

Dynamics of laboratory models of the wind-driven ocean circulation

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Except where otherwise indicated in the text, the research described in this thesis is my own original work.

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Abstract

This thesis presents a numerical exploration of the dynamics governing rotating flow driven by a surface stress in the “sliced cylinder” model of Pedlosky & Greenspan (1967) and Beardsley (1969), and its close relative, the “sliced cone” model introduced by Griffiths & Veronis (1997). The sliced cylinder model simulates the barotropic wind-driven circulation in a circular basin with vertical sidewalls, using a depth gradient to mimic the effects of a gradient in Coriolis parameter. In the sliced cone the vertical sidewalls are replaced by an azimuthally uniform slope around the perimeter of the basin to simulate a continental slope. Since these models can be implemented in the laboratory, their dynamics can be explored by a complementary interplay of analysis and numerical and laboratory experiments.

In this thesis a derivation is presented of a generalised quasigeostrophic formulation which is valid for linear and moderately nonlinear barotropic flows over large-amplitude topography on an f -plane, yet retains the simplicity and conservation properties of the standard quasigeostrophic vorticity equation (which is valid only for small depth variations). This formulation is implemented in a numerical model based on a code developed by Page (1982) and Becker & Page (1990).

The accuracy of the formulation and its implementation are confirmed by detailed comparisons with the laboratory sliced cylinder and sliced cone results of Griffiths (Griffiths & Kiss, 1999) and Griffiths & Veronis (1997), respectively. The numerical model is then used to provide insight into the dynamics responsible for the observed laboratory flows. In the linear limit the numerical model reveals shortcomings in the sliced cone analysis by Griffiths & Veronis (1998) in the region where the slope and interior join, and shows that the potential vorticity is dissipated in an extended region at the bottom of the slope rather than a localised region at the east as suggested by Griffiths & Veronis (1997, 1998). Welander’s thermal analogy (Welander, 1968) is used to explain the linear circulation pattern, and demonstrates that the broadly distributed potential vorticity dissipation is due to the closure of geostrophic contours in this geometry.

The numerical results also provide insight into features of the flow at finite Rossby number. It is demonstrated that separation of the western boundary current in the sliced cylinder is closely associated with a “crisis” due to excessive potential vorticity dissipation in the viscous sublayer, rather than insufficient dissipation in the outer western boundary current as suggested by Holland & Lin (1975) and Pedlosky (1987*b*). The stability boundaries in both models are refined using the numerical results, clarifying in particular the way in which the western boundary current instability in the sliced cone disappears at large Rossby and/or Ekman number. A flow regime is also revealed in the sliced cylinder in which the boundary current separates without reversed flow, consistent with the potential vorticity “crisis” mechanism. In addition the location of the stability boundary is determined as a function of the aspect ratio of the sliced cylinder, which demonstrates that the flow is stabilised in narrow basins such as those used by Beardsley (1969, 1972,

1973) and Becker & Page (1990) relative to the much wider basin used by Griffiths & Kiss (1999).

Laboratory studies of the sliced cone by Griffiths & Veronis (1997) showed that the flow became unstable only under anticyclonic forcing. It is shown in this thesis that the contrast between flow under cyclonic and anticyclonic forcing is due to the combined effects of the relative vorticity and topography in determining the shape of the potential vorticity contours. The vorticity at the bottom of the sidewall smooths out the potential vorticity contours under cyclonic forcing, but distorts them into highly contorted shapes under anticyclonic forcing. In addition, the flow is dominated by inertial boundary layers under cyclonic forcing and by standing Rossby waves under anticyclonic forcing due to the differing flow direction relative to the direction of Rossby wave phase propagation. The changes to the potential vorticity structure under strong cyclonic forcing reduce the potential vorticity changes experienced by fluid columns, and the flow approaches a steady free inertial circulation. In contrast, the complexity of the flow structure under anticyclonic forcing results in strong potential vorticity changes and also leads to barotropic instability under strong forcing.

The numerical results indicate that the instabilities in both models arise through supercritical Hopf bifurcations. The two types of instability observed by Griffiths & Veronis (1997) in the sliced cone are shown to be related to the western boundary current instability and “interior instability” identified by Meacham & Berloff (1997*b*). The western boundary current instability is trapped at the western side of the interior because its northward phase speed exceeds that of the fastest interior Rossby wave with the same meridional wavenumber, as discussed by Ierley & Young (1991).

Numerical experiments with different lateral boundary conditions are also undertaken. These show that the flow in the sliced cylinder is dramatically altered when the free-slip boundary condition is used instead of the no-slip condition, as expected from the work of Blandford (1971). There is no separated jet, because the flow cannot experience a potential vorticity “crisis” with this boundary condition, so the western boundary current overshoots and enters the interior from the east. In contrast, the flow in the sliced cone is identical whether no-slip, free-slip or super-slip boundary conditions are applied to the horizontal flow at the top of the sloping sidewall, except in the immediate vicinity of this region. This insensitivity results from the extremely strong topographic steering near the edge of the basin due to the vanishing depth, which demands a balance between wind forcing and Ekman pumping on the upper slope, regardless of the lateral boundary condition. The sensitivity to the lateral boundary condition is related to the importance of lateral friction in the global vorticity balance. The integrated vorticity must vanish under the no-slip condition, so in the sliced cylinder the overall vorticity budget is dominated by lateral viscosity and Ekman friction is negligible. Under the free-slip condition the Ekman friction assumes a dominant role in the dissipation, leading to a dramatic change in the flow structure. In contrast, the much larger depth variation in the sliced cone leads to a global vorticity balance in which Ekman friction is always dominant, regardless of the boundary condition.

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