Numerical modeling of low-pressure plasmas: applications to electric double layers

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Doctor of Philosophy of the Australian National University
and
Docteur de l’Université Paul Sabatier
by
Albert Meige

Research Supervisors
R. W. Boswell
J.-P. Boeuf

Research Advisors
C. Charles
G. J. M. Hagelaar

External Examiners
P. Chabert
J. Verboncoeur

Space Plasma Power and Propulsion (SP3),
Research School of Physical Sciences and Engineering,
the Australian National University,
Canberra, ACT, 0200, Australia

Centre de Physique des Plasmas et de leurs Applications de Toulouse (CPAT),
Université Paul Sabatier,
118 route de Narbonne, Toulouse Cedex 31062, France
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Albert Meige
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Abstract

Inductive plasmas are simulated by using a one-dimensional particle-in-cell simulation including Monte Carlo collision techniques (pic/mcc). To model inductive heating, a non-uniform radio-frequency (rf) electric field, perpendicular to the electron motion is included into the classical particle-in-cell scheme. The inductive plasma pic simulation is used to confirm recent experimental results that electric double layers can form in current-free plasmas. These results differ from previous experimental or simulation systems where the double layers are driven by a current or by imposed potential differences. The formation of a super-sonic ion beam, resulting from the ions accelerated through the potential drop of the double layer and predicted by the pic simulation is confirmed with nonperturbative laser-induced fluorescence measurements of ion flow. It is shown that at low pressure, where the electron mean free path is of the order of, or greater than the system length, the electron energy distribution function (eedf) is close to Maxwellian, except for its tail which is depleted at energies higher than the plasma potential. Evidence supporting that this depletion is mostly due to the high-energy electrons escaping to the walls is given.

A new hybrid simulation scheme (particle ions and Boltzmann/particle electrons), accounting for non-Maxwellian eedf and self-consistently simulating low-pressure high-density plasmas at low computational cost is proposed. Results obtained with the “improved” hybrid model are in much better agreement with the full pic simulation than the classical non self-consistent hybrid model. This model is used to simulate electronegative plasmas and to provide evidence supporting the fact that propagating double layers may spontaneously form in electronegative plasmas. It is shown that critical parameters of the simulation were very much aligned with critical parameters of the experiment.
Résumé

Un modèle *particle-in-cell / Monte Carlo collisions* (pic/mcc) unidimensionnel est utilisé pour simuler un plasma inductif. Un champ électrique radiofréquence (rf) est utilisé pour modéliser le chauffage inductif. L'amplitude du champ est non-uniforme et sa direction perpendiculaire à celle du déplacement des électrons. Ce modèle de plasma inductif permet de confirmer de récents résultats expérimentaux démontrant la possibilité de former des doubles couches électriques au sein de plasmas sans courant. Les doubles couches étudiées par le passé, aussi bien numériquement qu’expérimentalement, ont toujours été imposées par différence de potentiel ou en forçant un courant électrique dans le plasma. C’est en ce sens que les résultats présentés ici diffèrent de ceux précédemment reportés. La simulation prédit la formation d’un faisceau d’ions supersoniques résultant des ions accélérés par le saut de potentiel de la double couche. L’existence de ce faisceau d’ions supersoniques est confirmée par fluorescence induite par laser (nonperturbative laser-induced fluorescence). La simulation montre aussi qu’à basse pression, lorsque le libre parcours moyen des électrons est du même ordre de grandeur ou plus grand que le système, la fonction de distribution en énergie des électrons (eedf) est quasi-Maxwellienne, à l’exception de sa queue, dépeuplée pour des énergies supérieures au potentiel plasma. Ce dépeuplement est principalement dû à la perte aux parois des électrons les plus rapides.

Un nouveau schéma de simulation hybride (ions particulières et électrons particulières et Boltzmann), permettant de simuler des plasmas hautes pressions et hautes densités, en des temps de calculs relativement faibles, est proposé. Les résultats obtenus avec ce modèle hybride “amélioré” sont bien plus proches de ceux d’une simulation pic, que le sont ceux d’une simulation hybride classique. Ce modèle est appliqué à la simulation de décharges électronégatives et confirme des résultats expérimentaux démontrant la possibilité de formation de doubles couches propagatives. En particulier, les paramètres critiques contrôlant cette formation dans la simulation corroborent ceux de l’expérience.
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