

# Real Time Characterisation of the Mobile Multipath Channel

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The subfigures in the pdf version of this document sometimes do not quite match in size. But the file is a *lot* smaller than the postscript file. ☺

# Declaration

The contents of this thesis are the results of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work in this thesis has been published or has been submitted for publication as journal papers or conference proceedings. These papers are:

1. P. D. Teal, R. Raich, and R. G. Vaughan. Prediction of fading in the mobile multipath environment. *IEE Proceedings — Communications*, 2000. Submitted.
2. P. D. Teal, R. C. Williamson, and R. A. Kennedy. Error performance of a channel of known impulse response. In *Proc. IEEE Conference on Acoustics, Speech and Signal Processing*, volume 5, pages 2733-2736, Istanbul, June 2000.
3. R. Vaughan, P. Teal, and R. Raich. Prediction of fading signals in a multipath environment. In *Proc. IEEE Vehicular Technology Conference*, volume 1, pages 751-758, Fall, Boston, Sep 2000.
4. O. Nørklit, P. D. Teal, and R. G. Vaughan. Measurement and evaluation of multi-antenna handsets in indoor mobile communication. *IEEE Trans. Antennas & Propagation*, 49(3):429-437, Mar 2001.
5. P. D. Teal and R. G. Vaughan. Simulation and performance bounds for real-time prediction of the mobile multipath channel. In *Proc. IEEE Workshop on Statistical Signal Processing*, pages 548-551, Singapore, Aug 2001.
6. P. D. Teal, T. A. Abhayapala, and R. A. Kennedy. Spatial correlation for general distributions of scatterers. *IEEE Signal Processing Letters*, 2001. (to appear).
7. P. D. Teal, T. A. Abhayapala, and R. A. Kennedy. Spatial correlation in non-isotropic scattering scenarios. In *Proc. IEEE Conference on Acoustics, Speech and Signal Processing*, Orlando, Florida, 2002. (to appear).
8. P. D. Teal. Rough surface scattering and prediction of the mobile channel. *IEEE Communications Letters*, 2001. To be submitted.

The following informal presentations were also published.

1. P. Teal. Error performance of a channel of known impulse response. In *2nd Australian Communications Theory Workshop*, Adelaide, Feb 2001.  
<http://www.itr.unisa.edu.au/~alex/AusCTW2001/talks/teal1.html>.
2. P. Teal. Performance bounds for real time prediction of the mobile multipath channel. In *2nd Australian Communications Theory Workshop*, Adelaide, Feb 2001.  
<http://www.itr.unisa.edu.au/~alex/AusCTW2001/talks/teal2.html>.

The research represented in this thesis has been performed jointly with Professor Robert C. Williamson, Professor Rodney A. Kennedy, and Dr Rodney G. Vaughan. The substantial majority of this work is my own.

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Without any pretensions with regard to the relative importance of my work and Newton's I am grateful to God for the opportunity to explore his world, and to submit this contribution that "we may now more nearly behold the beauties of Nature, and entertain ourselves with the delightful contemplation; and which is the best and most valuable fruit of philosophy, be thence incited the more profoundly to reverence and adore the great Maker and Lord of all." [?]



# Abstract

In this thesis a new approach for characterisation of digital mobile radio channels is investigated. The new approach is based on recognition of the fact that while the fading which is characteristic of the mobile radio channel is very rapid, the processes underlying this fading may vary much more slowly. The comparative stability of these underlying processes has not been exploited in system designs to date.

Channel models are proposed which take account of the stability of the channel. Estimators for the parameters of the models are proposed, and their performance is analysed theoretically and by simulation and measurement.

Bounds are derived for the extent to which the mobile channel can be predicted, and the critical factors which define these bounds are identified.

Two main applications arise for these channel models. The first is the possibility of prediction of the overall system performance. This may be used to avoid channel fading (for instance by change of frequency), or compensate for it (by change of the signal rate or by power control). The second application is in channel equalisation. An equaliser based on a model which has parameters varying only very slowly can offer improved performance especially in the case of channels which appear to be varying so rapidly that the convergence rate of an equaliser based on the conventional model is not adequate.

The first of these applications is explored, and a relationship is derived between the channel impulse response and the performance of a broadband system.





# Notation and Symbols

$\cdot^*$	Complex conjugate
$\ \cdot\ $	Frobenius norm of a matrix or 2-norm of a vector
$\cdot^+$	Moore-Penrose generalised inverse
$\odot$	Element by element (Hadamard) product
$\otimes$	Convolution
$\equiv$	Equivalence
$\triangleq$	Definition
$\mathbf{x} \cdot \mathbf{y}$	Dot product between two spatial vectors $\mathbf{x}$ and $\mathbf{y}$
$\mathbf{A}$	Array steering matrix
$\mathbf{a}$	Array steering vector
$b(\cdot)$	Scattering function
$c$	Speed of light
$\mathcal{CN}(\cdot, \cdot)$	Complex normally distributed with given mean and covariance
$D, d$	Distances between source points and measurement points
$\mathbf{D}$	Diagonal matrix of eigenvalues
$E\{\cdot\}$	Expectation operator
$e^{\cdot}$	Exponential operator
$\mathbf{e}_i$	A vector of zeros with 1 in the $i$ -th position
$f$	Discrete channel impulse response co-efficients
$g(\cdot)$	Antenna gain pattern
$\cdot^H$	Complex Hermitian conjugate
$H_n^{(1)}(\cdot)$	Order $n$ Hankel function of the first kind
$h$	System or channel impulse response
$I_n(\cdot)$	Order $n$ Modified Bessel function of the first kind
$\mathbf{I}$	Identity matrix
$J_n(\cdot)$	Order $n$ Bessel function of the first kind
$j$	$\sqrt{-1}$
$j_n(\cdot)$	Order $n$ Spherical Bessel function
$\mathbf{J}$	Fisher Information matrix
$\mathbf{K}$	Row/Column reversing matrix
$k$	Wave number of signal carrier = $2\pi/\lambda$

$l$	Snapshot index
$L$	The number of terms of a finite impulse response channel model or the number of “snapshots” taken by an array
$M$	The number of sensors in an array
$m$	Array sensor index
$N$	The number of sources or scatterers in the environment of an array
$n$	Source or scatterer index
$\mathcal{N}(\cdot, \cdot)$	Normally distributed with given mean and covariance
$P$	dimension of a covariance or correlation matrix
$P_n^m(\cdot)$	Associated Legendre functions
$p(t)$	Transmit pulse shaping function
$s(t)$ $s[m]$	Transmitted signal or transmitted symbol
$T$	System symbol period
$\cdot^T$	Matrix or vector transpose
$t$	Time
$U(\cdot)$	Unit step function (Heaviside’s unit function)
$u(\cdot)$	Baseband transmitted signal
$\mathbf{v}$	an eigenvector
$\mathbf{V}$	Matrix of eigenvectors
$W$	Bandwidth of a baseband transmitted signal
$x, y, z$	3 dimensional spatial co-ordinates
$\delta(t)$	Dirac delta
$\delta_i, \delta_{i,j}, \delta[i]$	Kronecker delta
$\epsilon$	Error in decoded symbol
$\zeta$	Gain (attenuation) of a path between transmitter and receiver
$\eta$	Additive Noise process
$\theta$	Arrival angle of a signal (measured from broadside)
$\lambda$	Wavelength of signal carrier
$\sum$	Summation operator
$\tau$	Path delay
$\omega_c$	Carrier frequency
$\varpi_n$	Spatial frequency corresponding to path $n$
$\omega$	Baseband channel frequency
cdf	cumulative distribution function
pdf	probability density function
SNR	Signal to Noise Ratio

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