3. T3 flow facility and scramjet model

The experimental study of scramjet flows requires a means of generating a high speed, high enthalpy air flow, as well as a model scramjet to test. For this work, the flow was provided by a pulsed facility, a free-piston shock tunnel known as T3, and utilised a previously-developed scramjet model.

Despite requiring some modifications to allow the laser beam to access the flow, the use of an existing model reduced the mechanical design and manufacture requirements to the level that they were manageable for this project. This also meant that the flow was already relatively well understood, the fuel injection system was well calibrated and CFD efforts that were underway for other work were applicable to this project also.

This chapter describes the T3 shock tunnel, its operation and the flow conditions that the facility was configured to provide. Following from this, section 3.2 describes the scramjet model.

The discussion is relatively brief, since details of both T3 and the scramjet model—albeit in a variety of different configurations—have been presented elsewhere [35, 42, 66, 75, 78, 109].

3.1 The T3 shock tunnel

The T3 free-piston shock tunnel is the third tunnel of its type built. The free-piston shock tunnel was invented by Stalker [108] and T1 was built in 1962 to prove the operational concept.

As mentioned in section 2.2, pulsed facilities are one possible option for scramjet testing. A wide range of pulsed facilities exist with the general properties that the test time is short, $10^{-5}$ to $10^{-2}$ s [110], with higher enthalpy operation resulting in shorter test times. Free-piston shock tunnels are at neither extreme of the test conditions produced by the family of pulsed flow facilities and can produce flows that are applicable to a wide range of possible scramjet operating conditions, although not up to orbital insertion velocities.

Configuration

Modestly sized when compared with newer shock tunnels the T3 shock tunnel, shown in figure 3.1, has a total length of over 20 m. The facility consists of a series of interconnected chambers:
• A high pressure reservoir is located behind a piston which is initially stationary but free to move within the next chamber, the compression tube;

• The compression tube contains the driver gas and is terminated at its downstream end by a metal diaphragm, known as the primary diaphragm. The diaphragm is chosen according to the desired burst pressure and may be stainless steel or aluminium sheet with typical thickness ranging from 0.7 mm to 2.9 mm;

• The primary diaphragm is then followed by a narrower diameter tube, known as the shock tube, which terminates with a thin plastic diaphragm. This ‘secondary diaphragm’ is made from 0.05 mm thick mylar and separates the shock tube from the facility nozzle and test section;

• The test section has a cross section of 0.5 m and has windows allowing vertical or horizontal cross-stream optical access; and

• Following the test section is a large-volume dump tank.

Operation

Before operation, the compression tube, shock tube and dump tank are evacuated with oil-filled rotary vane roughing pumps to around 0.1 torr (13 Pa), before being filled with gases, as shown in figure 3.2a. The shock tube is filled with the test gas, which was dry air for this work, while the compression tube is filled with the driver gas. Helium was used as the driver gas, chosen for its high sound speed and because it is chemically inert. The piston is then held on a launcher as the high pressure reservoir is filled and, once the pressure here reaches the desired pressure, the tunnel is ready to fire.

From this state, the tunnel can be fired by releasing the piston from the launcher by opening a valve. The pressure behind the piston accelerates it down the compression tube, compressing the driver gas up to the burst pressure of the primary diaphragm. The initial pressure behind the piston is chosen so that the piston retains some forward momentum after the main diaphragm bursts. This is referred to as tuned operation [59] and helps to keep the pressure of the driver gas closer to constant than it would otherwise be.
3.1. The T3 shock tunnel

The flow within the shock tube after diaphragm rupture is described by Gaydon and Hurle [43]. With diaphragm rupture, a shock wave propagates down the shock tube compressing and heating the test gas, as shown in figure 3.2b. Upon arriving at the downstream end of the shock tube, the shock wave reflects off, and vaporises, the mylar diaphragm. The reflected shock then travels back up the shock tube again heating and compressing the flow, as well as slowing it so that the conditions upstream of the nozzle approximate a high temperature, high pressure stagnation region with conditions of the total pressure and temperature of the flow, around 15 MPa and 4500 K for the conditions used in this work. If the reflected shock brings the contact surface to rest, the condition is described as ‘tailored’, however other scenarios are possible [43, p 64]. The test gas then starts flowing out of this stagnation region through the hypersonic nozzle, shown in figure 3.2c.

As well as the shock produced at diaphragm rupture, an expansion wave is produced at the junction of the compression and shock tubes [43, p 60]. Its head travels into the compression tube, reflects off the piston and interacts with the flow structures at the far end of the shock tube. Provided that the shock tube is...
not too short, the primary shock reflects from the end of the shock tube before it is overtaken, and attenuated, by the expansion wave.

The nozzle has a throat-to-exit area ratio chosen for the desired Mach number in the test section. After a starting process is complete, quasi-steady flow exists in the test section, for around 3 ms at these operating conditions. This is shown in figure 3.2d. Keeping the dump tank at vacuum minimises the time required for the establishment of quasi-steady flow. The completion of a test is caused by the arrival of driver gas in the test section, indicated by the contact surface in figure 3.2b–d. Driver gas can arrive in the test section earlier than anticipated by consideration of the 1–dimensional flow due to driver gas ‘jetting’ along the edges of the shock tube [83, e.g.] so that identification of driver gas contamination is a major consideration in shock tunnel experiments. This is more of a concern at higher enthalpies than those used in this work. Previous work suggests that driver gas contamination happens sometime after 4 ms after the initial shock reflection for the present operating conditions [76].

Prior to the arrival of driver gas, the pressure in the stagnation region drops steadily as shown in figure 3.3. This pressure signal generally exhibits a plateau for part of the test time, however a significantly longer period of quasi-steady flow can be identified from pressure measurements. This is done by taking the ratio of pressure at a location in the test section to the stagnation pressure and identifying a period where this ratio is a constant. Such a comparison requires the transit time between the two pressure transducers to be taken into account, so that pressure is compared at the same point in the slug of test gas. A measurement of pressure in the stagnation region is also necessary to calculate the free-stream conditions at the exit of the nozzle, as explained next.

The test conditions that were used in this work were chosen to match those used in previous and ongoing work with this scramjet combustor, and tunnel
Table 3.1: Operating conditions for the T3 shock tunnel used for this work. The uncertainty in measured quantities is the shot-to-shot scatter while uncertainty in the effective area ratio, which is calculated from Pitot pressure, is due to Pitot pressure measurement uncertainty.

<table>
<thead>
<tr>
<th>Tunnel operating parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial shock tube fill pressure</td>
<td>50.0 ± 0.2 kPa</td>
</tr>
<tr>
<td>Primary shock speed</td>
<td>2510 ± 30 ms$^{-1}$</td>
</tr>
<tr>
<td>Stagnation pressure at 1.5 ms</td>
<td>14.3 ± 0.2 MPa</td>
</tr>
<tr>
<td>Stagnation pressure at 2.0 ms</td>
<td>12.9 ± 0.2 MPa</td>
</tr>
<tr>
<td>Physical nozzle throat-to-exit area ratio</td>
<td>12.96</td>
</tr>
<tr>
<td>Effective nozzle throat-to-exit area ratio</td>
<td>12.6 ± 0.4</td>
</tr>
</tbody>
</table>

operation parameters are shown in table 3.1. For the development of a new sensor, this is extremely useful as it allows measurements to be made in a flow that is relatively well understood from other work. This has the secondary benefit of saving the time it would have otherwise taken to calibrate the tunnel at new operating conditions.

Determination of free-stream conditions

For a number of years the free-stream conditions in both the T2 and T3 shock tunnels have been determined in a roughly similar manner, so there are numerous references describing this procedure [35, 38, 77]. A similar process was followed for this work as outlined below.

Initial pressure and temperature in the shock tube are known and two pressure transducers are used to determine the velocity of the primary shock. The delay between the initial rise of the two transducer signals is measured with a digital counter to within ±1 $\mu$s. The distance between the transducers was 1438 ± 5 mm and the primary shock speed was calculated for each shot.

The shock velocity and conditions in the shock tube are used to solve the conditions behind the shock, assuming 1–dimensional flow and equilibrium chemistry, using a Fortran code known as ESTC [65]. Using these new conditions as input, the same code is then used to solve for the conditions after the reflected shock with the additional constraint that the reflected shock brings the flow to rest.

The pressure measured in the stagnation region will, in general, differ from the conditions predicted by the two passes of ESTC due to the interaction of the reflected shock with the contact surface, so a correction is applied to adjust the conditions to match the measured pressure. This correction, which assumes that the interaction between the reflected shock and the contact surface is isentropic, is done by iteratively applying an isentropic expansion or compression to the model results until the measured and modelled pressure match, therefore fully specifying the conditions in the stagnation region.
A quasi-one-dimensional calculation then follows, using these stagnation conditions and the nozzle geometry as input. This is performed using the STUBE code [121] which incorporates finite-rate chemistry, and therefore chemical freezing, as well as vibrational non-equilibrium. This is necessary since, in a hypersonic nozzle flow, the flow can cool faster than the vibrational relaxation time so that vibrational temperature remains significantly higher than translational and rotational temperature.

Since this is an inviscid calculation, the effect of the boundary layer on the facility nozzle is not predicted. In order to correct for this, the area ratio of the nozzle at the test section is varied until Pitot pressure predicted by the model matches the measured Pitot pressure. Physically, this is justified by considering a boundary layer as a constriction of the nozzle and therefore a modification of the throat-to-exit area ratio.

For this work, this process resulted in the conditions shown in table 3.2 which are quoted at both 1.5 ms and 2 ms after shock reflection. The uncertainties given are calculated by repeating this process at the limits of the uncertainty in the input conditions.

Additional discussion of free-stream conditions

Sometime after this thesis was submitted for examination, a problem was discovered with the free-stream conditions. This issue was pointed out by Sean O’Byrne and is discussed, along with its implications, in this additional section.

As mentioned above, the test gas cools rapidly as it expands through the hypersonic nozzle. If the flow speed is great enough, the vibrational temperature of the species in the flow will no longer be in equilibrium with the rotational and translation temperature—it will be higher by an amount determined by the vibrational relaxation time. Since the relaxation time differs between species present in the flow, the vibrational temperature will be different for each species. In the nozzle flow considered here, N\textsubscript{2} is the only species for which vibrational non-equilibrium need be considered.

This rather complex situation is treated in STUBE by a so-called ‘sudden freezing’ approximation. As the flow progresses through the nozzle, it stays in equilibrium until the cooling rate is judged to be greater than the vibrational relaxation rate for a particular species. The vibrational temperature is then held constant for that species past this point. Measurements performed in the T3 facility show that this provides a reasonable approximation to the nozzle flow, although better for pure nitrogen flows than air flows [36].

The conditions shown in table 3.2 were inadvertently calculated with the vibrational temperature frozen at the reservoir condition, rather than at some lower-temperature condition part way down the nozzle. Since the vibrational temperature is too high, compared with the actual flow, energy conservation requires that the calculated free-stream temperature is low.

Because of this problem, the free-stream conditions have been re-evaluated and two additional cases appended to table 3.2.
### 3.1. The T3 shock tunnel

Table 3.2: Nominal free-stream conditions at the combustor intake. The ‘vibrational temperature frozen at reservoir temperature’ case shows the freestream conditions that were used in this thesis with the incorrect belief that the conditions had been calculated under the ‘sudden freezing’ approximation. The ‘vibrational equilibrium’ case shows the conditions that are the most appropriate for use with CFD and the ‘sudden freezing’ approximation is the most physically correct, but has been calculated only for 1.5 ms. See page 20 for further discussion.

<table>
<thead>
<tr>
<th>Freestream quantity</th>
<th>Value at 1.5 ms</th>
<th>Value at 2.0 ms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vibrational temperature frozen at reservoir temperature</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mach number</td>
<td>4.16 ± 0.09</td>
<td>4.18 ± 0.10</td>
</tr>
<tr>
<td>Static Pressure</td>
<td>80 ± 6 kPa</td>
<td>70 ± 6 kPa</td>
</tr>
<tr>
<td>Pitot Pressure</td>
<td>1750 ± 80 kPa</td>
<td>1580 ± 70 kPa</td>
</tr>
<tr>
<td>Density</td>
<td>0.23 ± 0.01 kgm$^{-3}$</td>
<td>0.21 ± 0.01 kgm$^{-3}$</td>
</tr>
<tr>
<td>Velocity</td>
<td>2790 ± 30 ms$^{-1}$</td>
<td>2760 ± 30 ms$^{-1}$</td>
</tr>
<tr>
<td>Rotational temperature</td>
<td>1160 ± 50 K</td>
<td>1130 ± 50 K</td>
</tr>
<tr>
<td><strong>Vibrational equilibrium</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mach number</td>
<td>3.76 ± 0.06</td>
<td>3.77 ± 0.08</td>
</tr>
<tr>
<td>Static Pressure</td>
<td>101 ± 7 kPa</td>
<td>91 ± 6 kPa</td>
</tr>
<tr>
<td>Pitot Pressure</td>
<td>1805 ± 80 kPa</td>
<td>1624 ± 75 kPa</td>
</tr>
<tr>
<td>Density</td>
<td>0.22 ± 0.01 kgm$^{-3}$</td>
<td>0.21 ± 0.01 kgm$^{-3}$</td>
</tr>
<tr>
<td>Velocity</td>
<td>2870 ± 30 ms$^{-1}$</td>
<td>2850 ± 30 ms$^{-1}$</td>
</tr>
<tr>
<td>Temperature</td>
<td>1560 ± 60 K</td>
<td>1510 ± 50 K</td>
</tr>
<tr>
<td><strong>Sudden freezing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mach number</td>
<td>3.74 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Static Pressure</td>
<td>88 ± 2 kPa</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>0.217 ± 0.007 kgm$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>2885 ± 20 ms$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Rotational temperature</td>
<td>1410 ± 30 K</td>
<td></td>
</tr>
</tbody>
</table>

1 provided by Sean O’Byrne. N$_2$ frozen at 2468 K, other species in equilibrium.
The first of the additional cases shows the free-stream conditions evaluated under vibrational equilibrium. The temperature of the physical free-stream flow should fall somewhere between the original prediction and vibrational equilibrium. This is seen to be the case for a third calculation showing the free-stream conditions evaluated with a sudden freezing approximation—the calculation that was thought to have been used initially.

Although the sudden freezing approximation is the most physically accurate of the three, there is still value to the vibrational equilibrium case. If computed nozzle exit conditions are to be used as inflow conditions in a CFD simulation, the equilibrium case should be used if the code does not support vibrational non-equilibrium. This approach conserves the total energy of the flow. Furthermore, the physical flow may well approach the equilibrium case inside the scramjet combustor after passing through the inlet shocks.

The use of incorrect free-stream conditions represents a major flaw in the CFD simulations presented later, and it would be interesting to revisit these simulations using vibrational equilibrium inflow conditions. Other results remain valid, however. In particular, mass flux changes by around 2.5% so that equivalence ratio calculations remain valid.

In my opinion, it would be worthwhile to explore alternatives to STUBE for future freestream calculations.

### 3.2 Scramjet combustor model

The scramjet model used in these experiments resembles a rectangular duct with an adjustable floor that can be set so that the duct diverges beyond the half-way point. Previously, this scramjet model has been configured with a strut injector for hyper-mixing studies [35, 66, 79]. The present configuration replaces the previously-used strut injector with injection from the rear face of a cavity. The performance of the cavity in a straight duct has been studied using pressure measurements [75] and OH-PLIF experiments [80].

Apart from the addition of the cavity and the removal of the strut injector, the scramjet model remains essentially the same as in previous work. In particular, the injection system and pressure transducer instrumentation remain the same.

The geometry of the scramjet duct is shown in figure 3.4. The scramjet model was designed with combustor studies in mind and therefore does not include an intake ramp. This allows a more homogeneous airflow to enter the combustor, simplifying the flow-field somewhat, as well as allowing for a larger combustor than would be possible if intake geometry were included.

Since the model does not compress the incoming airflow, the shock tunnel is configured to deliver an airflow that is analogous to the flow that would be present at the entrance to the combustor in flight. For the conditions used in these experiments, the free-stream conditions are equivalent to a vehicle flying at Mach 10.5, with details shown in table 3.2. Equivalent Mach number was calculated from the inlet conditions by applying conservation of enthalpy and
3.2. Scramjet combustor model

Assuming a free-stream temperature of 225 K. The high temperature, 1190 K, and pressure of around one atmosphere means that conditions in the combustor are sufficient to initiate combustion.

The flow from the nozzle of the facility is swallowed by the combustor intake, shown in figure 3.5. The intake has sharp leading edges so that, apart from relatively weak shocks attached to these intakes, the flow entering the combustor has similar properties to the free-stream. The flow that does not enter the scramjet is deflected by a bluff plate, also visible in figure 3.5. This plate protects the instrumentation located on either side of the scramjet, however it causes a major disturbance to the external flow. To verify that the plate does not influence flow entering the combustor duct, a photograph was taken of the flow relying on flow luminosity to identify areas of the flow affected by the plate. As shown by figure 3.6, the presence of the plate does not influence the interior flow, with the flow disturbance appearing downstream of the scramjet intake. Pressure measurements taken with and without the duct in place also supported the theory that the duct flow was unaffected by the exterior plate.

The flow within the combustor, meanwhile, travels down the first section of the rectangular duct before encountering a cavity in the floor of the duct. The cavity’s main purpose is to provide a recirculating zone holding a hot pool of radicals for flame-holding and stabilisation. Important features of the cavity are its length-to-depth ratio, the sloping rear face and location chosen for fuel injection [49].

The length-to-depth ratio, $L/D$, has a strong influence on the flow structure within the cavity which, in turn, determines drag and mass exchange with the
3. T3 flow facility and scramjet model

Figure 3.5: Scramjet inlet viewed from upstream. The duct is 52 mm wide and 25 mm high. The large bluff plate protects sensors mounted on the side of the scramjet and does not disturb the flow entering the duct.

Figure 3.6: False colour luminosity image of the scramjet intake from side-on taken to verify the integrity of the flow entering the combustor. Flow direction is left to right.
main flow. This cavity has $L/D = 4.8$, which puts it in the range of minimum induced drag, when fuel injection is off, according to Ben-Yakar and Hanson's review [9].

The rear face of the cavity is sloped in order to dampen the oscillations that would otherwise be produced in a rectangular cavity of a similar size. Flow oscillations within the cavity are generally undesirable because of the associated increase in drag and instability in the combustion process. Although the flow is certainly turbulent, the sloping back wall of the cavity stabilizes the re-attachment of the boundary layer and reduces acoustic wave reflection from the downstream end of the cavity. Both of these features result in reduced oscillations.

Hydrogen injection can also help stabilise the flow. Hydrogen was injected into the cavity from the downstream end, in the opposite direction to the free-stream flow. The model can also be configured for other injection schemes which are expected to be tested in the future. For the generation of thrust, the injection scheme used here is not considered optimal.

Having passed the cavity, the fuel–air mixture is expanded by a $15^\circ$ corner in the duct floor. Previous work with this scramjet model used a straight duct downstream of the cavity, but the presence of the expansion corner means that the downstream temperature and pressure is reduced, leading to a more suitable environment for spectroscopic measurements.

The laser beam for TDLAS measurements traverses the flow perpendicular to the main flow direction. A fibre optic collimator is located on one side of the combustor with a detector on the other side. These are mounted in a section of the scramjet side-plate that can be traversed vertically across the flow, with the extents of this traverse shown in figure 3.4. The alignment of the beam horizontally across the flow means that the laser is likely to probe a more homogeneous part of the flow than if aligned vertically. Due to the large vertical gradient in flow properties, however, the measurements produced by the sensor were expected to vary significantly as the beam was traversed vertically across the duct.

The laser beam could be located from 5.6 to 50.6 mm above the floor of the duct, covering half of the vertical extent of the duct, which had a total height of 97.6 mm at this location. This was expected to encompass most of the variation in water vapour concentration and temperature across the duct. Further details of how TDLAS was interfaced with the scramjet are given later in section 7.1, prior to the presentation of results.

The fuel injection subsystem is shown schematically in figure 3.7. A Ludwieg tube could be filled with hydrogen to a pressure between 350 and 2500 kPa before the tunnel was operated. A fast-acting valve was opened before the arrival of test flow in the scramjet, triggered from the recoil of the tunnel. The valve opened 30 ms before shock reflection and then shut between 50 and 150 ms after shock reflection. From the Ludwieg tube, the fuel flowed into a plenum chamber underneath the fuel injection ports and then, from the rear face of the cavity, into the scramjet.
3. T3 flow facility and scramjet model

Figure 3.7: Important components of the fuel injection system.

Figure 3.8: Details of the pressure transducer mounts in the scramjet floor (figure from [42]). Transducers are recessed to avoid damage.

Instrumentation

Besides TDLAS, the scramjet model is instrumented with multiple piezoelectric pressure transducers. The location of pressure transducers used in this work are shown in figure 3.4 with the details of the mounting shown in figure 3.8. The first of these transducers, referred to as the ‘inlet’ transducer, is mounted on the roof of the duct, upstream of the cavity and 70 mm downstream of the inlet. The results produced by this transducer were noisy compared with the other transducers. Since the flow should be more stable here than at other measurement stations, this is believed to be due to the different mounting configuration of this transducer compared with the others. This theory is supported by the improvement in data produced by this transducer after the vibration isolation of this mount was improved, however this improvement was made after data for this work had been collected.
The next transducer in the duct measured pressure on the floor of the cavity—the ‘cavity’ transducer. This transducer produced results that exhibited more fluctuations than the downstream transducers but less than the inlet transducer. Since this transducer was mounted in a similar manner to the downstream transducers, i.e. the configuration shown in figure 3.8, the fluctuations measured with the ‘cavity’ transducer were interpreted as being due to pressure fluctuations at this measurement location.

Downstream of the cavity, a section of the duct was instrumented with 10 pressure transducers spaced 20 mm apart, which was the closest spacing allowed by the size of the transducer mounts. These pressure transducers can be used to test for combustion in the duct by comparing pressure between subsequent experimental runs where fuel is injected into a nitrogen and then air free-stream. The increase in pressure between these two cases can be attributed to combustion.

Apart from the cavity transducer which was 10 mm from the centre-line, the pressure transducers were mounted on the centre-line of the duct and were PCB 113A21 model transducers with built-in amplification circuitry. The output from the transducers was amplified by model 483A02 and 483B03 PCB amplifiers and the results were stored on digital storage oscilloscopes.

As well as the transducers in the duct, the pressure in the plenum chamber was recorded over the operating time of the fast-acting valve. Pressure in the Ludwieg tube was recorded before and after operation of the tunnel, and these three quantities allow the calculation of the fuel–air equivalence ratio.

**Calculation of equivalence ratio**

The relative proportions of fuel and oxidiser in a combustible mixture are characterised by the equivalence ratio, $\phi$. The equivalence ratio relates the actual molecular fuel/oxidiser ratio to the stoichiometric ratio. By definition, for $\phi > 1$ the mixture is fuel-rich and unburned fuel will remain after combustion, whereas for $\phi < 1$ the mixture is fuel-lean and oxygen will remain after combustion.

The concept of an equivalence ratio is usually applied to closed systems where the fuel and air can fully mix and have time to react. This is not necessarily the case in a scramjet engine where the local equivalence ratio can be quite different from the global equivalence ratio. However, the concept is still a useful means of characterising the operating conditions.

For a hydrogen–air mixture,

$$\phi = \frac{n(H_2)}{2n(O_2)}$$

where $n(\ldots)$ represents the molar concentration of each species.

In a scramjet, this can be rewritten in terms of the input mass flow rates of hydrogen, $\dot{m}_{H_2}$, and air, $\dot{m}_{\text{air}}$. In doing so,

$$\phi = 34.23 \frac{\dot{m}_{H_2}}{\dot{m}_{\text{air}}}$$
assuming dry air which is 20.95% oxygen by volume, the molecular mass of hydrogen is 2.02 g/mol and the molecular mass of air is 28.97 g/mol.

The mass flow of air is known from the tunnel free-stream conditions, calculated in section 3.1, and the cross-sectional area of the combustor inlet. The calculation of hydrogen mass flow follows an unpublished method developed by Alan Paull and Russell Boyce [16], described briefly here. In this method, we measure the initial pressure in the Ludwieg tube, $p_i$, the final pressure in the Ludwieg tube after it has returned to room temperature, $p_f$, and record the pressure in the plenum, $p_m(t)$, while the fast-acting valve is open. From this, we can determine the total mass of fuel injected

$$m = (p_i - p_f) \frac{V}{R_{H_2} T_0},$$

(3.3)

where $V = 1.419 \text{ L}$ is the volume of the Ludwieg tube, $R_{H_2} = 4124 \text{ Jkg}^{-1}\text{K}^{-1}$ is the gas constant for hydrogen and $T_0$ is the Ludwieg tube temperature.

Previously, work at T3 [35, 42, 76] has assumed that the instantaneous mass flow was proportional to the plenum pressure. Using this assumption, the plenum pressure can be numerically integrated and equated to equation (3.3) to find the proportionality constant. The problem with this method is that it implicitly assumes that the temperature in the plenum is constant, which is not a good assumption since the gas cools as it expands out of the Ludwieg tube. Taking cooling into account, we must proceed as follows.

If we assume isentropic flow and treat the injectors as a sonic throat then the temperature at the injectors, $T^*$, is related to the initial temperature in the reservoir, $T_0$ equal to the total temperature, by

$$\frac{T_0}{T^*} = \left( \frac{p_i}{p^*} \right)^{\frac{\gamma - 1}{\gamma}}$$

(3.4)

where $p^*$ is the pressure at the throat and $\gamma$ is the ratio of specific heats ($\gamma = c_p/c_v$) for the fuel. This assumption allows the mass flow to be written as

$$\dot{m} = \sqrt{\frac{\gamma}{R_{H_2}}} A \frac{p^*}{\sqrt{T^*}}$$

(3.5)

since we know that the velocity here is equal to the local sound speed, $\sqrt{\gamma R_{H_2} T^*}$, and $A$ is the effective cross-sectional area of the injector ports. Equation (3.4) allows us to eliminate $T^*$ from equation (3.5) so that

$$\dot{m} = \left( \sqrt{\frac{\gamma}{R_{H_2}}} A p_0^* \right)^{\frac{\gamma - 1}{2\gamma}} p^*^{\frac{\gamma - 1}{2\gamma}}$$

(3.6)

where the term in parenthesis is a constant for a particular injector and fill pressure. Furthermore, this shows that the relationship between $\dot{m}$ and $p^*$ is non-linear. If we integrate equation (3.6) to obtain $\dot{m}$ and equate it with equation (3.3) then the constant term in equation (3.6) can be eliminated and we can write

$$\dot{m} = (p_i - p_f) \frac{V}{R_{H_2} T_0} \int_{-\infty}^{\infty} \frac{(p^*)^{\frac{\gamma - 1}{2\gamma}}}{p^*^{\frac{\gamma - 1}{2\gamma}}} \, dt$$

(3.7)
3.2. Scramjet combustor model

At this stage we still have the mass flow in terms of the unknown pressure at the injectors, $p^*$. However, if we assume that pressure in the plenum chamber, $p_m$, is proportional to $p^*$ and therefore substitute $p^* = C p_m$ into equation (3.7) then the proportionality constant, $C$, cancels between the numerator and denominator. This provides a useful expression for the mass flow;

$$ \dot{m} = (p_i - p_f) \frac{V}{R_{H_2} T_0} \frac{\int_{-\infty}^{+\infty} p_m^{\gamma+1} \gamma \gamma - 1 dt}{\int_{-\infty}^{+\infty} p_m^{\gamma+1} \gamma \gamma - 1 \gamma \gamma - 1 dt} $$

(3.8)

where $R_{H_2} = 4124 \text{ Jkg}^{-1}\text{K}^{-1}$, $V = 1.419 \times 10^{-3} \text{ m}^3$, $\gamma = 1.4$ and $T_0$ is typically 296 K.

Assuming that the isentropic relation holds, the ratio of local static pressure to pressure at the throat is a function of Mach number,

$$ \frac{p}{p^*} = \frac{(1 + \gamma^{-1} M^2)^{\gamma \gamma - 1}}{(1 + \gamma^{-1} M^2)^{\gamma \gamma - 1}} $$

(3.9)

then the assumption that $p_m \propto p^*$ will be accurate provided that the Mach number in the plenum is low.

An example of this calculation is shown in figure 3.9 where equation (3.8) was used to determine the mass flow from the injector pressure trace. A similar calculation assuming that $\dot{m} \propto p_m$, as assumed by several previous investigators [35, 42, 76], is also shown. Although the method outlined in this work is expected to be more precise, it produces almost identical results to the method used previously and therefore validates the previously-reported equivalence ratios. The equivalence ratios reported here do not match those reported by Neely et al. [75], despite the same operation conditions. This was due to an implementation error in Neely’s analysis so that, for hydrogen tests, the equivalence ratio was under-predicted by a factor of two.

Having calculated hydrogen mass flow, the equivalence ratio can be calculated from the mass flow of air predicted by STUBE/ESTC and using equation (3.2). This can be carried out repeatedly as stagnation pressure decays over the test period to obtain the result shown in figure 3.10. As well as showing the repeatability of the shock tunnel, this shows that equivalence ratio rises over the test period due to the stagnation pressure decaying faster than the plenum pressure. This has implications for the interpretation of the TDLAS results outlined later.

Recalling that the determination of the equivalence ratio required the scramjet inlet conditions to be known, the equivalence ratio is the parameter most likely to show shot-to-shot scatter. Since repeatability is good, it can be concluded that the test environment, comprising of the T3 shock tunnel and the scramjet combustor model, are well characterised and well suited to testing a new diagnostic technique.
3. T3 flow facility and scramjet model

Figure 3.9: Hydrogen mass flow calculated by the method outlined in the main text as well as the method used previously by others [35, 42, 76]. The results produced by the two methods are nearly identical at the time of tunnel operation.

Figure 3.10: Fuel-air equivalence ratio calculated from plenum pressure and stagnation pressure traces for fourteen shots, half with 950 kPa and half with 2500 kPa Ludwieg tube fill pressure. Good repeatability is evident between shots, and equivalence ratio always increases over the test.