Appendices

Appendix A: Extrapolating the Candidate Solution for Grelling's Paradox to Russell's Paradox

It is interesting to note that reflexivization is involved in Russell's paradox too. Were failure of reflexivization a solution for Grelling's, then such a solution should map to a solution to some other related paradoxes, in particular, Russell's paradox about the set of all sets that are not members of themselves.

Naively, members of sets satisfy some membership condition. Each member of Russell's set satisfies 'x is not a member of itself'. So,

Russell's set satisfies 'x is not a member of itself' iff Russell's set is not a member of itself.

Once again, the naive abstraction schema says: For any property, propositional function or predicate, P, there is a set, \{x: Px\}, of just those things that satisfy P. The schema and the proof of paradox can be more formally represented as follows:

1. \(y \in \{x: Px\}\) iff Py
2. \(y \in \{x: x \notin x\}\) iff \(y \notin y\)
3. \(\{x: x \notin x\} \in \{x: x \notin x\}\) iff \(\{x: x \notin x\} \notin \{x: x \notin x\}\)

On analogy with the proposed solution to Grelling's, in instantiating the comprehension schema for Russell's set, the use of membership in the schema is a dyadic relation, while the use of membership, when instantiating the variable for a predicate, is a monadic predicate. Again, I think this can be formally maintained and used to initially block the contradiction. Once again, however, reflexivization quickly regains the paradox. There is nothing ad hoc so far. I think at least this much is right.

If reflexivization were restricted in some way, perhaps similar to goundedness, this derivation of Russell's paradox could be avoided by invalidating the instance of reflexivization in the fourth line below. (The proof below uses notation in a non-standard, but intelligible way, for presentation purposes).
1. \( y \in^2 \{x: Px\} \iff Py \) \quad Abstraction schema (the \( \in^2 \)-schema)

2. \( y \in^2 \{x: x \notin^1 x\} \iff y \notin^1 y \quad \{x: x \notin^1 x\} / \{x: Px\} \)

3. \( \{x: x \notin^1 x\} \in^2 \{x: x \notin^1 x\} \iff \{x: x \notin^1 x\} \notin^1 \{x: x \notin^1 x\} \)

Universal instantiation

4. \( \{x: x \notin^1 x\} \notin^1 \{x: x \notin^1 x\} \iff \{x: x \notin^1 x\} \in^2 \{x: x \notin^1 x\} \)

Reflexivization

5. \( \{x: x \notin^1 x\} \notin^1 \{x: x \notin^1 x\} \iff \{x: x \notin^1 x\} \notin^1 \{x: x \notin^1 x\} \)

3, 4 SL.

If we reserve talk of 'self-membership' for the membership predicate, the third line can be read in some slightly stilted way as 'the set of all things that are not self-membered is a member of the set of all things that are not self-membered if, and only if, the set of all things that are not self-membered is not self-membered'. From which it follows, given reflexivization, that the set of all things that are not self-membered is self-membered iff it is not self-membered. Invalidating the fourth line, an instance of reflexivization, will not guarantee there is no other derivation of paradox, it merely invalidates this derivation. It is highly \textit{ad hoc}: it is, at best, a solution looking for a motivation. It is possible to motivate it, I believe.\(^1\) I think it bears some intuitive relation to the indeterminacy of these cases. And, I think it can be argued that the indeterminacy is in some sense \textit{prior} to paradox, that is, I think we can prove the indeterminacy of this case without deriving the paradox. However, for present purposes, I have simply mapped out the analogy with the candidate solution to Grelling's paradox.

Appendix B: A Further Note on Diagonalization and Paradox

Quine [1995, pp. 7-8] related the Liar and Grelling's paradoxes as paradoxes of truth; however, Simmons [1993] and Jacquette [2004] relate them as members of a broader grouping involving diagonalization. Nevertheless, I have challenged the generality of diagonalization because not all derivations of the Liar exhibit it, at least as self-intra-substitution. In this section, I add a brief comment to distinguish the way

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\(^1\) For example, impredicativity might be relaxed to validate line 3, but applied instead to restrict the general validity of reflexivization, and in particular, the validity of line 4. However, impredicativity itself would then need some motivation other than the \textit{Vicious Circle Principle}.
diagonalization becomes involved in the Liar, when it is, as compared with Grelling’s paradox.

In the derivations of the Liar I presented, it is the premise or the theorem which is the vehicle for entry of diagonalization into the derivation of the Liar. These take a term referring to a sentence and map it to a term referring to the negation of the truth of that sentence. Diagonalization does not enter into either derivation through the instance of the canonical T-schema. In contrast, diagonalization enters into my derivation of Grelling’s paradox through instantiating the $T^2$-schema — or, so it first appears — given what I have argued, however, reflexivization appears also to be necessary to achieve diagonalization.

While I have presented the Liar with a separate identity premise (and used a canonical T-schema that works just with canonical names), I have also noted that the general T-schema could have been applied relying perhaps only on a name for the sentence in the semantics, or even on the basis that the identity is a theorem. We may then have a shorter derivation of the Liar, where diagonalization seems to come in with the instance of the T-schema, as it seemed to come in with the instance of the $T^2$-schema for Grelling’s. However, the point remains that the identity can be separated out so that diagonalization would apply to it and not the instance of the T-schema. For the Grelling though, diagonalization emerges in the derivation through the instance of whichever T-schemata is used (in combination with reflexivization).