The Australian National University
Centre for Economic Policy Research
DISCUSSION PAPER

'This Arbitrary Rearrangement of Riches': an Alternative Theory of the Costliness of Inflation

William Coleman

DISCUSSION PAPER NO. 553
May 2007

ISSN: 1442-8636
ISBN: 1 921262 24 9

William Coleman, School of Economics, The Australian National University

Acknowledgements: The author is indebted to the comments of the participants of the seminar of the Economics Department of Victoria University of Wellington; the seminar of the Reserve Bank of New Zealand; and the seminar of the School of Economics of the Australian National University. The usual disavowals apply.

ABSTRACT

This paper develops a model of the costliness of inflation that places the locus of costs in the bond market, rather than the money market. It argues that inflation is costly on account on the contraction of the bond market caused by the riskiness of inflation. The theory is premised upon the social function of bond markets as consisting of the transference of technological risk from those economic interests where risk is most concentrated (and so most painful) to interests where it is less concentrated (and so less painful). Using a Ramsey-Solow model with decision-makers maximising expected utility from consumption and real balances, the paper argues that unpredictable inflation impedes this useful transfer in risks secured by the bond market. Unpredictable inflation makes debt most costly when income is the most needed by debtors (since when the *ex post* real interest is highest, the debtor is in consequence the poorest), and credit the most remunerative when income is the least needed by creditors (since when the *ex post* real interest is the highest, the creditor is as a consequence richest). The upshot of these disincentives to borrow and lend is that less risk is transferred. Thus unpredictable inflation reduces the socially beneficial transfer of risks that a bond market secures.

JEL codes: E31 E43 E61

Keywords: inflation cost, inflation risk, debt,

Many costs of inflation have been identified by economic theorists (see Dowd 1994 for a survey). This paper supplies another.

This paper (drawn from Coleman 2007) presents a theory of the cost of inflation that turns on the contraction of the bond market caused by the riskiness of inflation, and the reduction in the socially beneficial transfer of risk between economic interests that results.

The analysis rests critically on a certain property of Ramsey-Solow style model: that in such a model bond markets transfer technological risk from those factors of production where risk weighs heaviest (and so is most painful) to factors where risk weighs less heavily (and so is less painful), and thereby produce a social gain. To illustrate: if we suppose that capital-owners are more vulnerable to technological risk than workers, then debt allows workers to borrow from capital-owners and so purchase capital, and capital-owners to disburden themselves of capital and shift into bonds. This trade in risky capital secured by bond markets allows capitalists to hedge, and workers to speculate, to the mutual benefit of both. Capitalists reduce the risk on capital: workers get the premium of capital.

However, the successful transfer of technological risk by bond markets in a Ramsey-Solow type model does suppose that bonds are 'real' (ie inflation indexed). But real bonds are a marginal and atypical feature of the real world. We are left to ask, 'What if all borrowing and lending was conducted through money loans?' How successfully would risk be transferred if real bonds did not exist?

The paper demonstrates that if all borrowing and lending is conducted through money loans then the useful trade in risks is impeded by unpredictable inflation, on account of the disincentives that unpredictability creates for both borrowers and lenders. For capital-owners, lending to workers to finance worker's purchase of capital does not now reduce the capitalists 'total risk' as much as it did before, because the capitalist lender is now exposed to being injured by episodes of unexpectedly high inflation. At the same time, borrowing now increases the workers total risk more than it did before, because worker debtors are now exposed to being injured by episodes of unexpectedly low inflation.[1] This upshot is that less risk is traded. Less risk is transferred from where it is most painful to where it is least painful. Here lies a cost of inflation.

The paper begins by explicating the role of real bond markets in the welfare efficient management of technology shocks. It then assesses the performance of that role when bonds are nominal, not real. It concludes that risky inflation makes the effectiveness of the performance of that role by nominal bonds less than complete, and the incompleteness of that role is the cost of inflation. The paper then registers some qualifications and limitations to this claim, before co-ordinating its conclusions with other theories of costliness; outlining the congruence of the paper's analysis with some earlier understanding of the cost of inflation, and its incongruence with some other conceptions. The paper ends with an essay in the quantification

[1] Debtors also face the welcome 'risk' of unexpectedly high inflation. But the upsides and downsides of inflation are asymmetrical in impact, owing to the different marginal utilities associated with high and low inflation outcomes. For debtors, the nasty thing about inflation risk is that it makes debt most costly when income is the most needed by debtors; for when the *ex post* real interest is highest, the debtor is (as a consequence) the poorest. Similarly for creditors, inflation risk means credit is the most remunerative when income is the least needed by creditors; for when the *ex post* real interest is highest, the creditor is (as a consequence) the richest.

The redundancy of debt in the absence of risk.

In the standard interpretation, the pre-eminent social function of debt is to relocate consumption across time. In a phrase, 'consumption smoothing'. The present analysis contends that the social function of debt is not to smooth consumption; it is not to relocate consumption over time. The social function of debt is, instead, to deal with risk. It transfers risk from those more risk vulnerable, and to those less risk vulnerable, so that the exposure to risk is equalised across all members of society. This process might be called 'risk evening'.

The paper's argument for these contentions consists in drawing a contrast between the redundancy of debt under a perfect foresight Ramsey-Solow model, with the usefulness of debt under risk. This section, therefore, demonstrates that in a standard work-horse of economic analysis - a Ramsey-Solow growth model – debt is functionless the absence of risk.

Consider the familiar Ramsey-Solow with the standard array of assumptions: identical homothetic preferences; an exogenous labour supply; a two-factor, constant returns to scale production function; perfectly competitive factor and product markets; and general equilibrium. In the absence of risk, the model's foundations may be characterised by this suite of equations,

$$U = \frac{C^{\alpha}}{\alpha} + \frac{C_1^{\alpha}}{\alpha[1+\delta]} + \ldots \qquad \mathrm{Y = C + I} \qquad \mathrm{K_1 = K + I} \qquad Y = y(\frac{K}{L})L$$

The critical optimisation condition of decision-makers arises from the trade-off between consumption and capital, and implies a linkage between current marginal utility, future marginal utility, and the rate of profit, ρ,

$$U_C = [1+\rho]U_{C1} \qquad \text{capital}$$

where $$\rho = \frac{\partial Y_1}{\partial K_1}$$

Thus the explanation of the rate of profit is the familiar Fisherian one of the reconciliation of time preference and the marginal productivity of capital.

A theory of the rate of interest is easily constructed. Suppose there exists a one period bond that pays a contractual real rate of interest, r. The optimisation condition involving the trade-off between consumption and bonds would be,

$$U_C = [1+r]U_{C1}$$

A comparison of the optimisation conditions allows us to infer,

$$r = \rho$$

Thus the rate of interest equals the rate of profit, which is, in turn, a matter of time preference and productivity of capital.

But the key point is that there is no theory of debt in the riskless Ramsey-Solow model. The model provides no reason as to why anyone would wish own, or owe, debt. To press the point: what is the quantity of debt in the model? The quantity is completely indeterminate. It is indeterminate since bonds have no function, either socially or privately. There is no benefit to individuals in buying or selling them; anything a bond can do, capital can do just as well. Do you wish to transfer consumption into the future? Buy capital today. Do you wish to transfer consumption from the future to the present? Sell capital today. (And if you do not have any: short sell it).[2] Debt is not required for intertemporal trade in consumption. Bonds are a fifth wheel.

Neither does debt confer any benefit to 'society as whole'; the system is welfare efficient without it. The welfare efficiency is established from the equality of all persons marginal rate of intertemporal substitution with the marginal rate of intertemporal transformation, which may be obtained from the first order conditions for capital, without any reference to debt markets.

The upshot is that most extreme anti-usury laws would not have the smallest significance for the welfare efficiency of the model economy. If such laws completely wiped out any debt that might exist, the economy remains pareto-efficient.

---

2 Is there any difference between short-selling capital and selling a bond? There seems no operational difference in a riskless world. But we take this to say that in a riskless world short-selling capital can do the job of bonds.

Risk as the basis of debt

Debt acquires a usefulness in the presence of riskiness in technology.[3] We show that they, in certain circumstances, provide perfect insurance against profit risk caused by technology shocks.

To introduce the argument we note that technological change can be represented by labour augmenting and capital augmenting parameters;

$$Y = y(\frac{\kappa K}{\lambda L})\lambda L$$

To allow for riskiness in technology we suppose now that the state of technology in period 1 is not known but is a random variable; it changes unpredictably. So for any state of the world, s

$$\frac{{}^{s}Y_1}{{}^{s}\lambda_1 L_1} = y(\frac{{}^{s}\kappa_1 {}^{s}K_1}{{}^{s}\lambda_1 L_1})$$

There is, correspondingly, an *ex post* rate of profit, ${}^{s}\rho$, for each state s.

$${}^{s}\rho = \frac{\partial Y_1}{\partial K_1} + {}^{s}\dot{\kappa} = y'(\frac{\kappa_1^{s} K_1}{\lambda_1^{s} L_1}) + {}^{s}\dot{\kappa}$$

[3] All risk is technological risk. We assume bonds have no credit-risk.

$$\dot{\kappa}^{s} \equiv \frac{\kappa_1^s}{\kappa} - 1$$

This risk in technology does not alter the national income identities, and capital growth equations. But the equimarginal conditions are changed. Assuming that decision-makers know the probabilities of the states of the world, and maximise expected utility the equimarginal condition that matches the cost of acquiring capital (in terms of current consumption sacrifice) to the benefit (in terms of future consumption gain) is now ,

$$U_C = E[[1+\rho]U_{C1}] \qquad \text{Capital}$$

$$U_C = [1+r]EU_{C1} \qquad \text{Bonds}$$

Critically, the equimarginal condition for bonds that does not merely 'repeat' that for capital. This, we contend, is a signal of the functionality that bonds have acquired under risk. To argue that contention we will solve the model.

The Two Group: Two State: Two Period Model

We first present a solution in terms of a further simplified model. We suppose there are just two periods; Period 0 and Period 1.

We also suppose there are just two possible states of the world in Period 1: State 0 and State 1. State 1 is the high output state of the world, and State 0 is the low output state of the world. We also assume, for the time being, that the high output state is also the high profit rate state, and the low output state is also the lower profit rate state. (This assumption can be relaxed).The probability of State 0 is p.

We additionally suppose that there are two groups or ‘classes’ that make up the population, F and G. The two groups are completely homogeneous in composition, but differ from each other in their relative factor endowments. Group F has a relatively high amount of capital, and group G has a low amount of capital. The two groups have identical preferences.

Finally, we suppose, for the moment, that saving of the two groups in period 0 is exogenous. Consequently, the magnitude of the stock of capital in period 1 is exogenous. Further total saving in period 1 simply equals the negative of the capital stock, as there is no wish to have capital in period 2. Thus the consumption endowment of each group in period 1 equals their income from the factors they own, *plus* their endowment of capital.

The consumption endowment can be represented in an Edgeworth Box Diagram.

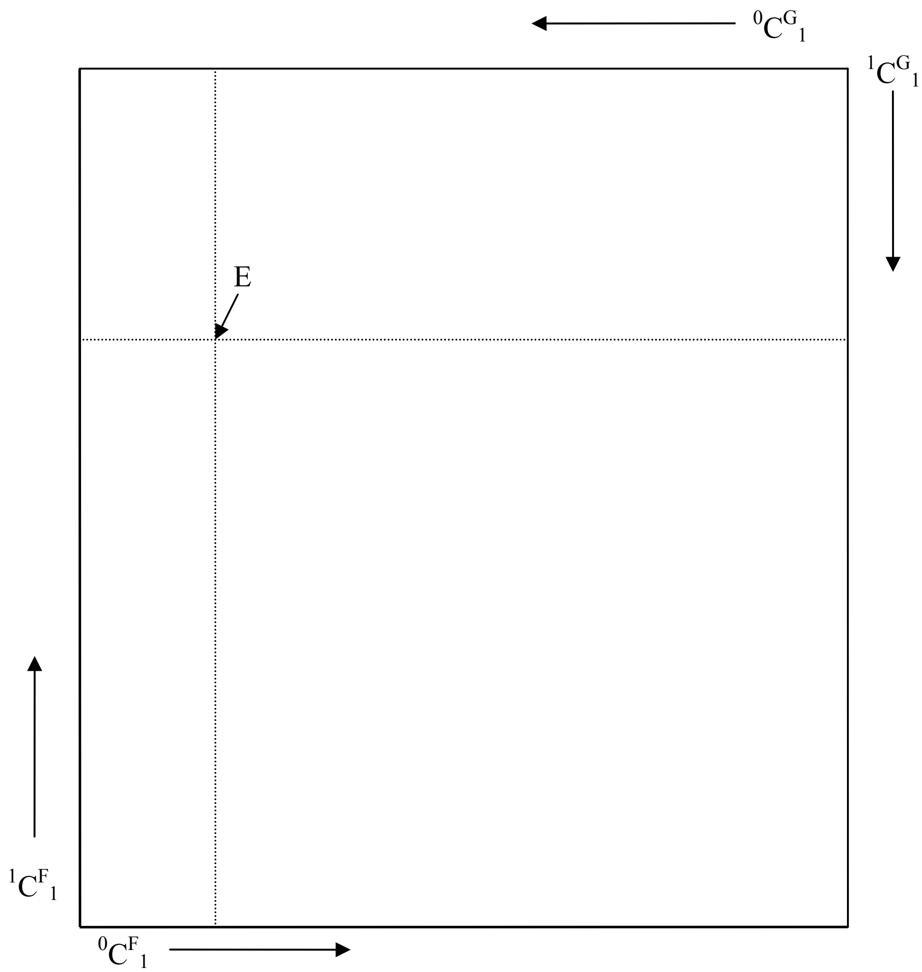

Figure 1: Social and private endowments in a two state model of risk

${}^{0}C_{1}^{F}$ = consumption of F in state of the world 1 in period 1 etc

As F and G are expected utility maximisers, it helps to plot on the Box the 'expected utility indifference curves'. The expected utility curves of F have a slope of p/[1-p] at the 45 degree ray from the south-west origin.[4]

---

[4] Given $EU^F = pU({}^{0}C_1^F) + [1-p]U({}^{1}C_1^F)$ then $dEU^F = p\,{}^{0}U'd\,{}^{0}C_1^F + [1-p]{}^{1}U'dC_1^F$. As ${}^{0}U' = {}^{1}U'$ along the 45 degree ray, if $dEU^F = 0$ along the 45 degree ray then

It also proves helpful to plot on the Box ‘iso expected consumption’ loci, each locus indicating the combinations of State 1 consumption and State 0 consumption which yield a certain level of expected consumption. These ‘iso expected consumption’ have a slope of p/[1-p].[5] Thus the expected utility indifference curves and ‘iso expected consumption curves’ are tangential at the 45 degree line.

$d\,{}^{0}C_1^F = -\frac{[1-p]}{p} dC_1^F$.

[5] Given $EC_1^F = p\,{}^{0}C_1^F + [1-p]{}^{1}C_1^F$ then $dEC_1^F = pd\,{}^{0}C_1^F + [1-p]d\,{}^{1}C_1^F$ .If $dEC_1^F = 0$ then $d\,{}^{0}C_1^F = -\frac{[1-p]}{p} d\,{}^{1}C_1^F$.

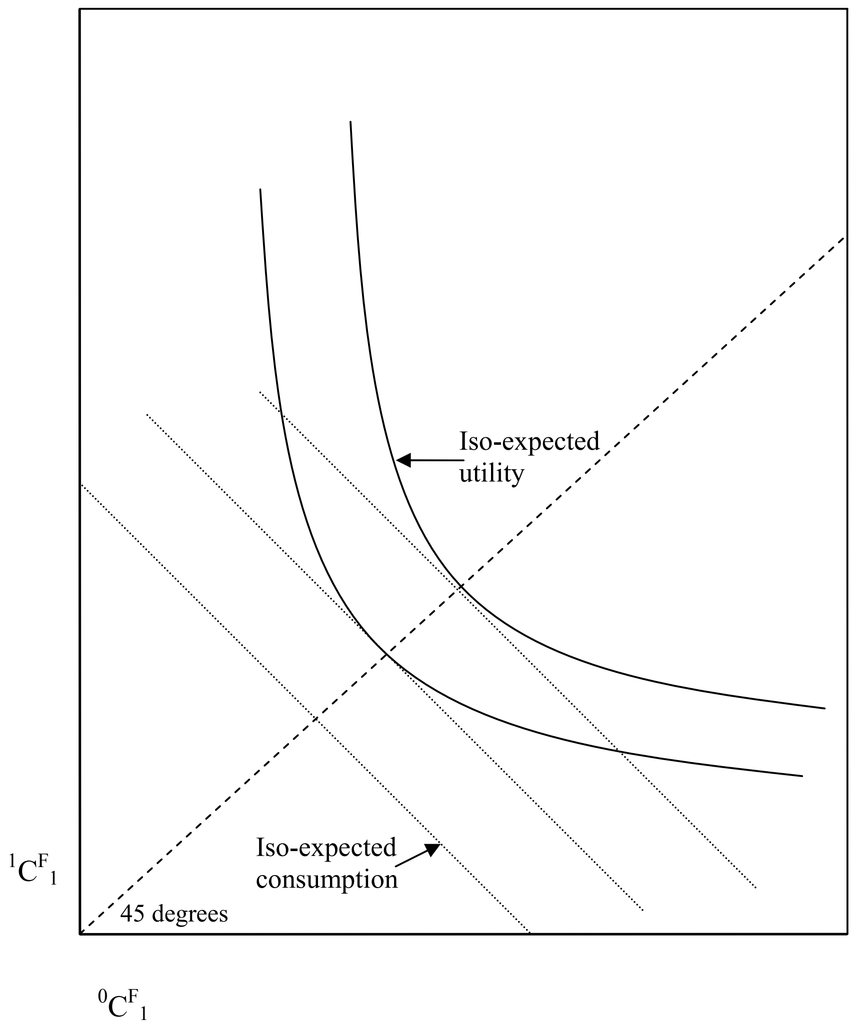

Figure 2: Preferences and expected consumption in a two state model of risk

G's expected utility indifference curves can also be plotted on the Edgeworth box. The expected utility curves of G have a slope of p/[1-p] at the 45 degree ray from the north-east origin.

Plainly, most distributions of consumption between the F and G are pareto inefficient.

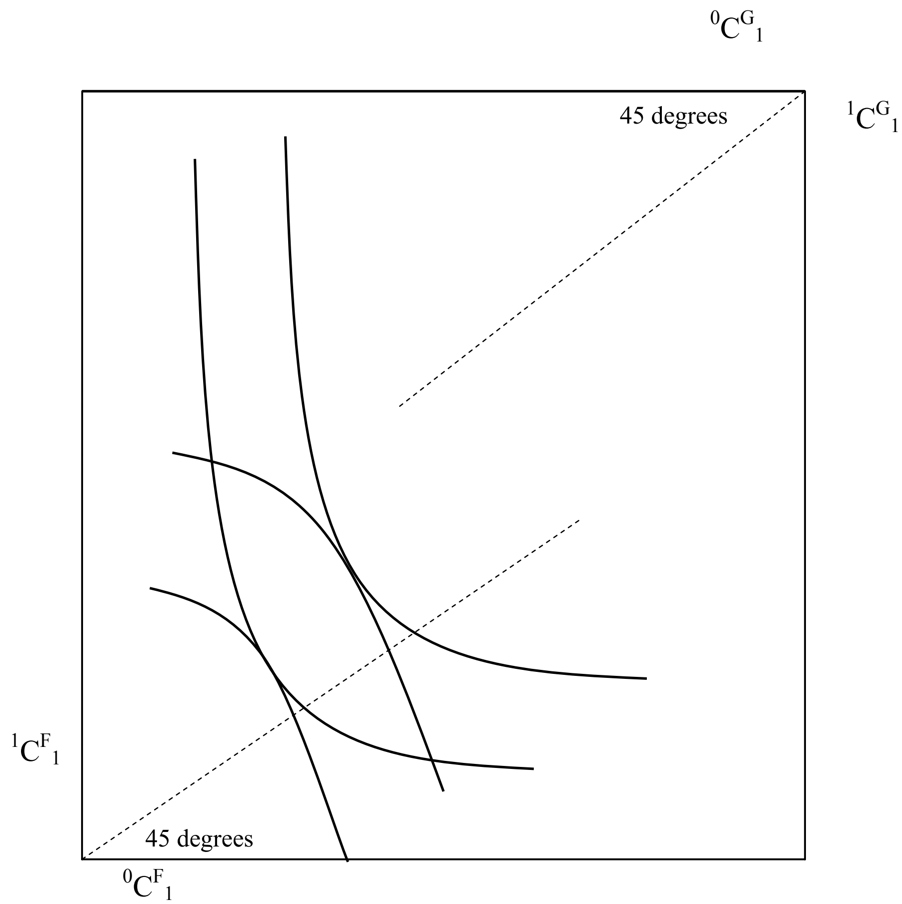

Figure 3: Most consumption points are pareto inefficient

But it is also plain that there exist efficient distributions, that are located at the tangency of the indifference curves of F and G. This equality of the slopes of the indifference curves may be written,

$$\frac{{}^0\upsilon^F}{{}^1\upsilon^F} = \frac{{}^0\upsilon^G}{{}^1\upsilon^G}$$

where

$$ {}^{s}\upsilon^{j} \equiv \frac{{}^{s}Uc_{1}^{j}}{EUc_{1}^{j}} $$

This condition allows a description of efficient consumption: an efficient distribution of consumption requires that the F's marginal utility in the 'up' state relative to F's marginal utility in the 'down' state (each normalised by the expectation of F's marginal utility) must equal G's marginal utility in the 'up' state relative to G's marginal utility in the 'down' state (each normalised by the expectation of G's marginal utility).

Since the marginal utility of consumption depends on the level of consumption, this requirement of welfare efficiency must imply some sort of relation between F's consumption in the up state, relative to the down state, with G's consumption in the up state, relative to the down state. If we assume homogeneous and identical preferences the implied relation is,

$$ \frac{{}^{0}C_{1}^{F}}{EC_{1}^{F}} = \frac{{}^{0}C_{1}^{G}}{EC_{1}^{G}} = \frac{{}^{0}C_{1}}{EC_{1}} $$

or

$$ \frac{{}^{0}C_{1}^{F}}{{}^{1}C_{1}^{F}} = \frac{{}^{0}C_{1}^{G}}{{}^{1}C_{1}^{G}} = \frac{{}^{0}C_{1}}{{}^{1}C_{1}} $$ [6]

[6] Proof:

The obvious diagrammatic corollary of the efficient management of risk is that efficient points lie upon the diagonal of the Box.

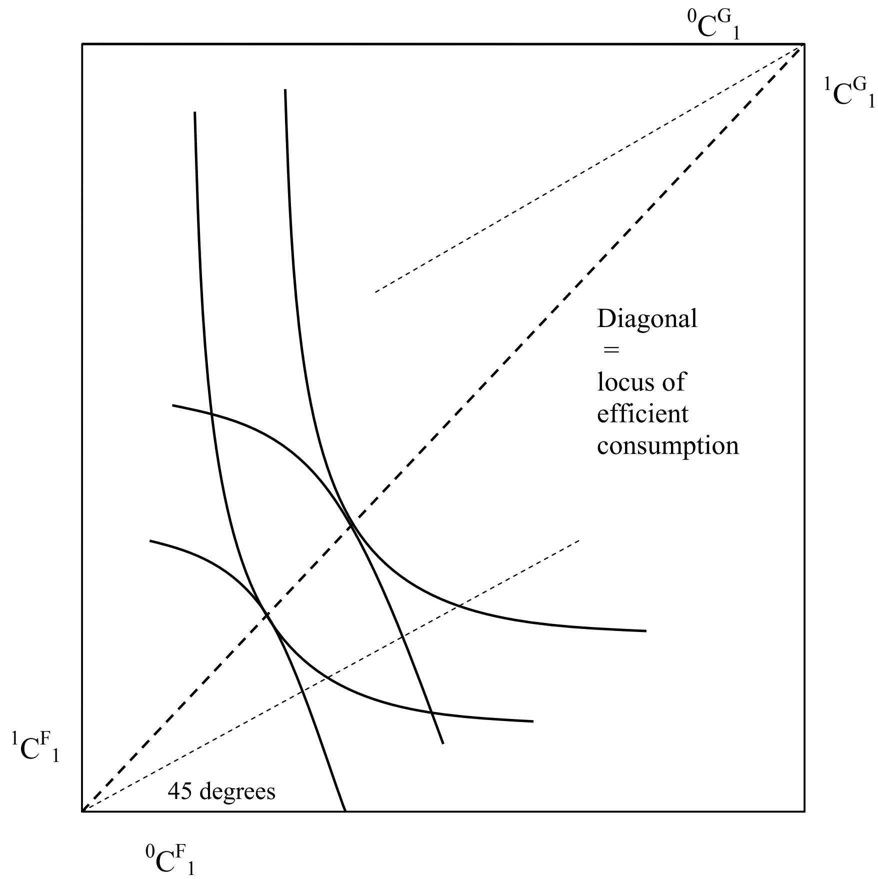

Figure 4: The locus of efficient distributions in consumption

---

$$ {}^{s}\upsilon^{F} = \frac{[{}^{s}C^{F}]^{\alpha-1}}{\sum {}^{s}p[{}^{s}C^{F}]^{\alpha-1}} $$

If $\frac{{}^{0}C_{1}^{F}}{{}^{1}C_{1}^{F}} = \frac{{}^{0}C_{1}^{G}}{{}^{1}C_{1}^{G}} = \frac{{}^{0}C_{1}}{{}^{1}C_{1}}$ then $\frac{C^{F}}{EC^{F}} = \frac{C^{G}}{EC^{G}}$. But if $\frac{C^{F}}{EC^{F}} = \frac{C^{G}}{EC^{G}}$ then

$$ {}^{s}\upsilon^{F} = \frac{[{}^{s}C^{G}\frac{EC^{F}}{EC^{G}}]^{\alpha-1}}{\sum {}^{s}p[{}^{s}C^{G}\frac{EC^{F}}{EC^{G}}]^{\alpha-1}} = {}^{s}\upsilon^{G} $$

Thus under identical and homothetic preferences, efficient management of social risk means that if society's consumption is x percent higher in the favourable state compared to the unfavourable state, then every individual's consumption is x percent higher in the favourable state compared to the unfavourable state. All boats must rise and fall with the tide, equally. That is the welfare efficient way of dealing with risk under identical and homothetic preferences.

We are now led to the question: What is the relation between the efficient allocation of consumption and the market allocation of consumption?

The analysis implies that the market for real bonds will, with some exceptions noted below, secure this welfare efficient allocation. The critical observation to sustain this contention is that F can shift their consumption point from their endowment point by selling bonds in period 0 in order to buy capital in period 0. If bonds are not to dominate capital, or *visa versa*, it must be that ${}^{1}\rho > r > {}^{0}\rho$. This implies that F's purchase of bonds, financed by the sale of equal value of capital, will *reduce* F's consumption by ${}^{1}\rho - r$ in state of the world 1, and *increase* F's by $r - {}^{0}\rho$ in state of the world zero. In other words, F's consumption point is sent 'south east' by F's purchase of bonds. Greater purchases of bonds by F will send F's consumption further 'south-east' along a line with a slope in absolute terms of $\frac{{}^{1}\rho - r}{r - {}^{0}\rho}$

Conversely, F's purchase of capital, financed by the sale of bonds, will shift the F's consumption point 'north west'.

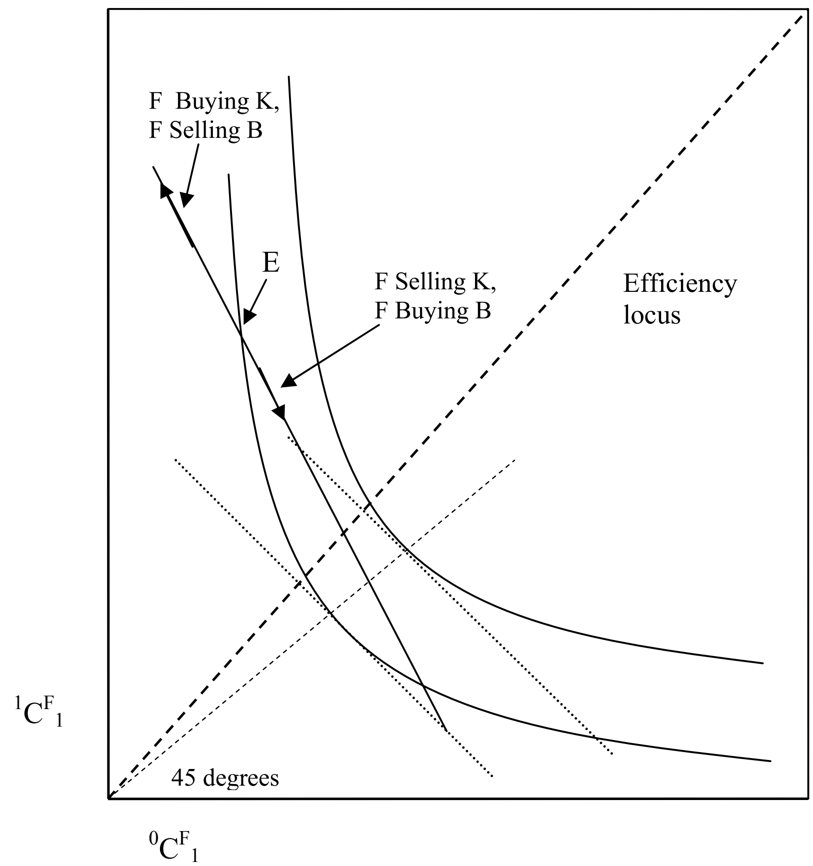

Figure 5: The consumption point is shifted by the exchange of bonds and capital

In the same way, G can shift themselves along the same frontier by either buying capital and selling bonds, or, selling capital and buying bonds.

The Box Diagram shows the general equilibrium, where F wants to buy the same number of bonds that G wants to sell.

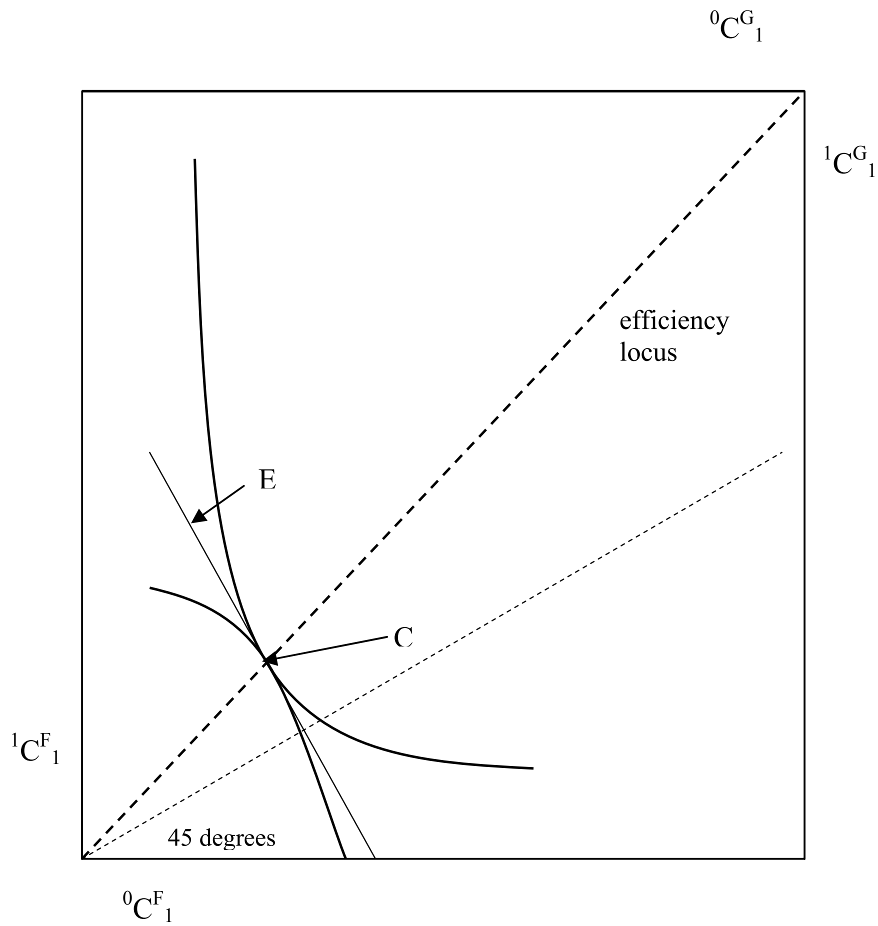

Figure 6: Equilibrium

At the maximizing point for, both G and F, the slope of the frontier $\frac{{}^1\rho - r}{r - {}^0\rho}$ equals the slope of the iso-utility frontier, $\frac{p\,{}^0Uc_1}{[1-p]\,{}^1Uc_1}$.

Evidently, F and G at the equilibrium point are on a higher level of expected utility than at the autarkic point E. There are social gains from trading bonds for capital. How can we understand these gains? Essentially, those burdened with risk have paid others to relieve them part of their risk. By moving south-east of their endowment, F has reduced their expected consumption; but, in approaching the 45 degree line, F has increased the security of their consumption. F in this situation is a ‘hedger’. At the same time, G, by shifting their consumption ‘south-east’ of their endowment; *increases* their expected consumption, but, by withdrawing further from their 45 degree line, *reduces* the security of their consumption. Group G are speculators.

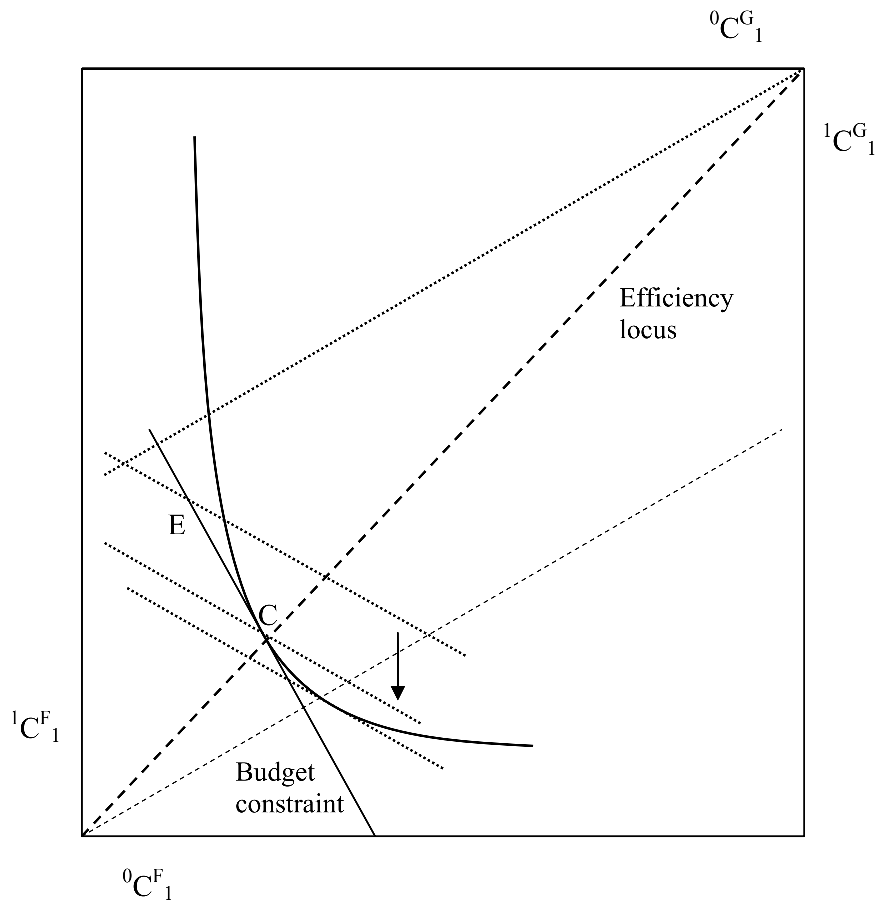

Figure 7: Hedging and Speculating

In the general equilibrium, one group will be Hedgers and the other Speculators: one group will be increasing their expected income at the cost to its security, and the other will be reducing their expected income, but to the benefit of their security. Hedgers pay Speculators to assume part of their risk.

The social function of debt is established. *Real debt allows the relatively risk insulated to supply 'insurance' to the relatively risk exposed over unpredictable fluctuations in factor prices.*

Debt provides an exchange of risk and expected consumption that is mutually improving. It gives consumption security to those who are consumption vulnerable, and consumption expansion to the (relatively) consumption invulnerable. It drives the variance of consumption of the two groups to equality. It shares out the risk between the two groups.

Hedgers and Speculators/Workers and Capitalists

We have analysed debt as a trade between those whose consumption endowments more risk vulnerable, and those less risk vulnerable. The more risk vulnerable experience disproportionately high incomes during the up, and disproportionately low incomes during the down. They become hedgers. Those less risk vulnerable experience income increases less than proportionate during the up, and decreases less than proportionate during the down. They become 'speculators'.

Can we relate the categories of Hedger and Speculator to other economic categories in the analysis? To the distribution of endowments of labour and capital? To the relative importance of labour augmenting and capital augmenting change? To the elasticity of technical substitution? With the help of these, can we identify 'who', and 'under what conditions' will be the Hedgers, and 'who', and 'under what conditions', will be the speculators? No. No sweeping simplicities govern the identification of Hedger and Speculator to with economic interest.

The identification of hedgers and speculators with the economies various economic interests cannot be secured with any confidence theoretically, but only empirically (see Coleman 2007 pp124-7 for a closer examination of this question). Experience teaches that the profit bill is more unstable over the business cycle than the wage bill: we will therefore henceforth assume that the interest with the higher capital/labour ratio, F, is the hedger.

A 'three state' exposition.

Is the analysis of the previous section simply a 'wonder' of its two state assumption? We do not think so. Diagrammatically it can be extended to the three state case.

Suppose there remain two interests, F and G, but there are now three states of the world; 0, 1 and 2. State 0 is 'low profit, low output'; State 1 is 'high profit, high output'; and State 2 is 'very high profit, very high output'. As before, F owns a large amount of capital relative to G, so its endowment rises relative to G in States 1 and 2.

The social, and their distribution, can be represented in an Edgeworth 'Cube'.

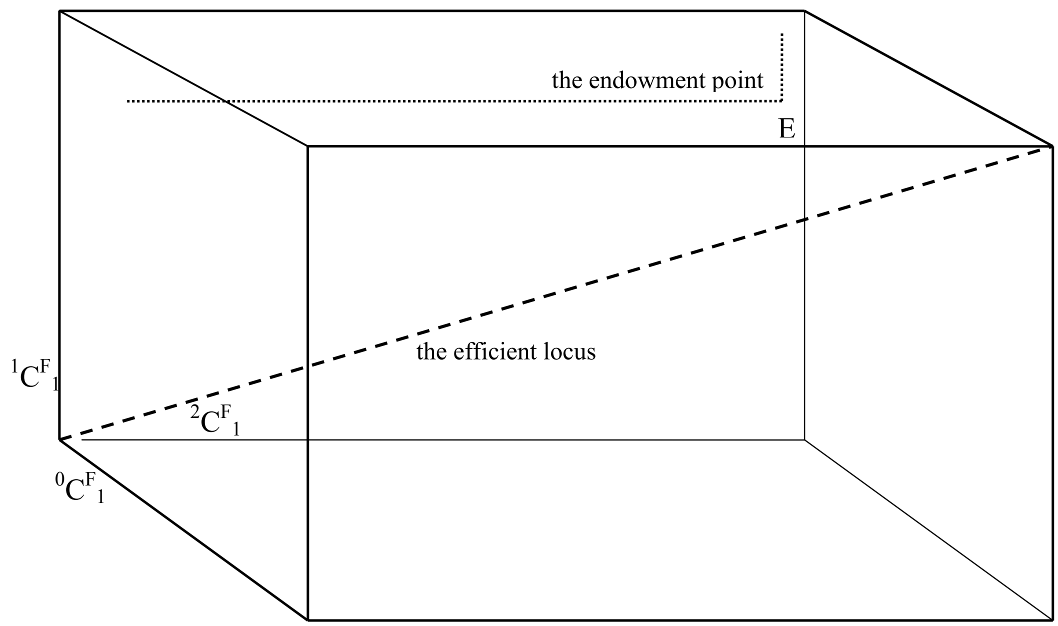

Figure 8: Endowments in a three state world.

The welfare efficient locus of consumption can be represented by a diagonal extending from the hindmost 'South West' corner to the foremost 'North East' corner. Evidently, the Endowment point, E, will (flukes aside) not lie on that diagonal, and not be welfare efficient.

An efficient consumption point can, nevertheless, be reached by means of F's sale of capital to G, and purchase of bonds from G. The magnitude of the real rate of interest on these bonds can be chosen so that it is lower than the rate of profit in state 1 and 2, but higher than the profit rate in state 0. Given this, every such capital-for bonds swap reduces F's consumption in states 1 and 2, but increases it in state 0. As can be seen from Figure 9, this drives the actual consumption point from E and towards the diagonal. A sufficiently large capital-for bonds swap will drive the consumption point to the diagonal.

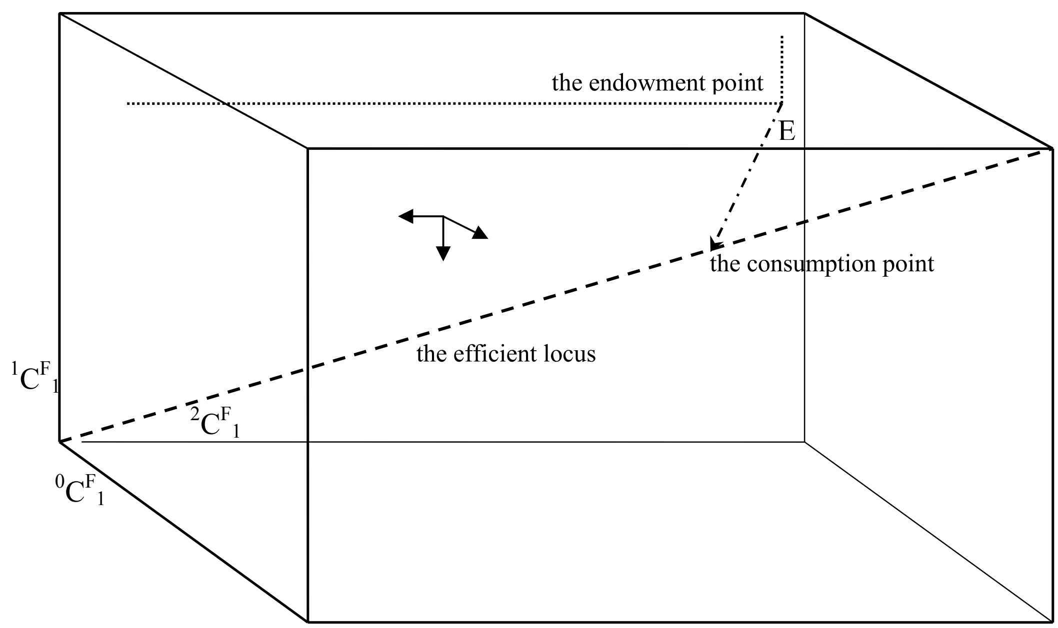

Figure 9: Efficient consumption in a three state world.

A supply and demand exposition of the transfer of risk by bond markets.

The Edgeworth Box is a useful tool for introducing the topic, but even in three dimensions it is too special. A supply and demand treatment has a superior generality.

The demand schedule and the supply schedule for bonds may be derived from the equimarginal conditions,

$$Uc^{j} = E[Uc_{1}^{j}[1+\rho]] \qquad \text{Capital}$$

$$Uc^{j} = [1+r]EUc_{1}^{j} \qquad \text{Real Bonds}$$

Substituting,

$$E[[1+\rho]Uc_{1}^{j}] = [1+r]EUc_{1}^{j}$$

Or,

$$E\rho - r = -\operatorname{cov}(\rho, \upsilon^{j})$$

where

$$\upsilon^j \equiv \frac{Uc_1^j}{EUc_1^j}$$

This equality is an example the familiar risk premium type expression in finance: the excess of the expected return on capital over bonds is the negative of the covariance of the profit rate with the normalised marginal utility of consumption.

It will prove convenient to define,

$$\omega_\rho^j \equiv -\operatorname{cov}(\rho, \upsilon^j)$$

and rewrite the condition as,

$$E\rho - r = \omega_\rho^j$$

ω_ρ^j, evidently, is a risk premium. It is the excess return over bonds that j must receive on their capital if they are not to prefer bonds to capital.

The LHS of this equality may be considered the 'price' of bonds; it equals the expected opportunity cost of owning bonds. The RHS is a magnitude that will change with the amount of bonds owned, or owed.

To advance the construction of demand and supply schedules, we again think in terms of two groups, F and G; two groups; one with a higher endowment of capital relative to labour, F; and one with a lower endowment of capital to labour, G.

$$E\rho - r = \omega_{\rho}^{F}$$

$$E\rho - r = \omega_{\rho}^{G}$$

At B = 0 the magnitudes of ω_{ρ}^{F} and ω_{ρ}^{G} are determined by the co-movement of each groups consumption endowment with the profit rate. The group with the more positive capital/labour ratio, F, will have the more negative co-movement in marginal utility with profit, and so a more positive ω_{ρ}^{j}. The group with the more smaller capital/labour ratio, G, will have a less negative co-movement in marginal utility with profit, and so a smaller positive ω_{ρ}^{j} (and possibly even a negative one). We can plot ω_{ρ}^{F} and ω_{ρ}^{G} as the vertical axis intercepts of a figure that plots $E\rho - r$ on the vertical axis, and bond issues on the horizontal axis.

As F's ownership of bonds becomes positive, the magnitude of ω_{ρ}^{F} falls, by the logic explained earlier by means of the Box: the bond holder, as 'hedger', is reducing the comovement of their consumption opportunity with the profit rate. We are tracing out a 'downward' sloping demand curve for bonds with respect to the risk premium.

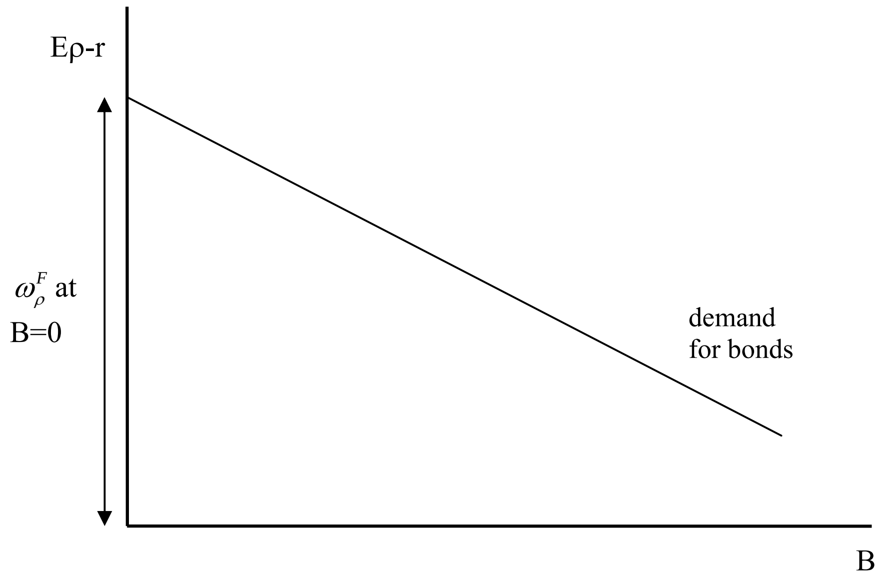

Figure 10: The demand curve for bonds

As G owes positive rather than zero, the magnitude of ω_ρ^G rises, also by the logic explained earlier: the bond seller, as ‘speculator’, is increasing the comovement of consumption with the profit rate. We are tracing out a ‘upward’ sloping supply curve for bonds with respect to the risk premium.

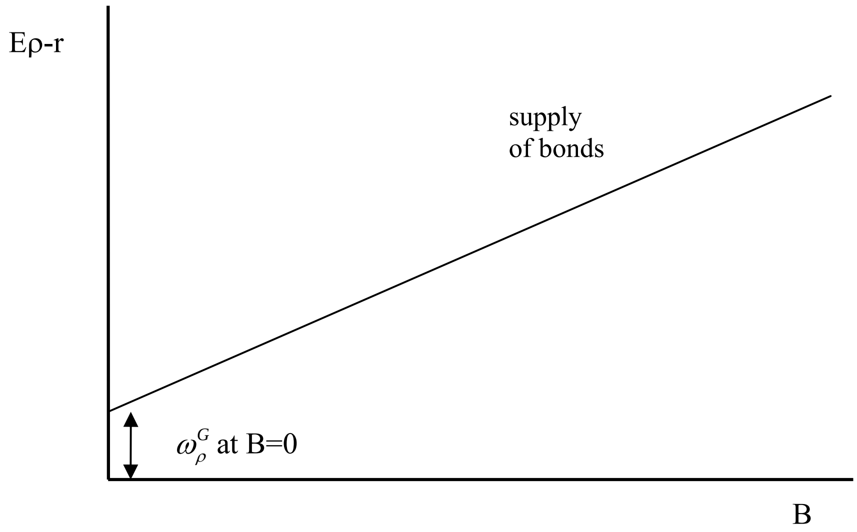

Figure 11: The supply curve of bonds

It is easy to persuade oneself that at the point where the demand curve for bonds intersects the supply curve for bonds, the negative of the covariance of the profit rate with the marginal utility of the two groups are the same.

$$\omega_\rho^F = \omega_\rho^G$$

But this is a signal of the efficient distribution of consumption. For this equality is satisfied if,

$$\upsilon^F = \upsilon^G \qquad \text{in all states of the world.}$$

which, in the two state, case implies

$$\frac{{}^{0}\upsilon^{F}}{{}^{1}\upsilon^{F}} = \frac{{}^{0}\upsilon^{G}}{{}^{1}\upsilon^{G}}$$

But we have already concluded, from the Box analysis, that this is condition welfare efficiency in the two state case, and a property of the market equilibrium.

The welfare efficiency of the market equilibrium is underlined by the fact that the intersection of the two schedules is interpretable as the quantity of debt that maximises the sum of 'creditors' surplus' and 'debtors' surplus'. Consider,

j's marginal expected utility of a unit of capital

$$= E[[1+\rho]Uc_1^j] = E[1+\rho]EUc_1^j + \text{cov}(\rho, Uc_1^j)$$

and

j's marginal expected utility of a unit of bonds

$$= [1+r]EUc_1^j$$

Then

marginal expected utility of a purchase of bonds by way of a sale of capital

$$= [1+r]E^{\,j}Uc_1 - E[[1+\rho]EUc_1^j] - \text{cov}(\rho, Uc_1^j)$$

$$= [r - E\rho - \text{cov}(\rho, \upsilon_j)]EUc_1^j$$

Thus current consumption equivalent of a purchase of bonds by way of a sale of capital is $[\omega_\rho^j - [E\rho - r]]\frac{1}{1+r}$ [7]. For any exchange of bonds for capital 'on the margin', the magnitude of this expression is obviously zero, since in equilibrium $E\rho - r = \omega_\rho^j$. But for any infra marginal exchange the magnitude is positive, and equals the excess of ω_ρ^j over $E\rho - r$, that equals the distance between the demand curve, and the 'price of bonds, $E\rho - r$.

Another way of looking at it, is that the creditor in buying bonds loses an amount equal to B[Eρ –r], but unburdens, on the debtor, a 'value' of risk equal to aa bb cc dd. The creditor's surplus in Figure 12 is the triangle aa bb ee.

[7] Thus current consumption equivalent of a purchase of bonds by way of a sale of capital is =

$$[r - E\rho - \text{cov}(\rho, \upsilon^j)]\frac{EUc_1^j}{Uc^j} \quad ^7$$

$$= \quad [-[E\rho - r] - \text{cov}(\rho, \upsilon_j)]\frac{1}{1+r}$$

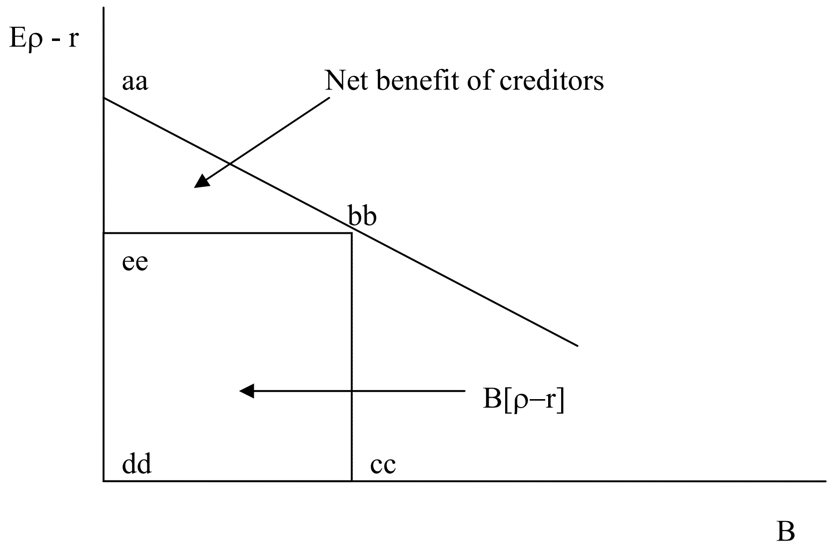

Figure 12: The 'surplus' of creditors

By a parallel logic the surplus of debtors is represented in Figure 11.

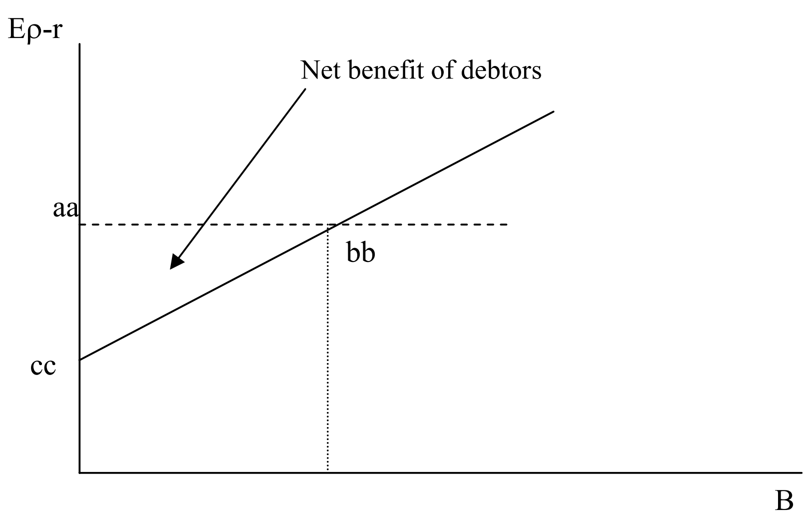

Figure 13: The 'surplus' of debtors

For borrowers, they receive a net income equal to B[ρ-r], but they pay a risk expense equal to the sum of inframarginal magnitudes of ω_ρ^G.

The implications of a missing real bond market for the evening of risk

Suppose now there are no real bonds.

The implications of a missing real bond market for the transfer of risk can be brought out by recapitulating the optimisation conditions,

$Uc = E[Uc_1[1+\rho]]$	Capital
$(Uc = [1+r]EUc_1$	Real Bonds)
$Uc = E[Uc_1 \frac{1+i}{1+\pi}]$	Money Bonds

If real bonds are present then, we may from infer from the capital and real bonds conditions,

$$E\rho - r = \omega_\rho^j$$

and so the magnitude of ω_ρ^j is the same for all j/

As explained in this condition (with some exceptions) secures efficiency in consumption with respect to technical shocks. So, if real bonds are present the only role of money bonds is redundant.

But if real bonds are absent then money bonds will assume the function of real bonds of transferring technological risk. But in place of the risk premium equality above, we infer from conditions for capital and money,

$$1+E\rho-E[\frac{1+i}{1+\pi}]=\omega_{\rho}^{j}-\omega_{-\Pi}^{j}$$

where $\omega_{-\Pi}^{j}\equiv-\mathrm{cov}(\rho,\upsilon^{j})$, and may be interpreted as the inflation risk premium on money bonds.[8]

Given the uniformity of the LHS across j one may infer that the magnitude of $\omega_{\rho}^{j}-\omega_{-\Pi}^{j}$ is the same for all j.

Clearly this equality does not insure ω_{ρ}^{j} is the same for all j. Indeed, as will be seen will ω_{ρ}^{j} generally be different for different j in the presence of risky inflation. This implies consumption is not shared out in the efficient manner.

---

[8] Consider the case when capital is riskless. Then, $\omega_{-\Pi}^{j}=E[\frac{1+i}{1+\pi}]-[1+\rho]$. Then $\omega_{-\Pi}^{j}$ is the excess real return on money bonds that money bonds must receive if capital is not to be preferred to money bonds.

The impact of inflation risk on the bond market

The impact inflation risk has on the demand and supply of bonds can be used to explicate the contention that inflation risk harms the efficient sharing of consumption

Demand and supply schedules for bonds can be derived from the equality,

$$1 + E\rho - E[\frac{1+i}{1+\pi}] = \omega_{\rho}^{j} - \omega_{-\Pi}^{j}$$

The LHS is the excess return capital is expected to earn over money bonds, and can be considered the price of money bonds. As both the terms on the RHS are a function of the quantity of money bonds, D, we can trace relations between the price of bonds and the quantity bonds.

To facilitate demand and supply analysis we assume (as was done before) that the population falls into two groups; one with a higher endowment of capital relative to labour, F; and one with a lower endowment of capital to labour, G. The first group, with a high exposure to risk, are 'hedger-capitalist-creditors'. The second are 'speculator-worker-debtors'. For each group there is an associated condition for the excess return on capital,

$$1 + E\rho - E[\frac{1+i}{1+\pi}] = \omega_{\rho}^{F} - \omega_{-\Pi}^{F}$$

$$1 + E\rho - E[\frac{1+i}{1+\pi}] = \omega_{\rho}^{G} - \omega_{-\Pi}^{G}$$

At D = 0 the magnitude of both $\omega_{-\Pi}^{F}$ and $\omega_{-\Pi}^{G}$ is zero, given our assumption that inflation is white noise. At D = 0 ω_{ρ}^{F} and ω_{ρ}^{G} are determined by the co-movement of each groups consumption endowment with the profit rate. As F has the more higher capital/labour ratio, F will have the more negative co-movement in marginal utility with profit, and so a more positive ω_{ρ}^{j} than G.

$$\text{At D = 0, } \omega_{\rho}^{F} > \omega_{\rho}^{G} \text{ and } \omega_{-\Pi}^{F} = \omega_{-\Pi}^{G} = 0.$$

We can represent ω_{ρ}^{F} and ω_{ρ}^{G} at D = 0 as the vertical axis intercepts of a figure that plots $E\rho - r$ on the vertical axis, and bond issues on the horizontal axis. These two intercepts may be interpreted as the demand price of money bonds when D = 0, and the supply price of bonds at D = 0, respectively.

To trace out the demand curve, we note that as D becomes positive then ω_{ρ}^{F} diminishes, by the process explained in an earlier section: risky capital is exchanged for (more) reliable bonds. But as D becomes positive there is also now a new, additional element in tracing out the demand curve; $\omega_{-\Pi}^{F}$ assumes a non-zero magnitude as D becomes positive. Once D is positive then for F (a creditor), an above average rate of growth in purchasing power must coincide with an above average consumption of F - simply because creditors are richest when the growth in purchasing power is highest. So an above average rate of growth in purchasing power must coincide with a below average marginal utility of creditors. Thus $\omega^F_{-\Pi}$ (≡-$\text{cov}(-\pi, \upsilon_F)$) becomes positive. And so the demand price for bonds is reduced. This reduction in the demand price for bonds can be understood as a manifestation of an undesirable property of money bonds in the circumstances we are analysing: for creditors money bonds give their best return when return is least 'needed' (that is, marginal utility is lowest). On account of this undesirable property the demand price for bonds falls.[9]

As D becomes larger $\omega^F_{-\Pi}$ becomes still more positive, as the largeness of D makes for a still stronger comovement of the growth of purchasing power and creditor consumption, and so demand price shifts down more.

[9] Evidently, it has been implicitly assumed that the magnitude of ω^F_ρ is unchanged by unpredictable inflation for any level of debt. This is reasonable. Unpredictable inflation will add noise to F's consumption, but will not change the covariance of their consumption with the profit rate.

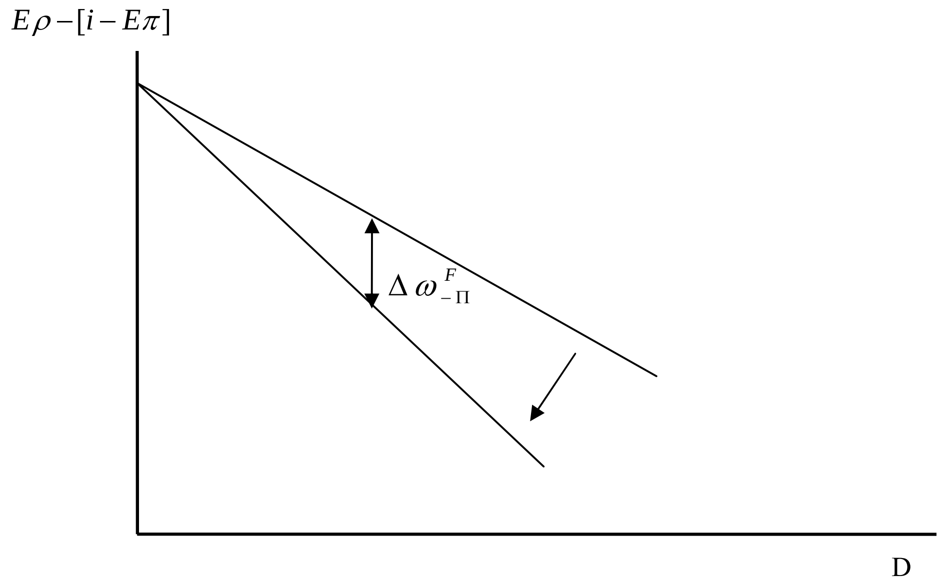

Figure 14: Inflation risk contracts the demand for bonds by capitalist hedgers

The construction of the supply curve for bonds follows a parallel logic. As D becomes positive then ω^{G}_{ρ} rises, by the process explained in earlier sections: risky capital is bought by the sale of relatively reliable bonds. But as D becomes positive there is also now a new, additional element in tracing out the supply curves. $\omega^{G}_{-\Pi}$ assumes a negative magnitude as D becomes positive. This is because for G (debtors), an above average growth rate in purchasing power must coincide with below average consumption - simply because debtors are poorest when the growth rate in purchasing power is highest. So an average rate of growth in purchasing power must coincide with an above average marginal utility of debtors. Thus $\omega^{G}_{-\Pi}(\equiv\text{-}\operatorname{cov}(-\pi,\upsilon^{G}))$ becomes negative. The 'supply price' of bonds is increased. This rise in the supply price for bonds can be understood as a manifestation of an undesirable property of money bonds in these circumstances: they are most costly to debtors when debtors are most in 'need' of income (when the marginal utility of consumption is the highest). And they are least costly debtors when debtors are the least 'need' of income (when the marginal utility of consumption is the lowest). On account of this undesirable property the supply price of bonds rises.[10]

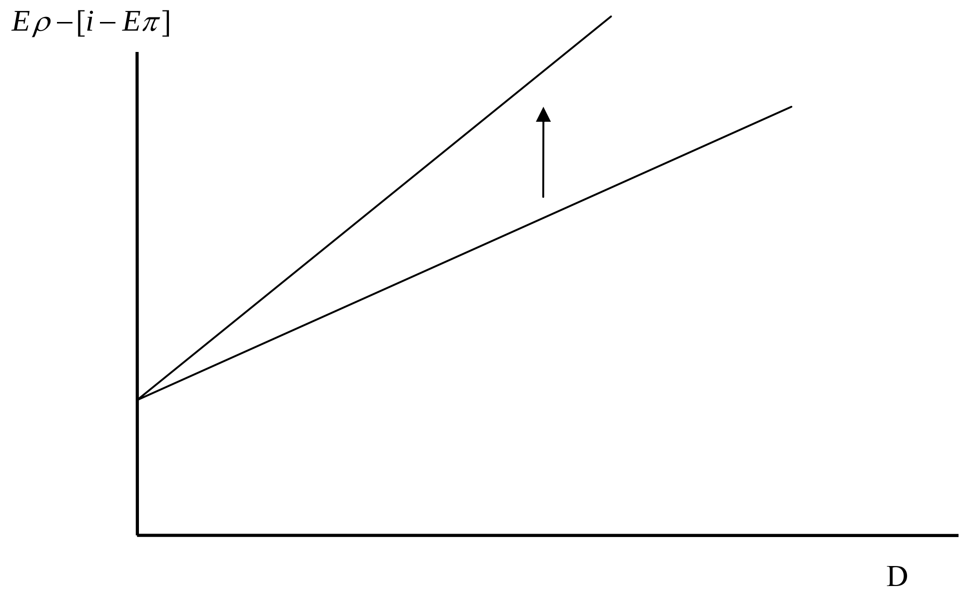

Figure 15: Inflation risk contracts the supply of bonds by worker speculators

Figure 16 illustrates the combined impact of the shift inwards in the demand for bonds, and the shift upwards in the supply of bonds.

[10] Inflation risk is bad for *both* capitalist creditors and worker debtors. Capitalist creditors seek a reduced opportunity cost in lending; that puts an upward pressure on the interest rate. Worker-debtors want a greater net gain from borrowing; that puts a downward pressure on the interest rate.

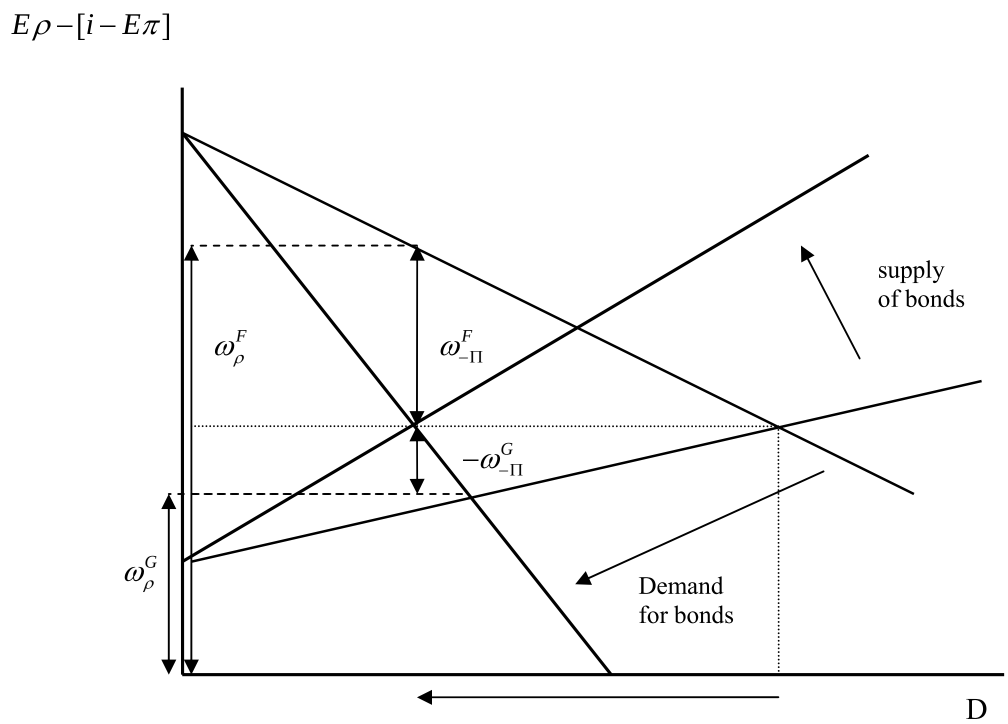

Figure 16: Bond market equilibrium under inflation risk

Three observations can be made about the impact of inflation risk:

First, the quantity of bonds unambiguously contracts.

Second, the expected real interest rate on money bonds is unchanged. The lack of response is result of the mutual cancellation of two contrary forces. Inflation risk has deterred capitalist-creditors from lending; an effect which would, operating alone, drive up the rate. But inflation risk has also deterred worker-debtors from borrowing, an effect which would, alone, drive down the rate. The two effects cancel each other out, and the impact of inflation risk on the expected the real interest rate, and the premium on capital is zero.

Thirdly, the magnitudes of ω_ρ^F and ω_ρ^G are driven apart. Consumption of the two interests is no longer synchronised in the face of technical shocks. There is a loss of welfare.

Figures 12 and 13 can be adapted to bring out the social inefficiency of noisy inflation. Under our assumptions, welfare in the absence of risky inflation was the area lay between the demand curve that prevailed in the absence of risky inflation, and the supply curve that prevailed in the absence of risky inflation. The contraction in these demand and supply curves signals a loss in welfare.[11] Welfare in the presence of noisy inflation is the contracted area underneath the demand curve, and above the supply curve, that prevail in the presence of noisy inflation; aa cc ee. A boomerang shaped area – aa ff ee cc – is the loss in consumer welfare.[12]

---

[11] Marginal Expected Utility of a Unit of Capital $= E[Uc_1^j[1+\rho]]$

Marginal Expected Utility of a Unit of Bonds $= E[Uc_1^j \dfrac{1+i}{1+\pi}]$

So,

Marginal Expected Utility of a Purchase of Bonds by way of a Sale of Capital

$$=[E[\frac{1+i}{1+\pi}]+\text{cov}(-\pi,\upsilon^j)-E[1+\rho]-\text{cov}(\rho,\upsilon^j))]E\,{}^jUc_1$$

$$[-\text{cov}(\rho,\upsilon^j)+\text{cov}(-\pi,\upsilon^j)-[E[1+\rho]-E[\frac{1+i}{1+\pi}])]\frac{1}{1+E\rho+\text{cov}(\rho,\upsilon^j)}]$$

So the downward shift in the demand curve is the loss of utility for bond buyers. Similarly for bond sellers.

[12] On an analogy with the welfare loss of tax on the demand and the supply of a good, it might be thought that the loss in welfare is the triangle bb dd ff. But the analogy does not hold. A tax on the demand and the supply of a good, yields revenue to the state (aa bb cc and ee cc dd) that the standard analysis assumes is used to provide tax relief. But in analysing the cost of unpredictable inflation there is no revenue. (It has been implicitly assumed here that any seigniorage is not used to annexe resources and so permit a reduction in taxes).

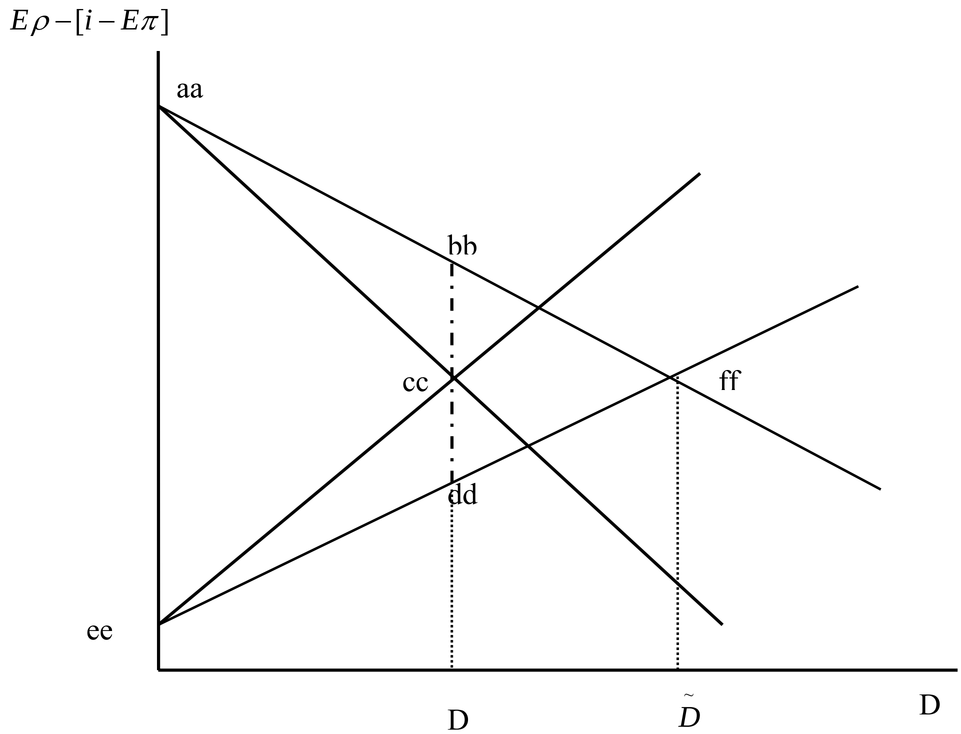

Figure 17: The impact of white noise inflation on welfare

In algebraic terms it can be shown (Appendices 2 and 3) that,

$$\text{the loss of welfare} = [\omega^F_{-\Pi} - \omega^G_{-\Pi}]\frac{\tilde{D}}{2} = [\omega^F_{\rho} - \omega^G_{\rho}]\frac{\tilde{D}}{2}$$

To summarise: risky inflation reduces welfare by impeding the synchronisation of consumption across interests. In the absence of unpredictable inflation a money bonds market would be capable of perfectly synchronising consumption, but under white noise inflation it cannot.[13] That synchronisation of consumption that would take place at the old volume of lending is now disturbed by inflation surprises, which impact on the wealth of creditors and debtors in an unpredictable fashion. Discouraged from lending and borrowing, the amount of lending and borrowing shrinks. Capitalist-creditors cannot obtain the security they want, and worker-debtors cannot obtain the return they want.

Inflation risk without cost.

The contention that risky inflation causes a welfare loss by discouraging the trade of risk must be qualified in several ways.

First, the contention assumes that there is some beneficial trade in risk to be done. This will not be the case where technological shocks are such that all persons are equally exposed to risk. This will be the case if there is an equal distribution of capital so that there are no ‘workers’ and ‘capitalists’ but simply worker-capitalists. It will also be the case if technical progress is of the Hicks neutral technical variety.

Secondly, the contention does rest on the assumption that inflation is white noise. If we suppose that inflation is not white noise but varies with complete predictablity with the rate of profit, then inflation risk will have no impact upon aggregate social welfare. Recall,

---

[13] If lending stayed at $\tilde{D}$ then ω_ρ^F and ω_ρ^G would remain equal to each other. Despite this, consumption would not be synchronized. The equality of ω_ρ^F and ω_ρ^G is a only necessary condition for synchronisation. It is not sufficient. The volatility imparted to real debts and credits by inflation shocks prevents synchronisation.

$$E[1+\rho]-E[\frac{1+i}{1+\pi}]=\omega_{\rho}^{j}-\omega_{-\Pi}^{j} \qquad \text{all j}$$

If there is perfect linear relation with the rate of profit

$$\pi=\gamma\rho$$

then $\gamma\omega_{\rho}^{j}=-\omega_{-\Pi}^{j}$ [14]

Therefore,

$$E[1+\rho]-E[\frac{1+i}{1+\pi}]=\omega_{\rho}^{j}[1+\gamma] \qquad \text{all j}$$

The equality of the LHS for all j ensures ω_{ρ}^{j} is equal for all j.[15] This is a signal that consumption is efficiently shared.

---

[14] $\omega_{\rho}\equiv-\text{cov}(\rho,\upsilon)=-E[\rho-E\rho]\upsilon=-\frac{1}{\gamma}E[\pi-E\pi]\upsilon=\frac{1}{\gamma}\text{cov}(-\pi,\upsilon)=-\frac{1}{\gamma}\omega_{-\Pi}$

[15] $\omega_{-\Pi}^{j}$ must be the same for all j, too. And this is puzzling. Should not the $\omega_{-\Pi}^{j}$s differ across interests? Would not workers (as debtors) lose from a high growth in purchasing power, and capitalists (as creditors) win? And would that not create a negative correlation between marginal utility and -π for capitalists (as creditors), and a positive correlation for workers (as debtors)? No. This is because (by assumption) high and low inflation states are linked to profit states. So while the capitalist does gain from a high growth in purchasing power augmenting the real value of loans, that does not make for a negative correlation between marginal utility and -π , as a high growth in purchasing power state is also a low profit state. The conjunction of low profit and high purchasing power growth leave the marginal utility of the capitalist uncorrelated with purchasing power growth. A parallel logic applies to the worker debtor.

But this qualification of the original proposition is unlikely to be strong, for inflation is unlikely to be perfectly correlated with the profit rate. Rather inflation is likely to have one component that is related to the rate of profit, and a second that is unrelated white noise.

$$\pi = \gamma\rho + \varepsilon$$

That part of inflation that is systematically related to profit has no social cost, but that which is white noise will be costly through discouragement of the trade in risk.

The costs of inflation reconsidered

Does the preceding theory of the cost of inflation change at all the appearance of the common account of the costs of inflation?

'this arbitrary rearrangement of riches'

The theory throws some light on the sentiment that it is the unpredictability in inflation, rather than its average level, that is harmful, as 'choices' are transformed into 'gambles'.

A more specific variant of this 'unpredictability thesis' – and one that is an old saw in the discussion of the costs of inflation – has it that that inflation is harmful on account of the unpredictable redistributions it secures between creditors and debtors. But how these redistributions are harmful to efficiency – as apart from equity - is barely touched.[16] The present analysis provides an answer. These unpredictable redistributions between creditors and debtors damage the welfare efficient by reducing the synchronisation of consumption across interests. Unpredictable redistributions reduce synchronisation directly and indirectly. Directly (through surprise inflations impact on the real value of credits and debts), and also indirectly by discouraging the transfer of risky capital from those with much to those with a little.

Unpleasant Surprises

The theory of the cost of inflation risk that has been advanced here is not only a cost in its own right, it is a cost that casts a shadow on some of supposed benefits of inflation. It puts a taint of those benefits.

It puts a taint on the benefits of inflation unpredictability that have been argued to exisr. It has been argued that the merit of preserving smooth tax rates validates unpredictable seigniorage, and a concomitant unpredictable inflation, as seigniorage should fluctuate in accordance with unpredictable temporary shocks to government spending so as to allow tax rates to be invariant in the face of such shocks. In this

---

16 The sight of this arbitrary rearrangement of riches strikes not only at security, but at the confidence in the equity of the existing distribution of wealth. Those to whom the system brings windfalls, beyond their deserts and even beyond expectations and desires, become 'profiteers'…As the inflation proceeds and the real value of the currency fluctuates wildly from month to month, all permanent relations between debtors and creditors, who form the ultimate foundation of capitalism, become so utterly disordered as to be utterly meaningless, and the process of wealth-getting degenerates into a gamble and lottery.

John Maynard Keynes, *The Economic Consequences of the Peace,* 1919.

On a more analytical plane, the notion of inflation risk obstructing the effective intermediation between borrower and lender is briefly aired in the *General Theory*. In section iv of chapter 11 Keynes discusses the impact that an 'adverse change in the monetary standard' has on what he calls 'lenders risk' and 'borrowers risk'. The upshot is a 'duplication' of risk margins, 'which has not hitherto been emphasised'.

logic a surprise increase in G should be financed by a (equally surprising) increase in S, relieving any need for taxes, and producing an a concomitant inflation surprise.

Whatever benefit that seigniorage may have, it will, according to the theory of this chapter, be mingled with a cost; the cost of the disruption of capital markets.

The argument for the cost of inflation risk is also an antidote to the recommendation of inflation risk that (we have seen) the theory of Shoe Leather costs implicitly makes in the presence of declining marginal shoe leather costs of inflation .

Minimising Shoe Leather Cost

The theory of the costs of unanticipated inflation also taints the benefits of deflation that is implied by the theory of Shoe Leather Costs. Recall that Shoe Leather Costs are minimised if,

$$\pi = -\rho$$

The present theory is well prepared to analyse the impact of such a policy on capital markets. The Shoe Leather Cost minimising rule is a special case of,

$$\pi = \gamma\rho$$

We have previously concluded that $\gamma = 1$ was no bar to efficient trade of risk. But $\gamma = -1$, or $\pi = -\rho$, is totally destructive of the risk shifting function of bond markets.

For if $\pi = -\rho$ then money bonds have the same return as capital in all states. F cannot shift their consumption point by lending because in both states the payoff is the same (zero). The risk trading function of money bonds is destroyed. There is, therefore, a strict conflict between minimising shoe leather costs and maintaining the risk managing properties of the bond market. To strictly minimise shoe leather costs is to destroy the trade in risk.

Quantifying the welfare loss of unpredictable inflation

How large is the welfare loss from the reduction in the trade of risk caused by unpredictable inflation? Theory gives very little indication. It could as low as zero: it will be zero if, for example, F and G had the same endowment of capital and labour. And if it can be zero it can also be minute.

Some help in quantifying its size can be derived from,

$$\text{welfare cost} = [\omega^F_{-\Pi} - \omega^G_{-\Pi}]\frac{\tilde{D}}{2} = [\omega^F_{\rho} - \omega^G_{\rho}]\frac{\tilde{D}}{2}$$

One could hazard numbers on the difference on risk premia and $\tilde{D}$, and draw the inference. So if we were willing to suppose the difference in risk premia is 0.01 (in decimals), and $\tilde{D}$ is twice C, then the cost of inflation risk is 0.01 of C, (or one percent).

A (perhaps) more reliable procedure would be to try to derive expressions for the risk premia, and then calibrate. These do not suggest the welfare loss from unpredictable inflation is large (Coleman 2007, pp238-9).

Appendix 1

This appendix provides an argument for the invariance of the interest rate to white noise inflation.

It is necessarily the case,

$$\operatorname{cov}(\rho,\frac{C^F}{EC^F})[1-\lambda]+\operatorname{cov}(\rho,\frac{C^G}{EC^G})\lambda=\operatorname{cov}(\rho,\frac{C}{EC})$$

and

$$\operatorname{cov}(-\pi,\frac{C^F}{EC^F})[1-\lambda]+\operatorname{cov}(-\pi,\frac{C^G}{EC^G})\lambda=\operatorname{cov}(-\pi,\frac{C}{EC})$$

where $\lambda=\frac{EC_G}{EC}$

If $\operatorname{cov}(\rho,\frac{C}{EC})$ and $\operatorname{cov}(-\pi,C)$ are unchanged by white noise inflation one may infer,

$$\Delta\operatorname{cov}(\rho,\frac{C^F}{EC^F})[1-\lambda]+\Delta\operatorname{cov}(\rho,\frac{C^G}{EC^G})\lambda=0=\Delta\operatorname{cov}(-\pi,\frac{C^F}{EC^F})[1-\lambda]+\Delta\operatorname{cov}(-\pi,\frac{C^G}{EC^G})\lambda$$

or

$$\Delta\operatorname{cov}(\rho,\frac{C^F}{EC^F})[1-\lambda]-\Delta\operatorname{cov}(-\pi,\frac{C^F}{EC^F})[1-\lambda]=-\Delta\operatorname{cov}(\rho,\frac{C^G}{EC^G})\lambda+\Delta\operatorname{cov}(-\pi,\frac{C^G}{EC^G})\lambda$$

But with homothetic and identical preferences

$$\upsilon^F-1=\frac{U_{C1}}{EU_{C1}}-1\approx[\alpha-1][\frac{C_1^F}{EC_1^F}-1]$$

or

$$\operatorname{cov}(\rho,C^j)\approx\frac{\operatorname{cov}(\rho,\upsilon^j)}{\alpha-1}$$

Thus

$$[1-\lambda][\Delta\operatorname{cov}(\rho,\upsilon^F)-\Delta\operatorname{cov}(-\pi,\upsilon^F)]=\lambda[-\Delta\operatorname{cov}(\rho,\upsilon^G)+\Delta\operatorname{cov}(-\pi,\upsilon^G)]$$

Using $1+E\rho-E[\frac{1+i}{1+\pi}]=\omega_\rho^j-\omega_{-\Pi}^j$, this can be written as ,

$$-[1-\lambda]\Delta[1+E\rho-E[\frac{1+i}{1+\pi}]]=\lambda\Delta[1+E\rho-E[\frac{1+i}{1+\pi}]]$$

The only way to satisfy this is $\Delta[1+E\rho-E[\frac{1+i}{1+\pi}]]=0$

Appendix 2

This appendix relates the algebraic and diagrammatic measures of the cost of inflation.

The welfare loss is the sum of three triangles: aa bb cc; ee cc dd; and bb ff dd.

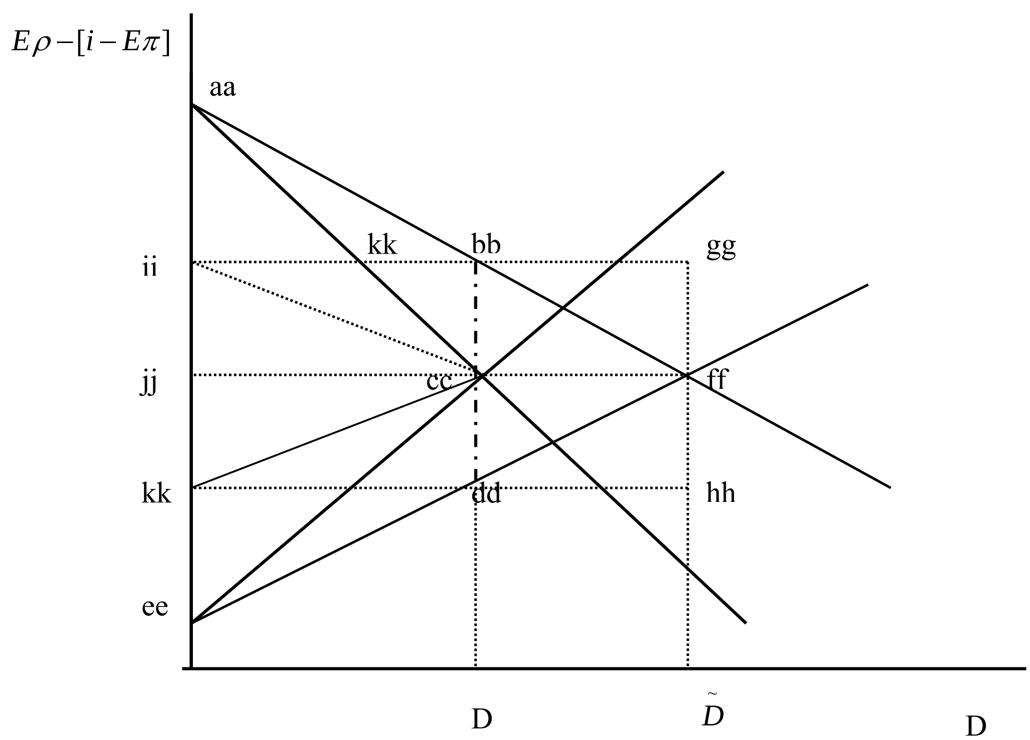

Figure 18: The welfare triangles analysis of the cost of unpredictable inflation

bb ff dd =bb ff cc + cc ff dd = $\frac{\text{bb gg ff cc}}{2}+\frac{\text{cc ff hh dd}}{2}=[\omega^F_{-\Pi}-\omega^G_{-\Pi}]\frac{[\tilde{D}-D]}{2}$

aa bb cc = aa bb kk + kk bb cc = ii kk cc + kk bb cc = ii bb cc = $\frac{\text{ii bb cc jj}}{2}=\omega^F_{-\Pi}\frac{D}{2}$

ee cc dd = $-\omega^G_{-\Pi}\dfrac{D}{2}$ (by parallel logic)

Welfare loss = aa bb cc + ee cc dd + bb dd ff =

$$[\omega^F_{-\Pi} - \omega^G_{-\Pi}]\frac{[\tilde{D} - D]}{2} + \omega^F_{-\Pi}\frac{D}{2} - \omega^G_{-\Pi}\frac{D}{2} = [\omega^F_{-\Pi} - \omega^G_{-\Pi}]\frac{\tilde{D}}{2}$$

Appendix 3

This appendix derives measures of $\omega^F_{-\Pi}$ etc.

$$\omega^F_{-\Pi} \equiv -\operatorname{cov}(\upsilon^F, -\pi) = E\upsilon^F[\pi - E\pi]$$

$$\omega^F_{-\Pi} = [\alpha - 1][\frac{C^F - EC^F}{EC^F}]E(\pi - E\pi)$$

The proportionate deviation in C from expectation caused by a surprise inflation equals the propensity to consume out of wealth times times the loss of wealth. Assuming the propensity to consume is the interest rate,

$$\frac{C^F - EC^F}{EC^F} = -rE(\pi - E\pi)\frac{D}{EC^F}$$

Therefore,

$$\omega^F_{-\Pi} = [1 - \alpha]\operatorname{var}\pi\ r\frac{D}{C^F}$$

and by the same logic,

$$-\omega^G_{-\Pi} = [1 - \alpha]\operatorname{var}\pi\ r\frac{D}{C^G}$$

But bb dd ff = $0.5[\omega^F_{-\Pi} + \omega^G_{-\Pi}][\tilde{D} - D]$

$$= 0.5[\operatorname{var}\pi\ r\frac{D}{C^F}[1-\alpha] + \operatorname{var}\pi\ r\frac{D}{C^G}[1-\alpha]][\tilde{D} - D]$$

$$= 0.5[\operatorname{var}\pi\ r\frac{D}{C}[1-\alpha]\frac{1}{s(1-s)}][\tilde{D} - D]$$

The area under the demand curve for bonds:

$$= \text{intercept}_0 D - \text{slope}_0\frac{D^2}{2}$$

aa bb cc = the area between the old and the new demand curves,

$$= \text{intercept}_0 D \text{ - } \text{slope}_0^{'} \frac{D^2}{2}$$

Therefore the area between the old and new demand curves is

$$= \text{-}[\text{slope}_0 - \text{slope}_0\,'] \frac{D^2}{2}$$

$$\text{slope}_0 - \text{slope}_0\,' = \frac{\omega_{-\Pi}^F}{D} = \frac{r \operatorname{var} \pi}{EC^F}[1-\alpha]$$

Therefore

aa bb cc = $\frac{r \operatorname{var} \pi}{EC^F}[1-\alpha]D^2 = \frac{r \operatorname{var} \pi}{C}[1-\alpha]D^2 \frac{1}{s}$

The area between the old and new supply curve,

$$= \text{-}[\text{slope}_0 - \text{slope}_0\,'] \frac{D^2}{2}$$

But

$$\text{slope}_0 - \text{slope}_0\,' = \frac{\omega_{-\Pi}^G}{D} = \frac{r \operatorname{var} \pi}{EC^G}[1-\alpha]$$

Therefore,

ee cc dd = $\frac{r \operatorname{var} \pi}{EC^G}[1-\alpha]\frac{D^2}{2} = \frac{r \operatorname{var} \pi}{C}[1-\alpha]\frac{D^2}{2}\frac{1}{1-s}$

Total cost = $0.5[\operatorname{var} \pi \; r \frac{\tilde{D}}{C}[1-\alpha]\frac{1}{s(1-s)}]D$

References

Coleman, William 2007 *The Causes, Costs and Compensations of Inflation. An Investigation of Three Problems in Monetary Theory*, Edward Elgar, U.K.

Dowd, Kevin 1994 'The Costs of Inflation and Disinflation' *Cato Journal*, vol. 14, no. 2, pp. 305-31