

Space-Time Coding and Space-Time Channel Modelling for Wireless Communications

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B.E. (Hons 1)(University of Adelaide, South Australia)

November 2006

A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
OF THE AUSTRALIAN NATIONAL UNIVERSITY



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Declaration

The contents of this thesis are the results of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work in this thesis has been published or has been submitted for publication as journal papers or conference proceedings. These papers are:

1. Tharaka A. Lamahewa, Marvin K. Simon, Thushara D. Abhayapala, and Rodney A. Kennedy, “Performance analysis of space-time codes in realistic propagation environments: A moment generating function-based approach, *International Journal on Communications and Networks*, vol. 7, no. 4, pp. 450–461, Dec. 2005.
2. Tharaka A. Lamahewa, Marvin K. Simon, Thushara D. Abhayapala, and Rodney A. Kennedy, “Exact pairwise error probability analysis of space-time codes in spatially correlated fading channels,” *Special issue of the Journal of Telecommunications and Information Technology*, vol. 1/2006, pp. 60–68, Apr. 2006.
3. Tharaka A. Lamahewa, Rodney A. Kennedy, Thushara D. Abhayapala, and Van K. Nguyen, “Spatial precoder design for space-time coded MIMO systems: Based on fixed parameters of MIMO channels,” in *Wireless Personal Communications*, DOI: 10.1007/s11277-007-9281-4 (to appear in 2007).
4. Tharaka A. Lamahewa, Thushara D. Abhayapala, Rodney A. Kennedy, Terence Betlehem, and Jaunty T. Y. Ho, “Space-time channel modelling in general scattering environments,” submitted to *IEEE Trans. Signal Processing*.
5. Tharaka A. Lamahewa, Tony S. Pollock, and Thushara D. Abhayapala, “Achieving maximum capacity from spatially constrained dense MIMO systems,” to be submitted to *IEEE Journal on Selected Areas in Communications*.

6. Tharaka A. Lamahewa, Thushara D. Abhayapala, and Rodney A. Kennedy, "Fading resistance of orthogonal space-time block codes under spatial correlation," in *IEEE Workshop on Signal Processing Advances in Wireless Communications, SPAWC*, Lisbon, Portugal, July 2004, pp 278–282.
7. Tharaka A. Lamahewa, Thushara D. Abhayapala, and Rodney A. Kennedy, "Effect of transmit antenna configuration on rank-determinant criteria of space-time trellis codes," in *IEEE International Symposium on Spread Spectrum Techniques and Applications, ISSSTA 2004*, Sydney, Australia, Sept. 2004, pp. 750 - 754.
8. Tharaka A. Lamahewa, Marvin K. Simon, Thushara D. Abhayapala, and Rodney A. Kennedy, "Exact pairwise error probability analysis of space-time codes in realistic propagation environments," in *Workshop on the Internet, Telecommunications, and Signal Processing, WITSP-2004*, Adelaide, Australia, Dec. 2004, pp 170–175.
9. Tharaka A. Lamahewa, Rodney A. Kennedy, and Thushara D. Abhayapala, "Upper-bound for the pairwise error probability of space-time codes in physical channel scenarios, in *Proc. 5th Australian Communications Theory Workshop*, Brisbane, Australia, Feb. 2005, pp. 26 - 32.
10. Tharaka A. Lamahewa, Rodney A. Kennedy, and Thushara D. Abhayapala, "Spatial precoder design using fixed parameters of MIMO channels," in *Proc. 11th Asia-Pacific Conference on Communications APCC 2005*, Perth, Western Australia, Oct. 2005, pp. 82–86.
11. Tharaka A. Lamahewa, Tony S. Pollock, and Thushara D. Abhayapala, "Achieving Maximum Capacity from a Fixed Region of Space," in *Workshop on the Internet, Telecommunications, and Signal Processing, WITSP-2005*, Noosa Heads, Brisbane, Australia, Dec. 2005, pp. 38–43.
12. Tharaka A. Lamahewa, Rodney A. Kennedy, Thushara D. Abhayapala, and Terence Betlehem, "MIMO channel correlation in general scattering environments," in *Proc. 6th Australian Communication Theory Workshop*, Perth, Western Australia, Feb. 2006, pp. 91–96.
13. Terence Betlehem, Thushara D. Abhayapala, and Tharaka A. Lamahewa, "Space-time MIMO channel modelling using angular power distributions," in *Australian Communication Theory Workshop*, Perth, Western Australia, Feb. 2006, pp. 163–168.

14. Tharaka A. Lamahewa, Van K. Nguyen, and Thushara D. Abhayapala, "Exact pairwise error probability of differential space-time codes in spatially correlated channels," in *IEEE International Communications Conference, ICC 2006*, Istanbul, Turkey, June 2006, Vol. 10, pp 4853–4858.
15. Tharaka A. Lamahewa, Thushara D. Abhayapala, Rodney A. Kennedy, and J. T. Y. Ho, "Space-time cross correlation and space-frequency cross spectrum in non-isotropic scattering environments," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Processing*, Toulouse, France, May 2006, vol. IV, pp. IV–609–612.
16. Tharaka A. Lamahewa, Van K. Nguyen, Thushara D. Abhayapala, and Rodney A. Kennedy, "Spatial precoder design for differential space-time coded systems: Based on fixed parameters of MIMO channels," in *IEEE Workshop on Signal Processing Advances in Wireless Communications, SPAWC'06*, France, July 2006.
17. Terence Betlehem, Tharaka A. Lamahewa, and Thushara D. Abhayapala, "Dependence of MIMO system performance on the joint properties of angular power," in *IEEE International Symposium on Information Theory, ISIT 2006*, Seattle USA, July 2006, pp. 2849–2853.
18. Rauf Iqbal, Thushara D. Abhayapala and Tharaka A. Lamahewa, "Information Rates of Time-Varying Rayleigh Fading Channels in Non-Isotropic Scattering Environments," in *Workshop on the Internet, Telecommunications, and Signal Processing, WITSP-2006*, Hobart, Australia, Dec. 2006 (ISBN: 0 9756934 2 5) .

The research work presented in this thesis has been performed jointly with A/Prof. Thushara D. Abhayapala (The Australian National University), Prof. Rodney A. Kennedy (The Australian National University), Dr. Marvin K. Simon (NASA Jet Propulsion Laboratory, USA), Dr. Tony S. Pollock (National ICT Australia), Dr. Van K. Nguyen (Deakin University, Australia), Dr. Terence Betlehem (The Australian National University) and Dr. Jaunty Ho (Monash University, Australia). The substantial majority of this work was my own.

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Acknowledgements

The work presented in this thesis would not have been possible without the support of a number of individuals and organizations and they are gratefully acknowledged below:

- My supervisors A/Prof. Thushara D. Abhayapala and Prof. Rodney A. Kennedy for their guidance, insight, support and encouragement throughout my PhD studies.
- Drs Marvin K. Simon (Jet Propulsion Laboratory, NASA), Tony S. Pollock (National ICT Australia), Terence Betlehem (The Australian National University), Van K. Nguyen (Deakin University) and Jaunty Ho (Monash University) for their collaboration on some of the work presented in this thesis. Also, Drs Leif Hanlen (National ICT Australia) Dhammika Jayalath (Australian National University) and David Smith (The National ICT Australia) for many fruitful discussions during my PhD studies.
- Everyone in the former Telecommunications Engineering group and NICTA Wireless Signal Processing (WSP) Group for their efforts in providing a friendly research environment. Special thanks to Ms Lesley Goldberg, the department administrator for all her assistance.
- The Australian Research Council (Discovery Grant DP0343804) and The Australian National University for the PhD scholarship. Special thanks to Thushara and Rod for their help in arranging my scholarship.
- My parents and siblings for everything they have provided for me in terms of education, guidance, encouragement and financial support.
- My wife Sankha and two sons Thimitha and Savitha for their understanding, support, encouragement and patience through out my PhD studies. Also, special thanks to my wife's parents for their continual support and encouragement.

Abstract

In this thesis we investigate the effects of the physical constraints such as antenna aperture size, antenna geometry and non-isotropic scattering distribution parameters (angle of arrival/departure and angular spread) on the performance of coherent and non-coherent space-time coded wireless communication systems. First, we derive analytical expressions for the exact pairwise error probability (PEP) and PEP upper-bound of coherent and non-coherent space-time coded systems operating over spatially correlated fading channels using a moment-generating function-based approach. These analytical expressions account for antenna spacing, antenna geometries and scattering distribution models. Using these new PEP expressions, the degree of the effect of antenna spacing, antenna geometry and angular spread is quantified on the diversity advantage (robustness) given by a space-time code. It is shown that the number of antennas that can be employed in a fixed antenna aperture without diminishing the diversity advantage of a space-time code is determined by the size of the antenna aperture, antenna geometry and the richness of the scattering environment.

In realistic channel environments the performance of space-time coded multiple-input multiple output (MIMO) systems is significantly reduced due to non-ideal antenna placement and non-isotropic scattering. In this thesis, by exploiting the spatial dimension of a MIMO channel we introduce the novel use of linear spatial precoding (or power-loading) based on fixed and known parameters of MIMO channels to ameliorate the effects of non-ideal antenna placement on the performance of coherent and non-coherent space-time codes. The spatial precoder virtually arranges the antennas into an optimal configuration so that the spatial correlation between all antenna elements is minimum. With this design, the precoder is fixed for fixed antenna placement and the transmitter does not require any feedback of channel state information (partial or full) from the receiver. We also derive precoding schemes to exploit non-isotropic scattering distribution parameters of the scattering channel to improve the performance of space-time codes applied on MIMO systems in non-isotropic scattering environments. However, these schemes

require the receiver to estimate the non-isotropic parameters and feed them back to the transmitter.

The idea of precoding based on fixed parameters of MIMO channels is extended to maximize the capacity of spatially constrained dense antenna arrays. It is shown that the theoretical maximum capacity available from a fixed region of space can be achieved by power loading based on previously unutilized channel state information contained in the antenna locations. We analyzed the correlation between different modal orders generated at the transmitter region due to spatially constrained antenna arrays in non-isotropic scattering environments, and showed that adjacent modes contribute to higher correlation at the transmitter region. Based on this result, a power loading scheme is proposed which reduces the effects of correlation between adjacent modes at the transmitter region by nulling power onto adjacent transmit modes.

Furthermore, in this thesis a general space-time channel model for down-link transmission in a mobile multiple antenna communication system is developed. The model incorporates deterministic quantities such as physical antenna positions and the motion of the mobile unit (velocity and the direction), and random quantities to capture random scattering environment modeled using a bi-angular power distribution and, in the simplest case, the covariance between transmit and receive angles which captures statistical interdependency. The Kronecker model is shown to be a special case when the power distribution is separable and is shown to overestimate MIMO system performance whenever there is more than one scattering cluster. Expressions for space-time cross correlations and space-frequency cross spectra are given for a number of scattering distributions using Gaussian and Morgenstern's family of multivariate distributions. These new expressions extend the classical Jake's and Clarke's correlation models to general non-isotropic scattering environments.

List of Acronyms

AOD	angle of departure
AOA	angle of arrival
AWGN	additive white Gaussian noise
BER	bit-error rate
BPSK	binary phase shift keying
CSI	channel state information
MGF	moment generating function
MISO	multiple-input single-output
MIMO	multiple-input multiple-output
OFDM	orthogonal frequency-division multiplexing
PEP	pair-wise error probability
PSD	power spectral density
QPSK	quadrature phase shift keying
SIMO	single-input multiple-output
SISO	single-input single-output
SNR	signal to noise ratio
STBC	space-time block code
STTC	space-time trellis code
UCA	uniform circular array
ULA	uniform linear array
UGA	uniform grid array

Notations and Symbols

$\mathbf{A}^\dagger$	complex conjugate transpose of matrix $\mathbf{A}$
$\mathbf{a}^\dagger$	complex conjugate transpose of vector $\mathbf{a}$
$\mathbf{A}^T$	transpose of matrix $\mathbf{A}$
$\mathbf{a}^T$	transpose of vector $\mathbf{a}$
$\mathbf{A}^*$	complex conjugate of matrix $\mathbf{A}$
$\mathbf{a}^*$	complex conjugate of vector $\mathbf{a}$
$\overline{f(\cdot)}$	complex conjugate of scalar or function $f(\cdot)$
$\ \mathbf{a}\ $	euclidian norm of vector $\mathbf{a}$
$\ \mathbf{A}\ ^2$	squared norm of matrix $\mathbf{A}$
$ \mathbf{A} $	determinant of matrix $\mathbf{A}$
$\text{tr}\{\mathbf{A}\}$	trace of matrix $\mathbf{A}$
$\text{vec}(\mathbf{A})$	matrix vectorization operator: stacks the columns of $\mathbf{A}$
$\otimes$	matrix Kronecker product
$\delta(\cdot)$	Dirac delta function
$\lceil \cdot \rceil$	ceiling operator
$\mathcal{E}\{\cdot\}$	mathematical expectation
$\mathbf{I}_n$	$n \times n$ identity matrix
$\mathbf{1}$	vector of all ones
$\mathbb{S}^1$	unit circle
$\mathbb{S}^2$	unit sphere
$Q(x)$	Gaussian Q -function: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du$

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