Chapter 1 Reappraising the Epimenides

The Epimenides paradox has been classed as a mere version of the Liar; meanwhile, Curry's paradox is talked about as a kind of paradox or a characteristic that cuts across paradoxes of truth and membership, and even entailment. In this chapter I show that we have sufficient reason to treat the Epimenides in the same way as we do Curry's. In relation to the rest of the work, I hope this also demonstrates a need for a more detailed classification of paradoxes. Our current ways of individuating paradoxes would benefit from some reflection, and some revision or at least clarification.

My structure would be a simple paralleling of variations of Curry's and the Epimenides but I want to use this chapter to do three additional things. Firstly, I want to introduce the Liar. It is the relationship of the Epimenides and Curry's paradoxes to the Liar that is after all topical. Secondly, I want to give some of the intellectual history of the paradoxes; partly because I think it is interesting, and partly because there has already been some reappraisal of the Epimenides paradox. Thirdly, I want to give some brief comments on some of proposed solutions. A proposed solution should identify some common characteristic for paradoxes addressed by the proposed solution. So, I am as interested in solutions as I am in what they presuppose would make a difference to the type of paradoxes they address. So my structure is a little more complex than simply paralleling variations of Curry's and the Epimenides.

In section 1, The Liar is introduced historically, and given some brief analysis. In the second section, I give a brief intellectual history of the Epimenides, and relate this to proposals concerning it and the Liar. In the third section, I give some brief history on Curry's paradox. In section 4, I present analogous variations of the Epimenides and Curry's paradoxes in parallel, including a new variation of the Epimenides involving entailment. I conclude by summarizing the argument for re-appraising the Epimenides.

1.1 The Liar and the Truth-teller

The Liar Paradox itself was formulated by Eubulides in the 4th Century BCE along these lines:

(1) A man says he is lying. Is what he says true or false?[3]

If what he says is true, then what he says is false. Furthermore, if what he says is false, then he is telling the truth. So, what he says is true if and only if it is false.

None of Eubulides' original work survives. We know about his formulation of the Liar Paradox indirectly through Cicero. Eubulides was a contemporary of Aristotle. However, Aristotle (384-322 BCE) makes only fleeting and somewhat cryptic reference to the Liar in his *De Sophisticis Elenchis* (*On Sophistical Refutations*) ch25:

> ... as regards the problem whether the same man can at the same time say what is both false and true: but it appears to be a troublesome question because it is not easy to see in which of the two connexions the word 'absolutely' is to be rendered with 'true' or with 'false'. There is, however, nothing to prevent it from being false absolutely, though true in some particular respect or relation, i.e. being true in some things, though not 'true' absolutely.

Logical interest peaked within a generation or two of Aristotle. Aristotle's disciple, Theophrastus, wrote three books on the Liar [Bochenski 1956, p. 151], and Chrysippus (c. 280 – c. 206 BCE) wrote at least seven books on it [Sedley 1998]. Clearly, the Liar was seen as presenting a significant problem; however, since these works are lost; one does not know how the problem presented itself to them. The Liar presents a challenge to the consistency of logical reasoning, and therefore its effectiveness. I imagine the ancient logicians were more focused on the Liar as posing a problem to the efficacy of reason than on a theory of truth.

Interest in the Liar then subsided to such an extent that none of these works has been preserved. The rigorous logic of Chrysippus, in particular, was trivialised, neglected, and largely forgotten. From a fragment, it appears that Chrysippus thought Liar sentences were meaningless in a sense that perhaps anticipates the

---

[3] Cicero presenting the Eubulidean Liar, *De Divinatione,* ii. 11, *Academica*, ii. 96, cited in Kneale & Kneale [1962 / 1984, p. 114].

mediaeval *cassatio* approach discussed below [Spade 1973, p. 308]. This pattern of intense work, followed by trivialisation and neglect was, we shall see, repeated after the Liar was rediscovered in later mediaeval times.

Modern presentations of the Liar can break the argument down. Consider my favourite sentence, which is 'My favourite sentence is not true', and a paradoxical argument:

1 My favourite sentence = 'My favourite sentence is not true'

Premise, by stipulation or observation

2 'My favourite sentence is not true' is true iff my favourite sentence is not true

T-biconditional

3 My favourite sentence is true iff my favourite sentence is not true

1, 2 Substitution of Identicals

4 My favourite sentence is true & not true

3 Sentential Logic (SL)

This particular derivation follows Tarski [1935/1983, p. 158], particularly in respect to line 2, which I will discuss shortly.[4]

Line 1 in this example is given as a contingently true premise. My favourite sentence just happens to be 'my favourite sentence is not true'. Other examples use a stipulation as a first premise. Still others use other means to establish similar statements in place of the first premise. If the Liar argument were a *reductio* of this and similar first premises, the premise would be necessarily false; yet in this case it is a contingent truth. Putative solutions to the paradoxes involve restrictions or distinctions (or both).[5] On the one hand, the following is an example of a restriction with respect to line 1. Goldstein [1999 and elsewhere] believes the Liar is a reductio of such identity statements, a 'case of mistaken identity'. Goldstein would relate

[4] Tarski [1935 / 1983, p. 157] attributes the version of the Liar he uses to Łukasiewicz.

[5] Cf. van Benthem [1978, p. 62], who contrasts strategies of *weakening* a system with *making a distinction.*

line 1 to a circular definition and would restrict it from making a statement.[6] On the other hand, as an example of a distinction, Gupta & Belnap [1993] want to introduce a distinction motivated by line 1's analogy with a circular definition. Their distinction introduces revision levels on either side of the biconditional in line 2. On scant anecdotal evidence, I think history favours the making of distinctions. The proof that the square root of two is irrational would have, in my opinion, presented itself as a paradox to Pythagoreans.

In line 2, and throughout my thesis unless signalled otherwise, 'iff' is a biconditional connective such that line 4 follows from 3. Line 2 itself is a *T-biconditional.* It can be derived in the usual two ways, either as an instance of an axiom schema or by using rules of inference. For example, as an instance of Tarski's [1935] famous T-schema:

> α is true iff $@$
>
> where α is a name of the sentence that is substituted for '$@$',

or by using rules such as Truth Introduction (TI) and Truth Elimination (TE):

> TI: $@ \vdash T\langle @\rangle$
>
> TE: $T\langle @\rangle \vdash @$

where angle brackets represent a canonical name-forming device, like quotes, such that the expression named can be effectively recovered from the name.[7]

There is more to say about Tarski's T-schema in terms of object- and meta-language, but that is more about Tarski's response to the Liar than his formulation of it. Logicians often use natural language extended with technical terms as a meta-language to talk about a formal object-language. In some cases the meta-language itself is formalised. Tarski's object-language did not contain its own truth-predicate, it being defined in a separate formal meta-language. Kripke's [1975] object-language does contain its own truth-predicate, but it is still defined in the meta-

---

[6] As another example of a restriction affecting line 1, Ryle [1954] would take issue with line 1 as he requires a *namely-rider* on references to other statements, effectively, a requirement that one should be able to in principle eliminate references to other statements. I note that both Goldstein and Ryle would distinguish the (primary) bearers of truth from sentences.

[7] More formally, the angle brackets are a meta-language device representing an object-language canonical name forming functor, like quotes; but as natural language is its own meta-language, the angles brackets represent a canonical name-forming device.

language. My task is not to define truth, but to classify paradoxes. I therefore allow the naive use of the T-schema, just as one uses naive set-theory to derive Russell's paradox. Since we are investigating how the paradoxes arise in natural language, we will work within a theoretical extension of natural language itself.[8]

The T-schema attributes truth to sentences as Tarski took truth to be a predicate of syntactic sentences. It seems natural to think of truth as an attribute of an interpreted sentence or some semantic entity; however, I follow Tarski in this matter for the time being and will offer some defence of this later on. Nevertheless, I want to qualify the sense of 'true' at issue, because just about anything can be described as true: a coin, a friend or a home. The truth at issue here is that preserved by a valid argument. Talk of bearers of truth must respect this constraint. In another subsection below, I show how to generalise the form of the Liar to range over the variety of possible bearers of truth.

Line 3 follows from 2 by substitution of identicals (also known herein as 'Leibniz's law' or '=E', being short for '=Elimination'), in this case substituting 'my favourite sentence' for " 'my favourite sentence is not true' ".[9] Skyrms [1970 and elsewhere] suspected this inference. Skyrms still considers it *valid* in that it avoids inferring something false on his account; but thinks, nevertheless, that it has not preserved truth. He thinks a sentence with a truth-value gap can be derived from truths. However, one could use the T-schema to go direct from line 1 to line 3 without the need for line 2.

Line 1 is the only premise in this representation of the paradox; but it is common to many modern solutions to represent the T-biconditional at line 2 or 3 as an additional premise and use the argument as a *reductio* of that premise (rather than a paradoxical argument). I shall leave the vinculum where it is until chapter 5, and accept the T-schema while I am classifying paradoxes.

Line 4 is derived from 3 using just sentential calculus. Some contradictions

---

[8] Many of the concepts used to analyse the paradoxes can themselves be used in *revenge* variations of the paradoxes. If this possibility eventuates with respect to my classification, it would not necessarily undermine the classification. It is conjectured that emergent *revenge Liar* paradoxes are subject to the same classification.

[9] Although the identity at line 1 is true, the ultimate reference of 'my favourite sentence' is *ungrounded* in a sense, and so, I believe, somewhat indeterminate. As such, I personally doubt that the substitution at line 3 has preserved sameness of reference. I believe it is invalid. Nevertheless, as I will note above, one could use the T-schema to go from line 1 direct to line 3.

have the form @ & ~@; but really any negation of a tautology is a contradiction. Furthermore, if the biconditional represents an equivalence, then intuitively @ ↔ ~@ is a contradiction. There are logics in which (@ ↔ ~@) does not entail (@ & ~@); but it seems very costly to restrict our thinking in such fundamental ways just to avoid the paradoxes.

At this point, I would like to introduce my second favourite sentence. My second favourite sentence is 'My second favourite sentence is true'. This is an example of the Truth-teller, a relative of the Liar. No contradiction follows from assuming my second favourite sentence is true or false. It is not closely associated with an argument. It can consistently be assumed true or false, but there is the lack of a good reason to say whether it is true or false; it is *hypodoxical.*

1.1.1 Three Problems associated with the Liar and Truth-teller

An approach or solution to the Liar must address a number of problems, starting with the following three. The first is its semantic value (and the semantic value of the Truth-teller). The second is the validity or invalidity of its paradoxical argument. The third is to avoid or at least minimize further problems emerging in addressing the first two issues, such as so-called "revenge" problems or "strengthened Liars".

In attempting to resolve the first problem of the Liar systematically, one faces an underlying issue, the pathology question: What, if anything, is wrong with the Liar sentence?[10]

I suggest there are material and modal aspects to each of the first two problems. From a compositional point of view, the valuation of the Liar and the Truth-teller both seem to lack sufficient basis for a valuation. However, in terms of considering possible truth values, the Liar's truth value seems over-determined and the Truth-teller's seems under-determined. I will argue for these claims in chapter two.

There is a related problem that the Liar itself does not pose. Our concept of paradox seems adequate for the Liar. Given the premises, if any, are apparently true, paradoxes are dual *reductio ad absurdum* arguments, both of which seem sound. For example, the identity in line 1 of the Liar is true; so, if the Liar sentence

---

[10] I think it was Keith Simmons [1999] who first applied the description 'pathology' to this issue about what is wrong with Liar-like sentences.

is true, it is false; and, if the Liar sentence is false, it is true. In this way the Liar conforms to this definition of paradox. However, not all versions of the Epimenides and Curry's paradox that will be presented in subsequent sections conform to this definition.

1.1.2 Significance of the Problems associated with the Liar

The significance of the Liar in modern times was at first associated with inconsistencies in logical systems (including set-theory) intended to provide foundations for mathematics. At least this was Russell's [1908] view. Peano [1906], however, distinguished the semantic from the set-theoretic paradoxes. At the time, the latter were called the 'logical' paradoxes. Ramsey [1925] supported Peano's distinction, and it was an orthodox view for some time.

Meanwhile, Tarski [1935] addressed the Liar as a problem for a consistent theory of truth, and ever since the problem of giving the Liar a semantic valuation has been strongly associated with philosophical work towards a theory of truth. It is particularly this first problem of the Liar, its semantic valuation, that seems to be the primary concern for theories of truth; but the second problem deserves attention too. As truth is preserved by validity, ultimately, a solution to the Liar may affect our concept of validity as well.

Indeed, truth relates to the Liar in five ways. Firstly, it may appear in the Liar sentence. (This, as earlier examples have shown, is optional.) Secondly, an instance of the T-schema (or a weakened schema or related inference) relates the Liar statement to the Liar argument. Thirdly, as truth is preserved by validity, it is preserved by the Liar argument if it is valid. Fourthly, the question of what are the bearers of truth may be relevant to the Liar statement and argument. Fifthly, possible truth values constrain the semantic valuation of the Liar statement, and which truth values are preserved (or avoided) by a good argument underpins semantic validity and soundness.

For some time, Russell's [1908, and Whitehead & Russell 1925] and Tarski's [1935] theories were the orthodox views on the Liar. There were others, but as Kripke [1975] pointed out, few others had been worked out in detail. Kripke [1975] took up on ideas present in the work of van Fraassen, Herzberger and Prior; Kripke clarified issues, and made a critical point in modern thinking about the paradoxes, about the involvement of empirical matters in whether a sentence was paradoxical. Kripke [1975] went on to outline a definition of truth that he argued was more in

line with our intuitions about truth than Tarski's. Since 1975 a number of theories of truth and associated solutions to the Liar have been worked out in detail. There are now a number of what might be called orthodox views: hierarchical views, still represented by Russell [1908, and Whitehead & Russell 1925] and Tarski [1935], now compete with theories taking account of Kripke's point about material facts, such as Herzberger [1982], and Gupta and Belnap [1993]; truth-value gap theories, represented by, Kripke [1975], and Martin and Woodruff [1975]; situational semantics such as Barwise and Etchemendy [1987]; modern approaches making essential use of sentence tokens such as Simmons' [1993] singularity approach; and paraconsistent theories with truth-value gluts as opposed to gaps, such as Priest [1987 / 2006]. This list is by no means exhaustive.

Orthodox views hold that pathology is engendered into the Liar through use of the T-biconditional (or associated inferences involving truth). Even those who think the Liar is a *dialethia*, a true contradiction, think this is associated with its use of truth.

There are some views that do not locate the source of pathology in the use of the T-biconditional. Among these, some cite examples of the paradox that do not use truth explicitly. These examples are somewhat contentious; nevertheless, I shall mention some in my investigation. Even so, the matter with the Liar still relates to truth in a more general sense; because validity (naively understood at least) preserves truth.

I want now to consider a relative of the Liar, the Epimenides.

1.2 A History of Ideas in relation to Variations on the Epimenides

Liar-like statements in fact precede the Liar itself, and date back to at least one statement of Epimenides, a native of Knossos in Crete, *circa* the 6th century BCE, who wrote:

(2) Cretans are always liars.

There are a number of logical conundrums associated with this statement. The Epimenides has been portrayed as though it were a lesser paradox, or as if it were only semi-paradoxical. For, on the one hand, if it is true that all Cretans are liars (and we assume liars always speak falsely), then Epimenides is lying and what he

says is false; so it's not true that all Cretans are liars. Yet, on the other hand, if his statement is false, a contradiction does not follow unless we add an extra premise that there are no other true Cretan statements. If being a liar is equated with (always) making false statements, then if all other Cretan statements are false, then this one is paradoxical. This is similar to the Liar. Undoubtedly, there are other Cretan statements which are true; but Epimenides' statement is still puzzling, because without knowing about any other Cretan statements but just given that a Cretan said this one, we can prove that it is false. Moreover, the falsity of (2) entails there is a true Cretan statement! Yet, 'Either no Cretan said Cretans are always liars or some Cretan statement is true' does not present itself as a necessary truth.

Here is a schematic, but still informal representation of the argument:

1 | 'All Cretan statements are false' is a Cretan statement
2 | All other Cretan statements are false
3 | If 'all Cretan statements are false' is true, then, it is false
4 | It is not the case that all Cretan statements are false
5 | Some Cretan statement is true, and it is not the statement embedded in (1)
6 | Some other Cretan statement is true (in contradiction of (2))

Yet there is more that is paradoxical in some sense about the Cretan Liar Paradox than this, for we can reach the conclusion in line 6 without the second premise. Granted, it is not a contradiction, but it does seem surprising that we can reach this contingent conclusion that does not seem to be contained in the premise. From "Epimenides, the Cretan, says that all Cretans are liars", it follows that "Something said by a Cretan is true". And if we add the premise "Epimenides always lies", we can even infer the existence of another Cretan who does tell the truth! It seems a contingent matter whether all Cretan statements are false. Yet if a Cretan says so, what he says is provably false, and therefore, there must be some true Cretan statement. And as it cannot be the universal statement we know about, there must be some *other* true Cretan statement. This conundrum was noted by Alonzo Church [1946][11] and discussed among other paradoxes by Arthur Prior

[11] Church [1946, p. 131] says "But it is said [by Koyre whose article he is reviewing] that Epimenides statement is self-destructive, in the sense that its truth is disproved by the given fact Epimenides made it. - Apparently M. Koyre is untroubled by a consequence which, though not outright antimony, might well be classed as paradox. Namely, without factual information about

[1958, 1961 and 1971]. Church's conundrum is like Curry's Paradox, which I'll present soon, in that it seems to prove a contingent statement.

We can represent a derivation of the Epimenides formally using 'T' to represent the truth predicate, '… is true'; 'D' for a predicate like '… is a Cretan statement', and the inference rules for TI and TE.

Here is a proof of a contradiction. I note that the second premise is not required until after Church's conundrum is proven at line 18.

1 |	$D\langle(\forall x\ (Dx \supset \sim Tx)\rangle$	1st premise
2 |	$\underline{\sim\exists x\ [(Dx\ \&\ x \neq s)\ \&\ Tx]}$	2nd premise
3 |	$\underline{\mid \forall x\ (Dx \supset \sim Tx)}$	assumption
4 |	$\mid D\langle\forall x\ (Dx \supset \sim Tx)\rangle \supset \sim T\langle\forall x\ (Dx \supset \sim Tx)\rangle$	3 ∀E
5 |	$\mid \sim T\langle\forall x\ (Dx \supset \sim Tx)\rangle$	4, 1 MP
6 |	$\mid \forall x\ (Dx \supset \sim Tx) \supset T\langle\forall x\ (Dx \supset \sim Tx)\rangle$	(CP, TI)
7 |	$\mid \sim\forall x\ (Dx \supset \sim Tx)$	6, 5 MT
8 |	$\sim\forall x\ (Dx \supset \sim Tx)$	3-7 ~I
9 |	$T\langle\forall x\ (Dx \supset \sim Tx)\rangle \supset \forall x\ (Dx \supset \sim Tx)$	(CP, TE)
10 |	$\sim T\langle\forall x\ (Dx \supset \sim Tx)\rangle$	9, 8 MT
11 |	$\exists x\ (Dx\ \&\ Tx)$	8, QN, Impl., DeM.
12 |	$\mid \underline{Db\ \&\ Tb}$	Assumption
13 |	$\mid \quad \mid \underline{b = \langle\forall x\ (Dx \supset \sim Tx)\rangle}$	Assumption
14 |	$\mid \quad \mid \sim Tb$	10, 14 =E
15 |	$\mid \quad \mid Tb$	12 &E
16 |	$\mid b \neq \langle\forall x\ (Dx \supset \sim Tx)\rangle$	13-15 ~I
17 |	$\mid \exists x\ [(Dx\ \&\ x \neq \langle\forall x\ (Dx \supset \sim Tx)\rangle)\ \&\ Tx]$	12, 16 &I, ∃I
18 |	$\exists x\ [(Dx\ \&\ x \neq \langle\forall x\ (Dx \supset \sim Tx)\rangle)\ \&\ Tx]$	11, 12-17 ∃E
19 |	$\exists x\ [(Dx\ \&\ x \neq \langle\forall x\ (Dx \supset \sim Tx)\rangle)\ \&\ Tx]\ \&$ $\sim\exists x\ [(Dx\ \&\ x \neq \langle\forall x\ (Dx \supset \sim Tx)\rangle)\ \&\ Tx]$	18, 2 &I

The Epimenides has not enjoyed the same status as the Liar. Quine somewhat disparaged the Epimenides:

---

other statements by Cretans, it has been proved by pure logic (so it seems) that *some other* statement by a Cretan, not the famous statement of Epimenides, must once have been true." I take some encouragement from this comment.

> Actually, the paradox of Epimenides is untidy; there are loopholes. Perhaps some Cretans were liars, notably Epimenides, and others were not; perhaps Epimenides was a liar who occasionally told the truth; either way it turns out that the contradiction vanishes.
>
> [Quine 1962, p. 6]

Haack wrote in a similar vein:

> If a liar is someone who always says what is false, then if what Epimenides said is true, it is false. The Epimenides is, however, somewhat *less* paradoxical than the Liar, since it can be consistently supposed to be false, though not to be true …
>
> [Haack 1978, p. 136]

However, the Epimenides was given more of its due status in 1975, when Kripke made clear the involvement of empirical circumstances in many instances of paradox.

Saul Kripke [1975] begins his *Outline of a Theory of Truth* with the Cretan Liar Paradox as follows:

> The Cretan example illustrates one way of achieving self-reference. Let P(x) and Q(x) be predicates of sentences. Then in some cases empirical evidence establishes that the sentence '$(x)(P(x) \supset Q(x))$' [or '$(\exists x)(P(x) \wedge Q(x))$', or the like] itself satisfies the predicate P(x); sometimes the empirical evidence shows that it is the *only* object satisfying P(x). In this latter case, the sentence in question "says of itself" that it satisfies Q(x). If Q(x) is the predicate 'is false', the Liar paradox results. As an example, let P(x) abbreviate the predicate 'has tokens printed in copies of the *Journal of Philosophy*, November 6, 1975, p. 691, line 5'. Then the sentence:
>
> $$(x)(P(x) \supset Q(x))$$
>
> leads to paradox if Q(x) is interpreted as falsehood.
>
> [Kripke 1975, pp. 690-691]

Here, the paradoxical statement falls within the extension of an empirical predicate, P(x); in the case of 'All Cretan statements are false', it happens to be one among many Cretan statements. In Kripke's example, a definite description uniquely identifies the statement in question; but it is still an empirical matter that

the definite description picks out a statement in which the description itself occurs.

I would like to return to the origin of the Epimenides statement and trace some more of the intellectual history of the paradox and associated conundrums.

1.2.1 Origins of the Epimenides

Epimenides is an enigmatic character. He lived on the cusp of legend and history. According to Diogenes Laertius (3rd century. CE.), he is variously reported to have lived to 154, 157 or 299 years, of which he is said to have spent 57 asleep in a cave after wandering off to find one of his father's sheep.[12] He is said to have received religious and philosophical instruction while asleep in the cave, and to have been able to see the future. He was a poet-philosopher, priest, and reformer [Demoulin 1901]. His phenomenal sleep patterns earned him quite a reputation in Crete and Greece as one favoured by the Gods. And he was invited to purify Athens after the massacre of Cylon's supporters [Freeman 1949]. (Some years prior to 600 BCE, Cylon and his followers had seized the Acropolis; but the coup failed and Cylon fled. His followers were 'suppliants at an altar' when they were slaughtered. The Athenians eventually came to believe that this sacrilege was responsible for something they called 'The Taint', possibly associated with an outbreak of plague. Hence, Epimenides was invited to purify Athens.)

Despite claims by professional philosophers to the contrary, we do know something of the context in which Epimenides made his famous statement. It has apparently been known to historians for some time, as explained in this passage on *Prehistoric Crete* by R.W. Hutchinson:

> The most startling heterodoxy of the Cretans in classical times was that the Zeus they worshipped had been born as a baby in Crete, had grown to manhood, and had finally been buried there. The Greeks did not so much mind the story of Zeus's birth, which was accepted by Hesiod, but the legend of his death and burial was regarded as downright blasphemy even by the Cretan Epimenides in his poem on Minos quoted by Saint Paul. The fragment preserved and brilliantly restored by Rendel Harris from a passage in a Syriac [Biblical] commentary has been translated as follows:

---

[12] Diogenes Laertius [1959 trans., pp. 115-21].

The Cretans carved a tomb for Thee, O Holy and High,
Liars, noxious beasts, evil bellies
For thou didst not die, ever Thou livest and standest firm
For in thee we live and move and have our being.

[Hutchinson 1968, p. 200]

The second line of this fragment is Epimenides famous statement, quoted in part by Callimachus and later the whole line is quoted by the author of the *Epistle to Titus*. (The forth line is also quoted in Acts 17, verse 28.)[13]

It seems that Epimenides was not concerned with the logical conundrum of him, as a Cretan, trying to assert this statement. He was evidently saying that other Cretans were lying (in saying they had entombed the body of Zeus, as it was acceptable for a Hellene God to be born, but dying was considered distinctively mortal). Diogenes Laertius attributes to Epimenides a number of prose and poetic works including a theogony, all of which seem to be of a serious tone. West [1966, p. 162] and Demoulin [1901 / 1979, p. 130] suggest the line imitates line 26 of Hesiod's *Theogony* (circa 750 - 650 BCE). The similar line in Hesiod occurs in an introduction where the Muses distinguish between truth and plausible fiction [West 1966, p. 162]. If Epimenides' quote has the same sort of context, then possibly he is saying that though all Cretans are story-tellers, the *theogony* he is introducing is the truth.

The phrase 'Cretans are always liars' may even pre-date Epimenides.

---

[13] Elsewhere these four lines of Epimenides are said to be translated from a fragment in a Syriac Biblical commentary, edited and translated by Margaret Dunlop Gibson [1913, p.40 in Syriac.] as:

They fashioned a tomb for thee, O holy and high one
The Cretans, always liars, evil beasts, idle bellies!
But thou art not dead: thou livest and abidest forever,
For in thee we live and move and have our being.

Gibson's reference and this translation are cited in 'Epimenides Paradox', Wikipedia, http://en.wikipedia.org/wiki/Epimenides_paradox , downloaded 31 December 2007. This translation is more like its quotation by Callimachus and in the Bible, which I present directly. I have validated the existence of Gibson's work, but have not been able to verify that this is her translation or that the page reference to the fragment in Syriac is accurate. The Wikipedia article cited says that the fragment from Epimenides is from a work entitled 'Cretica'. This is not a work listed by Diogenes Laertius, and I do not know whether this title is given in the Syriac commentary. Gibson entitles her book 'The Commentaries of Isho'dad of Merv', but Wikipedia, as at 31 December 2007, refers to this commentator as 'Isho'dad of Mero'.

According to A.W. Mair, 'the explanation by Athenodorus of Eretria ... is that Thetis and Medea, having a dispute as to which of them was the fairer, entrusted the decision to Idomeneus of Crete. He decided in favour of Thetis, whereon Medea said "Cretans are always liars" and cursed them that they should never speak the truth'.[14] (Idomeneus, leader of the Cretan contingent in the Trojan war, was thought by some to have divided the spoils of Troy unfairly.) However, the point is not whether Epimenides was the original author of the phrase, but whether he as a Cretan asserted it.

By the time of Callimachus, the Liar itself was well-known, and there is some evidence that an implication of a Cretan saying that Cretans always lie was understood. Callimachus, (circa 305 – 240BCE), an early librarian of the library at Alexandria, quoted Epimenides in line 8 of his *Hymn to Zeus*:

> "Cretans are ever liars." Yea, a tomb, O Lord, for thee the
> Cretans builded; but thou didst not die, for thou art for ever.
>
> [Callimachus, *Hymn to Zeus*, lines 8 & 9, trans. 1977, p. 37.]

As G. R. McLennan [1977, p. 35]comments that ...

> ... Callimachus humorously makes use of this Cretan claiming all Cretans are liars; this makes him a liar....

It seems that Callimachus was aware that a Cretan stating 'all Cretans are liars' would entail the falsity of this statement. Callimachus also seems aware that if he or some other non-Cretan asserts it, that does not entail that what he says is false: what Callimachus says may be true or false, depending on whether or not all Cretan statements are false.

So, Callimachus' conundrum associated with the Epimenides is that if a Cretan says that all Cretan statements are false, then what he says is false. It is strange to think that saying so makes it not so. (The formal proof of this conundrum was given at the beginning of this section. It is proven a step before Church's conundrum.)

The most famous endorsement of Epimenides' statement is in the Bible. A Biblical reference to Crete and its inhabitants actually quotes Epimenides. In the New Revised Standard Version, Titus 1:12-13 reads as follows:[15]

---

[14] Callimachus, *Hymns and Epigrams*, trans. A. W. Mair [1977, p. 37 in Mair's commentary].

[15] Anderson [1970, pp. 1-2] quotes five other versions, including a Greek and Latin Vulgate version.

> [12]It was one of them, their very own prophet, who said,
>
> "Cretans are always liars, vicious brutes, lazy gluttons."
>
> [13]That testimony is true.

The attribution of this line of verse quoted in the Bible to Epimenides was made by Clement of Alexandria (circa 125 – c. 212 CE) as follows:

> 59(1) The Greeks say that after Orpheus and Linus and their oldest poets, the first to acquire a high reputation for wisdom were the so-called Seven Sages. Four came from Asia: Thales of Miletus, Bias of Priene, Pittacus of Mytilene and Cleobulus of Lindos; two from Europe: Solon of Athens and Chilon of Sparta. For the seventh, some adduce Periander of Corinth, some Anacharsis of Scythia, (2) others Epimenides of Crete, whom the apostle Paul mentions in his letter to Titus [1.12] in the words: "One of their own number, a prophet of theirs, has said,
>
> Cretans are always liars, evil beasts, lazy gluttons,
>
> and his evidence is true." (3) Do you see how he grants a measure of truth to the prophets of Greece as well and is not ashamed, in a discussion designed to build them up and direct them to self-examination, to use Greek poems?[16]

I take the Biblical author (and Clement) to be serious, even though if what Epimenides said is true, then it follows, he being a Cretan himself, that he was lying; and so, if lying in this instance involves saying things that are not true, what he says is not true. In this way, there is a contradiction. What Epimenides says is provably false, and yet St Paul says it is true. This is a contradiction, not a paradox, strictly speaking; so it is naively false.

Yet the intuition that this is a paradox is common. It would be some sort of paradox, if one held a fundamentalist belief that every sentence of the Bible is necessarily true. Most people do not take this fundamentalist view, but there is a folk belief that the Biblical version is paradoxical (and not just a case of the author contradicting himself). To substantiate this intuition, we would have to treat one of the conundrums associated with the Epimenides as paradoxical. (On analogy with

---

[16] Clement of Alexandria, *Stromateis: Books One to Three*, trans. Ferguson [1991], p. 66: Book 1, 59(1)-(3).

Curry's paradox, which I explore later, this may well be reasonable.)

It may reasonably be objected that lying is not simply always speaking falsely; however, the philosophical debate is focused on the objective interpretation involving truth.[17]

In summary, approximately, 2,500 years after Epimenides made his famous statement and 2,200 years after Callimachus made his conundrum explicit, Alonzo Church made explicit the other associated conundrum I have mentioned in the previous section. For a Cretan saying that all Cretan statements are false entails that *some other* statement by a Cretan is true [Church 1946, p. 131]. Undoubtedly, some Cretan statements are true, but, if this argument is valid, then so are a lot of similar arguments with false conclusions. When we combine this second conundrum associated with the Epimenides statement, under unfavourable circumstances – there being no other Cretan truths – the Epimenides paradox results. That is Epimenides statement entails that some other Cretan statement is true; but if it is given that there are no other Cretan truths, then a contradiction results. One expects the paradoxical variation was originally derived from the contradiction in the Bible, if it was not already present in the lost works of the ancient logicians. The connection was not explicitly drawn by Scholastics; I suggest in the next section that they were well aware of something like this type of paradox.

1.2.2 The Philosophy of the Liar and the Epimenides in the Middle Ages

Interest in the Liar Paradox revived at about the time Aristotle's *De Sophisticis Elenchis* was introduced to Europe from Arabic sources. The Liar was included among collections of so-called *insolubles* (*insolubilia*) discussed in books on logic. *Insolubles* were classified as a type of sophism or fallacious argument. The Liar appears as early as 1132, in a work of Adam of Balsham; and work on *insolubilia* proliferates in the fourteenth century. Spade doubts that Aristotle's work alone could account for the mediaeval renewal of interest in the Liar. Scholastics were aware of *the Epimenides paradox*; but not perhaps with reference to Epimenides himself. Spade [1973, p. 296] attests the Biblical inclusion of Epimenides statement in Titus is not referenced in mediaeval *insolubilia* literature. There must be another or additional source that is perhaps lost to us. One imagines monks might avoid

---

[17] Even on the intensional interpretation of lying, Epimenides' statement still poses conundrums. See Eldridge-Smith [2004, pp. 76-77] wherein I at least demonstrate how confusing it can be.

referring to a contradiction in the Bible. Sorensen [2003] gives a plausible explanation of this blind spot in terms of established divisions between the spiritual and intellectual labours of Scholastics. I think another part of the mystery is that the contingent nature of these paradoxes was not fully appreciated until 1975. Nevertheless, Buridan's seventh sophism from Chapter 8 of his *Sophismata* is a version of the Epimenides paradox.

Buridan (c. 1300 – c. 1358) was an arts master at the University of Paris, and rector of that University in 1328 and again in 1340. There are few historical records of his personal life, though he is known to have been one of the earliest to ascend Mount Ventoux – this at a time when mountain climbing was unusual. Legend has it that he was thrown into the Seine in a sack for a dalliance with the Queen of France.

Buridan's seventh sophism reads as follows:

> (3) Every proposition is false.
>
> The case being posited is that all true propositions have been annihilated and only false ones have survived, and that then Socrates says 'Every proposition is false,' and nothing more. The question is whether his proposition is false or true.
>
> [Buridan, trans. Hughes 1982, p. 61]

The sophism is made up of a Liar statement (3), *the case* or circumstances in which it was uttered, and the puzzling question to be answered.

For Buridan propositions are temporal entities. They persist at least for the duration of his cases, so this case is plausible. Although 'proposition' is the correct translation, 'sentence token' would be a better paraphrase into modern parlance. I will use 'propositions' as Buridan did in this subsection.

There is, of course, a paradox associated with the sophism: if Socrates says all propositions are false and all other propositions are false, then what Socrates says is provably false, i.e. (3) is false. (The reasoning up to this point is analogous to that used in Callimachus' conundrum.) However, in this case if (3) is false, then all propositions are false; so (3) is the case – on the basis of which, one would normally think (3) is true. In which case, (3) is both true and false, a contradiction.

Many early Scholastics held that Liar statements such as the one expressed by (3) were meaningless:

> To someone who says that he is lying, we ought to reply 'You say

nothing.'

[Spade 1973, p. 307]

Cassatio was the mediaeval name for this view, one that has been explicitly borrowed from mediaeval work and defended in modern times.[18] The most common objection to this approach is that there is evidence that (3) is meaningful – if it were meaningless, how could it be correctly translated into another language? This may not be the level of meaning at issue; but then the onus is on advocates of the approach to clarify and independently motivate levels of meaningfulness.

A second approach accepts (3) as meaningful but maintains 'that no *insolubile* is true or false, because nothing of this kind expresses a proposition'.[19] So (3) is meaningful, but is neither true nor false. This is similar to modern 'truth-value gap' theories, such as Kripke's. The most common objection to this approach is that it entails that (3) and statements like it are not true, in which case the statement 'This sentence is not true,' is still problematic.[20]

Buridan, in his own discussion of his seventh sophism, summarises four further mediaeval approaches to the Liar Paradox, finds fault with each, and then presents his view.

A third grouping of mediaeval approaches, and the first that Buridan explicitly discusses and rejects, are various theories restricting self-reference, as for example the notion that 'terms that can stand for propositions never stand for the propositions in which they themselves occur, but only for other ones'. Ockham, famous for Ockham's Razor, held a variant of this [Spade 1974, pp. 298-300] – and a number of modern approaches to the Liar Paradox have involved restrictions of one sort or another. The most common objection to such theories is the lack of an independent reason as to why there should be any restriction to self-reference. In Buridan's words:

> But this solution certainly will not do. Whatever we can think about we can also speak about… we can think about all propositions indifferently, present ones, past ones, future ones, our own as well as

---

[18] Cf. the articles of Laurence Goldstein on the Liar in various philosophical journals.

[19] This quote is translated from Paul of Venice's *Logica Magna*, cited in Kneale & Kneale [1984, p. 228].

[20] Truth-value gap theories usually use an object- and meta-language distinction to avoid this problem.

> those of others; and therefore we can speak about them all. It is obvious, too, that I can say that the proposition I am now actually uttering is an affirmative one, and it can be my intention to speak about it; but the term 'proposition' that occurs in it stands for it itself.
>
> [Buridan, trans. Hughes 1982, p. 65]

A fourth mediaeval approach to Liar statements considers they are both true and false. (3) is true if and only if it is both true and false. This approach has a modern counterpart too, but the modern version draws inspiration from Hegel.[21] Intuitions over the law of non-contradiction are deeply entrenched. So it is interesting that Buridan did actually put up an objection, along the following lines.

Propositions should be able to be denied in a standard formal way. If we take (3) as being both true and false, then it is true, and ought to be contradicted by someone stating 'Some proposition is not false' in the circumstances given by the sophism. However, according to Buridan, this would not contradict (3); so Socrates' proposition cannot be true, let alone both true and false.

Two pieces of theory are required to understand Buridan's reasoning as to why this would not contradict (3). Firstly, we recall that for Buridan propositions are temporal, and (3) is such a temporal proposition. Secondly, 'Some proposition is not false' would have to refer to a proposition *from the same set* of propositions for it to contradict (3). Buridan illustrates as follows. Assume only two propositions are in existence: one which is clearly false ('A man is a donkey') and Socrates' proposition (3).

> Now in the given case Socrates' proposition stands for only two propositions, ... but if you were to state its contradictory (namely 'Some proposition is not false') then *it* would immediately stand for three – itself and the other two – and so it would not really contradict Socrates' proposition after all [since it would not talk about exactly the same collection of propositions].
>
> [Buridan, trans. Hughes 1982, pp. 65 & 67]

The idea is simply that (3) refers to two propositions, but its putative contradiction 'Some proposition is not false' would refer to one of three, not two. Whether we agree with it or not, it is a clever argument.

---

[21] Cf. Graham Priest [2006].

Buridan presents a fifth approach to the Liar statement – that of *self-stultification* – in this way:

> ...every proposition, by its very form, signifies or asserts itself to be true, and as a result any proposition that either directly or indirectly asserts itself to be false, *is* false. For although the facts are as such a proposition says they are in so far as it says it is *false*, yet they are not as it says they are in so far as it says it is *true*; so the proposition is false, not true, because in order for it to be true it is necessary not merely that the facts should be as it says they are but that they should be as *in every way* it says they are.
>
> [Buridan, trans. Hughes 1982, p. 67]

The argument turns on the idea that (3) is put forward as true because that is what asserting a sentence does. So, if (3) says of itself that it is false, its assertion is self-defeating, after the manner of

This space is intentionally left blank.

Notice that this statement is not *self-stultifying* purely in terms of its semantics. A statement of the same form outside a blank box and with reference to a blank box would have the same semantics but not be 'self-stultifying'.

Buridan has a complex argument for rejecting the 'self-stultification' approach. I have a simpler rebuttal.

By way of background: it is important to distinguish between a paradoxical *argument* and a paradoxical *statement*. The first is an argument (a proof) of a surprising or even contradictory conclusion from plausibly true premises using apparently valid reasoning. The second, like (3) above, is a statement on which a paradoxical argument *turns* – e.g., in the paradoxical argument associated with statement (3), if (3) is true, then it is false. In the subsection on the problems associated with the Liar, there were two related issues – one to determine the semantic value of the Liar sentence, another to address the validity or invalidity of the argument. My rebuttal will show that even if the self-stultification account resolves the issue of the semantic value of the Liar statement it does not affect the

soundness of the proof of a contradiction.

Here is my rebuttal: Basically, (3) itself ('Every proposition is false') is not used as a premise in the paradoxical argument associated with Buridan's seventh sophism. The premises of the paradoxical argument are '*Socrates says that* all propositions are false,' and '*All other* propositions are false'. Therefore, (3) is not *asserted* for the purposes of the proof of a contradiction. (Premises count as 'asserted'. But hypothetical assumptions within a proof are not asserted – they are like the antecedents of conditional statements). Hence, even if the *self-stultification* or *metalinguistic* view proves that statement (3) is false, a contradiction is still provable in this case without using (3) as a premise. So the *self-stultification* line of reasoning does not resolve the paradox.

The sixth approach to the Liar paradox discussed by Buridan relates, I believe, to Thomas Bradwardine (c. 1295 – 1349) and the idea that propositions *signify* (that is, they *mean*, in a certain technical sense) whatever follows from them.[22]

Both Bradwardine and Buridan require more for a statement to be true than things being as the statement says they are. Although unusual, a statement like (3) may be the case and yet not be true. Usually, statements only refer to things that are or are not the case, or other statements that ultimately depend solely on what is or is not the case. However, the truth or falsity of Liar statements would seem to depend on some criterion in addition to what is the case.

On scant evidence available to me, I reconstruct Bradwardine's solution as follows.[23] Bradwardine stipulated these criteria for true propositions:

1. A proposition is true if everything it entails is the case.
2. A proposition is false if and only if something it entails is not the case.

---

[22] Bradwardine was Archbishop of Canterbury when he died in 1349, presumably of plague. He wrote on *Insolubilia* some time between 1321 and 1324. See Spade [1981].

[23] I worked out this reconstruction in 2002, which was published in my [2004]. My acquaintance with Bradwardine's thought was merely through Spade [1981]. Since then I have become aware of Stephen Read's [2002] work on Bradwardine's solution. Both Spade and Read have accessed surviving manuscript copies of Bradwardine's work. Most likely, their accounts are more faithful to Bradwardine's theory; Read's [2002] is far more sophisticated than my reconstruction. Nevertheless, Read's work has confirmed my opinion that it was Bradwardine's theory that Buridan refers to in this section of his *Sophismata*.

3. A proposition is either true or false but not both.

And he reasoned along these lines: (3) entails that (3) itself is false and that all other propositions mentioned in the circumstances of sophism 7 are false. That (3) is false entails by Bradwardine's definition that something (3) entails is not the case; so (3) entails that either (3) is not false or that some other proposition is not false. But (3) entails that all other propositions are false; so (3) entails that (3) is not false but true.

Therefore, (3) entails both that it is false and that it is true. No proposition is both true and false; so in entailing this (3) entails something that is not the case; so (3) is false.

However, Bradwardine gives insufficient reason to stop at this conclusion. It seems that his reasoning can go round and round like a logical merry-go-round and there is no clear reason to stop and get off at the point where we conclude that (3) is false, rather than go around a bit further using similar logic to conclude again that (3) is also true. So all is not quite right.

Finally, there is Buridan's own view that the case is possible, but that Socrates' proposition (3) cannot possibly be true. Buridan, as noted above, explicitly distinguishes between a proposition being the case and its being true or false. This can be understood with reference to three conditions required for a proposition to be true:

1. The proposition must exist (as an asserted sentence, thought, etc.).
2. Things must be as it says they are.
3. Nothing entailed by the proposition is false. (I believe this is a notion derived from Bradwardine's requirement).[24]

The case may be as the seventh sophism says it is, in that every currently existing proposition could happen to be false, but as soon as Socrates asserts (3), *then* it can be proved that (3) is false. This uses a simple *reductio* argument: if (3) is true, then it is false; so (3) is not true, i.e. (3) is false.

In terms of the three conditions above, Buridan claims:

i. Proposition (3) exists.

ii. Everything is as (3) says (all existing propositions are false).

---

[24] The exegesis of Buridan's own view is difficult. This third requirement is my own reconstruction. For other interpretations, cf. Spade [1978] and the Introduction of Hughes [1982].

iii. (3) entails both that it is true and that it is false, which is a contradiction. Therefore, (3) entails something false.

Unfortunately, Buridan's third claim is incorrect. Buridan has not shown that (3) entails (3) is true. (3) does entail both that it is the case and that it is false. Yet in terms of Buridan's own distinction between being the case and being true, it is possible to be the case but be false.

I propose to salvage Buridan's theory by modifying his third condition for a proposition to be true, as follows:

3. Nothing entailed by assuming the truth of the proposition is false.

Again, as a result I can claim i and ii above, and iii. That (3) is true entails both that (3) is true and (3) is false, which is a contradiction. Therefore, that (3) is true entails something false.

Consequently (3) fails the third criterion for truth, so (3) is false. Furthermore, given Buridan's criteria for truth (with my modification), it does not follow from (3) being false that (3) is true. Therefore, the paradox is resolved, at least in the terms proposed by Buridan.

It seems odd that in this case Socrates could say 'All propositions are false', and that on Buridan's theory this can be proven false; so that the case is as Socrates says it is, but this is not sufficient to make his proposition true. Buridan's theory goes on to block the inference of 'Some proposition is true' from the falsity of 'All propositions are false'.

Alan Hazen [1987] documents a counter-example to Buridan's theory with the same form as the Epimenides. Since Buridan's talk of 'propositions' means much the same as what we mean nowadays by 'sentence tokens', Hazen's counter-example is 'All sentence tokens equiform with the sentence token on the blackboard are false'. A token of this is, in fact, the sentence token on the blackboard. From the above, we can see that the one on the blackboard is certainly false. It also follows from the above that on Buridan's theory any token equiform with the one on the blackboard is false. So, we are able to prove that all sentence tokens equiform with the sentence token on the blackboard are false; that is, we can prove a token as the conclusion of a sound argument that is false. This is not an outright counter-example because Buridan's definition of validity is adjusted to match his solution; however, Hazen's point is that it is bad for a theory to allow that a sound argument can prove a false conclusion. Hazen puts this case in the most damaging way

possible, as he should; but the point is perhaps more easily understood as a "revenge" problem about whether to assert 'All sentence tokens equiform with the sentence token on the blackboard are false'. Under the circumstances, on Buridan's account, all sentence tokens equiform with the sentence token on the blackboard are false. However, Buridan cannot assert that without asserting a false sentence token. Modern theories face similar problems.

1.2.3 Some Modern Approaches to the Epimenides Paradox

Interest in the Liar was rekindled around 1900 – this being brought on by the discovery of paradoxes in set theory, which sparked a perceived crisis in the foundations of mathematics. Collections of paradoxes have re-emerged, e.g. Smullyan [1981], and even whole books dedicated to paradoxes, e.g. Sainsbury [1995], and Clark [2002].

Bertrand Russell [1908] thought the paradoxes of set theory and paradoxes like the Liar were essentially similar in that they all violated a principle that arose in discussion between himself and Henri Poincaré. Articulating this principle proved almost as elusive as solving the paradoxes. Here is one of Russell's attempts to define what he called *the Vicious Circle Principle*:

> If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total.
>
> [Russell 1908, p. 225]

Basically, Epimenides makes his famous statement (2) with reference to all Cretan statements including (2) itself – and accordingly Russell thinks (2) is nonsense, like some Scholastics, because there is no statement made by (2). In order for such a statement to exist, it would have to be defined in terms of a collection that includes itself, which, for Russell, would be viciously circular.

Actually, talk about reference in connection with Russell's theory is confusing, as his theory restricts the *range of applicability* (RA) of open sentences, rather than restricting reference. An expression's range of applicability, the set of things which it can take as arguments to form true or false sentences, is constrained by *impredicativity*, which basically requires that any element of a predicate's or relation's RA must be specifiable independently of the predicate or relation itself.

Tarski's [1944] famous 'levels of language' restriction is not actually a

restriction on self-reference either but also a restriction on application of the truth predicate. For pedagogic purposes, I will introduce Tarski's approach with reference to 'category mistakes'. Here are some examples of category mistakes (mismatched attributions to an object):

Triangles are thatched.

Colourless green ideas sleep furiously.

Tarski's restriction is that it is not appropriate to predicate 'true' or 'false' of just any sentence. We should have levels of true and false, e.g. $true_1$, $true_2$, $true_3$, etc. Then sentences which do not contain 'true' or 'false' and do not refer to other sentences that do, will be $true_1$ or $false_1$. Sentences containing $true_1$ or $false_1$ will be $true_2$ or $false_2$, etc. For example, if Jill says 'It's raining' and Bob says, 'What Jill said is false' and Brian says, 'What Bob said is true' and it is raining, then Jill's sentence is $true_1$, Bob's is $false_2$, and Brian's is $false_3$.

The suggestion that we actually talk in this way is counter-intuitive, and Tarski did not intend that this restriction be imposed on natural usage. Rather he was concerned with formal logical languages.

Nevertheless, Quine has suggested that Tarski's theory could be applied to natural language. (And this approach was pursued by Tyler Burge [1979])

Arthur Prior [1958] was a modern exponent of the 'no statement' theory, but he also pointed out the integral dependence on empirical circumstances as to whether a Liar statement is true or problematic. If all Cretan statements were false, a non-Cretan could successfully make a statement to that effect. However, even if Epimenides utters the words, 'Cretans always lie,' under these circumstances, Prior says Epimenides cannot succeed in making a statement.

Prior's point of view seems to have been that it is impossible that both the world is a place where a Cretan says 'All Cretan statements are false' and all other Cretan statements are false. And Prior's response was that it was therefore impossible that a Cretan could make such a statement under these circumstances. If Epimenides intended to put it forward as being the case, it seems he cannot succeed because it must be false if a Cretan says it; so, it seems logic prevents a Cretan from successfully asserting this statement. Then, 'If all Cretan statements are false, then a Cretan does not say that all Cretan statements are false' is a necessary truth. So, by propositional logic, 'If Some Cretan says that all Cretan statements are false, then some Cretan statement is true' is a necessary truth. Therefore the argument is valid.

However, if Prior were right, we would have to revise our concept of "contingent" to allow for such necessary conditions between what people might say and what could be the case if they said so, and this seems absurd. However, what Prior intended was that the contingent limits what we can truly say.

Prior presented the Cretan Liar paradox without using the truth predicate. Here is a natural language argument that does not use an explicit truth-predicate.

> A Cretan says anything said by a Cretan is not the case. Suppose that is the case. Then this is something said by a Cretan; so it is not the case and our supposition results in a contradiction. Therefore, it is not the case that anything said by a Cretan is not the case, i.e. something said by a Cretan is the case; but not this Cretan saying; so, something else said by a Cretan is the case. If it is also given that nothing else said by a Cretan is the case, the case is paradoxical.

It is clear that the argument avoids use of an explicit truth-predicate or operator by using "is not the case". Prior [1971] equated this with a truth operator, because he held that "it is not the case that" was equivalent to "it is false that". However, most modern logicians use the former as an operator and the latter as a predicate. In the former case they treat the 'that' as Prior does as part of a complex verb. In the latter case, however, they treat the 'that' as attached to the noun clause that follows it, together forming a grammatical substantive usually taken as referring to a sentence. There is nothing obviously wrong with the use of universal instantiation in the above argument; but its formal exposition uses substitutional quantification. In this case, Prior quantifies over sentential expressions rather than over objects. Nevertheless, the usual problem with substitutional quantification – that there are more objects than there are names – does not seem to pose an issue in this case, as Epimenides' statement is about Cretan statements anyway.

There is another paradox that Prior attributes to Peter Geach, and that Prior thought was a variation of the Epimenides but is actually a distinguishable conundrum and paradox. This time the premise is 'A Cretan says that some Cretan statement is false', and this is provable, so that something else a Cretan says is false. A paradox can be generated if it is given that all other Cretan statements are true.

Here is the formal proof that given that a Cretan says this, it is the case. I follow Prior [1961, p. 18] in this proof and use delta is an operator for 'A Cretan says that', and use substitutional quantification. (The proof can easily be transposed

to use a predicate for '… is said by a Cretan', and objective quantification with the truth predicate, and incorporating the TI and TE rules where appropriate.)

1 |	$\delta(\exists p\ (\delta p\ \&\ \sim p))$	premise
2 |	| $\sim\exists p\ (\delta p\ \&\ \sim p)$	assumption
3 |	| $\forall p\ (\delta p \supset p)$	2 QN, defn
4 |	| $\delta(\exists p\ (\delta p\ \&\ \sim p)) \supset \exists p\ (\delta p\ \&\ \sim p)$	
		4 $\forall$E p / $\exists p(\delta p\ \&\ \sim p)$
5 |	| $\exists p\ (\delta p\ \&\ \sim p)$	4, 1 MP
6 |	$\exists p\ (\delta p\ \&\ \sim p)$	2-5 ~E

With reference to line 1 of this proof, Prior notes "It is, however, something that one cannot consistently suppose false" [1961, p. 18] If some Cretan says "Some Cretan statement is not the case", then if it is false, it is true. So naively it cannot be false and must be true. It seems to be a case of (a Cretan) saying so, makes it so. This is a different conundrum to the Epimenides.

Prior [1958, p. 265] points out, his delta operator can be generalised. It can be interpreted using various verbs of indirect speech, or "propositional attitudes".[25]

The idea is simply to generate Liar statements like these:

> Someone fears, doubts, wishes, or believes that all the things he fears, doubts, wishes or believes are not the case

It does not really matter whether this is formalised using an operator or a predicate. Either way the proofs go through to the conclusions:

> There is some fear, doubt, wish or belief that this person has that is the case.

This is similar to the Preface paradox. The premise is usually an existential epistemic statement to the effect that author believes something in his or her book is false. Yet the author has asserted every statement in the book. Prior [1971] discussed it and pointed out its similarity to Geach's paradox. Geach's paradox is not a version of the Epimenides, but actually more closely related to Curry's paradox, and I will return to it in discussing Curry's paradox.

Kripke [1975] made a virtue of avoiding contradictions between contingency

---

[25] I believe it could also be interpreted using a non-truth-functional operator like 'possibly'.

and logic using a different conception of truth, and at the same time achieved a practical simplification of Tarski's approach. Kripke articulated an issue with the Tarskian approach that it would restrict our usage in unexpected ways. For it seems quite legitimate for Jones to say:

(4) The majority of Nixon's statements about Watergate are false.

There is a small problem that Jones may not know which truth-predicate she is using. She may believe Nixon has made between 7,000 and 10,000 statements about Watergate, and that at least 5,001 of them are false; and she may believe this without knowing what was the highest-level truth-predicate Nixon used. Say '$true_n$', is the highest-level truth-predicate Nixon used in his statements about Watergate, then Jones' statement would appear to use '$true_{n+1}$'.

Now, it may turn out that one of Nixon's statements about Watergate was

(5) Everything Jones says about Watergate is true.

Now, we are unable to give the truth-predicates in Jones' statement and Nixon's statement Tarski-levels because each refers to the other, and so neither can be taken as necessarily lower or higher. Of course, we could make some arbitrary convention like the one spoken first is lower, but then we could just complicate the case, say by stipulating they were asserted at the same time.

The fact is, as Kripke points out, we can often make sense of such cases. If, independently of Nixon's statement about Jones' statements, the majority of his statements about Watergate are false, then Jones' statement, and even Nixon's about Jones', can be determined true. Tarski cannot account for this. Tarski's account is at best incomplete.

Kripke's example together with extra factual premises entails a contradiction. Nevertheless, even without these extra factual premises there is a somewhat surprising consequent.[26]

1 | Jones says the majority of Nixon's statements about Watergate are false

2 | Nixon says All Jones' statements about Watergate are true

3 | <u>All Jones' other statements are true</u>

4 | If what Nixon says is true, Jones' statement is true, and Nixon

[26] This example is rather like one of Cohen's that Prior [1961, p. 20] discusses.

must have made at least two more statements about Watergate.

5 | If what Nixon says is false, Jones' statement is false, and Nixon must have made at least one more statement about Watergate.

6 | Therefore, Nixon must have made at least one more statement about Watergate

I grant that this is not very surprising, but it is still a contingent conclusion that did not seem to be contained in the premises, reminiscent of our conundrums over Epimenides statement.

Kripke's main point about this example, nevertheless, is that:

> ... many, probably most, of our ordinary assertions about truth and falsity are liable, if the empirical facts are extremely unfavourable, to exhibit paradoxical features. ...it would be fruitless to look for an intrinsic criterion that will enable us to sieve out – as meaningless, or ill-formed – those sentences which lead to paradox.
>
> [Kripke 1975 / 1984, pp. 54 & 55]

Take the case where (4) is Jones' only assertion about Watergate and Nixon's assertions about Watergate are evenly balanced between true ones and false ones except for (5). Then:

> "[4] and [5] are both paradoxical: they are true if and only if they are false ... Yet no syntactic or semantic feature of [4] guarantees that it is unparadoxical. Under the assumptions of the previous paragraph, [4] leads to paradox. Whether such assumptions hold depends on the empirical facts about Nixon's (and other) utterances, not on anything intrinsic to the syntax and semantics of [4]."
>
> [Kripke 1975 / 1984, p. 55].

Kripke defined the notion of groundedness – the idea that some sentences are *grounded* if and only if their truth depends on the truth of sentences that do not themselves contain 'true' or 'false' [Herzberger 1970]. Jill's, Bob's and Brian's statements given in the previous section are all grounded because their truth value relies on the truth value of Jill's statement which does not use 'true' or 'false'. Epimenides' statement is a different case, though. If there is some other true Cretan statement, then Epimenides' statement is *grounded* and false. If there is no other true Cretan statement, Epimenides' statement is *ungrounded* – its truth value cannot

be determined, it has a 'truth-value gap'.

Such paradoxical statements are a subset of "ungrounded" statements. Kripke gave a definition of the concept of groundedness that sieves out the unproblematic statements based on empirical facts of the actual world, as a base model as it were. Those left over, the ungrounded statements, include the paradoxical statements. If the linguistic facts of the matter are such that there are other true Cretan statements, 'All Cretan statements are false' will be sieved out as grounded and false; otherwise it will be ungrounded and not receive a truth value. In such circumstances, there is a truth-value gap. Kripke's theory entails that it is a contingent matter whether 'All Cretan statements are false' is ungrounded and paradoxical. And the same applies to 'All Australian statements are false', or 'All statements of the such-and-such select committee are false', etc.

In considering examples like the above, Kripke outlined a theory of truth that made sense of the notion of "groundedness", used as a sort or Eratosthenes' sieve for sentences given the empirical truths:

> In general, if a sentence such as [4] asserts that (all, some, most, etc.) of the sentences of a certain class C are true, its truth value can be ascertained if the truth values of the sentences in the class C are ascertained. If some of these sentences themselves involve the notion of truth, their truth value in turn must be ascertained by looking at *other* sentences, and so on. If ultimately this process terminates in sentences not mentioning the concept of truth, so that the truth value of the original statement can be ascertained, we call the original sentence *grounded*; otherwise, ungrounded.
>
> [Kripke 1975 / 1984, p. 57]

Kripke's sieve, then, is to start with the base statements and determine the truth value of all of them, then determine the truth value of statements about the truth of base statements, then determine the truth value of statements about the truth of those statements, and so on. Statements that do not ultimately refer to base statements are thus sieved out. (Here we should take reference to include both direct and indirect reference, in the following sense. A sentence may refer directly to a sentence that itself directly refers to one or more other sentences, which may make still further direct references. All these sentences ultimately referenced in this way are counted as being referred to by the initial sentence.) Among these are

troublesome Liar claims and Truth Teller claims like "This sentence is true". Also sieved out are sentences like (4) and (5), if the truth values of the base sentences are such as described in the paradoxical case above.

Kripke formalised this intuition in a way that I'll characterise as progressively extending the application of a truth predicate between two sets. Start with all the sentences of a language not including the truth predicate. Allocate these between a set containing the true ones and a set containing the false ones. Now allocate all sentences including a truth predicate that refer to those sentences as true or false, according to whether they actually are true or false. Next find sentences that refer to the newly determined sentences and allocate those a truth value. And so on, progressively, until further rounds do not add any more sentences to the sets of true and false sentences. Those left over have a truth-value gap. They do not necessarily lead to paradox, but among them are those that do.

Whether (4), (5) and 'All Cretan statements are false' are ungrounded depends on the truth values of the base statements, which depend on the empirical circumstances.

Say, for example, all other Cretan statements are of a simple factual kind, like 'Ariadne owns a shop', etc. If one of them is true, then Epimenides statement is grounded and false. If, however, all other Cretan statements are false, Epimenides statement remains ungrounded and never receives a truth value.

Ungroundedness is an important modern explication of what is wrong with Liar statements. Indeed, it seems to me that the concept has a more general application, providing, for example, analogous insight into paradoxes of time travel [Eldridge-Smith 2007].

Kripke does not discuss validity in his article, but we can make some extrapolations. His theory implies that whenever the Epimenides statement leads to paradox, if the premises are true, Epimenides' statement is ungrounded; but Epimenides' statement is not used as a premise. With reference to the derivations in Section 1.2, the first derivation seems *prima facie* valid. Had it used the T-schema, it would have been invalid, as the T-schema is not generally true on Kripke's semantics; nevertheless, the inference rule TI would seem to be justified by Kripke's very construction. The inference rule TI seems at first to capture the way in which sentences become included in the extension of 'is true'. However, if the first derivation is valid, it is also sound. For its premise is that Epimenides said that

Cretans always lie, and he did. That means we have a proof that some Cretan statement is true. Now, say all other Cretan statement are false. Then we have a proof that Epimenides' statement is true; but in the first derivation we proved it was not the case. We seem to have proved:

$$T\langle\forall x\ (Dx \supset \sim Tx)\rangle\ \&\ \sim\forall x\ (Dx \supset \sim Tx).$$

While this is not contradictory; it is not possible on Kripke's semantics, as his semantics is monotonic. The error in this line of reasoning was to think that TI represents Kripke's construction for populating the extension of truth. In fact what seems to be required is some modified version of this rule that only infers the truth of a sentence that has been proven. So, a theorem of a theory (complete with axioms for each 'fact' in the base model) can be used to infer a statement of the truth of that theorem:

$$\text{If } \vdash @, \text{ then } \vdash T\langle @\rangle$$

This rule of inference cannot be used under an assumption in the way TI was used in the previous derivation; so, the derivation is invalidated by the Kripke semantics.

Some more recent theories of truth have followed Kripke's explanation of the apparent occasionally paradoxical nature of Epimenides statement. For example, according to Gupta & Belnap [1993, p. 6] contrast the Eubulidean Liar as 'intrinsically paradoxical' and the Epimenides as a 'Contingent Liar'.

This is not quite correct, for it is a contingent matter whether my favourite sentence happens to be 'My favourite sentence is not true' and which sentence has tokens printed in copies of the *Journal of Philosophy*, November 6, 1975, p. 691, line 5.

Nevertheless, I agree with Gupta and Belnap that whether some sentences are paradoxical is a contingent matter; but that presents a challenge to our conception of paradox as involving dual *reductios*. Somewhat ironically, this challenge first began to emerge with the discovery of Curry's paradox, when it would seem that the Epimenides could have been well put to the same purpose.

1.3 A Brief History of the Curry's Paradox

In this section I introduce Curry's paradox and give a very brief account of its intellectual history. Its status as a somewhat independent paradox can be contrasted with the status of the Epimenides as at best a variation of the Liar. Historically, Curry's paradox was put forward using lambda calculus and self-application [Curry 1942]. Curry gave two examples, one involving membership intended to be like Russell's paradox, and another involving a relation represented by 'T' intended to be like the Epimenides. Here is a version using the truth predicate:

(1) If this sentence is true, Sydney is in France.

At first the antecedent may look like the Truth-teller; but 'this' should be understood here as referring to the whole conditional.

The mere existence of this sentence, common principles of logic and the same principles of truth as used in the Liar are sufficient to prove Sydney is in France. With reference to (1), here is a Curry argument proving a contingently false statement:

> If this sentence is true, then Sydney is in France. Assume this sentence is true. Then Sydney is in France. So, if this sentence is true, Sydney is in France. So this sentence is true.
> Therefore, Sydney is in France, contrary to fact.

If 'Sydney is not in France' were added as a premise, a contradiction could be derived. And this could be done in every case where a paradoxical argument relates to a contingently false conclusion. We could simply add the negation of the conclusion as a premise. So, we should have no qualms about such arguments being paradoxical. I note that the material truth value of the consequent was not used in proving (1) is true; so that a similar proof can be given for every sentence of the form 'If this sentence is true, Q', whether Q is true or false and whether or not the sentence is paradoxical.

Curry's paradox's original point of distinction was that it can be proven without involving negation. This means it can be proven in a logical system without negation, and the Epimenides cannot. The significance of Curry's in this respect is that it shows that a solution merely in terms of negation will not address all variations of the (logical) paradoxes.

The subsequent significance of Curry's paradox is that it can prove an

arbitrary sentence, Q, without direct or indirect appeal to *explosion* (*ex falso sequitur quod libet*). However, the Epimenides can do this too. It does involve negation – it uses *modus tollens* and *double negation*; but it can prove an arbitrary sentence, Q, without proving or assuming or depending on a contradiction.

Being a modern paradox, one might think that the intellectual history of ideas about Curry's paradox is a lot clearer. To some extent it is, as the sources are readily accessible; but the associated ideas are closely tied to theories of paradox and in particular their classification.

1.3.1 Russell and Curry's Paradox

Curry's paradox was unknown to Russell in 1908, so this should be a short section; but when he went to interpret the Liar Russell did so as follows.

> When a man says "I am lying," we may interpret his statement as: "There is a proposition which I am affirming and which is false." All statements that "there is" so-and-so may be regarded as denying that the opposite is always true; thus "I am lying" becomes: "It is not true of all propositions that either I am not affirming them or they are true;" in other words, "It is not true for all propositions *p* that if I affirm *p*, *p* is true." The paradox results from regarding this statement as affirming a proposition, which must therefore come within the scope of the statement.
>
> [Bertrand Russell 1908, p. 224]

The trouble with this interpretation is that while 'I am lying' taken self-referentially entails a contradiction, 'There is a proposition which I am affirming and which is false' entails 'Some proposition I am affirming is true' rather than a contradiction. The case is parallel to Geach's conundrum in the previous section. 'Some Cretan statement is false' entails 'Some Cretan statement is true'. Russell's interpretation of the Liar does have strange consequences; but it does not entail a contradiction on its own.

I shall return to the correct classification of 'Some Cretan statement is false' at the end of this brief history of Curry's paradox.

1.3.2 Curry's original paradox

Curry refers to both Russell's paradox and the paradox of Epimenides in proving

his paradox [1942, p. 116]. As Curry uses self-application (of a term to a term), it is worth noting that his use of 'T' (whatever its intended interpretation) is relational. That is, it seems fair to interpret 'T' as a truth relation, representing something like 'x is true of y'. In the context of Curry's paper it is being used for self-application towards his paradox; so, it is representing a reflexive linguistic usage, 'x is true of itself'. This is the truth relation which I will later use in representing Grelling's paradox, that paradox about whether 'x is *not* true of itself' is true of itself. It therefore seems to me that his semantic paradox is actually more closely related to Grelling's paradox than the Liar. That is, Curry's semantic paradox is a Curried Grelling's paradox. From a sentence 'If x is true of x, then Q' one can prove Q, for any arbitrary sentence, Q.

1.3.3 Geach's Variations of Curry's paradox

If Curry had in mind a Curried Liar but only gave a Curried Grelling's, Geach [1955] certainly filled this gap, as did Löb (and / or an anonymous referee of Löb's article). Löb [1955] gave a natural language version using an indexical expression and a stipulated name for that expression, that is, he used a self-referential indexical expression like:

If this sentence is true, Sydney is the capital of France

Geach not only made the use of the T-schema explicit; but also reduced the premise to a theorem by using quote name expressions with variables to do duty in the way arithmetization works.

I will refer to this sort of variation as 'the Self-referential Curry Paradox'. Distinguishing the Geach-Löb variation, *prima facie*, suggests self-reference is being distinguished from self-application. This will not be the basis of the distinction I will be making; nevertheless, I digress briefly to introduce this issue. Self-reference involves the appending an open sentence to a name that refers to the resulting sentence. 'My favourite sentence' is a term referring to a whole sentence of which it is a part; the predicate 'is not true' is appended to the term 'my favourite sentence', which refers to the resulting sentence, 'My favourite sentence is not true'. Self-application involves the application of an open sentence to an open sentence or appending the open sentence to a term for the open sentence (or a term for a function of an open sentence). In this way, it seems the two may be related; but it is not clear whether one is reducible to the other. Self-reference is not

necessary for the Liar. Visser [2004, p. 156] cites an example of a circular Liar, which I will introduce in Chapter 3. Nevertheless, the same could be said about self-application. I will give examples, of circular-application in Chapter 3 as well. It is equally unclear how circular-reference relates to circular-application. Even circular-reference is not essential for a Liar-like paradox. There are examples like Yablo's paradox, which I will give in Chapter 3, using infinite lists of sentences each referring to subsequent members of the list. Nevertheless, the question here is whether issues with reference, self-reference, circular reference or even infinite reference can be reduced to issues of application, whether self-application, circular-application or conceivably even infinite-application.

Even if a clear cut distinction between self-reference and self-application can be sustained for unquantified versions of the paradox; such a division *prima facie* merges under quantification and instantiation. If Epimenides claim is instantiated with itself, that is not self-reference as such on a Russellian analysis of the form of the general claim and its logical semantics. The general claim was formed by binding an open sentence; so, the instantiation is the application of the open sentence to its own bound sentence. While acknowledging this analysis, I will argue in Chapter 3 that self-application is insufficient to explain the inferences that individuate the Epimenides.

As mentioned previously, Geach also devised the existential statement (by which Russell had previously meant a Liar) as a paradox in its own right.

$$D\langle \exists x(Dx \ \& \ {\sim}Tx\rangle$$

This bears some analogy to Curry's, because both statements must be true and because they are true, some other contingent statement follows.

1.3.4 Meyer and Non-truth-functional Interpretations

Meyer, Routley and Dunn [1979] explicated Curry's paradox (with reference to set-theory) involving non-truth functional interpretations of the arrow connective or relation.

> For almost as long, and as was first noted by Dunn and independently by Routley, we have known that the standard relevant logics R and E, even if they avoided the ultimate *reductio ad absurdum* … of naive set theory by the route that leads through Russell's paradox … would get to [any arbitrary formula] anyway by Curry's

paradox.

[Meyer, Routley and Dunn 1979, p. 126]

They also introduced a proof using an axiom schema corresponding to *modus ponens* [p. 127], which Beall calls *pseudo-modus ponens*. I give a version of this proof in the next section when I parallel it with a variant of the Epimenides.

1.4 Parallel Variant Epimenidean and Curry Paradoxes

In this section I wish to put the case for the Epimenides being treated like Curry's by simply paralleling the arguments.

1.4.1 Epimenides and Curry's with Quantification

If Epimenides says all Cretan statements are false, then what he says must be false. Thus some other Cretan statement must be true. If no other Cretan statement is true, then a paradox arises.

If a Cretan says some Cretan statement is false, then what he says must be true. Thus some other Cretan statement must be false. If no other Cretan statements are false, then a paradox arises.

1.4.2 Epimenides and Curry's in Lists

Consider these statements and the reasoning that follows them. Characteristically, the Epimenides statement must be false and the Curry statement must be true. In these cases they can each be used to prove some arbitrary statement.

Both this and the next statement are false.
Ariadne taught Theseus a dance.

If the first statement is true, it is false; so the first statement must be false. The only way that the first statement could be false is if the second statement is true. Therefore, Ariadne taught Theseus a dance.

Consider the parallel with:

Either this or the next statement is false.
Ariadne taught Theseus a dance.

If the first statement is false, it is true; so the first statement must be true. The only way that the first statement could be true is if the second statement is false.

Therefore, Ariadne did not teach Theseus a dance.

1.4.3 Epimenides and Curry's without Quantification

There are truth functional unquantified variations of the Epimenides and Curry's as follows.

This conjunction is false and Ariadne taught Theseus a dance.

If this conjunction is true, both its conjuncts are true; but then this conjunction is false (as the first disjunct says); so, this conjunction must be false. Thus, the first conjunct must be true. The only way that this conjunction can be false is if the second conjunct is false. Therefore, Ariadne did not teach Theseus a dance.

Either this disjunction is false or Ariadne taught Theseus a dance.

If this disjunction is false, both its disjuncts are false; but then this disjunction is true (as the first conjunct says); so, this disjunction must be true. Thus, the first disjunct must be false. The only way that this disjunction can be true is if the second disjunct is true. Therefore, Ariadne did not teach Theseus a dance.

Once again, the Epimenides statement is false and the Curry statement true, and both can be used to prove some arbitrary statement.

1.4.4 Epimenides and Curry's with Membership

These truth-functional variants can both be mapped intuitively into set-theoretic paradoxes. For the Epimenides set we have $b = \{x: x \notin x \,\&\, Q\}$. For the Curry set we have $c = \{x: x \notin x \vee Q\}$. When we ask whether each is a member of itself we have the following arguments.

$b = \{x: x \notin x \,\&\, Q\}$	
$b \in \{x: x \notin x \,\&\, Q\}$ iff. $b \notin b \,\&\, Q$	Comprehension & Abstraction
$b \in b$ iff. $b \notin b \,\&\, Q$	Leibniz's law
$b \notin b \,\&\, {\sim}Q$	SL

And there is the parallel argument:

$c = \{x: x \notin x \vee Q\}$	
$c \in \{x: x \notin x \vee Q\}$ iff. $c \notin c \vee Q$	Comprehension & Abstraction
$c \in c$ iff. $c \notin c \vee Q$	Leibniz's law
$c \in c \,\&\, Q$	SL

1.4.5 Non-truth-functional Variations of these Paradoxes

Another reason to distinguish Curry's paradox is the way Curry's extends into paradoxes of entailment (and non-material conditionals). Curry's paradox can be extended for non-material conditionals which respect any of a number of small finite sets of logical principles [Meyer *et al.* 1979]. The principle of contraction, $A \rightarrow (A \rightarrow C) \vdash A \rightarrow C$, gets particular attention in the literature; but there are alternatives derivations. My present task is not choosing between axiom sets but the less fine-grained task of showing that there is an analogous version of the Epimenides.

Indeed, I have two tasks really. For Curry's paradox can be shown to have variations using non-truth-functional conditionals and variations using entailment. Once again, this need not be classical entailment, but any of a variety of entailment relations which respect a finite set of logical principles. The proofs for these can be conflated however by treating the arrow ambiguously as either a connective or representing an entailment relation (in which case it is also to be understood as taking the names of expressions on either side). These conventions are common place in the literature. I am simply using them for analogy, not endorsing the conflation of a conditional connective with an entailment relation.

First compare the following proofs of Curry's paradox and the Epimenides respectively.

Curry's:

1. $c = \langle Tc \rightarrow Q\rangle$	Premise
2. $\vert\ \underline{Tc}$	Ass
3. $\vert\ \ T\langle Tc \rightarrow Q\rangle$	Leibniz's law
4. $\vert\ \ Tc \rightarrow Q$	TE
5. $\vert\ \ Q$	2, 4 MP
6. $Tc \rightarrow Q$	2-5 CP
7. $T\langle Tc \rightarrow Q\rangle$	6 TI
8. Tc	7, 1, Leibniz's law
9. Q	6, 8, MP

Epimenides:

1. $b = \langle\sim(Q \to Tb)\rangle$	Premise
2. $\vert\ \underline{\sim Tb}$	Ass
3. $\vert\ \sim T\langle\sim(Q \to Tb)\rangle$	1, 3 Leibniz's law
4. $\vert\ \sim(Q \to Tb) \to T\langle\sim(Q \to Tb)\rangle$	T-Intro
5. $\vert\ \sim\sim(Q \to Tb)$	4, 3 MT
6. $\vert\ Q \to Tb$	5, DN
7. $\vert\ \sim Q$	6, 3 MT
8. $\sim Tb \to \sim Q$	2-7 CP
9. $Q \to Tb$	8 Contraposition (CP with MT)
10. $T\langle\sim(Q \to Tb)\rangle \to \sim(Q \to Tb)$	T-Elim
11. $\sim T\langle\sim(Q \to Tb)\rangle$	10, 9 MT
12. $\sim Tb$	11, 1 Leibniz's law
13. $\sim Q$	9, 12 MT; or 8, 12 MP

This proof of Curry's paradox relied on Conditional Proof, *modus ponens*, the T-schema and Leibniz's law. In parallel this proof of the Epimenides relies on Conditional Proof, *modus tollens*, the T-schema, Leibniz's law and double negation.

As remarked by Geach [1955, p. 72] the rule of absorption is implicit in the use of conditional proof. The duality between the proofs is perhaps even better presented by using *pseudo modus ponens*, and *pseudo modus tollens*.

Following Meyer *et al.* [1979, p127-8], we have this alternate proof of Curry's paradox:

1. $c = \langle Tc \to Q\rangle$	Premise
2. $T\langle Tc \to Q\rangle$ iff $(Tc \to Q)$	T-schema
3. Tc iff $(Tc \to Q)$	2, 1 Leibniz's law
4. $[(Tc \to Q)\ \&\ Tc] \to Q$	"*pseudo modus ponens*":
5. $(Tc\ \&\ Tc) \to Q$	4, 3 subs of equiv.
6. $Tc \to Q$	5 Idempotence of '&', subs of equiv.
7. Tc	3 (defn, &E), 6 MP
8. Q	6, 7 MP

By relatively straightforward analogy but making dual use of *modus tollens* and contraposition, which itself is provable by conditional proof with *modus tollens*:

1. $b = \langle\sim(Q \rightarrow Tb)\rangle$	Premise
2. $T\langle\sim(Q \rightarrow Tb)\rangle$ iff $\sim(Q \rightarrow Tb)$	T-schema
3. $(Q \rightarrow Tb)$ iff $\sim Tb$	2, 1 Leibniz's law, defn and contraposition
4. $[(Q \rightarrow Tb)\ \&\ \sim Tb] \rightarrow \sim Q$ "*pseudo modus tollens*":	
5. $(\sim Tb\ \&\ \sim Tb) \rightarrow \sim Q$	4, 3 subs of equiv.
6. $\sim Tb \rightarrow \sim Q$	5 Idempotence of '&'
7. $Q \rightarrow Tb$	6 Contraposition
8. $\sim Tb$	3 (defn, &E), 7 MP
7. $\sim Q$	7, 8 MT; or 6, 8 MP

Let me finish this section with a proof of the Epimenides using strict implication, which was, of course, implicit in the more general proofs above.

1. $b = \langle\sim\Box(Q \supset Tb)\rangle$	Premise	
2. | $\underline{\sim Tb}$	Ass	
3. | $\sim T\langle\sim\Box(Q \supset Tb)\rangle$	Leibniz's law	
4. | $T\langle\sim\Box(Q \supset Tb)\rangle$ iff $\sim\Box(Q \supset Tb)$		T-schema
5. | $\sim\sim\Box(Q \supset Tb)$	3, 4, Substitution of Equivalents	
6. | $\Box(Q \supset Tb)$	5, DN	
7. | $Q \supset Tb$	6, Necessitation	
8. | $\sim Q$	2, 7 MT	
9. $\sim Tb \supset \sim Q$	2-8 CP	
10. $Q \supset Tb$	9 Contraposition	
11. $\Box(Q \supset Tb)$	10 is a theorem and therefore necessary	
12. $\sim Tb$	11, DN, T-schema, defn, MT, Leibniz's law	
13. $\sim Q$	12, 10 MT; or 12, 9 MP	

1.5 Conclusion

The classification of the Epimenides paradox, as a mere version of the Liar or at best a contingent variant of the Liar, contrasts with the classification of Curry's paradox, which is considered to cut across the semantic and set-theoretic paradoxes in some way. The Epimenides is thought of as a lesser version of the Liar [Quine 1962, Haack 1978] or at best as a contingent semantic paradox [Gupta & Belnap,

p.6], while Curry's was from the outset thought to cut across the set-theoretic and semantic paradoxes [Curry 1942].

I hope to have demonstrated the relationship between the Epimenides and its unquantified variants. In any case, these unquantified variants bear direct analogy with Curry paradoxes, i.e., variants of Curry's paradox. Moreover, Curry's seems to relate to a paradox involving quantification, perhaps first identified by Geach.

On the one hand, if one thinks of the distinction between semantic and set-theoretic paradoxes as being a primary distinction, then one ought perhaps to think of the Epimenides and Curry paradoxes as sharing a particular characteristic. This characteristic can be used to produce distinct variations of either the semantic or set-theoretic paradoxes. On the other hand, if one thinks of the paradoxes in a uniform way, then one ought perhaps to think of the Epimenides and Curry paradoxes as kinds of paradox, with set-theoretic and semantic versions (instances or productions) or variants (types). Either way, some reclassification of the Epimenides paradox (among others) seems in order.