On Limits of Multi-Antenna Wireless Communications in Spatially Selective Channels

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Declaration

The contents of this thesis are the results of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work in this thesis has been published or has been submitted for publication as journal papers or conference proceedings. These papers are:

1. T.S. Pollock, T.D. Abhayapala, and R.A. Kennedy, "Fundamental limits of constrained array capacity," in *Australian Communications Theory Workshop*, Melbourne, Australia, 2003, pp. 7–12.
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The research represented in this thesis has been performed jointly with Professor Rodney A. Kennedy and Dr Thushara D. Abhayapala. The substantial majority of this work is my own.

Tony Steven Pollock
The Australian National University
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To
Kirstie

by all appearances, I am one person, but in reality I am two

Acknowledgements

> *The real voyage of discovery consists not in seeing new land*
> *but in seeing with new eyes* - Marcel Proust

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Abstract

Multiple-Input Multiple-Output (MIMO) communications systems using multi-antenna arrays simultaneously during transmission and reception have generated significant interest in recent years. Theoretical work in the mid 1990's showed the potential for significant capacity increases in wireless channels via spatial multiplexing with sparse antenna arrays and rich scattering environments. However, in reality the capacity is significantly reduced when the antennas are placed close together, or the scattering environment is sparse, causing the signals received by different antennas to become correlated, corresponding to a reduction of the effective number of sub-channels between transmit and receive antennas.

By introducing the previously ignored spatial aspects, namely the antenna array geometry and the scattering environment, into a novel channel model new bounds and fundamental limitations to MIMO capacity are derived for spatially constrained, or spatially selective, channels. A theoretically derived capacity saturation point is shown to exist for spatially selective MIMO channels, at which there is no capacity growth with increasing numbers of antennas. Furthermore, it is shown that this saturation point is dependent on the shape, size and orientation of the spatial volumes containing the antenna arrays along with the properties of the scattering environment.

This result leads to the definition of an intrinsic capacity between separate spatial volumes in a continuous scattering environment, which is an upper limit to communication between the volumes that can not be increased with increasing numbers of antennas within. It is shown that there exists a fundamental limit to the information theoretic capacity between two continuous volumes in space, where using antenna arrays is simply one choice of implementation of a more general spatial signal processing underlying all wireless communication systems.

Notation and Symbols

AWGN	additive white Gaussian noise
BER	bit error rate
CDF	cumulative distribution function
CSI	channel state information
UCA	uniform circular array
UGA	uniform grid array
ULA	uniform linear array
MISO	multiple-input single-ouput
MIMO	multiple-input multiple-output
SISO	single-input single-output
SIMO	single-input multiple-output
SNR	signal-to-noise ratio
SDOF	spacial degrees of freedom
$\lceil \cdot \rceil$	ceiling operator
$\lfloor \cdot \rfloor$	floor operator
$\overline{f(\cdot)}$	complex conjugate of scalar or function f
$\boldsymbol{A}^{\dagger}$	complex conjugate transpose of matrix or vector $\boldsymbol{A}$
$\lvert \boldsymbol{A} \rvert$	determinant of matrix $\boldsymbol{A}$
$\lVert \boldsymbol{a} \rVert$	euclidian norm of vector $\boldsymbol{a}$
$E_X \{\cdot\}$	Expectation operator over random process X
$\delta(i-j)$	Kronecker delta
$\langle \cdot, \cdot \rangle$	inner product
$\boldsymbol{a}'$	transpose of matrix or vector $\boldsymbol{a}$
η	signal-to-noise ratio (SNR)
$\mathbb{S}^1$	1 sphere (unit circle)
$\mathbb{S}^2$	2 sphere (unit sphere)
$\boldsymbol{I}_n$	$n \times n$ identity matrix
$\mathbf{1}_n$	$n \times n$ matrix of ones

Contents

Declaration iii

Acknowledgements vi

Abstract viii

Notation and Symbols x

List of Figures xxii

1 Introduction 1

- 1.1 Motivation and Background 1
- 1.2 Wireless Communication Channels 3
  - 1.2.1 Diversity 5
- 1.3 Fundamental Limits to Wireless Communication Systems 9
  - 1.3.1 MIMO Fading Channel Model 9
  - 1.3.2 Channel Capacity 11
  - 1.3.3 Single-Input Single-Output (SISO) System 12
  - 1.3.4 Spatial Diversity Systems 13
  - 1.3.5 Multiple-Input Multiple-Output (MIMO) System 15
- 1.4 Capacity of MIMO systems 16
  - 1.4.1 Channel Capacity 16
  - 1.4.2 Channel Unknown at Transmitter 18
  - 1.4.3 Channel Known at Transmitter 18
  - 1.4.4 Partial Channel Knowledge 20
  - 1.4.5 Achieving Capacity: Space-Time Codes 21
- 1.5 Structure of this Thesis 22
  - 1.5.1 Questions to be Answered in this Thesis 22
  - 1.5.2 Content and Contribution of Thesis 23

2 Introducing Space into MIMO Capacity Calculations 27
2.1 Convergence of Ergodic Capacity 28
2.1.1 Capacity Scaling Limits 31
2.2 Receiver Spatial Correlation for General Distributions of Farfield Scatterers 31
2.2.1 Channel Model 31
2.2.2 Correlation of the Received Complex Envelopes 33
2.2.3 Two Dimensional Scattering Environment 36
2.2.4 Non-isotropic Scattering Environments 38
2.3 Capacity Results 42
2.4 Summary and Contributions 47

3 Saturation Effects of Spatially Constrained MIMO Channels 51
3.1 Eigen-analysis of MIMO Capacity 52
3.2 Uniform Circular Array 54
3.2.1 Eigenvalues of Spatial Correlation Matrix $\boldsymbol{R}$ 54
3.2.2 Capacity Scaling Limits 57
3.3 Arbitrary Arrays in General Scattering Environments 60
3.3.1 Spatial Correlation Matrix Decomposition 61
3.3.2 Capacity Limits: Constrained Aperture 65
3.3.3 Capacity Limits: Limited Angular Spread 68
3.3.4 Fixed Received Power 72
3.3.5 Constrained 3D Apertures 75
3.4 Summary and Contributions 77

4 Spatial Characterization of MIMO Channels 79
4.1 Modal Truncation of Plane Waves 81
4.1.1 Plane Waves 81
4.1.2 2D Plane Wave Propagation 82
4.1.3 3D Plane Wave Propagation 85
4.2 2D Channel Model 89
4.2.1 Channel Matrix Modal Decomposition 92
4.3 3D Channel Model 94
4.3.1 Channel Matrix Modal Decomposition 97
4.4 Comments on the Channel Model 99
4.4.1 Spatial Degrees of Freedom (SDOF) 100
4.5 Summary and Contributions 101

5 Capacity of Spatially Selective Channels 103
5.1 MIMO Model and Channel Rank 103
5.2 Capacity - Aperture Effects 106
5.2.1 Antenna Saturation 106
5.2.2 Aperture Size 109
5.3 Capacity - Scattering effects 110
5.3.1 Discrete Channel Representation 110
5.3.2 Angular Spread 115
5.4 Summary and Contributions 119

6 Intrinsic Capacity of Continuous Space Channels 123
6.1 Mode-to-Mode Communication 123
6.1.1 Mode Excitation 125
6.1.2 Properties and Statistics of Scattering Channel Matrix $\boldsymbol{H}_{\mathrm{S}}$. 128
6.1.3 Modal Correlation in General Scattering Environments . . . 131
6.2 Sampling Effects on Capacity 133
6.3 Communication Between Arbitrarily Shaped Apertures 139
6.4 Spatial Information and Communication 144
6.4.1 Dimensionality of Spatial Apertures 147
6.4.2 Communication Strengths Between Apertures 149
6.5 Summary and Contributions 152

7 Conclusions and Future Research 155
7.1 Conclusions 155
7.2 Future Directions of Research 156

References 159

List of Figures

1.1 Multipath Scattering Environment. (reprinted with permission from Dino Miniutti ©2002.) 4

1.2 Example of a Rayleigh fading channel. (a) Signal power as a function of time for a single receive antenna. (b) Signal power as a function of time for two receive antenna with maximum ratio diversity combining 5

1.3 Example of spatial fading. Signal power over a $3\lambda \times 3\lambda$ region in a multipath scattering environment. 6

1.4 A MIMO wireless transmission system with n_{T} transmit antennas and n_{R} receive antennas. The transmit and receive signal processing (S/P) includes coding, modulation, mapping, etc. and may be realized jointly or separately. 10

1.5 Cumulative Distribution Function (CDF) of the channel capacity for different numbers of transmit and receive antennas for an i.i.d. Rayleigh fading environment with SNR of 10dB. For each curve, the values at the top and bottom of the vertical scale gives an indication of the ergodic and outage capacities respectively. 13

1.6 Ergodic channel capacity with increasing SNR for different numbers of transmit and receive antennas for an i.i.d. Rayleigh fading environment. 14

1.7 Illustration of parallel eigen-channels of a MIMO system for the singular value decomposition $\boldsymbol{H} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^{\dagger}$. The width of the line indicates the different eigen-channel power gains λ_n. 19

2.1 Convergence error of ergodic capacity C_{erg} (2.1) to bound C (2.5) with increasing number of transmit antennas for various numbers of receive antennas and SNR 10dB. 30

2.2 Scattering model for a flat fading MIMO system. $g_t(\widehat{\boldsymbol{\psi}})$ represents the effective random complex gain of the scatterers for transmitted signal x_t arriving at the receiver array from direction $\widehat{\boldsymbol{\psi}}$ via any number paths through the scattering environment. The sphere surrounding the receive antennas contains no scatterers and is assumed large enough that any scatterers are farfield to all receive antennas located within. 34

2.3 Capacity of 2D and 3D isotropic scattering environments for fixed length aperture (1λ) ULA and UCA for increasing number of receive antennas. Insert: Spatial correlation between two antennas against spatial separation for the 2D and 3D isotropic scattering environments. 37

2.4 Multipath signal energy modelled as a non-isotropic scattering distribution $\mathcal{P}(\psi)$ with mean AOA ψ_0 and angular spread σ (defined as the standard deviation of the distribution). 38

2.5 Comparison of common scattering distributions: Uniform, Gaussian, von-Mises and Laplacian, for angular spread $\sigma = \{20°, 30°, 60°\}$. . . 41

2.6 Spatial correlation between two antennas for mean AOA 90° (broadside) against spatial separation for Uniform, Gaussian, von-Mises, and Laplacian scattering distributions and angular spreads $\sigma = \{1°, 5°, 20°\}$. 43

2.7 Spatial correlation between two antennas on the x-axis for mean AOA 30° (60° from broadside) against spatial separation for Uniform, Gaussian, von-Mises, and Laplacian scattering distributions and angular spreads $\sigma = \{1°, 5°, 20°\}$. 43

2.8 Capacity for non-isotropic distributed scattering with mean AOA $\psi_0 = \{0°, 45°, 90°\}$ and increasing nonisotropy factor, for the 8 antenna ULA and UCA of aperture width (length/diameter) 3.5λ. . . 44

2.9 Capacity scaling of the ULA and UCA with fixed aperture (length/diameter) $D = \{0.4\lambda, 0.6\lambda, 0.8\lambda\}$ in an isotropic scattering environment. . . . 45

2.10 Capacity scaling of the broadside uniform linear array with fixed aperture 4λ for angular spread $\sigma = \{1°, 5°, 20°\}$ of the various scattering distributions. 46

2.11 Capacity scaling of the UCA with fixed aperture 4λ for angular spread $\sigma = \{1°, 5°, 20°\}$ of the various scattering distributions. . . . 46

2.12 Capacity loss due to correlation of the broadside ULA for fixed aperture $D = \{0.5\lambda, 1.5\lambda, 2.5\lambda, 3.5\lambda, 4.5\lambda\}$ in an isotropic scattering environment. 48
2.13 Capacity loss due to correlation of the broadside ULA of fixed aperture 4λ for angular spreads $\sigma = \{1^\circ, 5^\circ, 10^\circ, 20^\circ\}$ of the various scattering distributions. 48

3.1 Example of a UCA of radius r_0, where d_ℓ denotes the distance between any antenna and its ℓ-th neighbor in a clockwise or anticlockwise direction. 53
3.2 High pass nature of the Bessel functions $J_n(z)$, for $n = \{5, 50, 500\}$ versus argument z in logarithmic scale. 56
3.3 The eigenvalues of the spatial correlation matrix for various UCA radii in a 2D isotropic diffuse scattering field. The dark solid line represents the theoretical eigenvalue threshold, and clearly shows the boundary between the significant and vanishing eigenvalues of the spatial correlation matrix for each array radius. 58
3.4 Capacity of MIMO systems for various antenna numbers of a UCA with radii $r_0 = \{0.1\lambda, 0.3\lambda, 0.5\lambda, 0.7\lambda\}$ in a 2D isotropic diffuse scattering field, along with the theoretical maximum capacity. As indicated by the dashed lines for each radii, the theoretical antenna saturation point gives a good indication where the MIMO system saturates and hence increasing antenna numbers gives only marginal capacity gain. 60
3.5 Two dimensional scattering model for a flat fading MIMO system. $g_t(\psi)$ represents the effective random complex gain of the scatterers for a transmitted signal x_t arriving at the receiver array from direction ψ via any number paths through the scattering environment. The receiver aperture of radius r_0 contains all receiver antennas, and is contained within a scatterer free region whose radius r_S is assumed large enough such that any scatterers are farfield to to all receive antennas. 62
3.6 Theoretical maximum capacity $C_\mathrm{max}(n_\mathrm{R}, r_0)$ for apertures of radius $r_0 = \{0.1\lambda, 0.7\lambda\}$ in an isotropic scattering environment for an increasing number of antennas. Vertical dashed lines indicate the theoretical antenna saturation point for each aperture size. Shown also is the capacity of the ULA and UCA within the same sized apertures. 68

3.7 The eigenvalues of the modal correlation matrix for various angular spreads. The dark solid line represents the estimated eigenvalue threshold, and clearly shows the boundary between the significant and vanishing eigenvalues of the modal correlation matrix for each angular spread. 70

3.8 Theoretical maximum capacity $C_{\max}(n_{\mathrm{R}}, \Delta)$ for aperture of fixed radius $R = 2.5\lambda$ for an increasing number of antennas. Vertical dashed lines indicate the estimated antenna saturation point for each angular spread. Shown also is the capacity of the broadside ULA and UCA within the same size aperture. 71

3.9 Theoretical maximum capacity of unconstrained aperture MIMO systems for various angular spreads $\Delta = \{5^\circ, 20^\circ, 45^\circ, 90^\circ\}$, along with the theoretical maximum capacity $C_{\max}$ corresponding to $\Delta = 180^\circ$. 71

3.10 Theoretical maximum normalized capacity $C_{\max}$ of an aperture of radius $r_0 = 1\lambda$ for angular spread $\Delta = \{20^\circ, 180^\circ\}$ with increasing number of antennas. Vertical dashed lines indicate the theoretical antenna saturation point for each angular spread. Shown also is the normalized capacity of the ULA and UCA within the same sized apertures. 75

4.1 Absolute truncation error $\epsilon_N(\boldsymbol{x})$ (4.5) for increasing number of terms $N+1$ of the 2D plane wave expansion (4.4). Dashed vertical lines indicate the number of terms given by critical value $\mathcal{N}(\boldsymbol{x})+1$ for each $\boldsymbol{x}$. 85

4.2 Absolute truncation error $\epsilon_N(\boldsymbol{x})$ (4.22) for increasing number of terms $N+1$ of the 3D plane wave expansion (4.21). Dashed vertical lines indicate the number of terms given by critical value $\mathcal{N}(\boldsymbol{x})+1$ for each $\boldsymbol{x}$. 88

4.3 Scattering model for a 2D flat fading narrowband MIMO system. r_{T} and r_{R} are the radii of circular apertures which contain the transmit and receive antenna arrays, respectively. The radii r_{TS} and r_{RS} describe scatterer free circular regions surrounding the transmit and receive apertures, assumed large enough that any scatterer is farfield to all antennas. The scattering environment is described by $g(\phi, \psi)$ which gives the effective random complex gain for signals departing the transmit aperture from angle ϕ and arriving at the receive aperture from angle ψ, via any number of scattering paths. 90

4.4 Scattering model for a 3D flat fading narrowband MIMO system. r_{T} and r_{R} are the radii of spherical apertures which contain the transmit and receive antenna arrays, respectively. The radii r_{TS} and r_{RS} describe scatterer free spherical regions surrounding the transmit and receive apertures, assumed large enough that any scatterer is farfield to all antennas. The scattering environment is described by $g(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\psi}})$ which gives the effective random complex gain for signals departing the transmit aperture from direction $\widehat{\boldsymbol{\phi}}$ and arriving at the receive aperture from direction $\widehat{\boldsymbol{\psi}}$, via any number of scattering paths. 96

5.1 Spatial model interpretation. Dark grey circles represent apertures and light grey represents scattering: (a) full rank, (b) loss in aperture rank, (c)–(e) loss in scattering rank 105

5.2 Ergodic capacity with increasing SNR for various channel scenarios for 6 antenna UCAs. 107

5.3 CDF of channel capacity for various channel scenarios for 6 antenna UCAs with SNR 10dB. 107

5.4 Antenna saturation of capacity for the ULA and UCAs constrained within transmit and receiver apertures of radius r_{T} and r_{R}, respectively. The scattering environment is modelled as isotropic and the received SNR is 10dB. Also shown is the unconstrained aperture capacity corresponding to i.i.d. channel gains. 108

5.5 Capacity growth with increasing aperture size for 6 antenna ULA and UCAs in isotropic scattering and SNR 10dB. 109

5.6 PDF's of the ordered singular values of $\boldsymbol{H}$ for $n_{\mathrm{T}} = n_{\mathrm{R}} = 6$ antenna ULAs within fixed radius apertures within isotropic scattering. μ_k represents the k-th largest singular value represented in dB. (a) $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.01\lambda$. (b) $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$. (c) i.i.d. channel, $r_{\mathrm{T}} = r_{\mathrm{R}} = \infty$. . . 111

5.7 PDF's of the ordered singular values of $\boldsymbol{H}$ for $n_{\mathrm{T}} = n_{\mathrm{R}} = 6$ antenna UCAs within fixed radius apertures within isotropic scattering. μ_k represents the k-th largest singular value represented in dB. (a) $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.01\lambda$. (b) $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$. (c) i.i.d. channel, $r_{\mathrm{T}} = r_{\mathrm{R}} = \infty$. . . 112

5.8 Mean of the ordered singular values of $\boldsymbol{H}$ for $n_{\mathrm{T}} = n_{\mathrm{R}} = 6$ antenna ULAs for increasing radius apertures within isotropic scattering. . . 113

5.9 Mean of the ordered singular values of $\boldsymbol{H}$ for $n_{\mathrm{T}} = n_{\mathrm{R}} = 6$ antenna UCAs for increasing radius apertures within isotropic scattering. . . 113

5.10 Capacity of 6 antenna ULA and UCA within apertures of radius $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$ for increasing number of multipaths n_{S}, with SNR 10dB. 116

5.11 Mean ordered singular values of scattering environment matrix $\boldsymbol{H}_{\mathrm{S}}$ for increasing number of multipaths n_{S}, for apertures $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$. 116

5.12 Ergodic capacity with increasing SNR for scattering scenarios for 6 antenna UCAs within apertures of radius $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$. 120

5.13 CDF of channel capacity for various scattering scenarios for 6 antenna UCAs within apertures of radius $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$ with SNR 10dB. 120

5.14 Capacity of 6 antenna UCAs in apertures $r_{\mathrm{T}} = r_{\mathrm{R}} = 0.5\lambda$ for increasing transmitter angular spread Δ_{T} for various receiver angular spread Δ_{R}, and SNR 10dB. Scattering is modelled by uniform limited field and full rank $\boldsymbol{H}_0$. 121

5.15 CDF of channel capacity for various angular spreading at the transmitter Δ_{T} and receiver Δ_{R} for the same scenario as Fig. 5.14 with rank one $\boldsymbol{H}_0$. 121

6.1 CDF of capacity for mode-to-mode communication for circular apertures of increasing radius $r = r_{\mathrm{T}} = r_{\mathrm{R}}$ with SNR 10dB. 126

6.2 CDF of capacity for mode-to-mode communication for spherical apertures of increasing radius $r = r_{\mathrm{T}} = r_{\mathrm{R}}$ with SNR 10dB. 126

6.3 Ergodic capacity for mode-to-mode communication for circular apertures of various radii $r = r_{\mathrm{T}} = r_{\mathrm{R}}$ with increasing spatial richness κ_S and SNR 10dB. 127

6.4 Ergodic capacity for mode-to-mode communication for circular apertures of various radii $r = r_\mathrm{T} = r_\mathrm{R}$ with increasing spatial richness κ_S and SNR 10dB. 127

6.5 Radiation patterns of the first six modes of a circular aperture, $\Re\{e^{in\phi}\}^2$. 129

6.6 Radiation patterns of the first six modes of a spherical aperture $\Re\{Y_n^m(\widehat{\boldsymbol{\phi}})\}^2$. 130

6.7 Modal correlation versus angular spread Δ of a uniform limited power density surrounding the aperture. 132

6.8 Capacity versus angular spread at the transmitter for mode-to-mode communication (modes), uniform linear array (ULA), uniform circular array (UCA), and uniform grid array (UGA), within spatial regions of radius 0.8λ and isotropic receiver scattering. 134

6.9 Capacity versus mean angle of departure for 20° spread at the transmitter for mode-to-mode communication (modes), uniform linear array (ULA), uniform circular array (UCA), and uniform grid array (UGA), within spatial regions of radius 0.8λ and isotropic receiver scattering. 134

6.10 Average power assigned to each mode for the ULA, UCA, and UGA, within an aperture of 0.8λ, relative to 0dB in each mode for ideal spatial-to-mode coupling. 136

6.11 Average power assigned to each mode for the UCA of radii $r = \{0.8\lambda, 0.75\lambda, 0.7\lambda\}$, within an aperture of 0.8λ, relative to 0dB in each mode for ideal spatial-to-mode coupling. 137

6.12 Capacity versus angular spread at the transmitter for mode-to-mode communication (modes), and a uniform circular array (UCA) of radii $r = r_\mathrm{T} = r_\mathrm{R} = \{0.8\lambda, 0.75\lambda, 0.7\lambda\}$, within spatial regions of radius 0.8λ and isotropic receiver scattering. 137

6.13 Continuous spatial channel model for communication between two arbitrary apertures. $g(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\psi}})$ is the effective complex random scattering gain of the scattering environment for signals leaving the transmit aperture Ω_T in direction $\widehat{\boldsymbol{\phi}}$ and arriving at the receive aperture Ω_R along $\widehat{\boldsymbol{\psi}}$. All scatterers are considered external to apertures and exist in $\mathbb{R}^3 \setminus \{\Omega_\mathrm{T}, \Omega_\mathrm{R}\}$. 141

6.14 Generalized continuous spatial channel model for communication between two arbitrary apertures. $g(\boldsymbol{x}, \boldsymbol{y})$ is the resultant function generated at $\boldsymbol{y} \in \Omega_{\mathrm{R}}$ due to the source function $u(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega_{\mathrm{T}}$. All scatterers are considered external to apertures and exist in $\mathbb{R}^3 \setminus \{\Omega_{\mathrm{T}}, \Omega_{\mathrm{R}}\}$. 145