Conclusions

I hope to have advanced the classification of paradoxes (and hypodoxes) of truth, satisfaction and membership in six ways. Firstly, I have distinguished two types among them, the Liar-like and the Grelling-like. Secondly, I have coined the term hypodox in relation to an intuitive distinction between Liar sentences and Truth-tellers. I have characterised the nature of a hypodoxical sentence as being able to consistently take either semantic value but for which there is a (logical, not just epistemic) lack of any reason determining which a hypodox takes. I extended the usage of the term hypodox to relate to these types and families of paradoxes, and have claimed that it extends to others. Thirdly, I have analysed at least some further elements or aspects of these paradoxes that in combination identify variations, such as the Unquantified Epimenides, Self-referential Curry, and the ESP among the Liar-like paradoxes. Fourthly, I have related variations of the Liar-like paradoxes via collections or list structures. Fifthly, I have used this classification and naive intuitions about truth to compare with classifications entailed by the theories of truth given by Kripke, and Gupta and Belnap. Sixthly, I have identified some new paradoxes (and their hypodoxes).

Let me briefly summarize what each chapter has contributed to these results, and then add some comments about each of these results.

In the Introduction and Chapter 1, I demonstrated the need for a classification of these paradoxes. In the Introduction, I illustrated the types of lists that have been commonplace in the literature. In chapter 1, I demonstrated the need to have a common way of classifying versions or variations of the Epimenides and Curry’s paradoxes. On the one hand, in some of the literature the Epimenides is classified as a mere version of the Liar. On the other hand, the treatment of Curry’s paradox is more varied. Some literature refers to Curry’s paradox as a separate paradox, or refers to ‘Curry paradoxes’, some of which are treated at least as distinct variations of the Liar and Russell’s paradoxes, if not separate paradoxes. Thus, the need for an even-handed treatment of these paradoxes was demonstrated.

In Chapter 2, I discussed a number of matters concerning the inter-relationships between truth and paradox. I introduced the T-schema in relation to a naive theory of
truth, preferring the canonical form of it (associated with which, Leibniz’s law is then used explicitly in deriving Liar-like paradoxes). I introduced two methods of semantic evaluation, in general terms, by compositional means and by considering possible truth values. I showed how these related to the semantic evaluation of the Liar and the Truth-teller. Both are compositionally indeterminate, but when their possible truth values are considered, assuming bivalence, then the result for the Liar can be described as over-determined and the result for the Truth-teller can be described as under-determined. I used these descriptions as valuations, but not as additional values and not as the lack of a value. The naive position still holds bivalence as a tenet in classifying the paradoxes; I simulate this position as my thesis is to classify the paradoxes, not resolve them. When pushed for specifics on how the valuations are to be made, I fall back on a version of Kleene’s strong valuation scheme and supervaluation. These are to be applied in sequence. Both schemes are supplemented with valuation rules analogous to the T-schema. The compositional valuations are not to be thought of as having a third value or a gap, when a value cannot be determined compositionally, it is just compositionally indeterminate. For such a sentence, one then considers its possible truth values, and I use a version of supervaluation to carry this out.

In Chapter 3, I adopted a working definition of a family of paradoxes, which did not prejudice later decisions about distinct types of paradoxes, and progressed a detailed classification of the nuclear family of Liar paradoxes. I note here that there is some risk of using ‘Liar-like’ ambiguously. The Liar family in Chapter 3 were identified as paradoxes using just the T-schema. All these variations may be said to be Liar-like, and the Liar-like paradoxes may be extended to paradoxes of satisfaction, and even some set-theoretic paradoxes. It is just fortunate that the paradoxes that use just the T-schema (and not in combination with additional schemata, such as the T2T-schema) are all of the one type. The family of satisfaction paradoxes has members of both types of paradox, as Chapter 4 shows.

In Chapter 4, I analysed the rationalizations for the Peano-Ramsey distinction between semantic and set-theoretic paradoxes. As Priest [1994] has pointed out, it is of merely historical interest that the semantic paradoxes were not originally formal. The Grelling’s paradox rendered as a paradox of satisfaction using canonical names is a counter-example to the assertion (found, for example, in Mendelson [1979, p. 3]) that the set-theoretic paradoxes require fewer logical principles. The point that the paradoxes use different notions is a more interesting rationalization, more interesting than Priest [1994] gives it credit. However, prima facie this is a basis for the families
as I have given them with respect to the truth predicate, the satisfaction or truth
relation, and the membership relation. I have given reasons to discount the assumption
that paradoxes of the truth predicate and truth relation are all members of the one
family, in the sense of being a type. As discussed in chapter 4, there are *prima facie* at
least two types of paradox among the paradoxes of satisfaction. In chapter 5, when the
relationship between the truth predicate and the truth relation, which I have represented
by the T2T-schema, is used to map Grelling’s paradox to a paradox of truth, the result
is not the Liar paradox; and when conversely one so maps the Liar to a paradox using
the truth relation, the result is not Grelling’s paradox. Even if one defines the
paradoxes of truth and the truth relation as a family, one can still prove there are two
types within the family in these ways, and, as shown in chapter 4, these are types that
cut across this semantic family and the family of the paradoxes of membership.

So I have argued for a distinction between Liar-like and Grelling-like paradoxes
and hypodoxes. I have argued for it informally in Chapter 4 by appealing to intuitions
about differences and exemplifying how many paradoxes would conform to an
ordering based on this primary division (and then further classified in the way I
outlined in Chapter 3). I have then formally grounded the primary distinction between
the paradoxes in Chapter 5 based on differences in the logical principles necessary to
prove a paradox of one type or the other. I have not been able to formally rule out the
possibility of hybrids of the two types; although the examples I have found only make
use of extra logical principles redundantly. Nevertheless, the division between Liar-
like and Grelling-like paradoxes is I believe exhaustive across the domain of paradoxes
of truth, satisfaction and membership.

I add the following discussion with respect to my six results. Firstly, being a
formal distinction, the Liar-like and Grelling-like distinction cuts across Priest’s
[1994] common structure for the paradoxes, at least for those directly covered in my
thesis. This does not so much undermine Priest’s structure, which might apply at a
higher level. However, Priest’s requirement was that a difference in the treatment of
the paradoxes should have a formal basis; this he called ‘the Principle of Uniform
Solution’ – a common form for the paradoxes indicates a common solution. As I have
shown two formal types of paradoxes, the Principle of Uniform Solution might be used
to predict two solutions.

I have argued that the Liar-like and Grelling-like distinction is more fundamental
than the semantic and set-theoretic distinction. The Grelling’s paradox is no less
formal than Russell’s, and it uses no more logical machinery. One can accept that
distinct notions are involved in the paradoxes, and that is some basis for distinguishing so-called ‘families’. The paradoxes can be converted to reductions of the implications of the naive conceptions of those notions. As the naive notions are inconsistent, one might disavow Priest’s Principle of Uniform Solution; as the notions are inconsistent one needs other notions, perhaps predicative notions, perhaps the grounded conception of truth, or the iterative conception of set. However, as I mentioned, I have shown that there are two distinct types among the paradoxes of satisfaction. One parallels Russell’s paradox, the other the Liar. Furthermore, examples of the two types of paradox are found among the conundrums and paradoxes in naive set-theory. Most Liar-like paradoxes were ruled out by free-variable constraints on circularity; nevertheless, some by-passed this constraint and are Liar-like paradoxes in set-theory. Moreover, I have shown that when the relationship between the T-schema and the corresponding schema for the truth relation is used to map the Liar into a satisfaction paradox and Grelling’s into a truth paradox, the two do not marry up. The simplest explanation is that they are not the same type of paradox. As there are then two formal types of paradox to be distinguished in semantics and set-theory, the simplest explanation is that the formal distinction is the more primary.

Having reached that result, the import of the Peano-Ramsey distinction is brought into question. The Peano-Ramsey distinction was used to suggest different solutions. The new formal distinction suggests different solutions, but ones of a radical nature, which I merely list as candidate solutions. In the case of the Liar-like paradoxes, a failure of substitution of identicals (or the like) is not without precedence, although it is hard to conceive in a non-opaque context. Nevertheless, the reference involved is indeterminate in a sense; so, perhaps, there is some basis for thinking the results of such a substitution would be indeterminate. In the case of the Grelling-like paradoxes, failure of reflexivization seems almost inconceivable and unprecedented in any similar context. One therefore merely lists it as a candidate solution, and notes that it extends to Grelling-like paradoxes, such as Russell’s paradox.

Secondly, I have made and investigated a conjecture about the relationship between paradoxes and hypodoxes. Duality was not an option without naivety; it was only conceivable from the point of view that equates being false with being not true. While there are some issues for the conjecture, it seems to hold pretty well among subsets of the paradoxes, and it still seems plausible that there is a mapping that puts the Liar-like paradoxes in one-to-one correspondence with hypodoxes.

Thirdly and fourthly, the elements of my classification have been discussed as
they have come out during the course of my investigation; so, that a structure was not given to start with, but analysis of which factors or elements individuate paradoxes has been an ongoing theme. Combinations of these elements individuate categories of paradoxes. They do not provide for the sort of exhaustive division the two types do, and the classification is only complete for subsets. In summary, within the Liar-like paradoxes are a number of families, in particular those of the Liar itself relying on the truth predicate, and those of one branch of the satisfaction family, the Liar-like branch, and some members of the set-theoretic paradoxes. Within this type for each of the families, paradoxes are related as collections or list structures, among which I have focused on the infinite and circular. (Among these, I also constructed a new infinite variation.) The Liar-like variations using self-reference or Grelling-like variations using self-application are a limit case of the circular. Twisted loops could surely also be constructed by applying sufficient ingenuity. Further ways of complicating the use of lists are exemplified by Sorensen [1998], who has examples of infinite lists where not all members are really involved in the referential chain that is used towards paradox, but only every odd or third member. One hopes that these are mere versions that can be mapped back to basic variations in terms of infinite or circular lists. Nevertheless, my main point in introducing lists was to show they provide the relationship between the infinite and circular versions. Quantificational variations relate to collections, which are clearly unordered, and, for finite collections, these can be related to conjunctions or disjunctions. So, some of the paradoxes have list structures where others relate to collections.

Categories of paradox can be identified in relation to the following elements: a structure: a collection or list, a schema or schemata, and possible semantic valuations. In relation to the last of these elements, I identified a subset of twelve unquantified, self-referential paradoxes using the T-schema. For this subset, I originally used identity statements to individuate seven of particular interest; but it became apparent that sets of possible semantic valuations were a way of individuating these variations that gained some measure of completeness. It is still convenient to use the identity statements, provided it is realised that a number of identity statements would relate to the same set of possible semantic valuations.

I would not say that my classification is complete. Nevertheless, an unbounded number of Liar-like paradoxes and hypodoxes can be reduced to a finite classification. I hope this classification is robust, extensible and relatively independent of particular theories of truth. Compounds and hybrids remain possibilities. I have given particular
attention to truth-functional variations in the hope of demonstrating a complete subset. I have mentioned some non-truth-functional cases; but have not attempted to give a complete subset there.

At yet a further level of detail, I have briefly canvassed methods of reference and methods involving quantification, which might be thought of as involving plural reference.

Fifthly, the naïve classification of these paradoxes can also be used to compare with the classifications that follow from various theories. I used it to compare with Kripke’s [1975] theoretical results, and those of Gupta and Belnap’s [1993]. In the case of Kripke’s theory, there was agreement down to a certain level, but the theory was not sensitive to distinctions between a Self-referential Curry paradox or an Unquantified Epimenides paradox, all these are either paradoxical or they are not, relative to a base model, which is not varied. Kripke provides a clear distinction between paradoxes and Truth-tellers. Nevertheless, there is an open issue about how it is implemented using monotonic constructions in relation to the treatment of some paradoxical sentences. The issue was whether one could add some intuitively paradoxical sentences as though they were hypodoxes. As discussed in Chapter 3, the Liar cannot be added to the initial ground model, because of Liar-like reasoning in the semantics. However, it is not clear what prevents ‘T(Ta)’, where $a = \langle \neg Ta \rangle$, from being consistently added in a monotonic construction, nor is it clear what would prevent ‘T(Th)’, where $h = \langle Th \equiv Q \rangle$ being added even when $\neg Q$. In Chapter 3, I specified a monotonic construction that results in a consistent set of sentences that is closed under classical consequence. This set satisfies the definition of a fixed point, relative to this construction, but not every sentence in the fixed point satisfies the Substitution Thesis introduced in Chapter 2, that $A$ and $T(A)$ are inter-substitutable. It is quite likely I have missed a relevant feature in Kripke’s outline, but, as I say, this is a monotonic construction that results in a consistent fixed point. It contains a sentence that is intuitively paradoxical, and yet it appears it would be theoretically classified as hypodoxical in this way consistent with Kripke’s outline.

Gupta and Belnap’s [1993] theory is also relative to a fixed base model, and therefore also not sensitive to distinctions between a Self-referential Curry paradox or an Unquantified Epimenides paradox. Gupta and Belnap’s theory relies on reinterpreting the biconditional, so that the Substitution Thesis does not hold. They argue against the naïve Substitution Thesis in favour of a circular definition of truth, for which they provide a theory of circular definitions. Their reasoning is plausible.
Conclusions

Presumably corresponding modifications could be made to intuitions, like Principle T, behind the Equivalence Thesis. Nevertheless, they also motivate their theory as a classification, distinguishing at least between what they call ‘Intrinsic Liars’, like the Eubulidean version, and ‘Contingent Liars’, like the Epimenides. Given that they do not vary the base model it is not clear how they implement this. The Epimenides is either paradoxical or it is not. Variations in the initial guess do facilitate fine grained classifications in terms of compounds of Liars and Truth-tellers, but these are not contingent expressions, and their valuations under the theory, though difficult to compare with naïve intuitions, seem counter-intuitive. Gupta & Belnap’s theory would readily disconfirm the dual conjecture for Liar-like paradoxes and hypodoxes. I count this as a counter-intuitive aspect of their theory.

Finally, I hope some new paradoxes are of interest, three in particular. The Unquantified Non-truth-functional Epimenides, which demonstrates that Curry’s paradox is not alone in having non-truth-functional extensions. The ESP, which I think nicely complements the Liar, the Epimenides and the Self-referential Curry paradox. The Liar sentence itself is paradoxical whatever the circumstances, the Epimenides sentence is naively at least false and entails some sentence is true, Curry’s sentence is naively at least true and entails some arbitrary sentence, but the ESP may naively be true or false and still entails an arbitrary sentence. My Unsatisfied Paradox can be derived from my favourite predicate, which just happens to be ‘does not satisfy my favourite predicate’, and any object in the range of its applicability. It is Liar-like, but distinct from Grelling’s paradox.