CHAPTER 2

Doppler Spectroscopy

And God said, Let there be light: and there was light.
– Genesis 1:3

This chapter provides an understanding for the basis of—and the information that can be derived from—spectroscopy where the dominant spectral lineshape broadening mechanism is the Doppler effect. In particular, a treatment is given for the measurement and interpretation of line-integrated measurements. It outlines the theory and reasoning for the use of a Fourier-transform spectrometer in making these measurements and gives a conceptual description of the Modulated Optical Solid State spectrometer. Consideration is given to the properties and characteristics of this Fourier-transform instrument. This chapter also establishes the framework—including the concepts, definitions and nomenclature—for the experimental work described later in this thesis. Throughout this and subsequent chapters, the terms ion and atom are often used interchangeably in the description of electronic transition processes.

2.1 Introduction

The transition of an electron, bound to a nucleus, from an upper fixed energy state $k$ (with energy $E_k$) to a lower level $i$ emits radiation with energy given by

$$E_{ki} = E_k - E_i = h\nu = \frac{hc}{\lambda_{vac}}$$

(2.1)

where $\nu$ is the frequency of the radiation and $\lambda_{vac}$ its wavelength in vacuum. $h$ and $c$ are the Planck and speed of light constants, respectively. However because the radiation process is affected by other influences the observed emission does not have the single frequency $\nu$.

The spectral lineshape—the energy distribution as a function of frequency—has a natural width $\Delta\nu = \Delta E / h$ (also called the linewidth) due to the Heisenberg uncertainty principle which is given by $\Delta\nu \Delta t \sim 1/2\pi$. That is, the energy spread
is inversely proportional to the radiative lifetime\(^\dagger\) of the electron in that upper energy state. The measured spectral lineshape may not be narrower than this natural width but can be broadened by different effects such as the Doppler, Zeeman and Stark (also called collisional or pressure broadening). Natural linewidth broadening can become significant when observing the high-energy lines from highly charged ions and yields information on the lifetime of the energy states.

The Doppler effect is the apparent shift in the emitted wavelength due to motion of the radiating source away (increase in wavelength or ‘red-shift’) or towards (wavelength decrease or ‘blue-shift’) the observer. Doppler broadening of the observed line arises from the distribution in ionic velocities which creates a corresponding distribution in the observed wavelengths.

Doppler broadening is dominant in many laboratory plasmas [Hutchinson, 2002] and the linewidth provides a quite direct measurement of the ionic temperature [Hilliard and Shepherd, 1966]. Additionally, the observation of a shift in the mean (or centre) wavelength of a spectral lineshape gives a measurement of the bulk flow velocity of the emitting species [Rees and Greenaway, 1983]. Localised turbulence can distort the interpretation of temperature information and régimes in which this may occur require closer scrutiny. Doppler spectroscopy can also be used to infer the flow velocities of very hot, fully-ionised majority species plasmas by observation of impurity ions [Benjamin et al., 1990] or through charge-exchange with injected particles.

The Stark effect is the perturbation to the energy levels of a bound electron in the presence of an external electric field. This perturbation alters the emitted wavelength of the radiation. In a plasma, nearby electrons generally provide the perturbing electric field. Thus for high pressures (or densities) or collisionality, the linewidth can be broadened by this effect. Stark broadening becomes an important consideration for lower-temperature, higher density plasmas and in such circumstances allows the determination of the electron density.

The Zeeman effect results in the splitting of a spectral line into three (or more) components in the presence of a magnetic field. The unshifted \(\pi\) component is polarised parallel to the magnetic field direction while the equally shifted \(\sigma\) components are orthogonally polarised. The change in the potential energy of the electron is \(q_e\hbar B/2m_e\) so that the spectral line splitting caused by the Zeeman effect becomes non-negligible in high magnetic field strength plasmas. Technically, this effect allows the magnetic field strength and direction to be determined, though the splitting is often obscured by Doppler broadening.

For plasma conditions typical of low-field argon discharges in the H-1NF stellarator, the spectral lineshapes are dominated by Doppler broadening, as will be discussed in detail below.

\(^\dagger\)Which is the inverse of the sum of the probabilities of all transitions from that excited energy state.
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2.2 Doppler Broadening

While Doppler spectroscopy is a powerful technique for sensing gross features of the emitting medium, the interpretation of the spectrum is complicated by the fact that the measurement is intrinsically a line-integrated one. This means that the inferred velocity can only be known in the direction of view and that the spectral lineshape is the result of intensity-weighted contributions from all points along the line-of-sight for an extended and inhomogeneous medium.

For a medium where the velocity distribution function is in local thermal equilibrium, the three plasma properties are conveyed by the spectral lineshape which is dominated by Doppler-broadening. They are the emitted radiance (derived from the area under the lineshape), the species temperature (linewidth) and the flow velocity (line centre shift). The physical mechanisms underlying each property and how the quantities are inferred are discussed below.

2.2.1 Emission Intensity

The local species emission for a given spectral line depends on the electron energy-state populations and the transition (emission and absorption) probabilities. The population of the energy states is governed by the particle density, the thermal conditions and dynamic processes. These processes include transitions which are both upward—such as collisional excitation and radiative absorption—and downward—such as radiative decay (both spontaneous and stimulated) and collisional de-excitation. The probabilities of the radiative transitions are intrinsic properties of the species and can, in theory, be calculated if the quantum wave functions are known.

Consider the spectral line of central wavelength $\lambda_{ki}$ (and frequency $\nu_{ki}$), related to the transition between energy levels $E_k$ (upper) and $E_i$ (lower). In the presence of radiation, a stimulated (also called induced) radiative decay may occur with probability per unit time of $B_{ki}\rho(\nu_{ki})$ where $\rho(\nu_{ki})$ is the energy density per unit frequency of the radiation at the ion. The ion may also absorb an incident photon causing a transition from state $i$ to $k$ with probability per unit time of $B_{ik}\rho(\nu_{ki})$.

The total emission co-efficient $\epsilon_{ki}$, which describes the power radiated per unit volume and per solid angle by the ions in the upper state $k$ due to spontaneous radiative decay, is given by

$$\epsilon_{ki} = \frac{h\nu_{ki}}{4\pi} A_{ki} n_k$$

(2.2)

where $A_{ki}$ is the spontaneous transition probability per unit time (the Einstein co-efficient) and $n_k$ is the population density of the energy state $k$.

If the mean free path of an emitted photon is greater than the dimensions of
the emitting medium, that medium is said to be optically thin. That is, radiation from any part of the medium will escape without being reabsorbed. For optically thin media, induced radiative decay can be neglected. Except for possibly strong transition lines in very dense (or large) plasmas [Hutchinson, 2002, p. 252], most laboratory plasmas are considered to be optically thin.

The population density \( n_k \) is determined by the balance between collisional excitation and de-excitation and radiative decay. A model is sought, whether time-dependent or static, which describes this balance in plasmas such as are studied in this thesis.

For a system in complete radiative thermal equilibrium (CTE), the radiation energy density for all possible transitions equates to the blackbody spectrum for the system temperature. Since CTE really only applies to stellar interiors, a less comprehensive régime is offered in local thermal equilibrium (LTE). In LTE the energy state populations follow the form of the Boltzmann distribution, as is also the case for CTE, but differ in that the radiation is not necessarily thermal. Additionally, collisional transitions dominate the radiative transitions and usually this requires quite a high density. The density is considered high enough if it satisfies the following rule-of-thumb inequality [McWhirter, 1965].

\[
\left( \frac{n_e}{q_e} \right) \gg 10^{19} \left( \frac{T_e}{q_e} \right)^{\frac{1}{2}} \left( \frac{\Delta E}{q_e} \right)^{\frac{3}{2}} \tag{2.3}
\]

where \( \Delta E/q_e \) and \( T_e/q_e \) are the difference in energy levels and temperature, respectively, in electron volts. \( q_e \) is the electron charge and the unit of the electron density \( n_e \) is \( m^{-3} \).

For typical H-1NF argon plasmas (\( T_e \sim 10 \text{ eV}, \Delta E = 2.54 \text{ eV} \) for the 488 nm ArII transition and \( n_e \sim 10^{18} \text{ m}^{-3} \)), the right-hand side evaluates to \( \sim 5 \times 10^{20} \) and, since this condition is not satisfied, LTE as a model régime must be rejected.

Since thermal equilibrium models are unable to determine the energy state populations, a rate-equation model is used. For a low density plasma, the coronal equilibrium model—which assumes that all downward transitions are spontaneous radiation and all upward transitions are the result of collisions—is an appropriate choice. If the upper energy state \( k \) is populated by collisions from the ground state \(^{\dagger}\), then the large ground state population will make this transition the dominant process. In steady-state conditions, the rate of collisional upward transitions can be equated with the photon emission rate due to spontaneous decay (see (2.2)). This balance leads to an expression for the population density of state \( k \). That is,

\[
n_k = n_1 n_e \langle \sigma_{1k} \nu_e \rangle \sum_i A_{ki} \tag{2.4}
\]

\(^{\dagger}\)Denoted as energy state 1.
where $\langle \sigma_{1k} v_e \rangle$ is the collisional excitation rate coefficient for the 1 to $k$ transition and the angled brackets denote an averaging over the velocity distribution function. Here $v_e$ denotes the electron velocity. For a Maxwellian electron distribution at temperature $T_e$, it can be shown that (as an approximation for optically-allowed dipole transitions)

$$\langle \sigma_{1k} v_e \rangle = \frac{f_{1k}K_A}{E_{1k}\sqrt{T_e}} \exp \left( \frac{-E_{1k}}{k_B T_e} \right)$$

(2.5)

where $f_{1k}$ is the 1 $\rightarrow$ $k$ transition oscillator strength, $K_A$ is a constant and $k_B$ is the Boltzmann constant.

Noting (2.2), and substituting (2.4) and (2.5) gives

$$\epsilon_{ki} = \frac{h\nu_{ki}}{4\pi} A_{ki} \frac{n_1 n_e}{\sum_i A_{ki} E_{1k} \sqrt{T_e}} \frac{f_{1k}K_A}{E_{1k}\sqrt{T_e}} \exp \left( \frac{-E_{1k}}{k_B T_e} \right)$$

(2.6)

In terms of plasma parameters, this may be written more simply as

$$\epsilon_{ki} \propto n_1 n_e \chi_{1k}(T_e)$$

(2.7)

Assuming quasi-neutrality ($n_e \approx Z_{\text{eff}} n_i$, where $Z_{\text{eff}}$ is effectively the ‘charge’ on the ions), this leads to

$$\epsilon_{ki} \propto n_e^2 \chi_{1k}(T_e)$$

(2.8)

Hence a measurement of the local emission co-efficient, via tomographic means, can yield information on the electron density or temperature if the other parameter is known.

2.2.2 Ion Temperature and Flow Velocities

As the following discussion deals only with a single electronic transition, the ‘rest frame’ emitted frequency is now denoted as $\nu_0$. Recall that the Doppler effect describes the shift in radiation frequency of a particle radiating in its own frame with frequency $\nu_0$ and moving with velocity $v$, relative to the observer. For small shifts, the observed frequency is given by

$$\nu = \nu_0 \left(1 + \frac{|v|}{c}\right).$$

(2.9)

For observation of an extended radiating medium, along a line in the direction $\hat{l}$, only the component of $v$ in the direction of the line-of-sight contributes to the shift in frequency. Then $|v| = v \cdot \hat{l}$ and (2.9) gives

$$\frac{\nu - \nu_0}{\nu_0} = \frac{v \cdot \hat{l}}{c} = \xi$$

(2.10)
where $\xi$ is a normalised frequency co-ordinate. Figure 2.1 shows the viewing geometry where the observation line is denoted $L$. The distance $p$ from the origin to $L$ is known as the impact parameter and is in the direction $\hat{p} = (\cos \phi, \sin \phi)$, which is orthogonal to $\hat{l}$ as shown.

![Figure 2.1: The viewing geometry for an ideal line-integral through an extended medium.](image)

Consider an emitting plasma with an inhomogeneous, isotropic distribution of ion velocities $v$ specified by the function $f(r, V)$ where $r$ is a position vector in the plasma and $V(r) = v(r)/c$ is the normalised velocity. Note that the distribution may be drifting with velocity $v_D$ (normalised as $V_D$) which is the first moment of the velocity distribution function. The non-drifting function is denoted as $f_0 = f(r, V - V_D)$. The normalised local spectral lineshape at position $r$, as viewed in the direction $\hat{l}$, is given by an integral in velocity-space over the velocity distribution [Shaw, 1987],

$$g(r, \hat{l}; \xi) = \int_{-\infty}^{\infty} f(r, V) \delta(\xi - V \cdot \hat{l}) dV$$  \hspace{1cm} (2.11)

where $dV = dV_x \, dV_y \, dV_z$. When observing in the direction $\hat{l}$, through an arbitrary velocity distribution, the contribution at frequency $\nu$ to the observed intensity spectrum $g(\nu)$ is given by particles with velocities which satisfy (2.10). Thus, if the velocity distribution function $f(r, V)$ is specified in three dimensions, then
the emitting particles which contribute to the local emission spectrum \( g(\xi) \) have velocities which are found in the plane a distance \( \xi \) from the origin and which is orthogonal to \( \hat{l} \). Figure 2.2 illustrates this concept.

![Diagram of Doppler Spectroscopy](image)

Figure 2.2: Only velocities which lie on the plane a distance \( \xi \) from the origin of the three-dimensional velocity distribution function \( f \) contribute to the spectral lineshape \( g \) observed in the direction \( \hat{l} \). Figure courtesy of J. Howard.

The delta function in (2.11) selects the velocities in the distribution that contribute, in the \( \hat{l} \) direction, to the emission. It is apparent from (2.11) that the line-shape is directly and simply related to the ion velocity distribution. However, the measured emission spectrum is line-integrated and is the sum of many varying local lineshapes weighted by the local intensity (or brightness) function \( I_0(r) \) (see (2.6)). If the line-of-sight is regarded as an ideal line-integral, the measurement \( y \) of the emission for the viewing line \( L(p, \phi) \) is given by

\[
y(p, \phi; \xi) = \int_{-\infty}^{\infty} I_0(r) g(r, \hat{l}; \xi) \delta(p - r \cdot \hat{p}) dr \equiv \int_L I_0(r) g(r, \hat{l}; \xi) dl \quad (2.12)
\]

where \( dr = dx dy \). Here, \( I_0(r) \) is the radiated power into a fixed collection solid angle from an infinitessimal volume element of projected area \( \delta A \) along the line-of-sight and in the direction of the detector element. This quantity is colloquially referred to in this thesis as the ‘radiance’ or ‘local emission intensity’.

If the velocity distribution is an isotropic Maxwellian, as is the case for local thermal equilibrium of velocities, and is shifted by some bulk flow \( v_D(r) \), then
the local lineshape is described by a Gaussian profile. That is,

\[ g(r, \hat{l}; \xi) = \frac{1}{\sqrt{\pi V_{th}^2}} \exp \left( -\frac{(\xi - V_D \hat{l})^2}{V_{th}^2} \right) \]  

(2.13)

where \( V_D = v_D(r) / c \) and is related to the Doppler-shift of the central frequency. The local linewidth is described by the normalised thermal velocity of the particle \( V_{th}(r) = \frac{v_{th}}{c} \) which is given by

\[ V_{th} = \sqrt{\frac{2 k_B T_i}{m_i c^2}} \]  

(2.14)

where \( T_i \) and \( m_i \) are the temperature and mass of the ion respectively. However, since the measured emission spectrum is line-integrated, it is not itself a Gaussian but is the sum of many local Gaussians of various linecentre shifts and widths.

For a velocity distribution as just described, it can be shown that, with a sufficient number of line-integrals covering the region of interest, the spatial distribution of the parameters \( I_0, T_i \) and \( v_D \) can be recovered. Scalar tomography techniques, as discussed in Chapter 3, are used for the first two parameters while the solenoidal component only of the intensity-weighted drift velocity vector field can be recovered using vector tomography [Howard, 1996]. §2.3 below outlines the optical methods used in this thesis to measure the parameters of Doppler-broadened emission lines.

2.3 Fourier Transform Spectroscopy

Fourier transform spectroscopy (FTS), where the spectrometer measures the Fourier transform of the spectral lineshape, has many advantages [Bell, 1972]. These include high light throughput and the ability to encode the spectral information in the time domain. This allows a single detector to record the spectrum for a single sight-line as opposed to a grating spectrometer, which requires a detector array to record the dispersed lineshape. A single detector per spatial channel gives scope to perform two-dimensional imaging of an extended light source using a detector array. This is not possible using traditional frequency-domain instruments.

The following discussion is based on [Born and Wolf, 1959; Howard et al., 2003] and deals with Doppler-broadened emission lines. Consider the analytic representation of the field of a quasi-monochromatic light wave as given by

\[ u(t) = A(t) \exp(-i2\pi v_0 t) \]  

(2.15)

where the complex amplitude \( A(t) \) varies slowly compared with \( 2\pi v_0 t \). If a wave is delayed by a time \( \tau \) with respect to a second wave a phase difference
\( \phi (=2\pi \tau v_0) \) is created between the waves. The intensity pattern formed when
the waves are recombined is known as an interferogram. If the waves are nom-
inally of the same amplitude, the intensity of the interferogram as recorded at a
square-law detector can be written as

\[
S(\phi) = \frac{I_y}{2} (1 + \Re[\gamma \exp(i\phi)])
\]  
(2.16)

where \( I_y \) is the line integral of the spectrally-integrated local emission intensity
(zeroth moment of the measured spectrum) and is given by

\[
I_y = \int L I_0(r)dl
\]  
(2.17)

The symbol \( \Re \) in (2.16) denotes that the real part of the complex number is being
taken. The quantity \( \gamma \) is termed the complex degree of optical coherence† (or just
the optical coherence) and is given by \( \gamma = \langle A(t)A^*(t+\tau) \rangle / I_y \) where the angle
brackets denote a time average. The optical coherence is related to the measured
spectrum via the Wiener-Khintchine theorem [Wiener, 1930; Khintchine, 1934] as

\[
\gamma(\phi; \hat{l}) = \frac{1}{I_y} \int_{-\infty}^{\infty} y(\xi; \hat{l}) \exp(i\phi \xi) d\xi
\]  
(2.18)

The local optical coherence is obtained by taking the Fourier transform of the
local lineshape \( g(r, \hat{l}; \xi) \) (defined in (2.11)):

\[
G(r, \hat{l}; \phi) \equiv \mathcal{F}[g(r, \hat{l}; \xi)] = \exp(i\phi V_D \hat{l}) G_0(r, \phi)
\]  
(2.19)

where the contributions from the drift velocity \( V_D \) and the body of the distri-
bution are separated using the shift theorem of the Fourier transform. \( G_0(r, \phi) \) is
a central slice of the Fourier transform of the spherically symmetric distribution
function \( f_0 \) (that is, the distribution function in the local (non-drifting) frame of
reference.)

As the time delay \( \tau \) increases beyond the coherence time of the original wave
(see [Hecht, 1987, p. 264]), there is a decrease in the visibility (or contrast) of the
interference fringes. This loss of coherence is due to the finite spectral bandwidth
of the light and is related to the linewidth. The visibility of the fringes is equi-
valent to the modulus of the optical coherence. Utilising (2.11), (2.12), (2.19) and
(2.18) it can be shown that the fringe visibility can be written as

\[
\zeta = |\gamma(\phi; \hat{l})| = \frac{1}{I_y} \int_{L} G_0(r, \phi)dl
\]  
(2.20)

†\( \gamma \) is equivalent to the cross-correlation function in the theory of stationary random processes.
Note that, due to the Fourier transform given in (2.19), the fringe visibility is independent of the spatially varying drift $V_D$. For a Maxwellian velocity distribution (LTE), the Fourier transform of the lineshape in the local drifting frame becomes
\[
G_0(r, \phi) = \exp \left[ -\frac{\phi^2 V_{th}^2}{4} \right] = \exp \left[ -\frac{T_i(r)}{T_c} \right]
\]
(2.21)
where $T_i(r)$ is the local species temperature. $T_c$ is an instrumental ‘characteristic temperature’ and is given by
\[
k_B T_c = \frac{1}{2} m_i v_c^2
\]
(2.22)
with the ‘characteristic velocity’ $v_c$ set by the total interferometer phase delay $\phi$:
\[
v_c = \frac{2c}{\phi}.
\]
(2.23)
The fringe visibility (2.20) can now be expressed in terms of the temperature as
\[
\zeta = \frac{1}{I_y} \int_L I_0(r) \exp \left[ -\frac{T_i(r)}{T_c} \right] dl
\]
(2.24)
Information regarding the drift velocity is found in the Doppler shift of the central frequency and is manifested in the phase of the interferogram. Provided that $\phi V_D < 1$—which is true if the drift velocity is less than the thermal velocity and a suitable phase delay is chosen—then the change in the interferogram phase is given by
\[
\phi_D = \frac{\phi}{\zeta I_y} \int_L I_0(r) \exp \left[ -\frac{T_i(r)}{T_c} \right] V_D \hat{l} dl
\]
(2.25)

2.3.1 Measurement Principle

The phase delay $\phi$ in (2.16) can be obtained by splitting an input light beam and having the resulting two beams then traverse separate paths, one of which may have a different length, before recombining the beams at a detector. The basic layout of this scheme (a Michelson interferometer) is shown in Figure 2.3, with the displacement of the mirror in the measurement arm creating the time delay $\tau$. The interferometer is desirable as the instrument used to create the phase delay due to its high étendue (optical throughput) [Jacquinot, 1954, 1960; Van Heel, 1967].

To measure the contrast $\zeta$ and phase shift $\phi_D$ of the interferogram the phase delay $\phi$ is modulated sinusoidally about some initial delay $\phi_0$—which is set by the path length delay $\tau$—with a frequency $\Omega$ and amplitude $\phi_1$. This gives the total delay as
\[
\phi = \phi_0 + \phi_1 \sin \Omega t
\]
(2.26)
Using some simple trigonometric identities, (2.16) then becomes
\[ S(\phi) = I_y [1 + \tilde{\gamma}_c \cos(\phi_1 \sin \Omega t) - \tilde{\gamma}_q \sin(\phi_1 \sin \Omega t)] \] (2.27)
where \( \tilde{\gamma} = \tilde{\gamma}_c + i \tilde{\gamma}_q = |\gamma| \exp(i\phi_0 + i\phi_D) \). The Bessel expansion shows that \( \tilde{\gamma}_c \) and \( \tilde{\gamma}_q \) are proportional to the even and odd harmonics of the modulation frequency, respectively. This approach allows line-integrated versions of the velocity distribution function parameters \( I_0, T_i \) and \( v_D \) to be directly related to \( I_y, \tilde{\gamma}_c \) and \( \tilde{\gamma}_q \), which are carried by the DC level and the even and odd harmonics of \( \Omega \).

Figure 2.4 illustrates the measurement concept, where the visibility of the interferogram fringes is recorded by tracing out the fringe amplitude via the modulation and the shift in the interferogram phase is apparent by a shift in the operating point of the modulation. The signals generated and the harmonics of those signals are also shown.

2.4 Modulated Optical Solid State Spectrometer

This section describes a robust, compact implementation of the concepts discussed in §2.3 and which can easily be adapted for multi-channel operation.

Michelson interferometers operating in the visible spectrum can be very susceptible to vibrational and acoustic noise interference due to the small wavelength and the separation of the light beams before recombination. The Modu-
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Figure 2.4: Simulated interferograms showing the effect on the interferogram phase of a shift in line-centre frequency (exaggerated for illustrative purposes). The dashed vertical line is the operating point and corresponds to the interferometer delay while the bold section is the portion of the interferogram traced out by the modulating path length. Information regarding $v_D$ is carried in the odd harmonics. The fringe contrast (visibility) varies with changes in the temperature of the emitting species with the even harmonics conveying this information.
lated Optical Solid-State (MOSS) spectrometer is essentially a Michelson interferometer that uses a solid-state birefringent optical element to produce the phase delay $\phi_0$.

The extraordinary (‘$E$’) and ordinary (‘$O$’) axes of the birefringent material are at right angles to each other, have different refractive indices and are usually oriented normal to the light propagation direction. The phase speed for light propagating through a material of refractive index $n$ is $v_\phi = c/n$. The $E$ axis is known as the ‘fast’ (or Z) axis due to its lower refractive index. For monochromatic light propagating perpendicular to the fast axis of a birefringent plate of thickness $L$ there is a phase difference between the $E$ and $O$ waves given by

$$\phi_0 = \frac{2\pi v_0}{c} (L n_E - L n_O)$$

(2.28)

where $n_E$ and $n_O$ are the extraordinary and ordinary refractive indices respectively. In terms of the crystal birefringence,

$$B = n_E - n_O$$

(2.29)

(2.28) can be written more succinctly as

$$\phi_0 = \frac{2\pi v_0}{c} L B = 2\pi v_0 \tau = 2\pi N$$

(2.30)

where $\tau$ is the time delay introduced between the waves and $N$ is the order of interference or the total difference in the number of waves.

By polarising the incident light at 45° to the $E$ axis the electric field is resolved into equal amplitude $E$ and $O$ waves by the birefringent element and are phase-shifted according to (2.30). A second polariser, placed after the birefringent material, transmits the components of the orthogonal $E$ and $O$ waves that are in the same polarisation state. These components then interfere to produce the intensity variations at the detector. The polarisation sequence is depicted in Figure 2.5.

It is a property of some birefringent materials that they are also electro-optic, meaning that the refractive index changes in the presence of an electric field. Thus, the effective path length difference between the $E$ and $O$ waves can be varied by modulating the birefringence using an oscillating voltage applied across the crystal (parallel to the $E$ or $O$ axis) and transverse to the light propagation direction. This gives a robust method, not reliant on critical alignment, for producing the required optical path length modulation of (2.26).

The electro-optic, birefringent element is a $Y$-cut lithium niobate (LiNbO$_3$) crystal whose fast axis is at 45° to the plane of polarisation. LiNbO$_3$ is an artificial crystal which—in addition to the two mentioned properties—also exhibits pyro-, piezo- and ferroelectric effects as well as photo-elasticity [Weiss and Gaylord,
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Plane polarised light is resolved into components at 45°. A slow wave is delayed with respect to a fast wave in a birefringent wave plate. Components interfere at a final polarising cube, and one component is selected.

Figure 2.5: The polarisation sequence to produce an interferogram using an electro-optic and birefringent element with suitable polarisers. Figure courtesy of C. Michael.
It has $n_E = 2.25$ and $n_O = 2.35$ for a birefringence of $B \approx -0.1$. The typical crystal aperture is 40 mm, while the length $L$ is in the range 5-100 mm depending on the expected coherence of the spectral line. Narrow lines require a larger delay in order to produce a significant change in the interferogram fringe contrast.

The additional small delay modulation $\phi_1 \sin(\Omega t)$ of amplitude $\phi_1 = \pi/2$ (see (2.26)) is obtained by electro-optically modulating the crystal at a frequency well above or below its natural mechanical resonant frequency (which is typically around 100 kHz). This is achieved by placing electrodes across the $Z$ faces of the crystal, transverse to the light direction. The phase difference $\phi_1$ is given by [Kaminow, 1974]

$$\phi_1 = \frac{\pi L V_0}{d c} (n_E^3 r_{33} - n_O^3 r_{13})$$

(2.31)

Here $d$ is the width of the crystal (see Figure 2.5). $r_{33} = 28.8 \times 10^{-12}$ m/V and $r_{13} = 7.7 \times 10^{-12}$ m/V are electro-optic tensor co-efficients. Voltages in the range 1.5–2.5 kV are required to produce a change in the refractive indices. It can be shown that a phase modulation of $\sim 140^\circ$ gives optimal signal-to-noise ratio for this particular encoding method. It is also possible to apply a DC field to fine tune $\phi_0$.

In practice, the optics are enclosed in a solid light-tight enclosure which allows a number of ports for input and output of various light beams. The input light can be coupled directly from the source through appropriate apertures or by using optical fibres and a collimating lens to produce parallel light. An interference filter isolates the desired emission line from the remainder of the visible spectrum before the interferogram is detected using a photomultiplier tube. A polarising beamsplitter cube is used as the initial polariser. This allows the transmission of the horizontally polarised component of the input light and the reflected component can be absorbed or relayed to another device (e.g., another MOSS or detector). Through the opposite port of the polarising cube another light source, such as a calibration laser, can be introduced. The second polariser is a high efficiency, thin film type. A typical layout of the optical elements of a MOSS spectrometer is shown in Figure 2.6.

The MOSS spectrometer can be adapted easily for imaging applications by the addition of a multi-channel detector and imaging lens after the final polariser. §4.2.1 describes in more detail the implementation of the spectrometer as used for experiments covered in this thesis.

## 2.5 MOSS Spectrometer Properties

It is important to know the spectral response of the spectrometer as well as more detailed information including the instrument dispersion, the instrument contrast (analogous to a grating spectrometer’s slit function) and the absolute
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Figure 2.6: The optical layout for the Modulated Optical Solid-State (MOSS) spectrometer.

delay (wavelength calibration). The light throughput, or étendue, as well as the sensitivity of the linewidth and lineshift estimates to photon noise, are also considered. The following discussion is based largely on [Howard, 2002].

2.5.1 Spectral response

The spectral response shown in Figure 2.7 was measured with white light input using a high resolution grating spectrometer. The ArII 488 nm plasma emission line is also shown. The spacing between fringe peaks is a rough measure of the spectral resolution $\nu_0/\Delta \nu = N$ and is proportional to the phase delay $\phi_0$. Recall that the delay is chosen to be comparable to the linewidth. The fringe depth (visibility or contrast) is governed by the instrument contrast and the spectral width of the radiation (in this case, the grating spectrometer resolution). The apodizing envelope is a result of the interference filter that is used to reject light outside the passband of interest. Modulating the phase delay shifts the interference pattern and allows the intensity variations, due to changes in the centre frequency and/or spectral width, to be determined.
Figure 2.7: Overlay of plasma spectrum in the vicinity of the 488 nm ArII line and the Moss spectral response function. The interferogram is apodized by the narrow bandpass interference filter that is used to isolate the spectral line from the plasma background. The period of the interferogram is inversely proportional to the MOSS interferometer delay (in this case, the crystal thickness is 24 mm) and is chosen to be comparable with the Doppler-broadened width of the emission line.
2. Doppler Spectroscopy

2.5.2 Phase delay dispersion

In general, the crystal birefringence is a function of frequency $B(\nu)$, and, because interferometers integrate over all frequency components (see (2.18)), it is necessary to take account of the dispersive properties when evaluating the effective (or group) phase delay. The delay at frequency $\nu = \nu_0 + \delta\nu$, where $\delta\nu$ is a small shift, is given approximately by

$$\phi(\xi) \approx \phi_0 + \kappa \phi_0 \xi$$

where

$$\kappa = 1 + \frac{\nu_0}{B_0} \frac{\partial B}{\partial \nu}$$

(2.33)

accounts for the frequency dependence of the birefringence. The Sellmeier equations give $n_E$ and $n_O$ as functions of wavelength and for lithium niobate are [Weiss and Gaylord, 1985]

$$n_E^2(\lambda) = 4.5820 + \frac{0.099169}{(\lambda^2 - 0.04443)} - 0.021950 \lambda^2$$

$$n_O^2(\lambda) = 4.9048 + \frac{0.11768}{(\lambda^2 - 0.04750)} - 0.027169 \lambda^2$$

(2.34)

where the wavelength $\lambda$ is in $\mu$m. Thus the phase delay due to the crystal must be scaled by the factor $\kappa$ and at 488 nm the correction term is $\kappa = 1.54$. This has a flow-on effect for measured contrast and phase shifts such that

$$v_c \Rightarrow \frac{v_c}{\kappa}$$

$$\phi_D \Rightarrow \kappa \phi_D$$

(2.35)

Figure 2.8 shows the wavelength dependence of the birefringence and the group birefringence $\kappa B$. Experimental estimates of the group birefringence determined by fitting the spectral response† at a number of points in the visible spectrum are also shown.

2.5.3 Instrument contrast

Including the delay dispersion, the signals at the output of the interferometer (2.27) become

$$S_\pm = I_0 \pm I_0 \xi \cos [\phi_0 (1 + \kappa V_D, \hat{l}) + \phi_1 \sin (\Omega t)]$$

(2.36)

where $\phi_0 + \delta\phi = \phi_0 (1 + \kappa V_D, \hat{l})$ and $V_D, \hat{l}$ is the component of the normalised flow velocity in the direction of view as specified in (2.13). The total fringe visibility is given by $| \gamma | \equiv \xi = \tilde{\xi} \xi$. The term $\tilde{\xi} \equiv \exp (-T_i / T_c)$, where $T_c$ is given by (2.22) and uses the modified $v_c$ in (2.35), is the change in fringe contrast related

†As for Figure 2.7 but with the interference filter removed.
2. Doppler Spectroscopy

Figure 2.8: Plot of the wavelength dependence of the birefringence for lithium niobate obtained using the Sellmeier equations. The dashed curve is the monochromatic birefringence while the solid curve shows the calculated effective birefringence brought about by inclusion of the refractive index dispersion. The square symbols are measurements obtained as discussed in [Howard, 2002].

to the source temperature. The instrumental fringe contrast $\zeta_I$, which is analogous to the familiar slit function for grating spectrometers, is determined by the solid angle of light collected and optical imperfections in the spectrometer. It is a constant factor and can be represented with an equivalent ‘instrument temperature’ $T_I$ where $\zeta_I = \exp \left( -T_I/T_c \right)$. Because the interferogram is monitored at a single fixed delay, the source temperature $T_i$ can be obtained from the measured fringe contrast through subtraction of exponents which are proportional to the measured and instrumental temperatures. This compares favourably to grating spectrometers which require noise-prone deconvolution of the instrument profile.

For $T_i = T_c$ the contrast reduces to $1/e$ of its maximum. This condition is used as a measure of the instrument resolving power $R_C$ [Thorne, 1988] and (2.22) gives

$$R_C \equiv \frac{\nu_0}{\Delta \nu} = \pi N_{\text{eff}}$$

(2.37)

where $N_{\text{eff}} = \phi_{\text{eff}}/2\pi = \kappa\phi/2\pi$.

The variation of fringe contrast $\zeta_s$ with phase delay $\phi_0$ for argon ion temperatures in the range 10 to 100 eV is shown in Figure 2.9. The vertical line corresponds to the delay introduced by the LiNbO$_3$ crystal at 488 nm and represents a characteristic temperature $T_c \approx 14$ eV. Observe that a 90% variation in fringe
contrast represents roughly an order of magnitude variation in temperature.

The instrument fringe contrast (or temperature) is limited fundamentally by the range of angles subtended by the source, or the ‘field-of-view’. For small angles of incidence \( \theta \), the delay difference between the characteristic \( E \) and \( O \) waves after propagation through the birefringent crystal is given by [Steel, 1967]

\[
\phi \approx \phi_0 \left[ 1 - \frac{\theta^2}{2n_O} \left( \frac{\cos^2 \theta}{n_O} - \frac{\sin^2 \theta}{n_E} \right) \right]
\]

(2.38)

where \( \theta \) is the azimuthal angle from the optic axis. The mutual coherence of the recombining waves at the detector is obtained by integrating over the source irradiance distribution (van Cittert-Zernike theorem). For a uniform, monochromatic circular source with solid angle \( \Omega = \pi \theta_{\text{max}}^2 \), for \( \theta \) small, the resulting instrument coherence is given by

\[
\gamma_I = \left( 1 - \frac{\Omega^2 N^2}{12n^4} + \ldots \right) \exp \left[ -i\phi_0 \left( 1 - \frac{B\Omega}{8\pi n^4} + \ldots \right) \right]
\]

(2.39)

meaning that the instrument temperature evaluates to

\[
T_I \approx T_c \left( \frac{\Omega^2 N^2}{12n^4} \right)
\]

(2.40)
where $\bar{n}$ is the mean refractive index. The reduction in contrast due to the integration over the solid angle is negligible provided the parameter $\varepsilon = \Omega N/\bar{n}^2$ is small. For the stated assumptions this parameter can be re-written as

$$\varepsilon = \frac{\pi N \theta^2}{\bar{n}^2}$$ \hspace{1cm} (2.41)

Hence there is the usual trade-off between resolving power $R_C(\sim N)$ and collection solid angle, but it is mitigated by a gain factor of $\bar{n}^2/\sqrt{2}$ compared with a free-space Michelson interferometer. Widening the field-of-view also gives a further gain [Steel, 1967; Michael et al., 2001]. Note the strong variation in the phase delay as a function of incident angles which is shown in the top left of Figure 2.10. To improve the contrast a second crystal is placed after the first and with its fast axis orthogonal to that of the initial crystal. A half-wave plate is inserted between the crystals, as shown in Figure 2.11, to rotate the polarisation states emerging from the first crystal to match the orientation of the second. Crossing the patterns in this way results in a very flat phase variation.

![Figure 2.10: The addition to the phase spatial variation of its complement (90° rotation) creates a flat phase shift for a large field of view. Figure courtesy of C. Michael.](image)

For a typical implementation of the spectrometer as discussed in §4.2.1, the use of crystals of total thickness 50 mm produces an effective delay $N_{\text{eff}} \sim 16000$
waves (from (2.30)) and a nominal wavelength resolution of 0.01 nm at 488 nm from (2.37). Light input, via optical fibres of diameter 1 mm and numerical aperture NA $\simeq 0.3$, is collimated by an $f/# = 1.8$ lens with focal length of 85 mm. Noting (2.42) gives the resulting collimated beam a half angle of $\theta_{\text{max}} \sim 0.006$ giving an evaluation of (2.41) as $\varepsilon \approx 0.34$. Thus (2.40) yields $T_I$ as $\approx 1\%$ of $T_c$ and the decrement in fringe contrast due to the field-of-view is small. However, imperfections in the crystals degrade the contrast further and may include such things as unequal surface reflection losses for different polarisation components, crystal birefringence inhomogeneities and non-parallelism of surfaces. Other factors such as non-unity extinction ratio for the polarising cubes and misalignment of the optical components may also contribute to a measured instrument contrast which is lower than (2.40) indicates. It is generally in the range $\zeta_I = \exp\left(-\frac{T_I}{T_c}\right) \sim 70 - 90\%$ at 488 nm corresponding to $T_I/T_c \sim 0.35 - 0.1$.

The instrument contrast can be compromised further by the combined action of piezo-electric and acousto-optic effects at high electric field strengths applied to the crystal. These distortions, which become significant for modulation frequencies near the crystal acoustic resonances, result in a variation of the instrument contrast at the modulation frequency with the effective decrease being a function of $\phi_0$. However, for the typical drive voltages used in experiments described in this thesis, measurements of the instrument contrast variation with $\phi_0$ have shown this to be a small effect (standard deviation of 1% of the mean $\zeta_I$) [Howard, 2002].
2.5.4 Light throughput

To better understand the Jacquinot (or light throughput) advantage the implementation of the MOSS spectrometer, discussed in §4.2.1, is compared with a grating spectrometer of equivalent resolving power. The étendue of an optical component is given by

\[ \hat{E} = A\Omega \]  

(2.42)

where \( A \) is the area of the element aperture and \( \Omega \) is the collection solid angle subtended by the source at the optic axis.

A standard \( F/5 \) grating spectrometer of focal length 0.5 m equipped with a 2400 lines/mm grating operating in first order and using a 25\( \mu \)m slit width will give \( R_C \sim 5000 \). For a slit height equal to the 40 mm clear diameter of the lithium niobate crystal\(^\dagger\) the grating spectrometer étendue is \( \hat{E}_G = A_G\Omega_G = 0.016 \text{Sr mm}^2 \), where \( A_G = 25\mu m \times 40 \text{mm} \) is the slit area and \( \Omega_G \) is the collection solid angle set by the spectrometer F-number. The limiting field-of-view for the MOSS spectrometer is obtained by setting \( T_i = T_c \) in (2.40) to obtain for the MOSS étendue \( \hat{E}_M = 0.9 \text{Sr mm}^2 \sim 120\hat{E}_G \). Note that with field widening and larger apertures, this étendue disparity is greater than previously reported [Howard, 2002]. Ultimately though, the amount of light collected from the source may be limited by spatial resolution considerations rather than the étendue of the spectrometer.

2.5.5 Instrument delay

For absolute flow velocity measurements, the calibration of the interferogram phase requires comparison with light from a standard source. This requirement is because of uncertainties in the determination of the absolute crystal delay \( \phi_0 \) due to, for example, thermal drifts of the refractive indices. Preferably the source should generate an identical emission, with lasers or spectral calibration lamps particularly suitable. This ‘instrument phase’ \( \phi_I \) also requires that light from both plasma and calibration sources have a common geometry through the spectrometer so that this phase offset is compensated. This can be arranged, for example, by using lens-coupled optical fibres to relay light from the standard source to the spectrometer. In the absence of a suitable calibration source, or careful temperature stabilisation, only relatively fast temporal changes in the mean normalised flow speed \( \langle V_D \rangle \) can be measured accurately. The absolute value of these changes is easily calculated from the measured phase shift and the estimated initial phase offset \( \phi_0 \).

\(^\dagger\)The primary limiting aperture for this implementation of the MOSS spectrometer.
2. Doppler Spectroscopy

2.5.6 Noise sensitivity

The sensitivity of the MOSS signal to changes in the spectral linewidth relies on the instrument temperature. Without regard to the details of the demodulation scheme, a rough estimate of the sensitivity of the signal described in (2.36) to temperature changes is given by

\[ \alpha_T \equiv \max_\phi \left( \frac{\partial S}{\partial T_i} \right) = -\frac{I_0}{T_c} \xi \]  

(2.43)

The sensitivity deteriorates when the instrument fringe contrast is poor and attains a maximum with respect to \(T_c\) when \(T_i + T_1 = T_c\) (provided \(T_1 < T_c\)).

The ability to distinguish changes in light intensity arising from temperature variations is determined, at best, by the Poisson noise of the intensity. The signal sensitivity to this noise, averaged over a modulation period, is given by \(\bar{n}_I \equiv \langle \frac{\partial S}{\partial I_0} \rangle = 1\). The sensitivity of the derived temperature to photon shot noise is estimated by setting \(dS = \bar{n}_I dI_0 + \alpha_T dT_i = 0\) to obtain

\[ \frac{dI_0}{I_0} = \xi \frac{dT_i}{T_c}. \]  

(2.44)

Thus, for a given light noise level, the absolute uncertainty in the derived source temperature increases as fringe visibility decreases. (2.44) can be re-written as

\[ \frac{dT_i}{T_i} = k_T(r) \frac{dI_0}{I_0}, \]  

(2.45)

where \(r = T_i/T_c\) and \(k_T(r) = \exp r/r\) is the factor relating the fractional light noise level to the relative uncertainty in the inferred species temperature. The behaviour of \(k_T(r)\) as a function of normalised species temperature \(r\) is shown in Figure 2.12.

The MOSS spectrometer is most sensitive to relative changes in \(T_i\) when \(T_i = T_c\). For the H-1 system, and at \(T_i = T_c = 36\) eV, intensity noise on the order of 5% corresponds to temperature uncertainties of about 4 eV. The divergence when \(r\) is small is due to the exponential relationship between measured contrast and source temperature. The dynamic range can be increased effectively by using multiple birefringent plates mutually aligned at 45°, as is the case for a field-widened system.

To maintain an optimum signal-to-noise ratio (SNR), higher source temperatures require a higher value for \(T_c\). Combining (2.22), (2.23) and (2.30) indicates that \(T_c\) is inversely proportional to \(L^2\) so that the voltage required to obtain a modulation depth of at least \(\pi/2\) is proportional to \(T_c^{1/2}\). This difficulty can be overcome through a reduction of the crystal dimensions perpendicular to the optic axis, but at the cost of light throughput. An alternative approach is to combine
crystals so that their individual delays subtract while the modulation voltages are of opposite polarity. In this case, however, the instrument temperature is also increased unless a half wave matching plate (field widening) is used.

The sensitivity to flow speed changes is estimated by

\[
\alpha_v \equiv \max_\phi \left( \frac{\partial S}{\partial v_D} \right) = I_0 \zeta \kappa \phi
\]

With respect to the characteristic temperature \(T_c\), the maximum sensitivity to flow speed variations occurs when \(T_i + T_i = T_c/2\). This optimum value is obtained because the increase in sensitivity to \(v_D\) with delay \(\phi\) is opposed by a decrease in the contrast \(\zeta\).

When the temperature is constant, the ability to resolve small flow velocity changes depends on the light signal to noise ratio. Setting \(dS = \bar{\alpha} dI_0 + \alpha_v d\nu_D = 0\) gives

\[
\frac{d\nu_D}{\nu_{th}} = k_v(r) \frac{dI_0}{I_0}
\]

where the scaling factor \(k_v = \exp(r/2\sqrt{r})\), which is analogous to \(k_T\), is also shown in Figure 2.12. For argon at \(T_i = T_c/2 = 18\,eV\), \(\nu_{th} \approx 9 \times 10^3\,m/s\) which implies a velocity resolution \(\delta \nu_D \approx 600\,m/s\) for \(\delta I_0/I_0 = .05\). Simulations [Howard, 2002] confirming the form of the noise sensitivity factors displayed in

Figure 2.12: Dependence of the scaling functions \(k_T(r)\) and \(k_v(r)\) relating the fractional light noise and the relative uncertainty in the inferred temperature \(T_i\) and drift velocity \(v_D\) as a function of the normalised temperature \(r = T_i/T_c\).
Figure 2.12 also indicate a slow dependence on the modulation depth $\phi_1 \gtrsim \pi/2$ which is optimum for $\phi_1 \sim 130^\circ$ [Sasaki and Okazaki, 1986].

In summary, the MOSS is an ideal instrument for measuring the ion emissivity, ion temperature and bulk flow velocities in the radiation-dominated low-temperature plasmas of the H-1NF heliac.