The Australian National University
Centre for Economic Policy Research
DISCUSSION PAPER

A Microfoundation for Increasing Returns in Human Capital Accumulation and the Under-Participation Trap

Alison L. Booth and Melvyn Coles

DISCUSSION PAPER NO. 543
December 2006

ISSN: 1442-8636
ISBN: 1 921262 14 1

Alison L. Booth, Economics Department, University of Essex, Wivenhoe Park UK and Economics Program, Research School of Social Sciences, Australian National University, ACT 0200 Australia

Melvyn Coles, I.C.R.E.A. and I.A.E., Campus UAB, 08139 Bellaterra, Barcelona, Spain

Acknowledgements
Helpful comments on an earlier draft were received from the Editor, two anonymous referees, Patricia Apps, Raquel Fernandez, Pietro Garibaldi, Thor Gylfason, Ray Rees, Etienne Wasmer, Gylfi Zoega and participants of the 2004 IZA / SOLE Transatlantic Meetings at Amersee, the CEPR Conferences at Bergen and the European Central Bank, the ESPE 2004 Conference, and a seminar at the Australian National University. We are grateful to the ARC for financial support under Discovery Project Grant No. DP0449887 "Modelling the Impact of Home and Market Productivities on Employment Status, Part-time and Full-time Wages". Melvyn Cole also thanks the Barcelona Economics Program of CREA for support.

ABSTRACT

This paper considers educational investment, wages and hours of market work in an imperfectly competitive labour market with heterogeneous workers and home production. It investigates the degree to which there might be both underemployment in the labour market and underinvestment in education. A central insight is that the ex-post participation decision of workers endogeneously generates increasing marginal returns to education. Although equilibrium implies underinvestment in education, optimal policy is not to subsidise education. Instead it is to subsidise labour market participation which we argue might be efficiently targeted as state provided childcare support.

JEL Classification: H24, J13, J24, J31, J42.

Keywords: Education, home production, hours of work, imperfect competition.

1 Introduction

This paper considers educational investment, wages and hours of market work in an imperfectly competitive labour market with heterogeneous workers and home production. It investigates the degree to which there might be both underemployment in the labour market and underinvestment in education. A central insight is that the ex-post participation decision of workers endogeneously generates increasing marginal returns to education. This non-convexity can result in a large discontinuity in educational choice and labour market participation across workers. The paper shows that for some workers, a competitive labour market would imply they invest significantly in education and participate with a high probability in the labour market. But wages below marginal product (in a non-competitive labour market) and increasing returns to education together imply a non-marginal switch to low educational investment and home production. These large substitution effects yield large welfare losses and so corrective taxation plays an important role. Although there is underinvestment in education, optimal policy is not to subsidise education. Instead it is to subsidise labour market participation, which we argue might be efficiently targeted as state-provided childcare support.

The paper considers a hold-up problem where in the first phase of their lives, youngsters increase their future workplace ability by investing in general skills. Those investments are made prior to becoming employed in the workplace. Clearly some skill investments, such as primary school education in literacy and numeracy skills, are invaluable both in the home and in the workplace. But the focus here is on educational choice past the compulsory school level, by which time literacy and numeracy skills have presumably been well honed. Instead students might further invest in a university degree in mathematics or a qualification in information technology, imbuing them with expertise that is valuable in the workplace but is unlikely to increase their skills in the home.

A central feature of the model is that there are increasing marginal returns to education. We stress that these increasing returns do not arise because we assume a Mincer wage equation with increasing returns. Indeed the arguments are consistent with a Mincerian wage rate $w = a + e$ where the wage rate w depends on endowed ability a and is linearly increasing in education e. But such a wage equation does not describe the marginal return to education. For example, the person who intends to specialise entirely in child rearing and home-making has a zero financial return to investing in workplace skills, regardless of the size of the Mincer wage effect. The marginal return to education depends both on the Mincer wage effect and expected labour supply, where increased labour supply implies human capital investments are "used" more intensively in the workplace. We shall show there are three reasons for increasing returns to education. First, there are increasing marginal returns to education because of a *participation effect*. More highly skilled workers earn higher wages in the workplace and so are more likely to participate in the workplace, thereby raising the ex ante expected returns to human capital investment. Second, increasing returns arise through an increasing *labour supply effect*, where more educated workers may find it worthwhile to work longer hours. But with a frictional labour market there is a third reason for increasing marginal returns to education - an increasing *wage competitiveness effect*. We show that firms bid more competitively for the worker's services as the value of employment increases. As wage compression decreases at higher productivity levels, the marginal return to education increases as education increases.

A second important feature of the paper is that it assumes workers have different productivities both at home and in the workplace. We introduce this assumption not only because it is realistic, although that is clearly an advantage.[1] But more importantly, it allows us to demonstrate how expected home productivity affects optimal educational choice and labour supply, where home and workplace productivities vary across individuals. Specifically we show that the deadweight losses that arise through an imperfectly competitive labour market are not equally spread across all workers. Increasing returns to education coupled with an imperfectly competitive labour market generates an "under-participation trap". If the labour market were competitive, then workers in that trap would choose a high level of education and high expected labour supply in the workplace. But because the labour market is not competitive and so wages paid are below marginal product, they substitute instead to home production. The increasing returns to education, however, imply the substitution effect is non-marginal for workers in this "trap". Instead they make very low skills investments ex ante, and participate with low probability in the labour market ex post. This large substitution effect implies a correspondingly large deadweight loss.

[1] High returns to home productivity might be realized by those involved with care of young children or elderly parents, or for individuals with a taste for leisure or for home renovations, or for those with a strong aversion to workplace employment. For childless households, non-participation might be associated with pure leisure, although time use studies do show that - even in households without children - considerable time is devoted to home-related activities such as cooking and cleaning.

The next section describes the model and Section 3 determines equilibrium remuneration and participation rates of workers by productivity type. Section 4 examines the worker's optimal investment decision and Section 5 develops the implications for optimal childcare policies. We establish that a participation subsidy, paid to the worker, not only corrects the ex-post under-participation problem, but also corrects the ex-ante under-education problem.

2 The Model

Each individual is productive both at home and in the workplace. A representative person is born in the first period with ability a and has expectations of future home productivity b. In the first period, the individual at cost $\phi(k)$ can acquire k units of general human capital, whereupon the worker's second period productivity in the workplace is $\alpha = a + k$. Assume ϕ is continuously diffentiable, strictly convex and $\phi(0) = \phi'(0) = 0$. The discussion section considers a more general specification where higher ability types can become more productive at lower marginal cost; i.e., $\phi = \phi(a, k)$ with marginal cost $\partial\phi/\partial k$ decreasing in a (and so education and ability are complementary inputs in productivity). We shall show that this variation makes little material difference to the results.

A useful simplifying assumption is that human capital investment k does not affect second period home productivity b. Again in the discussion section we describe what happens if, in addition, skills investment k increases home productivity. The results presented below hold as long as the effect of skills investment on market productivity, α, is sufficiently large relative to its impact on home productivity b. For ease of exposition, however, we assume for now that home productivity is fixed.

In the second period, the worker has a unit time endowment which is allocated between time spent in home production (h) and in the workplace (l), so that $h+l = 1$. Note that home production can also be interpreted as leisure. There are diminishing marginal returns to home production. If the worker allocates time h to home production, assume the value of home output is $bx(h)$ where $x(.)$ is increasing, differentiable and concave with $x(0) = 0$.

There are constant marginal returns to labour in the workplace; a worker with workplace productivity α who supplies l units of labour to the workplace generates revenue αl. One could instead assume diminishing marginal returns to labour, but if the worker's output is small relative to the scale of the firm, the constant returns assumption seems a reasonable approximation. The critical ingredient for what follows is that this revenue function $R = \alpha l$ exhibits increasing returns to scale in productivity and labour supply. As this is an important feature of the model, it is worth discussing it a little. For example in the competitive case, one typically assumes given wage rate w, the worker earns income $E = wl$ by supplying l hours to the market. A Mincer type wage equation, where wage $w = w(a, k)$ depends on education k, then implies earnings $E = lw(a, k)$. Even if there are diminishing returns to education in wages (i.e. w is concave in k) note that earnings $E = lw(a, k)$ exhibit **joint** increasing returns with respect to labour supply **and** education k. It is this non-convexity which is fundamental to the results. One interpetation is that increased labour supply implies human capital investments are 'used' more intensively. For example zero labour supply implies zero human capital usage, and the marginal return to education is then zero regardless of the magnitude of the Mincer wage effect $\partial w/dk$.

The market failure is a hold-up problem: the worker invests in human capital in the first period, and wages are determined in the second period in an imperfectly competitive labour market. One modelling approach would be to specify an equilibrium search framework where wages are determined by Nash bargaining (e.g. Pissarides (2000)) or by wage posting with on the job search (e.g. Burdett and Mortensen (1998), Postel-Vinay and Robin (2002)). In those frameworks, equilibrium implies workers *ex post* earn less than their marginal product and the hold-up problem implies each worker *ex ante* underinvests in skills. But given our focus on optimal labour market policy, the monoposony framework as described in Bhaskar and To (1999) provides a simpler equilibrium framework. Like the Nash bargaining approach, the Bhaskar and To (1999) framework implies equilibrium wage compression; that wages need not increase one-for-one with an increase in labour market productivity. The central advantage to the Bhaskar and To (1999) framework, however, is that we need not specify matching functions, free entry conditions etc or describe equilibrium wage dispersion. The policy discussion is consequently clearer, as there are neither thick market nor congestion externalities to complicate matters.

In contrast to Postel-Vinay and Robin (2002) who assume Bertrand wage competition between firms should an employee receive an outside offer, Bhaskar and To (1999) assume workers have idiosyncratic preferences over employment at different firms, and those preferences are private information. Thus a firm's wage offer depends on how much he/she believes the employee prefers working there rather than elsewhere. Bhaskar and To (1999) cite various empirical studies supporting the assumption that workers have heterogeneous preferences for non-wage characteristics. Bhaskar et al (2002) further note that this assumption can usefully summarise the variety of reasons for imperfect competition in the labour market. Specifically equilibrium implies a firm offers a wage below marginal product, where the firm's trade-off is between offering an even lower wage and an increased probability that the worker chooses to work elsewhere.[2]

The assumed market structure is analogous to a Hotelling pricing game with $n \geq 2$ competing firms.[3] Consider a representative worker who is characterised by productivities (α, b) which are observed by all firms.[4] Firms differ in their nonpecuniary attributes, such as geographical location and other nonwage job characteristics. Workers have heterogeneous preferences where the more distant are the i-th firm's characteristics from the worker's preferred characteristics, the larger is the worker's disutility cost c_i associated with employment at that firm. Note that this cost c_i is a fixed cost to working at firm i and is analogous to a transport or commuting cost.[5] The representative worker's employment preferences c_i, $i = 1, .., n$ are private information and considered as i.i.d. draws from c.d.f. F. Assume F is twice differentiable and its density is decreasing over its support $[0, \overline{c}]$; i.e., F is concave. Each firm i simultaneously makes a contract offer (y^i, l^i), where y^i is the amount paid to the worker in return for providing l^i units of labour time. Given those contract offers, the worker either accepts one, say at firm i, and so obtains period 2 utility $U_2 = bx(1 - l^i) + y^i - c_i$, or rejects all and so obtains period 2 utility $U_2 = bx(1)$ through home production. Note the worker is risk neutral in consumption. Should the worker accept firm i's contract offer, firm i makes profit $\alpha l^i - y^i$, while the other firms obtain zero profit.

Throughout we shall only consider symmetric pure strategy equilibria. In the second period and given (α, b), each firm i offers contract (y^i, l^i) to maximise expected profit. The symmetric Nash

[2] In Burdett and Mortensen (1998) with on-the-job search, a lower wage also increases the quit rate of the worker. But given the asymmetric information friction adopted here, we simplify by assuming no search frictions.

[3] Our approach thus differs from labour supply theory with exogenously given market-wage rates. Important contributions to the literature considering the allocation of time to home production - albeit in a different context to ours - include Becker (1965), Gronau (1977) and Apps and Rees (1997).

[4] Inferences on home productivity b might be based on age and gender, though we abstract from such issues here.

[5] Although one could instead specify a disutility cost $c_i l$, where that loss is proportional to the amount of time spent working at the firm, this would then introduce screening issues - a firm posts a menu of contracts where part-time employment contracts are targeted to workers with high c_i and full time contracts for those with low c_i. The transport (fixed) cost approach adopted here abstracts from such issues.

equilibrium implies all offer the same contract $(y^*(\alpha, b), l^*(\alpha, b))$. Given those equilibrium contract offers in the second period, the worker in the first period computes expected second period utility, denoted $U_2^*(\alpha, b)$. The worker then invests in skills k to maximise $U_2^*(\alpha(a, k), b) - \phi(k)$.

In anticipation of the results, it is useful to define the competitive benchmark where the market wage rate equals marginal product; $w = \alpha$. Let V denote the value of employment in that case

$$V(\alpha, b) = \max_{l \in [0,1]} \left[\alpha l - b[x(1) - x(1-l)] \right],$$

which is the (maximised) value of earnings net of foregone home production. As the worker prefers pure home production if $c_i > V$ for all i, the worker's participation probability in a competitive labour market is

$$P(V) = 1 - [1 - F(V)]^n.$$

Conditional on labour market participation, let $l^*(\alpha, b)$ denote the optimal labour supply decision; i.e.

$$l^*(\alpha, b) = \arg\max_{l \in [0,1]} \left[\alpha l - b[x(1) - x(1-l)] \right].$$

and note that the Envelope Theorem implies $l^* = \partial V / \partial \alpha$. Claim 0 describes their basic properties.

Claim 0. Characterisation of V, l^*.

(i) $l^* = 0$ and $V = 0$ for $\alpha \leq bx'(1)$;

(ii) $l^* \in (0, 1)$ and $V > 0$ are both strictly increasing in α and strictly decreasing in b for $\alpha \in (bx'(1), bx'(0))$;

(iii) $l^* = 1$ and $V = \alpha - b[x(1) - x(0)]$ for $\alpha \geq bx'(0)$; .

Claim 0 follows from standard optimisation theory. We shall refer to $\alpha = bx'(0)$ as the full time margin and productivities $\alpha \in [bx'(0), \infty)$ as the full-time employment region, noting that $l^* = 1$ is optimal for such α. We shall refer to $\alpha = bx'(1)$ as the part-time margin, and the interval $(bx'(1), bx'(0))$ as the part-time employment region as $l^* \in (0, 1)$ is optimal for such α. Note that $\alpha \leq bx'(1)$ implies there is no gain to trade as home productivity strictly dominates workplace productivity.

3 Equilibrium Wages

Given the set of contract offers $\{(y^i, l^i)\}_{i=1,..,n}$ and idiosyncratic utility costs c_i, the worker's second period payoff is

$$U_2 = \max_{i=1,..,n} \{bx(1-l^i) + y^i - c_i, bx(1)\}$$

where the worker either accepts one firm's offer or rejects all. This section characterizes the (symmetric, pure strategy) Nash equilibrium where each firm i simultaneously makes a contract offer (y^i, l^i) to maximise expected profit, given the job acceptance strategy of the worker.

As productivities are observed, each firm's optimal contract offer implies $l^i = l^*$. Given the set of optimal contract offers, $\{(y^i, l^*)\}_{i=1,..,n}$, the worker's optimal job acceptance strategy is to accept employment at firm i if

$$y^i - c_i + bx(1-l^*) > \max_{j \neq i}\{y^j - c_j + bx(1-l^*), bx(1)\}$$

Note that firm i faces two margins: a participation margin and a poaching margin. The participation margin requires that the job offer must fully compensate for foregone home production; i.e. the worker considers firm i's offer only if $y^i - c_i > b[x(1) - x(1-l^*)]$. The poaching margin requires that firm i's offer is also preferred to all other wage offers; i.e. $y^i - c_i > y^j - c_j$ for all $j \neq i$. Theorem 1 now describes the symmetric Nash equilibrium to this contract posting game.

Theorem 1. Equilibrium Contract Offers.

For any (α, b) with $V > 0$, a pure strategy, symmetric contract-posting equilibrium implies each firm offers contract (y^*, l^*) where

$$y^* = b[x(1) - x(1-l^*)] + s^*$$

with $s^* = s^*(V)$ given by

$$\frac{1}{n}[1 - [1-F(s^*)]^n] = [V - s^*]\left[[1-F(s^*)]^{n-1} f(s^*) + (n-1)\int_0^{s^*} [1-F(c_1)]^{n-2}[f(c_1)]^2 dc_1\right]. \quad (1)$$

Proof is in the Appendix.

The equilibrium wage offer, y^*, fully compensates the worker for foregone home production and offers additional surplus s^*. The worker participates in the labour market (i.e. accepts a job offer) if and only if $y^* - c_i + bx(1 - l^*) > bx(1)$ for at least one i, which is equivalent to $c_i < s^*$ for at least one firm. Hence the above equilibrium wage offers imply the worker's participation probability is

$$P(s^*) = 1 - [1 - F(s^*)]^n.$$

The equilibrium surplus offered, s^* as defined in (1), depends on V, the value of employment, and on the number of competing firms. As n becomes arbitrarily large, competition between firms implies s^* converges to V and equilibrium converges to the competitive case. However, for finite n, firms shave those offers so that $s^* < V$. The equilibrium choice, described by (1), reflects the standard monopsony trade-off between lower wage offers and lower employment. Optimality requires that these two margins are equal. The left hand side of (1) is the probability of employment (given by $P(s^*)/n$) and describes the marginal loss in profit should, say, firm 1 offer slightly more surplus than the equilibrium offer. The right hand side describes the marginal increase in firm 1's profit by making a more attractive offer which increases the probability the potential employee will accept it. The first term in square brackets on the RHS, $f(s^*)[1-F(s^*)]^{n-1}$, is the measure of workers who are marginally attracted from non-participation, that is, workers whose $c_1 = s^*$ and $c_j > s^*$ for $j \neq 1$. The second term is the measure of workers marginally poached from a competing firm j, where the worker is indifferent between accepting firm 1's offer and a firm $j's$ offer (that is, $c_1 = c_j < s^*$ and $c_k > c_1$ for $k \neq 1, j$), and where this state potentially occurs with each of the $n - 1$ competing firms. Also note that (1) describes the optimal contract offer with pure monopsony, where $n = 1$, and there is no poaching margin.

The critical feature for what follows is that the equilibrium contract offer implies both wage compression and underparticipation in the labour market.

Claim 1. $s^*(V)$ is increasing and continuously differentiable in V with:

(i) $s^* = 0$ at $V = 0$;

(ii) $ds^*/dV < 1$ and $s^*(V) < \overline{c}$ for $V \in (0, \overline{c} + d)$,

(iii) $s^*(V) = V - d$ for $V \geq \overline{c} + d$ where

$$d = \frac{1}{n(n-1)\int_0^{\overline{c}}[1-F(c)]^{n-2}f(c)^2 dc}. \tag{2}$$

Proof is in the Appendix. Notice that d can be thought of as a measure of labor market stickiness, as described for example in Stevens (1994).

It can be shown that the same properties of s^* occur when F is only log concave; i.e. when $F''F < F'^2$, but the proof is both long and tedious.[6] Formally the equilibrium outcome described in Theorem 1 corresponds to an n-buyer first price auction, where the seller has private independent match values. Although assuming F is concave (or log concave) is sufficient to guarantee non-paradoxical comparative statics; i.e. more productive workers receive higher wage offers, establishing that a pure strategy symmetric equilibrium necessarily exists is less straightforward. The Technical Appendix describes the formal existence problem. In what follows, we simply assume a symmetric pure strategy equilibrium exists.

Section 4 describes optimal investment in the first period given workers anticipate contract offers as described in Theorem 1. Those results depend critically on the following market failures.

I. Equilibrium Wage Compression.

Imperfect competition in the labour market implies firms offer surplus $s^* < V$. Claim 1 establishes at low workplace productivities, where $0 < V(.) < \overline{c} + d$, that $ds^*/dV < 1$. Following Acemoglu and Pischke (1999) we describe this outcome as wage compression; that is, wage offers do not increase one-for-one with workplace productivity. An important feature for what follows is that wage compression disappears at high enough levels of workplace productivity. In particular, Claim 1 implies

(i) there is wage compression for (α, b) satisfying $V < \overline{c} + d$ as $ds^*/dV < 1$ in that region, while

(ii) there is no wage compression for (α, b) satisfying $V > \overline{c} + d$ as $ds^*/dV = 1$.

To understand why there is no wage compression at high V , recall that a firm faces two

[6] Establishing that $0 < \frac{ds^*}{dV} < 1$ in (9) in the Appendix requires showing

$$[1 - F(s^*)]^{n-1} f(s^*) + [V - s^*][1 - F(s^*)]^{n-1}[-F''(s^*)] > 0$$

where s^* is defined by (1). Using (1) to substitute out $(V - s^*)$ it is possible, but tedious, to show that log concavity of F, which implies $FF'' < F'^2$, is sufficient to imply the above inequality at s^*.

oligopsony margins: a poaching margin and a participation margin. By offering higher wages, a firm might not only attract an employee from a competing firm - the poaching margin - but also attract a non-participant into the market sector.

The participation margin does not bind for workers with sufficiently high V that, in equilibrium, they accept a job offer with probability one. As noted above, a useful analogy is the Hotelling pricing literature where we might interpret c_i as the worker's transport cost to work at firm i. The case "V sufficiently high that an offer is always accepted" is typically referred to as a "covered market". The equilibrium is that all firms offer a wage equal to the worker's value of output less "price" $d > 0$. Equilibrium d reflects the marginal probability that a small increase in the offered wage will poach the worker away from the competing firms and, in a symmetric equilibrium, d depends only on the number of competing firms and the distribution of transport costs. The lump-sum deduction implies there is no wage compression.

In contrast, the participation margin binds for workers with V less than $\overline{c} + d$. Such workers include low workplace-productivity workers and intermediate productivity workers with high home productivities. An important property of the Hotelling pricing structure is that, as the value of employment increases, wage competition at the margin becomes more intense. In particular, (9) in the Appendix implies $ds^*/dV = 0.5$ at $V = 0$, $ds^*/dV < 1$ for $V < \overline{c} + d$ and $ds^*/dV \to 1$ as $V \to \overline{c} + d$. Hence wages rise more quickly with productivity as the participation margin peters out, where $ds^*/dV = 1$ for all $V \geq \overline{c} + d$.

II. Equilibrium Underparticipation

The worker's participation probability is $P(s^*) = 1 - [1 - F(s^*)]^n$. Given the competitive outcome would imply $s^* = V$, Claim 1 implies:

(i) there is underparticipation for (α, b) satisfying $0 < V < \overline{c} + d$ as $P(s^*) < P(V)$ with $P(s^*) < 1$, while

(ii) there is efficient participation for (α, b) satisfying $V > \overline{c} + d$ as $P(s^*) = P(V) = 1$.

The underparticipation problem arises as worker preferences or disutility costs c_i are not observed and firms offer less than full surplus. If the value of workplace productivity is sufficiently high, however, that the worker participates with probability one, then the privately optimal participation decision coincides with the socially optimal one.

4 The Worker's Optimal Education Decision

To identify the privately optimal investment decision in the first period, Claim 2 now computes expected second period utility, which is denoted $U_2^*(\alpha, b)$.

Claim 2. For any (α, b) and offers as described in Theorem 1:

$$U_2^*(\alpha, b) = bx(1) + \int_0^{s^*} [1 - (1 - F(c))^n]\, dc. \tag{3}$$

Proof is in the Appendix.

Expected second period utility equals the option value of home production plus the expected surplus from employment, which depends on $V = V(\alpha, b)$ and labour market imperfections, as $s^* = s^*(V)$.

In the first period, given ability a and expected home productivity b, the worker's optimal investment decision solves:

$$\max_{\alpha \geq a} U_2^*(\alpha, b) - \phi(\alpha - a)$$

where the worker chooses second period productivity $\alpha \geq a$ at investment cost $\phi(k)$, where $k = \alpha - a$. The necessary condition for a maximum is

$$\partial U_2^* / \partial \alpha = \phi'(\alpha - a),$$

i.e., the worker sets the marginal return to education equal to its marginal cost, where (3) implies the marginal return to education, denoted MR, is

$$\begin{aligned} MR &\equiv \frac{\partial U_2^*}{\partial \alpha} = [1 - (1 - F(s^*))^n] \frac{ds^*}{dV} \frac{\partial V}{\partial \alpha} \\ &= P(s^*) \frac{ds^*}{dV} \frac{\partial V}{\partial \alpha}. \end{aligned} \tag{4}$$

Note, MR depends on three components: $P(s^*)$ is the probability the worker participates in the labour market; ds^*/dV is the rate at which offered compensation s^* increases with V; and $\partial V / \partial \alpha$ describes how V increases with productivity α.

In a competitive labour market with earnings function $E = \alpha l$, the Envelope Theorem would imply marginal return to education $\partial E/\partial\alpha = l^*$, which is simply expected labour supply. The above expression is more complicated as there are labour market imperfections. Nevertheless the interpretation is the same. The definition of V and the Envelope Theorem imply $\partial V/\partial\alpha = l^*$. Hence $[P(.)][\partial V/\partial\alpha]$ together describe expected labour supply. The marginal return to education is expected labour supply times the marginal increase in wage through higher productivity.

Figure 1 plots MR (with b fixed). Most importantly for what follows, note that there are increasing marginal returns. This occurs for three reasons:

(i) Participation effects: an increase in productivity implies firms offer better wages which increases the worker's participation probability; i.e. $P(s^*)$ increases as α increases. The higher participation probability increases directly the marginal return to education.

(ii) Increasing labour supply: $\partial V/\partial\alpha$ equals l^* and as an increase in workplace productivity implies an increase in labour supply l^* (Claim 0), this further increases the marginal return to education.

(iii) Increasing wage competiveness: as the value of employment V increases, firms at the margin bid more competitively for the worker's services. In particular, $ds^*/dV = 0.5$ at $V = 0$, while $ds^*/dV \to 1$ as $V \to \overline{c} + d$ (see Claim 1); i.e. wage compression decreases at higher productivity levels.

To plot MR (given b) define the efficiency frontier $\alpha = \overline{\alpha}(b)$ where

$$V(\overline{\alpha}, b) = \overline{c} + d$$

and note Claim 0 implies $\overline{\alpha}$ is strictly increasing in b. Also note that $V(\alpha, b) \geq \overline{c} + d$ if and only if $\alpha \geq \overline{\alpha}(b)$. The above implies the following results:

(a) $MR = 0$ for $\alpha < bx'(1)$ [Claim 0 implies $l^* = V = 0$ in this region and so $P(s^*) = 0$].

(b) the slope of MR is zero at $\alpha = bx'(1)$;

(c) suppose $\overline{c}$ is relatively large; specifically $b[x'(0) - [x(1) - x(0)]] < \overline{c} + d$. This implies that a person at the full time margin, one with productivity $\alpha = bx'(0)$, has value of employment $V < \overline{c} + d$ and so does not necessarily participate in the labour market. It follows that $\overline{\alpha}(b) > bx'(0)$ as drawn in Figure 1 and so $MR = 1$ for $\alpha \geq \overline{\alpha}(b)$.

Figure 1 here.

Although MR is continuous, its slope is not continuous at the full time margin (where $\alpha = bx'(0)$). In particular, labour supply $l^* \equiv \partial V/\partial\alpha$ is strictly increasing in α in the part-time employment region, where increasing labour supply generates increasing returns to education [see (ii) above]. At the full time margin, however, labour supply becomes constrained $l^* = 1$ and this source of increasing returns stops discontinuously at that point.

$\phi'(\alpha - a)$ is the marginal cost to skill accumulation and is denoted MC_a in Figure 1. The assumptions on ϕ imply $MC_a = 0$ at $\alpha = a$ and is strictly increasing in α. The optimal skills investment decision of a worker with ability a occurs where MC_a crosses MR. As demonstrated in Figure 1, there may be multiple intersections - the middle one describes a minimum, the other two describe local maxima. We now determine which of those local maxima describe the global maximum.

Consider the interesting case of a person of ability type $a = a_M$, as drawn in Figure 1. Because the two shaded areas are equal, this person is indifferent to investing to $\alpha = \alpha_2 > bx'(0)$ or investing to $\alpha = \alpha_1 < bx'(0)$. Now consider an increase in ability $a > a_M$. This implies the MC_a curve shifts to the right (and so marginal cost falls) while MR is unchanged. Thus workers with ability $a > a_M$ strictly prefer the right-side maximum and so train where $\alpha > \alpha_2 > bx'(0)$. Such workers have high V ex-post, have relatively high participation probabilities and work full time (choose $l^* = 1$). In contrast a decrease in ability $a < a^M$ implies the MC_a curve shifts to the left (and marginal cost rises) and so lower ability types strictly prefer the left-side maximum. Such workers train to $\alpha < \alpha_1 < bx'(0)$, they have low V ex-post, low participation probabilities and will only consider part-time employment. Increasing returns to education therefore leads to discontinuous investment decisions across ability a^M.

To see that this discontinuity generates large deadweight losses, consider the optimal investment and participation decisions in a competitive labour market. Recall that the private marginal return to investment is

$$MR = P(s^*)\frac{ds^*}{dV}\frac{\partial V}{\partial\alpha} = P(s^*)\frac{ds^*}{dV}l^*.$$

As previously explained, the competitive outcome implies $s = V$ and so the social return to education, denoted SR, is

$$SR = P(V)\frac{\partial V}{\partial \alpha} = P(V)l^* \tag{5}$$

which is expected labour supply. Hence $MR < SR$ if there is underparticipation, $P(s^*) < P(V)$, or if there is wage compression $ds^*/dV < 1$.

It follows that $MR = SR$ at very low productivities, where $\alpha < bx'(1)$, in which case $V = 0$ and so $MR = SR = 0$ (there is no gain to trade). It also follows that $MR = SR$ for very high productivities, where $\alpha > \overline{\alpha}(b)$, as there is efficient participation and no wage compression. For intermediate productivities, however, we have $MR < SR$ due to underparticipation and wage compression.

Figure 2 here.

Note, both MR and SR have a zero slope at the part-time margin, and both have discontinuous slopes at the full-time margin. Claim 1 implies $SR > MR$ for all $\alpha \in (bx'(1), \overline{\alpha}(b)]$.

Recall that the worker with ability a_M is indifferent between investing to α_1 or α_2. The shaded area describes the deadweight loss associated with the low investment decision. The socially optimal decision is that the worker invests to α_s. If the worker invests to α_2, the resulting deadweight loss corresponds to the Harberger triangle labelled DWL_2 in Figure 2. If the worker instead invests to α_1, the large substitution effect implies deadweight loss DWL_1 which is clearly much larger.

Increasing returns to education and an imperfectly competitive labour market can therefore lead to an *under-participation trap*. Workers with ability $a < a_M$ invest in skills where $\alpha < \alpha_1$. Having low V, they have low participation probabilities, and only participate in part-time employment (if at all). But the socially optimal decision for these workers may be that they invest to skills $\alpha_s > bx'(0)$ and participate in full time employment with a high participation probability. The discontinuity in investment behaviour leads to a large deadweight loss.

Figure 1 describes a_M for a particular value of home productivity b. More generally for any b, let (a_M, b) denote the worker who is indifferent to investing to high α and working full-time, or investing low α and working part-time with a low probability. As the value of employment V depends on b, then a_M varies with b. The following characterises $a_M = a_M(b)$.

An increase in b does not affect the MC curve. Now consider how an increase in b affects the MR curve. First note that a (small) increase in b implies an increase in $\overline{\alpha}(.)$ and a right shift in the part-time and full time margins. Second, fix an $\alpha \in (bx'(1), \overline{\alpha}(b))$. A (small) increase in b implies lower labour supply l^* (strictly lower in the part-time region), strictly lower V (Claim 0) and as $P(s^*) < 1$ in this region, MR falls in this region. Figure 3 draws two MR curves, denoted MR, MR$'$ corresponding to two different home productivities b, b' with $b < b'$.

Figure 3 here.

An increase in b to b' implies a fall in MR as drawn in Figure 3. The marginal worker as depicted in Figure 1, the one with ability $a = a_M(b)$ and home productivity b, now strictly prefers to choose low skills $\alpha < \alpha_1$ should home productivity increase to $b' > b$. Hence $a_M(b') > a_M(b)$; *i.e.* the undeparticipation trap is increasing in home productivity. It also follows that if home productivity is sufficiently small that $a_M < 0$, then the underparticipation trap disappears.

Of course, the above applies if the marginal cost curve, MC_a, is relatively flat. If the marginal cost curve is steep enough, then the part-time employment trap does not exist. Figure 4 depicts this case.

Figure 4 here.

As in the previous cases, the investment and participation decisions are distorted for those with intermediate ability. Those with very low workplace ability and high home productivity do not invest in general human capital and focus purely on home production. Those with very high workplace ability invest fully in skills, where $MC = 1$, and participate with a high probability in full time employment. The imperfect labour market distorts market behaviour for those with with intermediate participation probabilities. Although there are increasing marginal returns to education, a steep marginal cost curve (implying education choices are inelastic relative to endowed ability) implies relatively small substitution effects and the efficiency loss corresponds to standard Harberger triangles.

5 Discussion

It is well known that an imperfectly competitive labour market may lead to wage compression and underinvestment in general human capital.[7] A key insight here is that it also leads to underparticipation which acts as a multiplier effect - lower participation rates lower still further the marginal

[7] See for example Stevens (1994) and Acemoglu and Pischke (1999).

return to education.[8] Further, with heterogeneous workers and increasing returns to education, the corresponding welfare losses are largest for a particular subset of workers - those with high workplace ability a but whose home productivity is also relatively high, and so an imperfectly competitive labour market leads to a large substitution to home production. For workers in the "underparticipation trap" the efficient outcome (in a competitive market) implies large investments in human capital and high participation rates in the labour market. But as they do not receive the full return to those investments, they instead substitute to home production – they make low skills investments and participate with low probability in the labour market.

5.1 Extensions

Before turning to optimal policy we first discuss two variations on the model. The first relates to the cost of skills acquisition. Suppose that the cost of education is now $\phi(k, a)$ where marginal cost $\partial\phi/\partial k$ is decreasing in a and $\partial\phi/\partial k = 0$ at $k = 0$ as before. Thus higher ability types can accumulate greater skills at lower cost. Note then that an increase in ability implies MC_a not only shifts to the right, it also falls. But the overall effect is qualitatively identical to that already considered and so does not affect the insights.

The second variation relates to the assumption that home productivity b is independent of k. Relaxing this has less innocuous implications. Suppose that $b = b(k)$ and is an increasing function. Note that, given (α, b) in the second period, Claims 1 and 2 continue to hold. The expected marginal return to training in the first period, however, is now given by:

$$\frac{dU_2^*(\alpha, b)}{dk} = \frac{\partial U_2^*(\alpha, b)}{\partial \alpha}\frac{d\alpha}{dk} + \frac{\partial U_2^*(\alpha, b)}{\partial b}\frac{db}{dk}$$

as home productivity also increases with k. Noting that $d\alpha/dk = 1$ by assumption and that the Envelope Theorem implies $\frac{\partial V}{\partial b} = -[x(1) - x(1 - l^*)]$, we obtain

$$MR = P(s^*)\frac{ds^*}{dV}\frac{\partial V}{\partial \alpha} + \left[x(1)[1 - P(s^*)\frac{ds^*}{dV}] + x(1 - l^*)P(s^*)\frac{ds^*}{dV}\right]\frac{db}{dk} \tag{6}$$

[8] The approach of Acemoglu (1996) is quite different. In that model, while firms have constant returns to scale production functions, an interaction between ex ante human capital investments and bilateral search results in *social* increasing returns to average human capital. In contrast, we explicitly allow for home production and therefore capture the possibility of under-participation and wage compression generating mutually reinforcing effects on human capital investments.

Note the second term is positive (as $0 \leq P(s^*), \frac{ds^*}{dV} \leq 1$) and so, if home productivity is strictly increasing in k, this further increases the marginal return to education. We have already established the first term yields increasing marginal returns. The second term is ambiguous. If for example $b(k)$ is sufficiently large that $P(s^*) = 0$, then $b'' < 0$ implies decreasing marginal returns to education. As such types do not participate in the labour market, the only effect of education is its impact on home productivity. But it would appear reasonable to assume that university level degree schemes have a larger impact on potential earnings in the workplace than on one's capabilities as a home-maker. If $b'(k)$ is relatively small, the first term dominates in (6) and the insights obtained above go through.

5.2 Policy

Optimal policy requires increasing the return to participation in the labour market relative to non-participation. The obvious approach is either to (i) tax non-participants with a home production tax, or (ii) subsidise participation. The first approach - a tax on non-participation - is unlikely to be politically feasible and so we focus on the latter.

Suppose the government observes the worker's productivity parameters α, b and offers an employment subsidy $x = x(V)$ to workers who participate in the labour market, where $V = V(\alpha, b)$ as defined before. Repeating the analysis as before and given $x \geq 0$, it is straightforward to show that, in a pure strategy symmetric equilibrium, the equilibrium surplus offered by firms is $s^*(V + x) - x$. In other words, the firms extract the employment subsidy from the worker (the $-x$ term), but the equilibrium offer then reflects that the value of workplace employment is $V + x$. Given such offers, workers obtain net surplus $s^*(V + x)$.

To identify the optimal subsidy, note that the competitive outcome implies $s = V$. Hence implementing the competitive outcome implies optimal employment subsidy, x^*, where

$$s^*(V + x^*) = V.$$

This condition identifies the optimal employment subsidy. It follows from the Implicit Function Theorem and (1) in Theorem 1 that $x^*(0) = 0$, $x^*(.)$ is strictly increasing for $V < \overline{c} + d$, and $x^* = d$ for $V \geq \overline{c} + d$.

Thus guaranteeing efficient participation **and** efficient education requires an employment subsidy paid to workers. An education subsidy, in contrast, is inappropriate. Of course for many types the welfare gains through subsidising particpation may be small. Indeed the welfare gain is zero for high ability types who invest in large amounts of education and participate in the labour market with probability one. Instead as clearly demonstrated in Figure 2, the welfare gains are largest for those who are caught in the "underparticipation" trap. In the uncorrected market, these workers are characterised by relatively high home and workplace abilities, but they choose low education *ex ante* and have low labour market participation rates. It is well known from empirical studies using European data that women with children at home are characterised by low participation rates and relatively low education levels (see Petrongolo (2004) and references therein). This suggests that individuals most likely to be caught in the "underparticipation trap" are young women who expect to have children. An obvious employment subsidy which targets precisely this group is a state-subsidised childcare scheme, where childcare payments are made conditional on employment. Such a subsidy potentially generates large welfare gains, for it not only corrects the ex-post underparticipation distortion but also encourages women to invest more in education when young. [9]

6 Conclusion

It is surprising that the increasing returns argument presented here has received no attention in the profession. Possibly it has been missed as there are decreasing marginal returns to labour supply and, given labour supply, there are also decreasing returns to education. Of course this does not imply a concave programming problem as there are joint increasing returns. When decisions are sequential, as in the hold-up problem considered here, these joint increasing returns generate increasing marginal returns to education in the first period. We have shown that, in an imperfectly competitive labour market, increasing returns to education generate an under-participation trap. Optimal corrective policy is an employment subsidy, which we argue might be efficiently targeted as a public childcare program.

[9] For empirical estimates of the excess demand for subsidised childcare places by mothers of small children, see for example Wrohlich (2005) and the references surveyed therein.

A popular alternative model of an imperfectly competitive labour market assumes instead search frictions and that wages are determined by Nash bargaining. In particular given (α, b) and free entry of firms, the axiomatic Nash bargaining approach would imply the firm negotiates profit π and labour supply l as

$$\max_{\pi,l}[\pi]^{1-\gamma}[\alpha l - \pi + bx(1-l) - bx(1)]^{\gamma}$$

where $\gamma \in [0,1]$ is the worker's bargaining power, $bx(1)$ is the worker's threatpoint [i.e. the value of home production] and the firm's threatpoint is zero in a free entry equilibrium. By definition of V in the text, this reduces to

$$\max_{\pi}[\pi]^{1-\gamma}[V-\pi]^{\gamma}$$

and Nash bargaining implies worker remuneration y^* satisfies $dy^*/dV = \gamma$. As in Claim 1, this implies equilibrium wage compression and so one would anticipate the same effects on education and particpation as discussed here. But there are two main advantages to the Bhasker and To (1999) approach. One is that it rules out search externalities, such as thick market and congestion externalities, which would otherwise complicate the policy discussion. It also does not require solving for the steady state distribution of job seeker productivities which, in equilibrium, affects the vacancy creation decision of firms. The simpler approach shows clearly that underparticipation and wage compression generate mutually reinforcing distortions on human capital investment: wage compression implies workers tend to underinvest in workplace skills, and lower skills imply a lower participation probability which further reduces the expected return to human capital accumulation.

In currently ongoing work we examine how increasing returns to education interact with various other market distortions such as (i) endogenous household formation with matching frictions in the marriage market (Booth and Coles, 2005); and (ii) government tax policy, where increasing returns to education causes large substitution effects, and hence large deadweight losses, around the non-participant margin (Booth and Coles, 2006a). In a third paper, we show that increasing returns to education arise even in a perfectly competitive labour market (Booth, Coles and Gong, 2006b) and identify these effects empirically using individual-level data.

References

[1] Acemoglu, Daron (1996) "A Microfoundation for Social Increasing Returns in Human Capital Accumulation". *The Quarterly Journal of Economics*, Vol. 111, No. 3. (Aug., 1996), pp. 779-804.

[2] Acemoglu, Daron and Jörn-Steffen Pischke. "The Structure of Wages and Investment in General Training." *Journal of Political Economy*, 107:3 (June 1999), 539-572.

[3] Apps PF and R Rees (1997). 'Collective Labor Supply and Household Production', *Journal of Political Economy*, 105, pp. 178-190.

[4] Becker, GS (1965). 'A Theory of the Allocation of Time', *The Economic Journal*, 75, pp. 493-517.

[5] Bhaskar, V. and Ted To (1999), 'Minimum Wages for Ronald McDonald Monopsonies: A Theory of Monopsonistic Competition,' *The Economic Journal*, 109(455), pp. 190-203.

[6] Bhaskar, V., Alan Manning and Ted To (2002). 'Oligopsony and Monopsonistic Competition in Labor Markets'. *Journal of Economic Perspectives,* 16(2), pp.155-174, Spring.

[7] Booth Alison L and Melvyn G Coles (2005). "Education, Matching and the Allocative Value of Romance". CEPR Discussion Paper No. 5099. June.

[8] Booth Alison L and Melvyn G Coles (2006a). "Modeling the Impact of Fiscal Policy on Labor Supply and Education in an Economy with Household and Market Production". Mimeo, December.

[9] Booth Alison L, Melvyn G Coles and Xiaodong Gong (2006b). "Increasing Returns to Education: Theory and Evidence". Mimeo ANU.

[10] Burdett, Ken and Dale T. Mortensen (1998). "Wage Differentials, Employer Size and Unemployment," *International Economic Review,* 39, 257-273

[11] Gronau, R (1977). "Leisure, Home Production and Work – the Theory of the Allocation of Time Revisited", *Journal of Political Economy*, 85(6), pp 1099-1124, December.

[12] Petrongolo, Barbara (2004) "Gender segregation in employment contracts", *Journal of the European Economic Association*, Papers and Proceedings, 2 (2-3), pp. 331-345, April-May.

[13] Pissarides, CA (2000). *Equilibrium Unemployment Theory,* 2nd Edition, MIT Press, Cambridge MA.

[14] Postel-Vinay, F. and J.-M. Robin (2002). "Wage Dispersion with Worker and Employer Heterogeneity", *Econometrica*, 70(6), 2295-350.

[15] Stevens, Margaret (1994). "A Theoretical Model of On-the-Job Training with Imperfect Competition", *Oxford Economic Papers*, v.46, pp. 537-562.

[16] Wrohlich, K (2005). "The Excess Demand for Subsidized Child Care in Germany". IZA Discussion Paper No. 1515.

7 Technical Appendix

Proof of Theorem 1.

Consider a symmetric equilibrium where all firms post contract (y^*, l^*). Suppose firm 1 considers a deviating (but optimal) contract (y^1, l^*). Given the worker's optimal job acceptance strategy (as defined in the text), firm 1's expected profit by offering y^1, denoted π_1, is

$$\pi_1 = P(y^1 - c_1 \geq \max_{j \neq 1}[b[x(1) - x(1 - l^*)], y^* - c_j])[\alpha l^* - y^1],$$

where $P(.)$ is the probability that the worker accepts firm 1's job offer,[10] whereupon the firm makes profit $\alpha l^* - y^1$.

To compute this probability, note that for each c_1 satisfying $y^1 - c_1 \geq b[x(1) - x(1 - l^*)]$; i.e. for $c_1 \leq y^1 - b[x(1) - x(1 - l^*)]$, the worker prefers employment at firm 1 rather than pure home production. Further for such c_1, the worker also prefers firm 1's employment offer to firm $j's$ offer as long as $y^* - c_j \leq y^1 - c_1$; i.e. as long as $c_j \geq y^* - y^1 + c_1$ which occurs with probability

[10] As there are no mass points in F, by assumption, we can assume a weak inequality.

$1 - F(y^* - y^1 + c_1)$. Hence integrating over such c_1, the probability the worker accepts firm 1's contract offer is

$$\int_0^{y^1 - b[x(1)-x(1-l^*)]} [1 - F(y^* - y^1 + c_1)]^{n-1} f(c_1) dc_1.$$

Hence firm 1's expected profit is

$$\pi_1 = [\alpha l^* - y^1] \int_0^{y^1 - b[x(1)-x(1-l^*)]} [1 - F(y^* - y^1 + c_1)]^{n-1} f(c_1) dc_1.$$

Now define $s^* = y^* - b[x(1) - x(1 - l^*)]$ and so

$$y^* = b[x(1) - x(1 - l^*)] + s^*.$$

y^* is decomposed as full compensation for foregone home production plus additional surplus s^*. Similarly define $s^1 = y^1 - b[x(1) - x(1 - l^*)]$. Substituting out y^1, y^* in the above and using the definition of $V(\alpha, b)$, firm 1's profit reduces to

$$\pi_1(s^1, s^*; \alpha, b) = [V - s^1] \int_0^{s^1} [1 - F(s^* - s^1 + c_1)]^{n-1} f(c_1) dc_1 \tag{7}$$

with $V = V(\alpha, b)$. Hence given s^*, firm 1's best response for s^1 is defined by the first order condition $\partial \pi_1 / \partial s^1 = 0$ where the above implies

$$\begin{aligned} \frac{\partial \pi_1}{\partial s^1} \quad = \quad & - \int_0^{s^1} [1 - F(s^* - s^1 + c_1)]^{n-1} f(c_1) dc_1 \qquad (8) \\ & + [V - s^1] f(s^1) [1 - F(s^*)]^{n-1} \\ & + [V - s^1] \int_0^{s^1} (n-1) \left[f(s^* - s^1 + c_1) [1 - F(s^* - s^1 + c_1)]^{n-2} \right] f(c_1) dc_1. \end{aligned}$$

A pure strategy, symmetric equilibrium requires firm 1's best response $s^1 = s^*$, and so the above condition implies

$$\begin{aligned}\int_0^{s^*}[1-F(c_1)]^{n-1}f(c_1)dc_1 &= [V-s^*]f(s^*)[1-F(s^*)]^{n-1} \\ &\quad +[V-s^*]\int_0^{y^*}(n-1)[1-F(c_1)]^{n-2}f(c_1)^2dc_1\end{aligned}$$

is a necessary condition for a pure strategy symmetric equilibrium. The left hand side is integrable and this equation simplifies to (1). This completes the proof of the Theorem.

Proof of Claim 1. (1) immediately implies $s^*(0)=0$. Differentiating (1) w.r.t. V and rearranging yields:

$$\frac{ds^*}{dV}=\frac{[1-F(s^*)]^{n-1}f(s^*)+(n-1)\int_0^{s^*}[1-F(c)]^{n-2}f(c)^2dc}{2[1-F(s^*)]^{n-1}f(s^*)+(n-1)\int_0^{s^*}[1-F(c)]^{n-2}f(c)^2dc+[V-s^*][1-F(s^*)]^{n-1}[-F''(s^*)]} \tag{9}$$

Putting $s^*=V=0$ implies part (i).

Noting $V>0$ implies $s^*<V$ [a firm never offers $s^*>V$ as it implies a negative profit] then F concave over its support implies $0<\frac{ds^*}{dV}<1$ while $0<s^*<\overline{c}$. As F is twice differentiable, ds^*/dV is continuous for $s^*<\overline{c}$ and note $s^*\to\overline{c}^-$ implies $ds^*/dV\to 1$. Putting $s^*=\overline{c}$ in (1) implies $V=\overline{c}+d$ where d is defined in the Claim. Finally (1) implies $s^*=V-d$ for $s^*\geq\overline{c}$. This completes the proof of the Claim.

Proof of Claim 2. Theorem 1 implies

$$\begin{aligned}U_2^*(\alpha,b) &= E_{c_i}\max[bx(1),y^*-c_i+bx(1-l^*)] \\ &= bx(1)+E_{c_i}\max[0,s^*-c_i].\end{aligned}$$

Let $c=\min[c_1,c_2,..,c_n]$ and note this random variable has c.d.f. $G=1-(1-F)^n$. As

$$U_2^*(\alpha,b)=bx(1)+\int_0^{s^*}[s^*-c]dG(c),$$

integration by parts now implies the claim.

The Existence Problem.

Each firm offers a wage which fully compensates for home production and offers additional surplus s^* which depends on the value of workplace employment V. To address the existence issue, suppose each firm $j \neq 1$ announces $s^*$and suppose firm 1 deviates by announcing s. Let

$$L(s, s^*) = \int_0^y [1 - F(s^* - s + c_1)]^{n-1} f(c_1) dc_1$$

which is the probability the worker accepts firm 1's job offer. Hence

$$\pi_1 = L(s, s^*)[V - s].$$

Note that $\pi_1 \equiv 0$ for $s \leq s^* - \bar{c}$ (as $L = 0$) and $\pi_1 \leq 0$ for $s \geq V$. Hence define $\Gamma(V) = [\max[0, s^* - \bar{c}], V] \subseteq [0, V]$ where $s^* = s^*(V)$ is defined by (1). Note that Claim 1 implies $s^* \in \Gamma(V)$ and so Γ is non-empty. Without loss of generality we can restrict attention to $s \in \Gamma(V)$ - all other offers yield negative profit. As π_1 is not concave in s over this domain, a sufficient condition for existence of a pure strategy symmetric equilibrium is that π_1 is single peaked; i.e. that at any $s \in \Gamma(V)$ where $\partial \pi_1 / \partial s = 0$, then $\partial^2 \pi_1 / \partial s^2 < 0$. Using the above definition of π_1, a sufficient condition is that

$$L \frac{\partial^2 L}{\partial s^2} - 2 \frac{\partial L}{\partial s}^2 < 0 \text{ for all } s \in \Gamma(V). \tag{10}$$

Given the definition of L, (10) describes a restriction on F which guarantees existence of a symmetric, pure strategy Nash equilibrium (where Claim 1 implies s^* always exists). Unfortunately computing these terms yields long and unwieldy expressions. Although the restriction to F log concave (or the stronger condition that F is concave) guarantees sensible comparative statics, we have been unable to show it is sufficient to guarantee single peakedness as defined in (10).

It is well known in the Hotelling framework with linear transport costs that pure strategy equilibria may not exist. The problem there is that demand is discontinuous - a small price cut can imply a jump in demand. Such demand discontinuities do not arise here - idiosyncratic match values imply demand $L(.)$ is continuous in s. We believe the pure strategy symmetric equilibrium exists when F is log concave but have not been able to prove this formally.

Figure 1: Discontinuous Investment Choices

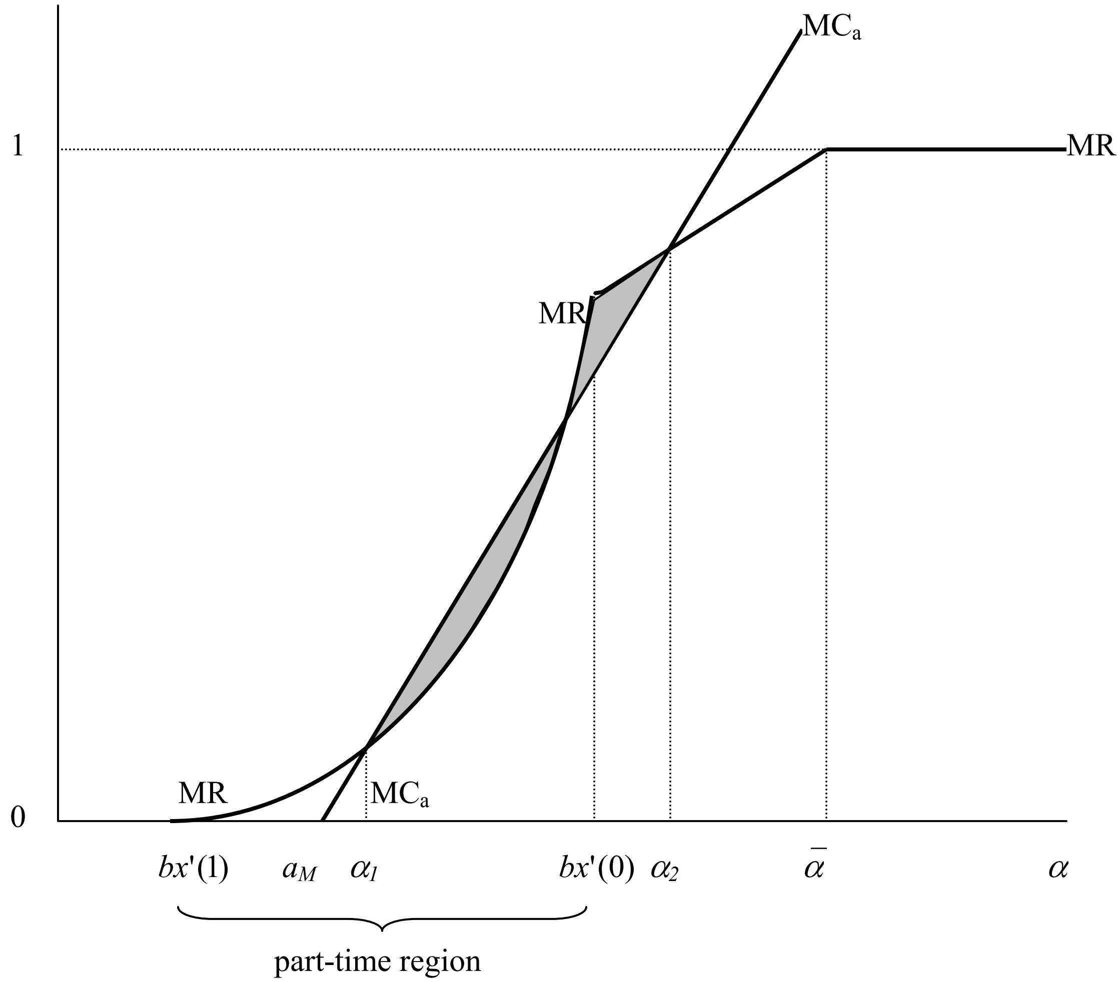

Figure 2: Deadweight Losses

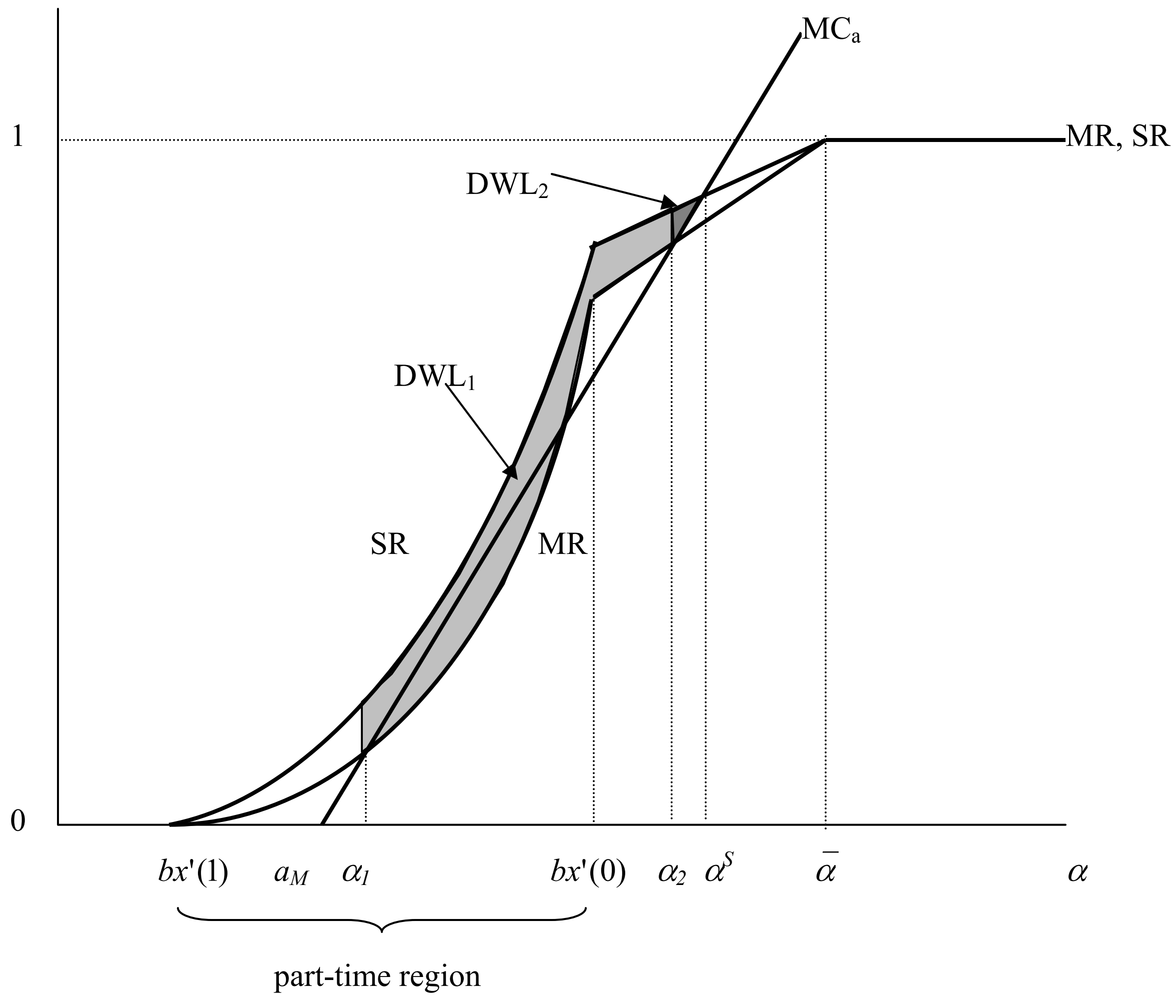

Figure 3: Comparative Statics on Home Productivity

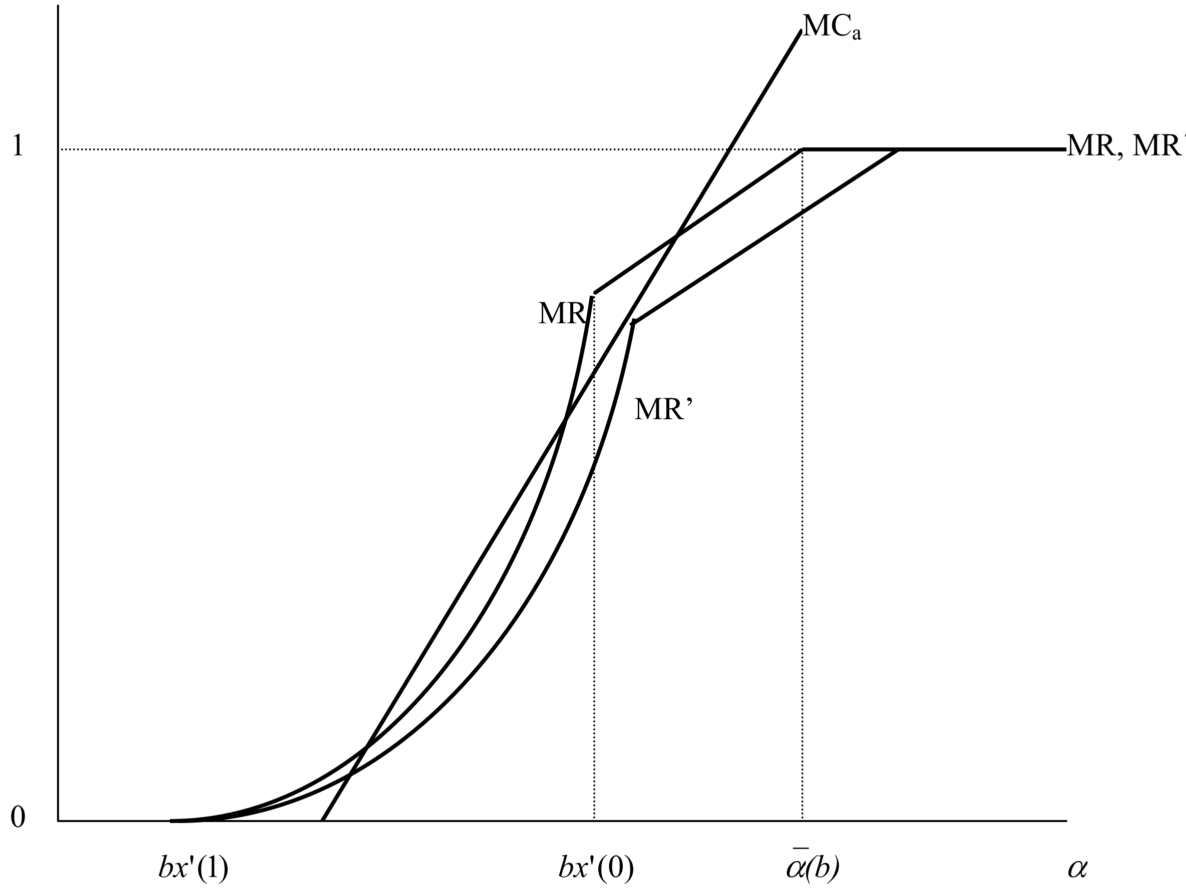

Figure 4: Non-Existence of a Part-Time Employment Trap