wild, as it were, as opposed to arguing for any particular theoretical move intended to avoid, inoculate or remediate them.

I assume bivalence naively. I do not use truth value gaps or gluts (having both truth values) as valuations in the classification, nor do I consider paradoxical derivations for many-valued logics. I accept intuitive interpretations of the biconditional, that it is at least such that \((\alpha \iff \neg \alpha)\) entails \((\alpha \& \neg \alpha)\). I accept the principle of absorption. I even accept a T-schema, at least for sake of argument. The form of the T-schema that I use will be discussed in Chapter 2. Naturally, it does not involve a strict object- meta-language distinction. (Even so, I reserve recourse to a meta-language to discuss these arguments.) In other words, the paradoxes I classify are genuine paradoxes; I am concerned with systematic and logical issues relating to truth and the paradoxes; I am concerned with philosophical, not philological issues. Thus I present a classification against which theory-laden classifications that are entailed by particular solutions can be compared. And it will be of some interest to compare with the classifications entailed by the theories of Kripke [1975], and Gupta and Belnap [1993].

I make some contributions towards clarifying the relation between
paradoxical Liar-like sentences and Truth-teller-like sentences, and the significant formal features of the arguments proving the Liar, the Grelling's and Russell's paradoxes, and provide elements that in combination individuate variations among these paradoxes.