Microwave Frequency Characterisation of Squeezed Light From an Optical Parametric Oscillator

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Declaration

This thesis is an account of research undertaken between February 2006 and October 2006 at The Department of Physics, Faculty of Science, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Roger J. Senior
November, 2006
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Abstract

The quantum statistics of a laser result in noise when measurements of the beam are made. This noise sets a classical limit beyond which a laser cannot be used with increasing sensitivity. This quantum noise limit is imposed on many of the uses of lasers currently, especially in power limited devices such as optical communications. The statistics of the laser photon field can be modified to produce a non-classical state resulting in lower noise than the quantum noise limit when detected appropriately. This state, called a squeezed state, has been measured previously from a cavity enhanced optical parametric oscillator (OPO) only at frequency sidebands within the linewidth of the cavity.

This thesis reports measurements of squeezing at microwave frequency sidebands on an optical beam produced by an optical parametric oscillator. This is the first reported measurement of squeezing at frequency sidebands at higher longitudinal modes of the cavity from an OPO. Noise reduction below the quantum noise limit is measured at sideband frequencies of 5 MHz, 1.7 GHz, 3.4 GHz and 5.1 GHz, corresponding to the zeroth, first, second and third longitudinal modes from the squeezed beam. These results are the highest frequency sideband measurements of squeezing to date. In addition to measuring squeezing at different longitudinal modes for the fundamental Gaussian spatial mode, non-classical noise reduction is measured at the same frequencies for a squeezed higher order spatial mode, TEM$_{10}$.

A single mode theoretical model of the OPO is presented, based on the work of ref. [1]. Computer simulations of the squeezing predicted by this model are developed and compared to the experimental results, showing excellent agreement between the different longitudinal modes for each of the two spatial modes measured.
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Chapter 1

Introduction

This thesis describes an experimental investigation of the quantum noise properties of non-classical light. The property of interest motivating this work is the spectral density of the quantum noise on the non-classical light, and in particular what this spectral density is at microwave frequencies. A very reliable source of non-classical light is an Optical Parametric Oscillator [2]. This work will describe the steps taken in building the experiment to measure the spectral density of non-classical light from an optical parametric oscillator and the results obtained. Finally, a detailed theoretical model of the results will be presented and compared to the measurements taken.

The field of quantum optics has enjoyed remarkable progress in the last two decades largely because it provides immediate access to a simple bosonic quantum system - coherent light. In comparison to experiments with more strongly interacting particles, such as nuclear physics, coherent light allows simple experimental setups to measure fundamental quantum properties of the system. Understanding the physical principles behind these quantum properties is the aim of quantum optics. Another strong driving force behind the understanding of light is the prospect of enhancing its use in technologies such as optical communications and precision measurement.

As sources of coherent light, lasers are finding an ever increasing number of uses in modern technology - industry, medicine, the military, data storage and communications are examples of areas where lasers are now used. Optical communications uses fiber optic cables to guide light between a transmitter and receiver so that the light may be encoded to carry information. Telephones, the internet and even secure communications systems [3] make use of lasers and fiber optic cables, making this field scale in size alongside the information technology market.

In any communications system (optical communications included) the ratio of the signal power (information) to the noise power is called the signal to noise ratio, or SNR, which must be greater than unity to be detected. Furthermore, the signal modulation frequency, in combination with the SNR, determines the information carrying capacity of the communications system.

One way in which quantum optics can improve the use of lasers in technology is by reducing noise, and hence improving the SNR, through the use of non-classical dynamics. Improving the SNR is a powerful means of improving a communications system, yet further improvement can be made by increasing the frequency bandwidth of the noise reduction.

Experiments in quantum optics have demonstrated noise reduction in bright beams at frequencies of several hundred megahertz (422 MHz in the case of ref. [4]), which is well below the modulation frequencies commonly used in communications systems today.
Introduction

Lasers have an inherent uncertainty in their amplitude and their phase \([2] \ [5]\), in addition to any technical noise sources. This uncertainty in the amplitude of a beam manifests as noise whenever the intensity of the beam is detected. This noise is referred to as *quantum noise* and, as will be demonstrated later in this work, has a Poissonian distribution. This distribution of the quantum noise makes it appear the same as electronic noise in a circuit when detected with a photodiode. Historically the electronic noise is called *shot noise*, therefore the quantum noise of a laser beam is also referred to as the shot noise, as it will be in this work. Note that the quantum noise, or shot noise, of a laser is a quantum feature of the photon field and has nothing to do with electronic noise - the usage of the term shot noise results only from the Poissonian nature of both the quantum and electronic noise distributions.

The proof of the inherent uncertainty between the amplitude and phase of a laser was one of the founding achievements of quantum optics \([6]\). This was done by improving the approximation of a laser from a classical electromagnetic plane wave to that of a quantum state, the coherent state. The corresponding amplitude and phase operators were shown to be non-commuting, therefore giving a Heisenberg uncertainty relationship. Since the quantum noise results from a quantum uncertainty it is not correlated in the classical sense when the beam is split. This is problematic because it cannot be subtracted out of the signal like technical noise can be - so the quantum noise limit is a fundamental noise floor in the use of lasers.

When investigating the noise on a laser beam (whether quantum in nature or not) we must qualify the frequency range where the noise is being analysed. Noise is analysed by considering how the frequency components symmetric about the carrier are correlated. These frequency components are called *sidebands*. In any system where a carrier beam is being used (radio for example), sidebands greater than the carrier frequency have no meaning, as amplitude and phase are defined only after one wavelength. Therefore we must limit our discussion of noise on a laser beam to sideband frequencies smaller than the frequency of the light being used. As microwave frequencies are five orders of magnitude smaller than the frequency of the lasers used in this work, this assumption is clearly valid.

One of the exciting developments of quantum optics is the ability to go beyond the quantum noise limit of lasers by producing a non-classical state, for example the *squeezed state*. A squeezed state differs from a coherent state only in the respect that the uncertainty relation between amplitude and phase is not symmetric, but is non-classical because it can only be consistently modeled with a quantum description of the photon field. A squeezed state reduces the uncertainty (and hence the measured noise) in one parameter at the expense of increasing the uncertainty in the other. By modulating information onto the parameter that has the reduced noise (amplitude or phase), an increase in the amount of information encoded per unit time is possible.

The state described above is, strictly speaking, a *quadrature squeezed state* - as the non-commuting operators in the Heisenberg relation of interest are the quadrature operators...
(directly related to amplitude and phase). There are other types of quantum noise limited observables of a coherent state, such as polarisation [2], which can be squeezed. In the case of polarisation the squeezed state would be called a polarisation squeezed state. This work will deal exclusively with quadrature squeezing, therefore the use of the term squeezing will be assumed to refer to quadrature squeezing.

There are many different experimental methods used to produce squeezed states. These include using the Kerr effect in optical fibers [7], four-wave mixing [4] and using an optical parametric oscillator (OPO) [8]. All of these methods make use of a weak non-linear interaction between an intense light field and a special material to produce the squeezed state. The frequency response of the non-linearities themselves, though, is effectively infinite compared to the frequency limits of modern electronics. The Kerr effect, for example, squeezes very short pulses of light - which have very high frequency components relative to the carrier frequency.

Since Kerr squeezing in optical fibers has very high frequency components, it is a legitimate question whether a high frequency characterisation of squeezing from an OPO is at all interesting. The key difference between these two methods is that Kerr squeezing uses very short, very high intensity pulsed beams while OPO squeezing uses continuous, much lower intensity beams. Therefore any application requiring continuous wave operation, such as precision measurements, that wants to make use of squeezed light at high frequency first requires the characterisation developed in this work.

Non-linear materials generally rely upon high intensity optical fields in order to produce squeezed light. Using short, high intensity pulses of light is one method of directly driving the non-linearity, but in the continuous wave regime this intensity requirement is achieved with an optical resonator - where the light bounces back and forth many times. This is a well established technique in four-wave mixing and OPO experiments. The time it takes light to make a round trip of the optical resonator introduces a timescale into the system, which is shown to determine the high frequency spectrum of squeezing from a resonator enhanced non-linear process.

An OPO can reliably produce strongly squeezed light in continuous wave mode with a relatively uncomplicated experimental setup [2], compared to four-wave mixing for example. High frequency squeezing from an OPO is something that has been overlooked in the past due to technical difficulties in detection at microwave frequencies, but provides novel access to noise-free squeezed light at frequencies appropriate for a host of engineering applications.

The most immediate potential applications of squeezed light are optical power limited devices. In such devices competing non-linearities become significant as the power is increased beyond some threshold, for example a biological sample being destroyed or some dispersive non-linearity occurring in a small optical fiber. The reason behind the usefulness of squeezing in these systems is that the SNR of a shot-noise limited beam improves as the optical power is increased, making an alternative method of improving the SNR viable when the power cannot be further increased.

This work demonstrates how measurements of squeezed light from an OPO at microwave frequency sidebands were made. In addition to this, the measurements are shown to agree very well with the simulations predicted by a theoretical model of the system.
1.1 Thesis Outline

This thesis is divided into two main sections - chapters 2, 3 and 4 review and describe the quantum theory of an OPO, while chapters 5, 6 and 7 detail the experiment and results of measuring the squeezed spectrum from an OPO.

Chapter 2 reviews the relevant material in quantum optics required to understand the material in the rest of the thesis - explaining number states, coherent states, quadratures, linearised quantum noise and the principles of squeezing.

Chapter 3 builds upon the previous material by detailing the theoretical model of an optical parametric oscillator and how it produces squeezed light. Additionally this chapter will review the classical optics relevant to optical cavities (resonators) as these cavities are an integral part of the experiment described in chapter 5.

Chapter 4 uses the methodology of quantum optics outlined in chapter 2 to derive a model describing the squeezed output of an OPO that is valid in a much wider frequency range than previous models, following the work in ref. [1]. This chapter will show how squeezing is produced at regularly spaced frequency intervals, with computer simulations of the theoretical model in support of this.

Chapter 5 lists the experimental requirements needed to actually measure the noise power spectrum of a squeezed state. In particular, this chapter explains the reasoning behind the design of the experiment and how the different components affect the measurements taken.

Chapter 6 details the significant developments made to certain elements of the experiment in order to improve the stability and reliability of the measurements obtained. This stability is an important property of the experiment as we want to compare results obtained for different frequencies - which must be collected at different times. This chapter describes improvements in the control systems for the optical resonators, the photodetectors used and the method used to produce a squeezed beam in different spatial modes.

Chapter 7 presents the results of the measurement of the squeezing spectrum, in addition to the method of analysing the measurements taken directly from the experiment. These results are then compared to computer simulations of the theoretical model, showing an excellent agreement. This chapter also lists the full set of details of the experiment used, including the non-linear crystal properties, the detector properties and the OPO cavity setup.

Finally, chapter 8 concludes the thesis, along with a discussion of the results relating the sources of experimental error to the conclusions that can be drawn from the results. Additionally, this chapter includes a comprehensive description of the future directions of this research in both the short term and the long term.
The goal of this chapter is to introduce the fundamental theory underpinning the field of quantum optics. These theoretical tools will later be used when presenting the model for the frequency spectrum of a squeezed state in chapter 4. A flow chart of the development of the theory in this chapter is presented in figure 2.1. The material presented is a review of many treatments on quantum optics (see for example [2], [9], [10]) while, together with chapter 3, focusing primarily on the material to the results of this thesis.

![Flow chart](image)

**Figure 2.1:** A flow chart depicting the underlying theory of quantum optics.

### 2.1 Quantizing the Electromagnetic Field

The theoretical framework of quantum optics begins by developing a quantum mechanical treatment for the electromagnetic field - a process called quantization. This process involves finding a basis of eigenmodes of the Hamiltonian that describes the electromagnetic field, and from there the operators which act on this basis.
2.1.1 Number States

By canonically quantizing the classical equations for an electromagnetic field in the radiation gauge we get the following Hamiltonian [10]

\[ H = \hbar \omega (a^\dagger a + \frac{1}{2}) \]  

where \( a^\dagger \) is the usual creation operator for a photon of frequency \( \omega \), \( a^\dagger a \) is the number operator and modes with different frequencies are assumed to be independent. Treatments of how to arrive at this Hamiltonian can be found in [5], [10]. At this point we note that the Hamiltonian is exactly analogous to that of a one dimensional harmonic oscillator. The most obvious set of eigenvalues of this Hamiltonian are \( \hbar \omega (n + \frac{1}{2}) \), with \( n \) being a natural number. We denote the eigenstates corresponding to these eigenvalues by \(|n\rangle\) and call them number states, since they are eigenstates of the number operator. Hence

\[ a^\dagger a |n\rangle = n |n\rangle \]  

The number states form a complete, orthogonal basis for a single mode radiation field [5]. As with the harmonic oscillator, the ground state (or vacuum state) is defined by

\[ a |0\rangle = 0 \]  

With this definition of the ground state the rest of the number states can then be determined

\[ |n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle \]  

The number states are a fundamentally useful basis in quantum optics since the probability of finding \( n \) photons in some state \(|\psi\rangle\) is given by

\[ P_{\psi}(n) = |\langle n|\psi\rangle|^2 \]  

This probability is a useful quantity to be able to calculate because photon number provides a measure of the intensity of a beam, which is an easily measurable quantity.

2.1.2 Operators

The creation and annihilation operators introduced in the previous section play a central role in determining the evolution of a photon field, because we can fully express the field in terms of these operators acting on the vacuum state. The above quantization procedure uses the assumption that the radiation field can be decomposed into standing modes - the photon states. The creation and annihilation operators in this case (\( a \) and \( a^\dagger \)) therefore operate on modes which have the dimensions of photon number. Experimentally this is valid when a light field is bounded, for example in an optical cavity (see chapter 3).

More often, though, in experiments we deal with beams that are propagating. Instead of having the property of a photon number, these beams have the dimensions of photon flux. It is important to be able to model the photon flux properly in quantum optics as this flux is then turned into an electric current when detected, which is what we measure. We therefore introduce propagating photon states, with units of photons per second, and the corresponding annihilation and creation operators \( A \) and \( A^\dagger \). These modes have the same
properties as their standing mode operator counterparts with the only difference being that they represent propagating modes with different dimensions (photons per second instead of just photons).

### 2.1.3 Quadratures

Amplitude and phase are two fundamental parameters required to describe a sinusoidal oscillation, which is the nature of an optical beam. The other parameters, such as polarisation and direction, can be ignored by holding the constant in an experiment. Amplitude and phase, of course, are polar coordinates - which complicates the direct comparison of these parameters. Waves and oscillations are more easily dealt with by changing coordinates from amplitude and phase into quadratures. If we suppose that the complex amplitude of a wave is given by \( \tilde{A} = A_0 \exp(i\psi) \) then the quadrature transformation is

\[
\begin{align*}
X_1 &= \tilde{A} + \tilde{A}^* \\
X_2 &= i(\tilde{A} - \tilde{A}^*)
\end{align*}
\quad (2.6)
\quad (2.7)
\]

From the transformation above we can immediately identify these quadrature variables as the cartesian counterparts of amplitude and phase. The two quadrature variables provide the axes for the usual phasor diagram representation of light in classical optics, an example of which is shown in figure 2.2.

![Figure 2.2: Phasor diagram depicting a classical state of light. The amplitude quadrature axis provides the arbitrary phase reference for the state, which in quantum optics is normally taken to be zero.](image)

Just as quadratures simplify calculations in classical optics, so too do they simplify matters in quantum optics. Using the same canonical quantization performed in the derivation of the results in the previous section, the transformation of the quantum mechanical operators \( a \) and \( a^\dagger \) to get the quadrature operators is the same as for the classical case

\[
\begin{align*}
X_1 &= a + a^\dagger \\
X_2 &= i(a - a^\dagger)
\end{align*}
\quad (2.8)
\quad (2.9)
\]
Quantum Optics and Squeezed Light

The notation used here is standard throughout this thesis, but there is another notation used in some quantum optics literature. For comparison with this, the relation between the notations is $X^+ = X_1$ and $X^- = X_2$.

2.2 Coherent States

While the number states are a useful basis set to work with, they themselves do not represent realistic states producible in an experiment. This can be easily seen by interrogating the phase of a number state. Consider the following phase operator

$$X^\theta = X_1 \cos(\theta) + X_2 \sin(\theta)$$ (2.10)

The expectation value of this operator looks at the mean value of the component of a particular state along a phase projection of $\theta$. Hence we can use this operator to analyse how well defined the phase of a state is by looking at the expectation values of the above operator for different values of $\theta$. In the case of a well defined phase only one value of $\theta$ would return a non-zero value, but in the case of a number state we get

$$\langle n|X^\theta|n\rangle = \langle n|(ae^{-i\theta} + a^\dagger e^{i\theta})|n\rangle = \langle n|a|n\rangle e^{-i\theta} + \langle n|a^\dagger |n\rangle e^{i\theta}$$ (2.11)

$$= 0$$ (2.12)

using the orthogonality of the number states. Since this expectation value is zero for all $\theta$ we can conclude that the number states have no definable phase at all.

All of the current processes used to generate electromagnetic radiation use a fundamental oscillation (either atomic, electronic or optical) from which the radiation is generated. This results in radiation with a phase that may noisy, but is certainly well defined by the fundamental oscillation phase. Hence we need to look for state representations that more closely resemble the light that is producible in experiments.

In 1963 Roy Glauber [6] published his work on the coherent state, which is a superposition of number states that closely approximates a laser. The state is denoted $|\alpha\rangle$ and is expressed in the basis of number states by

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \exp\left(-\frac{1}{2} |\alpha|^2 \right) |n\rangle$$ (2.14)

with $\alpha$ being complex in general. Plotted in figure 2.3 are photon number probability distributions for two coherent states, showing clearly the fact that the photon number of a coherent is not definite.

The photon number distribution for a coherent state is Poissonian, since the distribution has the form $P(n) = \alpha^{2n} \exp\left(-|\alpha|^2 \right)/n!$, as can be seen in figure 2.3.

2.2.1 Amplitude - Phase Uncertainty

The previous section demonstrated the fact that the photon number of a coherent state is not well defined. As a result of this the detected intensity will never be a single value but rather fluctuate randomly about the mean amplitude. This fluctuation is called quantum noise, setting a fundamental noise floor on the detection of lasers. More generally, given
that quadrature operators do not commute [2], there exists a Heisenberg uncertainty relation between these operators which holds true for any quantum state of light. Hence there is a Heisenberg uncertainty relation between the actual amplitude and phase of a state of light, meaning a laser has quantum noise not only in intensity measurements but phase measurements also.

A coherent state has a symmetric uncertainty between the quadratures which can be represented as a circle of uncertainty on a phasor diagram. A schematic illustrating this uncertainty area is displayed in figure 2.4.

### 2.3 Linearised Quantum Noise

The uncertainty relation between non-commuting observables is a purely quantum mechanical phenomenon. Exploring the uncertainty relationships between measurable attributes of light provides the field of quantum optics a means of accessing truly quantum features of a photon field. Uncertainty in an observable obviously manifests as noise when repeatedly detected, which has a distribution representative of the quantum uncertainty. Describing the uncertainty of a quantum state $|\phi\rangle$ is done in the standard quantum mechanics methodology by finding the term $\langle \phi | X | \phi \rangle$. For systems operating in a steady state, this term is more easily found by using the mean field approximation, as described in ref [2] - writing the annihilation operator as

$$\hat{A} = \alpha + \delta A$$  \hspace{1cm} (2.15)

The 'fluctuating' part of an operator $\delta \hat{A}$ does not imply noise on the operator, since noise is an outcome of measurement. Rather, it is a convenient way of calculating the uncertainty of a state. This is especially useful for a system which has a specified optical input - where the fluctuation operators of the input and output beams can be related.
Figure 2.4: This so-called "ball and stick" diagram depicts a quantum coherent state, which can be compared to the classical phasor in figure 2.2. The shaded region represents the different instantaneous values of amplitude and phase that are most likely to be measured.

and therefore the noise properties compared. The above equation leads to an important principle in quantum optics, which is that a vacuum state has the same uncertainty in amplitude and phase as a coherent state even though it has a mean field amplitude of zero.

In frequency space the fluctuation operator can be thought of as describing a continuum of sideband modes around the carrier. A sideband is a mode that is offset from a carrier beam by some frequency. A noise power spectrum measures the correlations between a negative and a positive sideband symmetric about the carrier, at each frequency starting from DC (Ω = 0).

If the correlations are between non-zero mean field amplitudes of the sidebands then a signal is present. On the other hand, the sidebands of a coherent state are uncorrelated - resulting in the measurement of quantum noise in this interpretation. If correlations are introduced between the sideband fluctuations then the noise power is reduced below the quantum noise limit - giving the non-classical squeezed state. Direct correlations between the sidebands gives phase squeezing and anti-correlations result in amplitude squeezing.

2.4 Quadrature Variances

The variance in an observable for a particular state provides a very useful measure of the noise that will result between repeated measurements of that observable. This work uses photodiodes for detection purposes, which output an electric current proportional to the intensity envelope of an incident beam in the rotating wave approximation. The rotating wave approximation is a transformation into the frame of the optical carrier beam through a division of all equations by $e^{i\nu t}$, where $\nu$ is the optical carrier frequency (about $10^{14}$ Hz in the case of light).

A detector photocurrent records the time domain intensity noise on the optical beam. The variance of the number operator can be obtained by taking the power spectrum of
this photocurrent, for example with a spectrum analyser.

The variances of the quadrature operators (denoted $V_1$ and $V_2$ for the amplitude and phase quadratures respectively) are defined in the usual manner (using the amplitude quadrature operator $X_1$)

$$V_1 = \langle \Delta X_1^2 \rangle = \langle X_1^2 \rangle - \langle X_1 \rangle^2 \quad (2.16)$$

If we now apply the mean field approximation to this equation ($X_1 = \langle X_1 \rangle + \delta X_1$) we get

$$V_1 = \langle (\langle X_1 \rangle + \delta X_1)^2 \rangle - \langle X_1 + \delta X_1 \rangle^2 \quad (2.17)$$

$$= \langle X_1 \rangle^2 + \langle X_1 \rangle \langle \delta X_1 \rangle + \langle \delta X_1 \rangle \langle X_1 \rangle + \langle |\delta X_1|^2 \rangle - \langle X_1 \rangle^2 - \langle \delta X_1 \rangle^2 \quad (2.18)$$

Since the mean field approximation was used, the fluctuation operator must have an expectation value of zero (taking the expectation value of the mean field decomposition equation gives $\langle X_1 \rangle = \langle X_1 \rangle + \langle \delta X_1 \rangle \implies \langle \delta X_1 \rangle = 0$). Therefore

$$V_1 = \langle |\delta X_1|^2 \rangle \quad (2.19)$$

An identical result is found for the phase quadrature variance, $V_2$.

### 2.4.1 Mixing Quantum Fields

Now that the concept of quadrature variances has been presented we can properly describe one of the most important results in quantum optics - the mixing of two quantum fields. The most basic method of envisaging this process is to consider two fields, $A_{in1}$ and $A_{in2}$, interfering on a beamsplitter with reflection coefficient $\epsilon$. Classically, the intensities of the input fields mix in the ratio of $\epsilon$ and $(1 - \epsilon)$ to give the output intensities. In the quantum description, the fluctuations of the input fields also mix, meaning the output quadrature variances are given by [2]

$$V_{1\text{out}} = \epsilon V_{1\text{in1}} + (1 - \epsilon)V_{1\text{in2}} \quad (2.20)$$

$$V_{1\text{out2}} = (1 - \epsilon)V_{1\text{in1}} + \epsilon V_{1\text{in2}} \quad (2.21)$$

Since vacuum modes have fluctuations this equation acquires a real significance, because even when the second input in the mixing process has a mean field amplitude of zero (so it is 'not there' classically), a vacuum mode occupies the unused port. The equation above (which this work will refer to as the beamsplitter equation) illustrates how loss brings a state closer to the quantum noise limit - as the vacuum noise is mixed in. Since squeezed states have a variance lower than the quantum noise limit this means that loss will quickly reduce the degree of squeezing present.

### 2.5 Squeezing a Coherent State

In order to produce squeezed light from a coherent state we need to be able alter the distribution of uncertainty between the quadratures. Ultimately the Heisenberg uncertainty principle sets the minimum product of uncertainty, but a squeezed state allows the noise in
one quadrature to become smaller than the standard quantum noise limit by making the noise in the other quadrature larger. This process of forming a state with an asymmetric uncertainty distribution is realised by many different processes available to experiments.

One such process is the Kerr effect, which uses an intensity dependent index of refraction in a material. An index of refraction has the effect of phase shifting light, which corresponds to a rotation on a phasor diagram. The Kerr effect will therefore rotate the higher intensity section of the uncertainty ball further than the lower intensity section. This results in a squeezed state as shown on the phasor diagram in figure 2.5, providing an example of squeezing that is intuitively simple to understand.

Another method of squeezing a coherent state is to parametrically amplify the state. This method produces squeezing in a very direct manner - by amplifying one quadrature of a state while de-amplifying the other quadrature. This leads to increased noise in one quadrature and decreased noise in the other quadrature. This method of producing squeezed light is the method entailed in the experimental section of this work and is described in the next chapter.

Figure 2.5: This ball and stick diagram illustrates how Kerr squeezing can be understood in a simpler manner as an intensity dependent phase shift of the state.
This chapter describes in more detail the theory behind one particular method of producing squeezed light - optical parametric amplification. This method will be employed in the experiment (see chapter 5) to produce the squeezed state which will have its microwave squeezing spectrum measured. The theory describing optical cavities will also be presented in this chapter, as a cavity will be used to enhance the squeezing produced in the experiment. We will find in chapter 4 that using a cavity in such a fashion significantly determines the resultant frequency spectrum of the squeezed light.

3.1 Parametric Amplification

Parametric amplification is a term describing the process of amplification in a phase sensitive manner. Based on chapter 2, the annihilation operator can be written as

\[ a = \frac{1}{2}(X_1 + iX_2) \]  

(3.1)

Now we want to parametrically amplify the field by amplifying one quadrature with a gain of \( \sqrt{G} \) while de-amplifying the other quadrature by the same amount. The output operator then has the form

\[ a_{out} = \frac{1}{2} \left( \sqrt{G} X_1 + i \sqrt{G} X_2 \right) \]  

(3.2)

It is straightforward to check that this output operator satisfies the required commutation relation

\[ [a_{out}, a_{out}^\dagger] = \frac{1}{4} \left[ \sqrt{G} X_1 + i \sqrt{G} X_2, \sqrt{G} X_1^\dagger - i \sqrt{G} X_2^\dagger \right] \]

(3.3)

\[ = \frac{1}{4} \left( G[X_1, X_1^\dagger] - i[X_1, X_2^\dagger] + i[X_2, X_1^\dagger] + \frac{[X_2, X_2^\dagger]}{G} \right) \]

(3.4)

from the definitions of the operators \( X_1 \) and \( X_2 \), and using the property that they are Hermitian, we have

\[ [X_1, X_2^\dagger] = 2i \left[ a, a^\dagger \right] + 2i(a^\dagger a^\dagger - aa) \]  

(3.5)
\[
\begin{align*}
\left[ X_2, X_1^\dagger \right] &= -2i \left[ a, a^\dagger \right] + 2i(a^\dagger a^\dagger - aa) \\
\text{Hence we have} & \\
\left[ a_{\text{out}}, a_{\text{out}}^\dagger \right] &= \frac{1}{4}(2 + 2) = 1
\end{align*}
\]

as expected. Therefore, as the commutator is intact it can be concluded that parametric amplification is a valid transformation of the field. By calculating the variances of the transformed state it can be found that the noise in one quadrature can be reduced below the quantum noise limit.

### 3.2 Optical Parametric Amplification

A device which produces squeezed light through parametric amplification is called an optical parametric amplifier, abbreviated OPA. Parametric amplification is a phenomenon observed in some special materials - providing a means to construct an OPA. Non-linear materials are those in which the higher order powers of the electric field strength are significant in the induced dipole polarisation caused by the electromagnetic field. If the dipole polarisation is written as

\[
P(E) = \chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots
\]

then normal materials have \( \chi E \gg \chi^{(2)} E^2 \gg \chi^{(3)} E^3 \gg \ldots \). In this case very large optical intensities are required to drive the non-linear effects, so they are not suitable experimentally. Some special materials have a more significant \( \chi^{(2)} \) coefficient, making this term experimentally accessible. This condition leads to the naming of these materials as second order non-linear materials.

Second order non-linear materials provide a coupling between an optical field at a fundamental frequency and its second harmonic, at twice the frequency. It is this coupling, as the next chapter will show, that gives these materials the ability to parametrically amplify the fundamental field. The coupling processes between the fundamental and harmonic fields are referred to as down conversion (harmonic to fundamental) and up conversion (fundamental to harmonic). Down conversion is not the only method of phase sensitive amplification, as four-wave mixing is a phase sensitive process, but this method of squeezing is used in this work due to its simplicity and stability.

While these second order non-linear materials can produce squeezing directly (a so called single pass OPA [11]), the degree of squeezing can be significantly increased by placing the non-linear crystal in an optical resonator, or cavity (see section 3.4). This system is known as an Optical Parametric Oscillator, or OPO. Similar to a laser (which is gain inside a resonator), an OPO operates in a bimodal fashion, with a threshold condition separating the two modes. The threshold is a pump power limit, above which stimulated down conversion dominates - producing a coherent output at the fundamental field frequency. Below threshold, stimulated down conversion does not build up in the cavity, meaning the spontaneous down conversion becomes dominant. The spontaneous term leads to the parametric amplification of the fundamental field - meaning that an OPO operates as a parametric amplifier below threshold. Therefore in the rest of this work, unless explicitly stated otherwise, the term 'OPO' will be used interchangeably with 'below threshold OPO' and 'OPA'.

\[ X_2, X_1^\dagger = -2i \left[ a, a^\dagger \right] + 2i(a^\dagger a^\dagger - aa) \]

(3.6)

Hence we have

\[ \left[ a_{\text{out}}, a_{\text{out}}^\dagger \right] = \frac{1}{4}(2 + 2) = 1 \]

(3.7)

as expected. Therefore, as the commutator is intact it can be concluded that parametric amplification is a valid transformation of the field. By calculating the variances of the transformed state it can be found that the noise in one quadrature can be reduced below the quantum noise limit.

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3.3 OPA Squeezing on a Phasor Diagram

The OPA process of producing squeezed light can be modeled on a phasor diagram as a symplectic transformation of the input state [12]. This process is illustrated in figure 3.1.

Two distinct modes of operation of an OPA with a bright seed beam can be seen in figure 3.1 - one where the mean amplitude of the seed beam is deamplified and one where the seed beam amplitude is amplified. Figure 3.1 indicates that deamplification corresponds to amplitude squeezing and amplification gives phase squeezing.

The phase of the pump beam relative to the phase of the seed beam governs the relative alignment of the input seed phasor to the symplectic transformation axes. Therefore, producing optimal squeezing from an OPO requires the pump beam to be kept either in phase or in quadrature ($\pi/2$ out of phase) with the seed beam.

3.4 Optical Cavities

A commonly used method to improve the squeezing produced by a $\chi^{(2)}$ non-linear material is to place the it in an optical resonator for the fundamental field. An optical resonator, or cavity, is a combination of mirrors that can be understood using classical Gaussian optics as supporting at least one resonant optical mode. Most texts on Gaussian optics detail the theory of optical cavities (for example ref. [13]), but a summary of this will be presented here for readers not familiar with this topic.

We can analyse the salient properties of cavity from the point of view of classical optics. Consider a group of mirrors arranged such that a beam can traverse a path, reflecting off...
all the mirrors, that forms a closed loop. Upon each round trip of the cavity a beam will undergo a phase shift, relative to its starting arrangement, as a result of the path length of the cavity. We can think of this graphically as infinitely many (classical) phasors adding up inside the cavity. For a loss-less cavity these phasors are all of equal length, so if the phase shift between successive phasors is non-zero then infinitely many of them will form a circle - giving the net intensity inside the cavity of zero. The phase shift, $\Delta \phi$, between successive round trips in the cavity is given simply by

$$\Delta \phi = \frac{\nu d}{c}$$

where $\nu$ is the optical frequency and $d$ is the optical length of the cavity. The only resonant modes inside a loss-less cavity are those where no phase shift (so $\Delta \phi$ is an integer multiple of $2\pi$) occurs between successive roundtrips. This condition implies that the only resonant modes of the cavity are standing waves, giving a discrete spectrum of resonant frequencies for an optical cavity. This is illustrated in part (a) of figure 3.2. The difference in frequency between these resonant modes is known as the free spectral range, or FSR, of a cavity.

Equation 3.9 dictates that resonant modes satisfy the following property

$$\nu = N \frac{c}{d}$$

for some integer $N$. This clearly illustrates the concept of the free spectral range, as this resonance condition illustrates a fixed frequency spacing of the standing modes in the cavity. Therefore the FSR of a cavity is calculated very simply by

$$\text{FSR} = \frac{c}{\text{pathlength}} \text{ [Hz]}$$

where $c$ is the speed of light inside the cavity, in meters per second, and the path length is the optical path length in meters.

The effect of loss (or transmission) inside a cavity is to increase the spectral width to the resonant lines. Going back to the phasor picture, a cavity with loss will see the phasors of successive roundtrips becoming increasingly shorter as the amplitude of the field is attenuated by the loss. As a result of this, a non-zero phase shift between phasors no longer adds up to become a circle - but rather a spiral with some non-zero net intensity. Hence lossy cavities support modes of all frequencies but with differing total amplitudes. The lower the phase shift between round trips then the more "stretched out" the spiral will be - giving a larger net intensity. Larger loss in the cavity increasingly broadens the resonant spectral lines, giving them a linewidth normally measured as the Full Width at Half Maximum, or FWHM. The finesse of a cavity is the ratio of the FSR to the FWHM. The transmission spectrum of a lossy cavity is illustrated in part (b) of figure 3.2.

The finesse of the cavity is dependent on the roundtrip amplitude attenuation factor $r$ and is given by

$$F = \frac{\pi \sqrt{r}}{1 - r}$$

Note that the intensity roundtrip loss (which is more easily determined) is $r^2$.

One final property of Gaussian optics that is an important consideration of optical cavities is the concept of a beam waist, or just waist. This is the cross-section (normal to the direction of propagation) of the beam that has the smallest diameter. For example, if
Figure 3.2: The transmission spectrum of (a) an infinite finesse cavity (no losses and perfectly reflecting mirrors) and (b) a finite finesse cavity. The vertical axis measures transmission as a fraction of the incident intensity and the horizontal axis measures frequency offset from some fixed optical frequency.

A laser that is not diverging is transmitted through a convex lens then the beam will come to a focus near the focal length of the lens. This focus point is the part of the beam with the smallest diameter - the waist. All stable optical cavities contain within them a beam waist [13].
Chapter 4

The OPO Squeezed State Spectrum

The theoretical framework of quantum optics will be applied to a model of a cavity enhanced OPO in this chapter. This will give theoretical predictions for what we will later measure as the squeezed state noise spectrum from an OPO. This theory has been done previously and published in ref. [1], but is re-derived here to provide the reader with a more complete picture of the experimental system. A discussion of the key assumptions behind this theory will also be presented. Furthermore, the theoretical expressions derived will then be used to generate simulations of the squeezing spectrum so that we can compare the model with the results obtained in chapter 7.

4.1 Problem Description

This section presents a model of the experimental OPO cavity on which the forthcoming theory will be based. Figure 4.1 depicts the OPO cavity which will be used in the experiment. Mirror 1 is a high reflecting coating on the non-linear crystal itself and mirror 2 is a partially transmitting mirror (called the output coupler).

![Figure 4.1: Schematic view of the OPO cavity with the internal and external operator modes marked.](image-url)
The theoretical model presented in figure 4.1 represents a singly resonant, back seeded OPO. Singly resonant means that the OPO cavity is resonant for the seed and not the pump. The back seeded nature of the model describes the fact that the bright seed beam is incident on the high reflecting face of the cavity, as opposed to the low reflecting side. A consequence of this is that we must consider the vacuum mode (at the seed frequency) that couples into the cavity from the low reflecting side. The quantum modes used in the model are described in table 4.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{A}<em>{in}, \hat{A}</em>{in}$</td>
<td>The bright input seed time and Fourier domain modes</td>
</tr>
<tr>
<td>$\hat{A}<em>{out}, \hat{A}</em>{out}$</td>
<td>The output time and Fourier domain modes - this is the mode of the &quot;squeezed beam&quot; that is measured later on</td>
</tr>
<tr>
<td>$\hat{A}_u, \hat{A}_u$</td>
<td>The vacuum time and Fourier domain modes that couple into the cavity through the low reflecting mirror</td>
</tr>
<tr>
<td>$B_{in}, B_{out}$</td>
<td>The input and output modes for the pump beam in the time domain</td>
</tr>
<tr>
<td>$b$</td>
<td>The intracavity pump mode in the time domain</td>
</tr>
<tr>
<td>$\hat{A}_c(t)$</td>
<td>The circulating mode propagating around the OPO cavity at the seed frequency in the time domain, at time $t$</td>
</tr>
<tr>
<td>$\hat{A}_c(t + \tau)$</td>
<td>The circulating mode propagating around the OPO cavity after one cavity round trip from time $t$</td>
</tr>
<tr>
<td>$a(\omega)$</td>
<td>The standing mode of the OPO cavity at the seed frequency in the Fourier domain</td>
</tr>
<tr>
<td>$\chi^{(2)}$</td>
<td>The second order non-linearity coefficient of the non-linear material</td>
</tr>
<tr>
<td>$R_1, R_2$</td>
<td>The reflectivities of the two OPO cavity mirrors</td>
</tr>
<tr>
<td>$\tau$</td>
<td>The time required for light to make one round trip of the OPO cavity</td>
</tr>
<tr>
<td>$\kappa_1, \kappa_2$</td>
<td>The decay rate of the classical amplitude of an intracavity mode due to leakage through cavity mirrors one and two</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>The phase shift of the intracavity mode after one round trip divided by the cavity round trip time, $\Delta = \phi/\tau$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>The angular frequency parameter offset from the seed angular frequency</td>
</tr>
</tbody>
</table>

Table 4.1: Explanation of operators and symbols in the OPO derivation

4.2 Deriving the Spectrum of OPO Squeezed Light

The aim of the following derivation is to obtain the transfer function of the OPO cavity, taking the variances of the input mode and calculating the variances of the output mode. This is done by analysing the field operators at different stages of the system, relating them to the input field operator and any vacuum fields. Hence the frequency spectrum of the squeezing produced by a cavity enhanced OPO can be calculated.

The steps in the derivation of the transfer function are as follows,

- Determine the circulating mode
- Fourier transform into the frequency domain
- Use the mean field approximation to calculate how operator fluctuations propagate through the cavity
- Obtain the variances of the output mode and hence the transfer function
4.2.1 The OPO Cavity Circulating Mode

The circulating mode of the OPO cavity is thought of as a propagating beam bouncing back and forth inside the cavity. This is a transient state that is useful to begin with since it is straightforward to determine the evolution of this mode. The standing mode of the cavity, or steady state solution, is then determined from the circulating mode. Note that the circulating mode is treated as a propagating mode dimensionally (a photon flux), while the standing cavity mode has the dimensions of a bound mode (photons).

The change to the circulating mode after one round trip of the cavity results from two main effects - parametric amplification in the non-linear crystal and the coupling to external fields due to the cavity itself. The effects of parametric amplification on the circulating mode will be derived from the interaction Hamiltonian. The derivation of the cavity effects on the circulating mode is quite involved, so the results of this from ref. [2] will be presented. We also need to allow the more general case where the circulating mode acquires a phase shift of \( \phi \) upon a cavity round trip, as this will happen for example when the cavity is detuned off resonance. Setting \( \Delta = \phi/\tau \) then the phase shift is given by \( \Delta \tau \).

In arriving at the total change to the circulating mode we assume that the Hamiltonians for the two interactions are independent. The validity of this assumption is discussed later in this chapter. Therefore, the total change to the circulating mode \( \hat{A}_c(t + \tau) - e^{i\Delta \tau} \hat{A}_c(t) \) is simply the sum of the changes from each of the interactions (since the Hamiltonians add together to give the total Hamiltonian), giving

\[
\hat{A}_c(t + \tau) - e^{i\Delta \tau} \hat{A}_c(t) = (\hat{A}_c(t + \tau) - e^{i\Delta \tau} \hat{A}_c(t))_{OPA} + (\hat{A}_c(t + \tau) - e^{i\Delta \tau} \hat{A}_c(t))_{CAV} \tag{4.1}
\]

The analysis of this system will be performed in the Heisenberg picture. This is because the experiment uses "bright" beams (in the order of microwatts or more of optical power), and the Schrödinger picture is most useful for single photon systems [2]. The time evolution of an operator in the Heisenberg picture is determined by the commutation of the operator with the interaction Hamiltonian \( \hat{H} \) [14]

\[
\dot{\hat{a}} = \frac{1}{i\hbar} [\hat{a}, \hat{H}] \tag{4.2}
\]

The interaction Hamiltonian of a below threshold OPO has the form [2]

\[
\hat{H}_{OPA} = i\hbar\chi^{(2)}(\hat{b}^{\dagger}\hat{a}^{2} - \hat{a}^{2}\hat{b}) \tag{4.3}
\]

We now assume that the pump field is undepleted, giving \( \hat{b} = 0 \). Therefore, we replace the operator \( \hat{b} \) with its mean field value \( \beta \)

\[
\hat{b} = \beta = \text{real} \tag{4.4}
\]

Therefore, the interaction Hamiltonian can be rewritten

\[
\hat{H}_{OPA} = \frac{i\hbar\chi^{(2)}}{2}(\hat{a}^{2} - \hat{a}^{\dagger 2}) \tag{4.5}
\]

with \( \chi = 2\beta\chi^{(2)} \). This gives the time derivative of the circulating mode, considering only the effect of the OPA, as
\[ \dot{\hat{A}}_c = \frac{1}{i\hbar}[\hat{A}_c, \hat{H}_{OPA}] \]  
\[ = \frac{1}{i\hbar} \hbar \chi (\hat{A}_c.0 + 0.\hat{A}_c - \hat{A}_c.1 - 1.\hat{A}_c) \]  
\[ = -\chi \hat{A}_c^\dagger \]  

It is the evolution of the cavity mode after one cavity round trip that we want to know. This can be determined by Taylor expanding the circulating mode to first order. The higher order Taylor terms are dropped as we assume the round trip change in the circulating mode amplitude is small. Taking into account the phase shift upon a round trip gives

\[ \hat{A}_c(t + \tau) = e^{i\Delta \tau} (\hat{A}_c(t) + \tau \hat{A}_c(t)) \]  
\[ = e^{i\Delta \tau} (\hat{A}_c(t) - \chi \tau \hat{A}_c^\dagger(t)) \]

Hence the change in the circulating mode due to the OPA after one round trip is

\[ (\hat{A}_c(t + \tau) - e^{i\Delta \tau} \hat{A}_c(t))_{OPA} = -\chi \tau e^{i\Delta \tau} \hat{A}_c^\dagger(t) \]

Next we consider the evolution of the circulating mode due to its coupling with field modes outside the cavity. The circulating mode after one cavity round trip is presented in detail in ref [2], with the salient result being

\[ \hat{A}_c(t + \tau) = e^{i\Delta \tau} \left( \sqrt{1 - T_1} \sqrt{1 - T_2} \hat{A}_c(t) + \sqrt{T_1} \sqrt{1 - T_2} \hat{A}_{in1} + \sqrt{T_2} \hat{A}_{in2} \right) \]

where again the field acquires a phase shift of \( \phi \), and \( \Delta = \phi / \tau \) as before. In the above expression \( T_1 \) and \( T_2 \) are the transmissions of the two cavity mirrors. \( \hat{A}_{in1} \) and \( \hat{A}_{in2} \) are the external fields which enter the cavity through the first and second mirrors respectively, either as a seed field or a vacuum field.

Now we set \( \hat{A}_{in1} = \hat{A}_m \) and \( \hat{A}_{in2} = \hat{A}_u \) (a vacuum mode) based on the description of the OPO cavity given in section 4.1. Since both \( T_1 \ll 1 \) and \( T_2 \ll 1 \), then in 4.12 we perform binomial expansions of the square root expressions and keep only the first order terms. This gives

\[ \hat{A}_c(t + \tau) = e^{i\Delta \tau} \left( (1 - \frac{1}{2}T_1 - \frac{1}{2}T_2) \hat{A}_c(t) + \sqrt{T_1} \hat{A}_m + \sqrt{T_2} \hat{A}_u \right) \]

Now set \( \kappa_n = T_n/2\tau \) and we get

\[ \hat{A}_c(t + \tau) = e^{i\Delta \tau} \left( (1 - \tau(\kappa_1 + \kappa_2)) \hat{A}_c(t) + \sqrt{2\tau\kappa_1} \hat{A}_m + \sqrt{2\tau\kappa_2} \hat{A}_u \right) \]

So, the change in the circulating mode due to the cavity after one round trip is

\[ (\hat{A}_c(t + \tau) - e^{i\Delta \tau} \hat{A}_c(t))_{CAV} = e^{i\Delta \tau} \left( -\tau(\kappa_1 + \kappa_2) \hat{A}_c(t) + \sqrt{2\tau\kappa_1} \hat{A}_m + \sqrt{2\tau\kappa_2} \hat{A}_u \right) \]
Based on equation 4.1 we can finally calculate the circulating mode after one cavity round trip to be

$$\hat{A}_c(t + \tau) = e^{i\Delta \tau} \left( -\chi \hat{A}_c^\dagger(t) + (1 - \tau(\kappa_1 + \kappa_2)) \hat{A}_c(t) + \sqrt{2\tau\kappa_1} \hat{A}_{in} + \sqrt{2\tau\kappa_2} \hat{A}_u \right)$$

(4.16)

### 4.2.2 Fourier Transforming the Circulating Mode

We now have an expression (equation 4.16) for the evolution of the circulating mode in the cavity after time $\tau$. By transforming to the frequency domain via the Fourier transform, the expression for the circulating mode (and eventually the standing mode) is simplified to the point where it is almost able to be solved.

A well known property of Fourier transforms is

$$F(\hat{A}_c(t + \tau)) = A_c(\omega) e^{i\omega \tau}$$

(4.17)

where $F(\hat{A}_c(t)) = A_c(\omega)$. We now assume that the field operators are a function of $\omega$, so the Fourier transformed circulating mode becomes

$$e^{i\omega \tau} A_c = e^{i\Delta \tau} \left( -\chi \hat{A}_c^\dagger + (1 - \tau(\kappa_1 + \kappa_2)) A_c + \sqrt{2\tau\kappa_1} A_{in} + \sqrt{2\tau\kappa_2} A_u \right)$$

(4.18)

$$\left( e^{i\omega \tau} e^{-i\Delta \tau} / \tau \right) A_c = \left( -\chi \hat{A}_c^\dagger + (1 - \tau(\kappa_1 + \kappa_2)) A_c + \sqrt{2\kappa_1 / \tau} A_{in} + \sqrt{2\kappa_2 / \tau} A_u \right)$$

(4.19)

While it is clear that the operators used here are in the frequency domain (and hence are a function of $\omega$), we must be careful with what we understand to be the conjugate of these operators as the Fourier transformation is an operation performed in the full complex domain. As a result of this, unless explicitly stated otherwise, it will be assumed that any conjugate operators in the frequency domain are the Fourier transform of the corresponding conjugate operator in the time domain. This is written as $a^\dagger(\omega)$. By contrast, it is possible to take the conjugate of an operator in the frequency domain, but this will be dealt with in section 4.2.3.

The standing mode of the cavity is determined from the circulating mode by canonically quantizing the classical result [13], giving

$$a = \sqrt{\tau} A_c$$

(4.21)

Note the consistency in dimensionality of the above equation between the propagating mode and the standing mode. From this equation, the standing mode is given by

$$\left( e^{i\omega \tau} e^{-i\Delta \tau} / \tau \right) a = \left( -\chi a^\dagger + \left( \frac{1}{\tau} - (\kappa_1 + \kappa_2) \right) a + \sqrt{2\kappa_1 / \tau} A_{in} + \sqrt{2\kappa_2 / \tau} A_u \right)$$

(4.22)

$$\left( (\kappa_1 + \kappa_2) - 1 - e^{i\omega \tau} e^{-i\Delta \tau} / \tau \right) a = \left( -\chi a^\dagger + \sqrt{2\kappa_1 / \tau} A_{in} + \sqrt{2\kappa_2 / \tau} A_u \right)$$

(4.23)

This allows us to write down an expression for the standing mode $a$.
The OPO Squeezed State Spectrum

\[ a = \frac{-\chi a + \sqrt{2\kappa_1} A_{in} + \sqrt{2\kappa_2} A_u}{(\kappa_1 + \kappa_2) - \frac{1-e^{i(\omega-\Delta)\tau}}{\tau}} \] (4.24)

We also want a similar expression for the conjugate of the standing mode, \( a^\dagger \). This is obtained by taking the conjugate of equation 4.16, and then performing the same operations on the resultant equation as we did on the original equation. This gives an expression for the conjugate of the standing mode to be

\[ a^\dagger = \frac{-\chi a^\dagger + \sqrt{2\kappa_1} A_{in}^\dagger + \sqrt{2\kappa_2} A_u^\dagger}{(\kappa_1 + \kappa_2) - \frac{1-e^{i(\omega-\Delta)\tau}}{\tau}} \] (4.25)

Now we have two independent equations for the standing mode operator and its conjugate so we can solve them simultaneously, giving the standing mode operator only in terms of the input and vacuum operators. By substituting the expression for \( a^\dagger \) found above into the expression for \( a \) we get

\[ a = \frac{-\chi a + \sqrt{2\kappa_1} A_{in} + \sqrt{2\kappa_2} A_u}{(\kappa_1 + \kappa_2) - \frac{1-e^{i(\omega-\Delta)\tau}}{\tau}} \] (4.26)

\[ a = \frac{-\chi a^\dagger + \sqrt{2\kappa_1} A_{in}^\dagger + \sqrt{2\kappa_2} A_u^\dagger}{(\kappa_1 + \kappa_2) - \frac{1-e^{i(\omega-\Delta)\tau}}{\tau}} \] (4.27)

\[ a = \frac{-\chi a + \sqrt{2\kappa_1} A_{in} + \sqrt{2\kappa_2} A_u}{(\kappa_1 + \kappa_2) - \frac{1-e^{i(\omega-\Delta)\tau}}{\tau}} \] (4.28)

Let \( \kappa_t = \kappa_1 + \kappa_2 \), then rearranging the above equation we get

\[ \left( \frac{\kappa_t - 1 - e^{i(\omega-\Delta)\tau}}{\tau} \right) \left( \frac{\kappa_t - 1 - e^{i(\omega+\Delta)\tau}}{\tau} \right) \chi^2 \right) a = \]

\[ -\chi(\sqrt{2\kappa_1} A_{in}^\dagger + \sqrt{2\kappa_2} A_u^\dagger) + \left( \frac{\kappa_t - 1 - e^{i(\omega+\Delta)\tau}}{\tau} \right) \left( \sqrt{2\kappa_1} A_{in} + \sqrt{2\kappa_2} A_u \right) \] (4.29)

So finally we have

\[ a = \left( \frac{\kappa_t - 1 - e^{i(\omega-\Delta)\tau}}{\tau} \right) \left( \frac{\kappa_t - 1 - e^{i(\omega+\Delta)\tau}}{\tau} \right) - \chi^2 \right) \]

\[ \left( \sqrt{2\kappa_1} A_{in}^\dagger + \sqrt{2\kappa_2} A_u^\dagger \right) \left( \sqrt{2\kappa_1} A_{in} + \sqrt{2\kappa_2} A_u \right) \] (4.30)

### 4.2.3 Calculating Operator Fluctuations

The standing mode solution found in equation 4.30 is a steady state result. It is therefore appropriate to apply the mean field approximation, allowing for the calculation of the operator fluctuations. Once these are known then the variances of the output mode relative to the input mode can be calculated. The mean field approximation is
\[ \hat{a} = \langle \hat{a} \rangle + \delta \hat{a} \] (4.31)

Notice that this approximation is made in the time domain. All the equations used in this section to derive the results so far have been linear in terms of the field operators. Since the mean field approximation is a linear decoupling of an operator, along with the fact that the equations so far have all been linear in the operators, then we can immediately write down the operator fluctuation term for the cavity standing mode

\[ \delta a = \left( \frac{\kappa_t - 1 - e^{i(\omega + \Delta)\tau}}{\tau} \right) \left( \sqrt{2\kappa_1} \delta A_{in} + \sqrt{2\kappa_2} \delta A_u \right) - \chi \left( \sqrt{2\kappa_1} \delta A_{in}^{\dagger} + \sqrt{2\kappa_2} \delta A_u^{\dagger} \right) \] (4.32)

In writing down the above equation all operators fluctuations are assumed to be a function of \( \omega \), giving \( \delta a(\omega) \), \( \delta A_{in}(\omega) \) and \( \delta A_u(\omega) \). One of the key operations in arriving at this equation was the Fourier transform. So, as before, we must be careful when with what we understand to be the conjugate of these operators in the frequency domain. Here we have used the convention established previously where \( \delta A^{\dagger} = \delta A(\omega) \).

Now we want to use the results of the intra-cavity field operator to determine what the output operator, and hence the output operator fluctuations, are. The output mode of the cavity is given by the sum of the transmitted mode from inside the cavity and the reflected mode at the mirror surface of interest. This boundary condition is

\[ A_{out} = \sqrt{2\kappa_2} a - \sqrt{R_2} A_u \] (4.33)

Again, using the mean field approximation we have

\[ \delta A_{out} = \sqrt{2\kappa_2} \delta a - \sqrt{R_2} \delta A_u \] (4.34)

Notice in the above equations the difference in dimensions between the bound mode operators and the propagating mode operators (units of photons compared to photons per second), as described in chapter 2. Using the above result we get

\[ \delta A_{out} = \left( \frac{1}{\kappa_t - 1 - e^{i(\omega + \Delta)\tau}} \right) \left( 2\sqrt{\kappa_1}\kappa_2 \delta A_{in} + 2\kappa_2 \delta A_u \right) - \chi \left( 2\sqrt{\kappa_1}\kappa_2 \delta A_{in}^{\dagger} + 2\kappa_2 \delta A_u^{\dagger} \right) \] (4.35)

\[ \delta A_{out} = \left( \frac{1}{\kappa_t - 1 - e^{i(\omega + \Delta)\tau}} \right) \left( 2\kappa_2 - \sqrt{R_2}(\kappa_t + \kappa_2) + \sqrt{R_2} \frac{1 - e^{i(\omega - \Delta)\tau}}{\tau} \right) \delta A_u \] + \sqrt{R_2} \chi^2 \delta A_u - 2\chi \left( \sqrt{\kappa_1}\kappa_2 \delta A_{in}^{\dagger} + \kappa_2 \delta A_u^{\dagger} \right) - \left( \frac{1 - e^{i(\omega + \Delta)\tau}}{\tau} \right) \left( 2\sqrt{\kappa_1}\kappa_2 \delta A_{in} \right) (4.36)
\[
\delta A_{\text{out}} = \frac{1}{\left(\kappa_t - \frac{1 - e^{i(\omega + \Delta)\tau}}{\tau}\right) \left(\kappa_t - \frac{1 - e^{i(\omega - \Delta)\tau}}{\tau}\right) - \chi^2} \left(\kappa_t - \frac{1 - e^{i(\omega + \Delta)\tau}}{\tau}\right) \left(2 - \sqrt{R_2}\kappa_t - \kappa_1 + \sqrt{R_2} \kappa_1 \frac{1 - e^{i(\omega - \Delta)\tau}}{\tau}\right) + \sqrt{R_2} \chi^2 \right) \delta A_u
\
- 2\chi(\sqrt{R_1}\kappa_2 \delta A_{\text{in}}(-\omega)^{\dagger} + \kappa_2 \delta A_{\text{in}}(-\omega)) - \left(\kappa_t - \frac{1 - e^{i(\omega + \Delta)\tau}}{\tau}\right) 2\sqrt{R_1}\kappa_2 \delta A_{\text{in}}(\omega)\right) (4.38)
\]

\[
\delta A_{\text{out}}(-\omega)^{\dagger} = \frac{1}{\left(\kappa_t - \frac{1 - e^{i(\omega + \Delta)\tau}}{\tau}\right) \left(\kappa_t - \frac{1 - e^{i(\omega - \Delta)\tau}}{\tau}\right) - \chi^2} \left(\kappa_t - \frac{1 - e^{i(\omega - \Delta)\tau}}{\tau}\right) \left(2 - \sqrt{R_2}\kappa_t - \kappa_1 + \sqrt{R_2} \kappa_1 \frac{1 - e^{i(\omega + \Delta)\tau}}{\tau}\right) + \sqrt{R_2} \chi^2 \right) \delta A_u(-\omega)^{\dagger}
\
- 2\chi(\sqrt{R_1}\kappa_2 \delta A_{\text{in}}(\omega) + \kappa_2 \delta A_{\text{in}}(\omega)) - \left(\kappa_t - \frac{1 - e^{i(\omega - \Delta)\tau}}{\tau}\right) 2\sqrt{R_1}\kappa_2 \delta A_{\text{in}}(-\omega)\right) (4.39)
\]

### 4.2.4 Amplitude Quadrature Fluctuations and Transfer Function

In order to calculate the variances of the output mode, the quadrature fluctuations need to be known. Upon transmission through (or reflection from) a detuned cavity, the mean field amplitude will pick up a different phase shift relative to uncertainty area [16]. This has the effect of rotating the squeezing ellipse relative to the mean field amplitude. This means that the amplitude and phase quadrature fluctuations are written as

\[
\delta X_1(\omega) = \delta A(\omega) e^{-i\phi_{\text{out}}} + \delta A(-\omega)^{\dagger} e^{i\phi_{\text{out}}}
\]

\[
\delta X_2(\omega) = i[\delta A(\omega) e^{-i\phi_{\text{out}}} - \delta A(-\omega)^{\dagger} e^{i\phi_{\text{out}}}] \quad (4.40)
\]

with \(\phi_{\text{out}}\) given by [1],

\[
\tan(\phi_{\text{out}}) = \frac{2\kappa_1 \sin(\Delta\tau)}{\kappa_t \chi^2 - \frac{4}{\tau^2} \left[1 - \cos(\Delta\tau)\right]} \quad (4.42)
\]

The output variances are calculated by \(V_1(\omega) = \langle|\delta X_1(\omega)|^2\rangle\) and \(V_2(\omega) = \langle|\delta X_2(\omega)|^2\rangle\). In order to simplify the derivation we now set \(\Delta = 0\). This will correspond with the results.
we will later measure, where the OPO cavity can only be locked on resonance with the
seed, so we have \( \phi_{\text{out}} = 0 \). In order to simplify the (lengthy) working to follow we will
introduce the following notational substitutions

\[
B = \left( \kappa_t - \frac{1 - e^{i\omega t}}{\tau} \right) \left( 2 - \sqrt{R_2} \right) \kappa_2 - \kappa_1 + \sqrt{R_2} \frac{1 - e^{i\omega t}}{\tau} + \sqrt{R_2} \chi^2
\]

\[
C = -2\chi \kappa_2
\]

\[
D = -\left( \kappa_t - \frac{1 - e^{i\omega t}}{\tau} \right) 2\sqrt{\kappa_1 \kappa_2}
\]

\[
E = -2\chi \sqrt{\kappa_1 \kappa_2}
\]

These substitutions give

\[
\delta A_{\text{out}}(\omega) = \frac{B \delta A_u(\omega) + C \delta A_u(-\omega)^\dagger + D \delta A_{\text{in}}(\omega) + E \delta A_{\text{in}}(-\omega)^\dagger}{(\kappa_t - F)^2 - \chi^2}
\]

\[
\delta A_{\text{out}}(-\omega)^\dagger = \frac{C \delta A_u(\omega) + B \delta A_u(-\omega)^\dagger + E \delta A_{\text{in}}(\omega) + D \delta A_{\text{in}}(-\omega)^\dagger}{(\kappa_t - F)^2 - \chi^2}
\]

Again, for brevity we write \( \delta A(\omega) = \delta A \) and \( \delta A(-\omega)^\dagger = \delta A^\dagger \). Therefore

\[
|\delta X_{\text{out}}(\omega)|^2 = |\delta A_{\text{out}} \delta A_{\text{out}} + \delta A_{\text{out}} \delta A^\dagger_{\text{out}} + \delta A^\dagger_{\text{out}} \delta A_{\text{out}} + \delta A^\dagger_{\text{out}} \delta A^\dagger_{\text{out}}|
\]

Using equations 4.48 and 4.49 we get

\[
|\delta X_{\text{out}}(\omega)|^2 = \frac{B^2 + 2BC + C^2}{(\kappa_t - F)^2 - \chi^2} (\delta A_u \delta A_u + \delta A_u \delta A_u^\dagger + \delta A_{\text{in}} \delta A_{\text{in}} + \delta A_{\text{in}} \delta A_{\text{in}}^\dagger)
\]

\[
+ \frac{D^2 + 2DE + E^2}{(\kappa_t - F)^2 - \chi^2} (\delta A_{\text{in}} \delta A_{\text{in}} + \delta A_{\text{in}} \delta A_{\text{in}}^\dagger + \delta A_{\text{in}}^\dagger \delta A_{\text{in}} + \delta A_{\text{in}}^\dagger \delta A_{\text{in}}^\dagger)
\]

\[
+ \frac{1}{(\kappa_t - F)^2 - \chi^2} (BE(\delta A_{\text{in}} \delta A_{\text{in}} + \delta A_{\text{in}} \delta A_{\text{in}}^\dagger + \delta A_{\text{in}}^\dagger \delta A_{\text{in}}) + BD(\delta A_{\text{in}} \delta A_{\text{in}}^\dagger + \delta A_{\text{in}} \delta A_{\text{in}}) + CD(\delta A_{\text{in}}^\dagger \delta A_{\text{in}}^\dagger + \delta A_{\text{in}} \delta A_{\text{in}}^\dagger))
\]

The last eight terms in this equation involve products of fluctuations of different quan-
tum operators (the input field and the vacuum field). These are uncorrelated sources, so
when we take the expectation value of these terms the result will be zero. Therefore we
will simply label all these terms as cross terms. So
\[ + \frac{\text{crossterms}}{|(\kappa_t - F)^2 - \chi^2|^2} \] (4.52)

Now we take the expectation value of this operator and we get

\[
\langle |\delta X_{1\text{out}}(\omega)|^2 \rangle = \left| \frac{B + C}{(\kappa_t - F)^2 - \chi^2} \right|^2 \langle \delta A_u \delta A_u + \delta A_u \delta A_u^\dagger + \delta A_u^\dagger \delta A_u + \delta A_u^\dagger \delta A_u^\dagger \rangle \\
+ \left| \frac{D + E}{(\kappa_t - F)^2 - \chi^2} \right|^2 \langle \delta A_{\text{in}} \delta A_{\text{in}} + \delta A_{\text{in}} \delta A_{\text{in}}^\dagger + \delta A_{\text{in}}^\dagger \delta A_{\text{in}} + \delta A_{\text{in}}^\dagger \delta A_{\text{in}}^\dagger \rangle \\
+ \frac{\langle \text{crossterms} \rangle}{|(\kappa_t - F)^2 - \chi^2|^2} (4.53)
\]

As indicated before, the expectation value of the cross terms is zero (as vacuum fluctuations are uncorrelated with everything). Also, we know that the amplitude quadrature variance of an operator is given by

\[
V_1 = \langle |\delta X|^2 \rangle = \langle \delta A \delta A + \delta A \delta A^\dagger + \delta A^\dagger \delta A + \delta A^\dagger \delta A^\dagger \rangle (4.54)
\]

Hence

\[
V_{1\text{out}}(\omega) = \left| \frac{B + C}{(\kappa_t - F)^2 - \chi^2} \right|^2 V_{1\text{u}}(\omega) + \left| \frac{D + E}{(\kappa_t - F)^2 - \chi^2} \right|^2 V_{1\text{in}}(\omega) (4.56)
\]

Substituting back in the expressions for \( C, D, E \) and \( F \) we finally get the transfer function for the amplitude quadrature

\[
V_{1\text{out}}(\omega) = \left| \frac{B - 2\chi\kappa_2}{(\kappa_1 + \kappa_2 - \frac{1-e^{-i\omega \tau}}{\tau})^2 - \chi^2} \right|^2 V_{1\text{u}}(\omega) \\
+ \left| \frac{2\sqrt{\kappa_1\kappa_2}(\kappa_1 + \kappa_2 - \frac{1-e^{-i\omega \tau}}{\tau} + \chi)}{(\kappa_1 + \kappa_2 - \frac{1-e^{-i\omega \tau}}{\tau})^2 - \chi^2} \right|^2 V_{1\text{in}}(\omega) (4.57)
\]

Note the substitution for \( B \) is not made as simply gives a long equation with no additional insight. Therefore

\[
V_{1\text{out}}(\omega) = \left| \frac{B - 2\chi\kappa_2}{(\kappa_1 + \kappa_2 - \frac{1-e^{-i\omega \tau}}{\tau})^2 - \chi^2} \right|^2 V_{1\text{u}}(\omega) \\
+ \left| \frac{2\sqrt{\kappa_1\kappa_2}}{\kappa_1 + \kappa_2 - \frac{1-e^{-i\omega \tau}}{\tau} - \chi} \right|^2 V_{1\text{in}}(\omega) (4.58)
\]

### 4.2.5 Phase Quadrature Transfer Function

Now we apply the methodology from the last section in order to calculate the phase quadrature transfer function. We have
\[ \delta \mathbf{X} = i [ \delta \mathbf{A} - \delta \mathbf{A}^\dagger ] \]  

(4.59)

and so

\[ |\delta \mathbf{X}_{\text{out}}(\omega)|^2 = \delta \mathbf{A}_{\text{out}} \delta \mathbf{A}^\dagger_{\text{out}} + \delta \mathbf{A}^\dagger_{\text{out}} \delta \mathbf{A}_{\text{out}} - \delta \mathbf{A}_{\text{out}} \delta \mathbf{A}^\dagger_{\text{out}} - \delta \mathbf{A}_{\text{out}} \delta \mathbf{A}_{\text{out}} \]  

(4.60)

Again using equations 4.48 and 4.49 we get

\[ |\delta \mathbf{X}_{\text{out}}(\omega)|^2 = \frac{B^2 - 2BC + C^2}{|\kappa - F|^2 - \chi^2} (\delta \mathbf{A}_{\text{in}} \delta \mathbf{A}^\dagger_{\text{in}} + \delta \mathbf{A}^\dagger_{\text{in}} \delta \mathbf{A}_{\text{in}} - \delta \mathbf{A}_{\text{in}} \delta \mathbf{A}^\dagger_{\text{in}} - \delta \mathbf{A}_{\text{in}} \delta \mathbf{A}_{\text{in}}) \]  

(4.61)

We note that \( V^2 = \langle |\delta \mathbf{X}|^2 \rangle = \langle \delta \mathbf{A} \delta \mathbf{A}^\dagger + \delta \mathbf{A}^\dagger \delta \mathbf{A} - \delta \mathbf{A} \delta \mathbf{A}^\dagger - \delta \mathbf{A}^\dagger \delta \mathbf{A} \rangle \), so when we take the expectation value of both sides of equation 4.61 we get

\[ V^2_{\text{out}}(\omega) = \left| \frac{B - C}{|\kappa - F|^2 - \chi^2} \right|^2 V^2_{\text{in}}(\omega) + \left| \frac{D - E}{|\kappa - F|^2 - \chi^2} \right|^2 V^2_{\text{in}}(\omega) \]  

(4.62)

Substituting in the expressions for \( C, D, E \) and \( F \) we obtain

\[ V^2_{\text{out}}(\omega) = \left| \frac{B + 2\chi \kappa_2}{(\kappa_1 + \kappa_2 - 1 - \frac{1}{e^{i\omega\tau}}) \kappa_1 \kappa_2 - \chi^2} \right|^2 V^2_{\text{in}}(\omega) \]  

(4.63)

Therefore, the transfer function for the phase quadrature variance is

\[ V^2_{\text{out}}(\omega) = \left| \frac{B + 2\chi \kappa_2}{(\kappa_1 + \kappa_2 - 1 - \frac{1}{e^{i\omega\tau}}) \kappa_1 \kappa_2 - \chi^2} \right|^2 V^2_{\text{in}}(\omega) \]  

(4.64)

### 4.2.6 Approximate Solutions

The experimental setup will have \( T_1 \ll T_2 \ll 1 \), meaning that \( \kappa_1 \) will not be a significant term in the expressions for the variances obtained. We therefore approximate \( \kappa_1 = 0 \) to obtain simplified expressions for the output variances. This approximation is entirely valid when the results it leads to are related to the final experiment, where the ratio \( T_2/T_1 \) is 40. Using this approximation we find the simplified transfer functions for the quadrature variances are
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\[ V_{1\text{out}}(\omega) = \left| \frac{B - 2\chi \kappa^2}{(\kappa^2 - 1 - e^{i\omega \tau})^2 - \chi^2} \right|^2 V_{1u}(\omega) \quad \text{(4.65)} \]

\[ V_{2\text{out}}(\omega) = \left| \frac{B + 2\chi \kappa^2}{(\kappa^2 - 1 - e^{i\omega \tau})^2 - \chi^2} \right|^2 V_{2u}(\omega) \quad \text{(4.66)} \]

Notice that the output variances show periodic behavior at \( \omega = 2\pi n/\tau \), which corresponds to the integer multiples of the free spectral range of the cavity (as \(1/\tau\) is the cavity round trip time for a beam of light). Simulations using these equations are presented in section 4.3 in this chapter, illustrating the squeezing spectrum expected from the experimental measurements. On any resonance of the cavity (where \( \omega = 2\pi n/\tau \)) the squeezing predicted by equations 4.65 and 4.66 agrees with the corresponding result of squeezing about the zeroth free spectral range in ref. [2].

4.2.7 Assumptions

The following important assumptions were made in the derivation of the theory in this chapter. The following list details which assumptions were made, how they relate to the experimental work and what the effects of the assumptions are when they are no longer valid.

- **The cavity and OPA Hamiltonians are independent** - This result is a manifestation of a very deep principle - the Markov approximation [5]. This assumes that the state of the quantum processes inside the OPO (including the cavity) are very fast relative to the state change rate of the photon field in the cavity (characterised by time \( \tau \)). This can be understood as the system not possessing any memory of the photon field state - so that the cavity Hamiltonian will not remember what the parametric amplification Hamiltonian did to the state. Hence there is no coupling term between these processes, so the Hamiltonians can indeed be treated as independent.

- **All terms are linear, apart from the \( \chi^{(2)} \) non-linearity** - The only processes modeled in the theory that interact with the photon field are the parametric amplification and the optical resonator. All other factors in the experiment are assumed to be linear, and so are ignored in the theory. This assumption would become invalid if a non-linearity such as thermal expansion of the crystal was significant in the system. Experimentally the optical power levels used are not sufficient to drive any of these processes, and so this assumption is valid to make.

- **The OPO cavity is high finesse** - The optical amplitude after one cavity round trip is assumed to be change only slightly. This means that the circulating mode after one round trip can be calculated in terms of the circulating mode before the round trip without having to consider spatial effects inside the cavity. In addition to requiring high reflecting mirrors for the OPO cavity, the high finesse approximation entail the following assumptions
  - **No internal losses in the cavity** - While this assumption is never experimentally valid its effects on the results are simply modeled using the beamsplitter equation, provided the loss is small and does not violate the high finesse approximation. The effect of loss inside the cavity is to couple in an additional
vacuum term - which is exactly analogous to loss outside the cavity. Any experiment will have loss external to the cavity, hence the process of fitting the predicted results to measurements will consider all these losses together. This is why internal cavity losses were ignored.

– The pump field remains undepleted - The degree to which the approximation is valid can be measured by the ratio of the pump power to the threshold power. As the pump approaches threshold this approximation breaks down - with the result that the theory predicts a divergence in the anti-squeezing spectrum. The usage of this assumption is in writing the effect of the parametric amplification process as a constant gain spatially throughout the cavity.

The high finesse approximation allows the effects of the cavity mirrors to be modeled as a decay rate of the intra-cavity field with $\kappa = T/2\tau$, and hence compared to the gain rate due to the non-linear process ($\chi$). Using a 96% output coupling mirror (as in the experiment and simulated results) this assumption leads to a small error in the predicted squeezing and anti-squeezing inside the linewidth of the cavity. In the case of the measured results though (see chapter 7), there is a very strong asymmetry between squeezing and anti-squeezing as a result of loss. This asymmetry is significantly larger than the error due to this assumption, and hence this model still remains valid for comparison with these results.

• A single spatial mode is used - The single spatial mode approximation, while very well satisfied experimentally, is discussed here to emphasize the exact nature of what the creation an annihilation operators represent. Each operator represents the creation or annihilation of a photon in a mode that has a specific frequency, direction, polarisation and spatial mode. The derivation in this chapter assumes all modes in the experiment have these parameters in common. Experimentally the main concern with this requirement is the overlap of the pump beam with the seed beam - where misalignment results in a significantly reduced level of squeezing.

### 4.2.8 Interpretation

The most illustrative interpretation of the results derived above is that of the correlated sideband picture (see chapter 2). About the seed at low frequency (the "0th free spectral range") the OPO produces pairs of correlated photons, one at frequency $\Omega$ above the seed frequency and one at frequency $\Omega$ below. If these photons are in phase then the beam will be phase squeezed at frequency $\Omega$, and if the photons are $\pi$ out of phase then the beam will be amplitude squeezed.

The OPO cavity can be thought of in a basic way as a spectral filter on these correlated sidebands. At integer multiples of the cavity free spectral range the correlated photons will build up in the cavity - photons at the $-N$th FSR and photons at the $+N$th FSR. Off resonance the phase shift between successive round trips of the correlated photons will cause destructive interference and destroy the quantum correlations - meaning no squeezing is observed.

This interpretation makes it clear that the high frequency squeezing spectrum which is to be measured is a property of the cavity and not the non-linear material inside the cavity. In fact, the model in this chapter relied upon a material with a non-linear coefficient that constant with frequency. This assumption is valid inside the phase matching bandwidth
The OPO Squeezed State Spectrum

(see chapter 5) of the non-linear material. This bandwidth is the seed beam frequency range in which, at a fixed temperature, the OPO will produce squeezed light.

4.3 Simulations of the Results

This section will display computer simulations of the approximate solutions for the output variances of an OPO cavity found in equations 4.65 and 4.66. These simulations are then fitted to the measured results in chapter 7 to provide a measure of the correctness of the derived theory.

Equations 4.65 and 4.66 are simulated using the computer program MATLAB® [17], and the results are displayed in figure 4.2. The simulations display both the wideband (spanning four free spectral ranges) and narrowband (focused about one FSR to look at the cavity linewidth) features of the predicted spectra of both squeezing and anti-squeezing.

Parts (a) and (b) of figure 4.2 are simulated without loss, while parts (c) and (d) simulate the spectra with a 50% total loss. All the simulations use a value for $\chi$ which is 20% that of the threshold value and an output coupler reflectivity of 96%. Additionally, all the simulations use a cavity roundtrip time of $\tau = 1 \text{ [GHz}^{-1}]$.

![Figure 4.2: Computer simulations of the output variances, both squeezed and anti-squeezed, of a cavity enhanced OPO. Parts (a) and (b) are simulations with no loss present while (c) and (d) simulate a 50% total loss.](image)
Chapter 5

Measuring High Frequency Squeezing

This chapter will describe the experimental requirements in achieving the goal of investigating the high frequency spectrum of a squeezed beam. This includes stabilisation of the optical cavities in the experiment, the production of squeezed light at different spatial modes from a cavity enhanced OPO and the detection of the squeezed light. Finally, a basic experimental layout is presented that will satisfy the aforementioned requirements. Chapter 6 then highlights the technical improvements made to this basic setup, giving an experiment that is capable of measuring the squeezing spectrum with reliability and precision.

5.1 Experimental Requirements

The experiment required to measure the frequency spectrum of a squeezed beam is conceptually very simple. A summation of the key elements of the experiment are presented in the following list. A discussion of the necessity of each element is presented in the proceeding sections of this chapter.

- A laser producing a quantum noise limited beam at the fundamental frequency of interest
- A source of coherent light at twice the frequency of the fundamental which is phase locked to the fundamental frequency beam
- A method of selecting the spatial mode of the fundamental beam
- A non-linear crystal inside an optical cavity with a high $\chi^{(2)}$ coefficient, forming the OPO cavity
- A method of stabilising the optical cavities in the experiment
- A method of measuring the noise of the squeezed beam at a specific frequency

Each of these elements is discussed in this chapter, detailing why they are required for the experiment and the technical considerations surrounding their use.
5.2 The Laser Source

The first consideration in the choice of a laser source is the decision of the wavelength of operation. This is a decision based on practical considerations more than theoretical limitations. The availability of optical equipment (such as lasers, mirrors and polarising elements), along with the availability of non-linear materials that will squeeze light at the desired wavelength determine the wavelength used. These are the reasons why the wavelength of the fundamental field used is 1064 nm.

The fundamental concept of this experiment is to measure the optical noise of a non-classical beam relative to the quantum noise limit. In order to do this we need to consider how technical noise on the laser will affect these measurements. The two areas which can be influenced significantly by technical noise on the laser are the OPO and the detection scheme.

As derived in chapter 4, an OPO is a device that modifies the noise properties of an input mode and produces squeezed light at the output, only if a state sufficiently close to the quantum noise limit is inputted. We therefore need to seed the OPO with a bright coherent state at the fundamental frequency so that we produce a squeezed beam at the output with a non-zero mean field amplitude. Of course there are many experiments that produce vacuum squeezed beams (zero mean field amplitude), but seeding an OPO with a bright field is much simpler to operate from a technical standpoint. The beam used to seed the OPO, though, is heavily attenuated from the main output from the laser. Based on the beamsplitter equation (see chapter 2) then, the attenuated seed will be very close to the quantum noise limit, even if there is technical noise on the laser. The back-seeded OPO itself therefore does not require a low noise laser source.

As described in section 5.7 the detection scheme for measuring squeezing uses a single detector, which is driven by a strong local oscillator beam. Measurement of the quantum noise limit experimentally corresponds to a measurement of the noise on the local oscillator. Since a homodyne scheme is not used technical noise will not be subtracted from this measurement, meaning we rely upon the local oscillator to be a quantum noise limited beam. This requirement means that we need a laser with as little technical noise as possible, or a means of eliminating technical noise on a beam.

Strictly speaking, the condition of a quantum noise limited beam can be relaxed slightly to a beam which is quantum noise limited at the frequencies where we want to observe squeezing. The converse of this condition is more significant for the experiment - we can only observe squeezing where the beam is quantum noise limited.

The most basic method of reducing the technical noise on a laser is to attenuate it, making use once again of the beamsplitter equation. A more sophisticated method of reducing the technical noise on the laser is to transmit the beam through a high finesse cavity. This is easily seen by looking, for example, at the transfer function (amplitude quadrature) of a cavity - which in the case of a lossless ring cavity is given by [2]

\[ V_{1\text{out}}(\omega) = \frac{\kappa^2 V_{1\text{in}}(\omega) + \omega^2 V_{1\text{u}}(\omega)}{\kappa^2 + \omega^2} \]  

(5.1)

where \( \kappa \) is the cavity decay rate due to transmission through the cavity mirrors and \( V_{1\text{u}} \) is the variance of the vacuum mode coupling into the cavity. Here, as usual, \( \omega \) is the angular frequency. For an input mode with excess technical noise (\( V_{1\text{in}} = 10 \), say) the spectrum of the variance of the output mode in this example is displayed in figure 5.1.

Clearly this is an effective method of producing a beam which is very nearly quantum
5.3 The Frequency Doubled Source

Second order non-linear materials parametrically amplify by coupling the fundamental beam to its harmonic. The theoretical concepts of the fundamental and harmonic fields are translated into the experimental realities of a seed beam and pump beam respectively. The amount of non-linear coupling, and hence the strength of parametric amplification, degrades rapidly as the pump frequency deviates from twice the seed frequency.

The easiest method of producing a pump beam, while ensuring that its frequency is twice that of the seed beam, is to take a portion of the seed beam and double its frequency. This process is known as second harmonic generation, or SHG. This process is generally a second order non-linear material above threshold, where it spontaneously produces light at the second harmonic.

SHG is an appealing method of producing the pump beam, as the resulting beam always sits at exactly twice the frequency of the seed beam. Any change or drift in the seed frequency will be mirrored in the pump beam, resulting in a constant coupling strength inside the OPO. This is important since we need the amount of squeezing produced by the OPO to remain constant in time so as to objectively compare the results obtained.

5.4 Higher Order Spatial Modes

The frequency spectrum of a squeezed beam is strongly dependent on the cavity properties of the OPO. One of the dominant properties of optical cavities is that they support not only the fundamental Gaussian mode, but also higher order spatial modes. For completeness we therefore want an experiment that can measure the frequency properties of squeezed
light in different spatial modes. The production of squeezed higher order spatial modes using an OPO is a recent advancement developed by Lassen et. al. [18] at the Australian National University. This technique will be used and improved upon (see chapter 6) to measure the squeezing spectra of different spatial modes.

5.4.1 Spatial Mode Theory

The propagation of electromagnetic radiation through space is governed by the Helmholtz equation [13]. The equation is solved for the complex amplitude of the radiation. For well localised and monochromatic beams, such as those produced by a laser, the paraxial approximation can be applied - giving the paraxial Helmholtz equation

\[ \Delta^2 T A - 2ik \frac{\partial A}{\partial z} = 0 \quad (5.2) \]

where \( A(x, y, z) \) is the amplitude of the beam and \( \Delta^2 T = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \).

This is a differential equation with many different solutions, describing the possible modes in which a beam of light can propagate through space. Certain basis sets of these solutions are easier to produce experimentally, and hence are most commonly used. These are the Hermite-Gauss and Laguerre-Gauss bases [13].

The solutions of the paraxial Helmholtz equation all have the form of a fixed intensity distribution shape in a plane perpendicular to the direction of propagation, which then shrinks or dilates with the propagation of the beam. Different spatial modes are therefore described quite generally by the intensity distribution of the mode normal to propagation. The Hermite-Gauss basis describes modes with a rectilinear symmetry, while the Laguerre-Gauss basis describes modes with a radial symmetry. The lowest order Hermite-Gauss (TEM) and Laguerre-Gauss (LG) modes are depicted in figure 5.2. The Hermite-Gauss and Laguerre-Gauss bases describes modes using two integers (TEM\(_{nm}\) or LG\(_{nm}\)) - one for each axis of symmetry describing the order of the mode in that axis.

\[ \begin{array}{ccc}
(a) & (b) & (c) \\
(d) & (e) & (f)
\end{array} \]

**Figure 5.2:** Images of the intensity distribution of different spatial modes perpendicular the the direction of propagation. The modes are (a) TEM\(_{00}\) / LG\(_{00}\) (b) TEM\(_{10}\) (c) TEM\(_{12}\) (d) LG\(_{10}\) (e) LG\(_{01}\) (f) LG\(_{11}\). Images used from ref. [19].
5.4.2 Spatial Mode Selection

The most significant technical consideration of using higher order spatial modes in the experiment is the means by which the higher order modes are produced. The problem of spatial mode selection therefore requires a device which can transform a beam from a TEM$_{00}$ mode (produced by the laser) into a TEM$_{nm}$ mode. As we will see, high finesse optical cavities are well suited in such an application.

We have seen (in chapter 3) that the phase shift inside a cavity upon a round trip is

$$\frac{\Delta \phi}{2\pi} = \nu \frac{d}{c}$$ (5.3)

with $\nu$ being the frequency of the mode inside the cavity and $d$ being the cavity path length. This gives the resonance condition for the cavity to be

$$\nu = n \frac{c}{d}$$ (5.4)

where $n$ is a natural number. The above result is based on simple ray optics. Using the more complete description of Gaussian optics (based on the solutions of the paraxial Helmholtz equation seen earlier) we find that beams acquire a phase shift, relative to a plane wave of the same frequency, upon propagation. This is known as the Gouy phase shift [13], which is usually denoted $\xi$, and modifies the cavity resonance condition to be

$$\nu = \left(n - \frac{\xi}{2\pi}\right) \frac{c}{d}$$ (5.5)

Importantly, the Gouy phase shift increases for higher order spatial modes. This means that the resonant frequencies for different spatial modes are displaced from each other by a fixed amount. Therefore, a properly designed cavity will support only a single spatial mode at one time, allowing the cavity to act as a spatial mode selection device by tuning the cavity length. It is this mode selection capacity of a cavity that is used to produce a squeezed higher order spatial mode.

The final consideration in the spatial mode selection is the method by which light is coupled from the laser (a TEM$_{00}$ mode) into the higher order mode of the selection cavity. There are numerous methods of doing this, though the simplest method is misalignment of the input beam to the cavity. This will couple a fraction of the incident power into higher order modes with the same symmetry of the misalignment. For example, misaligning the input beam horizontally will couple power into higher order modes with horizontal symmetry - the TEM$_{n0}$ modes ($n$ a natural number). As discussed in chapter 6, the stability of the experiment and optical power throughput when producing squeezed higher order spatial modes becomes a significant technical consideration.

5.5 The OPO

At the most basic level, the OPO consists of a second order non-linear crystal inside of an optical cavity. In the last two decades there has been a huge variety of methods by which these two elements are combined [8], [20]. The technical aspects of the OPO design that impact on the physics of the experiment are

- Cavity length
- Cavity shape and mirror properties
- Crystal material
- Crystal surface shape and coatings
- Crystal position inside the cavity
- Pump phase matching

The most basic requirement of an OPO is that the pump field and the seed mode inside the cavity are fully overlapped. This means that not only do the two beams need to travel the same path inside the cavity but that they must have the same beam size at all points as well. Misalignment, or a bad match of the beam sizes, leads to a rapid decrease in the observed squeezing.

The cavity length and shape (mirror geometries) determine the size and position of the smallest diameter part of the beam inside the cavity - the waist. This size is of critical importance to the successful operation of an OPO as the non linear interaction strength scales up with the magnitude of the electric field of the pump field. Since the seed and pump modes are matched in their beam sizes then the waist size of the seed will be the same as the waist size of the pump - which determines the strength of the non-linear interaction.

The other factor that cavity length affects in this experiment is the free spectral range of the OPO cavity. If the cavity length is too short then the free spectral range will be too large - meaning the region where we expect to see high frequency squeezing will be too high to detect.

The crystal material has two main properties that affect the observed squeezing - the intrinsic non-linearity and the loss in the material. Ideally a material would have as high a non-linearity as possible with a very low loss. The goal of this work is not to investigate the properties of different non-linear materials, so we restrict our discussion to the available material - Lithium Niobate.

The coatings on the crystal surfaces and the position of the crystal inside the cavity are closely linked. The crystal available in this experiment was designed to have one surface as a cavity mirror. This means the crystal is positioned at the end of the cavity and must be long enough such that the beam waist is inside the crystal at this point. Furthermore, the coating on the crystal surfaces controls how much light is reflected from that surface. The crystal surface that formed a cavity mirror needs to be high reflection coated, while the other surface needs to be anti-reflection coated, to minimise loss in the cavity.

In chapter 3 it was demonstrated that the phase of the pump beam needs to be kept locked to the phase of the seed beam to produce stable squeezing. This must be performed in addition with another process known as phase matching. Lithium Niobate is a dispersive medium, meaning that the refractive index of the crystal is not the same for different wavelengths of light. This means that the phase of the pump and seed beams can be locked at one spatial point inside the crystal but will change along the propagation axis because of the different indices of refraction. To get optimal squeezing the pump phase needs to maintain a constant phase relation with the seed beam throughout the entire crystal. In Lithium Niobate this can be done by using birefringence - if the pump and seed are at different polarisations they see different birefringent indices of refraction in addition to the dispersion relation. Tuning (and then fixing) the temperature of the crystal allows for these two effects to perfectly cancel each other out, giving equal indicies of refraction for the pump and seed beams. This is the phase matching condition.
5.6 Stabilising the Optical Cavities

Both the OPO and the device to perform mode selection use high finesse cavities which must be locked on resonance for a beam to be transmitted. This resonance condition requires maintaining the frequency of the incident light relative to the length of the cavity. The main two sources of noise that affect these two parameters are temperature fluctuations and mechanical vibrations. These noise sources can be reduced significantly by thermally insulating and mechanically isolating the laser and the optical cavity, but not to a level which is comparable with the strict resonance condition imposed by the use of optical wavelengths. Therefore, a control system is required to maintain the laser on resonance with the optical cavities in the experiment.

What follows is a brief review of control theory and how it is applied to achieve a locking system for optical cavities. The technical considerations of the control loops in the experiment are salient as the performance of these loops largely determines the stability (and hence reliability) of the experiment. As such, the performance of the control loops was one of the key developments made in the experiment - which is discussed in chapter 6.

5.6.1 Control Theory

A controller is a device which adjusts the input to a system to force the output of the system to follow a specified reference over time. There are two main classes of controller - open-loop and closed-loop, distinguished by the flow of information in the system. Open loop controllers have no information returning from the output of the system to determine how the input should change. Closed loop controllers, or feedback controllers, measure the difference of the output from the reference to determine what change in the input should be.

A feedback controller can operate in two different modes - positive or negative feedback. Positive feedback adds a fraction of the output of a system back to its input while negative feedback subtracts a fraction of the output from the input. Positive feedback devices tend to amplify the system output while negative feedback controllers stabilise the output of a noisy system. In this work we want to stabilise optical cavities from noise, hence we use negative feedback controllers.

A negative feedback controller requires the following components, which are illustrated in a block diagram in figure 5.3.

- An error signal, measured by the difference of the system output from the reference
- A controller, which changes the input of the system to a value based on the error signal
- An input to the system which can change the parameter of the system of interest

Each element in the feedback loop has a transfer function which, as a function of frequency, gives the output of the element based on the input. The roundtrip loop gain of the controller (the product of all the transfer functions in the loop) determines how the system under control will behave - for example if it will be stable or if it will oscillate. Ideally this roundtrip gain will have no phase shift and very large gain at all frequencies. This ideal is not achievable in any real world system - most control loops do not even approximate this ideal due to the technical constraints of electronic circuits and control
Figure 5.3: A flow diagram of a feedback controller. The aim of a negative feedback system is to stabilize the output of the system to be equal to the reference by adjusting a system input. The task of the controller is to determine the correct input to the system based on the error signal derived from the difference of the system output from the reference.

elements. Indeed it is the way in which controllers differ from this ideal that forms the rich subject of control theory.

The amplitude and phase characteristics of any electronic system are closely related. Indeed, control elements in systems most often have amplitude changes in their transfer function of orders of magnitude. Correspondingly, the phase change in the transfer function frequently involves multiple $\pi$ shifts from an in phase response. The correlation between the amplitude and phase response of a system can often be understood from the point of view of energy conservation - as a diminished amplitude transmission results in increased energy storage in the system, requiring a change of phase response of the system.

Control theory analyses the stability and performance of control loops by studying the amplitude and phase of the round trip loop gain together in the complex plane. Note that in this work the specification of the phase of the round trip loop gain of a controller is referenced to the ideal negative feedback gain. The key rules in understanding the performance of a control loop are

- **Phase** - The phase of the round trip loop gain at a particular frequency will determine if the loop operates in a negative or positive feedback mode. A phase shift that is within $\pi/2$ of an in phase response gives negative feedback at that frequency - meaning the loop is suppressing the noise there. A phase shift outside of the $\pm\pi/2$ range at some frequency means the loop operates in a positive feedback mode for that frequency. The noise in the system at those frequencies where the loop operates in a positive feedback mode will be amplified instead of suppressed.

- **Amplitude** - The amplitude response of a control loop describes the noise suppression ability of the system. A higher gain in the control loop at a certain frequency gives a better suppression (with negative feedback) of noise at that frequency. Therefore, the relative round trip amplitude change between frequencies measures the locking quality of the loop across it’s frequency range. Conversely, for frequencies where the loop is operating as a positive feedback system, the gain determines how much the noise is amplified.

- **Stability** - The stability criteria for a negative feedback loop is a combination of the amplitude and phase spectra of the round trip loop gain. If there is a frequency
which has a $\pi$ phase shift along with an amplitude greater than unity then the loop will oscillate at that frequency. This oscillation is unacceptable for an optical cavity, setting an upper limit on the amplitude of the transfer function for frequencies with a $\pi$ phase shift.

5.6.2 The Pound-Drever-Hall Locking Technique

Arguably the most difficult aspect of a feedback control loop for locking optical cavities is deriving the error signal. A very successful method of deriving this signal for an optical cavity is the Pound-Drever-Hall technique [22].

The error signal is obtained by first modulating the phase of the incident beam to the cavity sinusoidally (at frequency $\Omega$, say), creating small modulation sidebands on the carrier beam. The sidebands are $\pi$ out of phase, meaning they destructively interfere - so the modulation is not seen on the intensity of the beam. Upon reflection from the cavity the sidebands and carrier beam all acquire different phase shifts depending on where they are relative to the cavity resonance. The sidebands, no longer perfectly out of phase, now beat with the carrier giving an intensity modulation of the reflected beam at frequency $\Omega$. The magnitude of this modulation determines the amount of phase difference introduced between the sidebands.

The magnitude of the intensity modulation is measured by electronically mixing the measured intensity of the reflected beam with a sinusoid at frequency $\Omega$. Examples of error signals experimentally measured are plotted in figure 5.4. The two main types of error signal are displayed - the first is where the modulation frequency is larger than the linewidth of the cavity, and the second is where the modulation frequency is smaller than the linewidth.

5.6.3 Completing the Control Loop

The final two elements required for a complete control loop for an optical cavity are a means of changing the frequency of the laser relative to the cavity length and a controller. In order to implement suitable devices in these roles we need to know what the locking bandwidth of the loop should be. The locking bandwidth of the loop is the frequency range in which the round trip loop gain is significant (relative to unity) and determines where the loop has significant noise suppression. To know what bandwidth to implement in the control loop it is important to know which noise source is most dominant in the experiment.

The most significant technical noise sources identified in relation to locking optical cavities are mechanical and thermal in nature. These two noise sources are most dominant in very different frequency regions - thermal noise is dominant in the six decades below one Hertz while mechanical noise is dominant in the six decades above one Hertz. Thermal isolation of cavities and laser sources is more effective than mechanical stabilisation. This, together with the dominant frequency range of mechanical noise, makes mechanical noise the most significant term that a control loop needs to suppress.

A control loop for an optical cavity must therefore have a bandwidth approximately between 1 Hertz and 100 Kilohertz. Furthermore, with more than one cavity in the experiment (each with independent mechanical noise) the control element needs to be a device that changes the cavity length, instead of the laser frequency. Such a device needs to be able to provide length control on the scale of fractions of a wavelength in the bandwidth mentioned above, namely small movements at high frequencies. The most
Measuring High Frequency Squeezing

Figure 5.4: Plots of measured error signals overlaid with the cavity reflection intensity. The horizontal axis in the plots is proportional to the length of the cavity as this is scanned in time. Part (a) is measured from a high finesse (800) cavity, zoomed in about the resonance, while part (c) is measured from the same cavity zoomed out to display a full FSR. Part (b) is measured on a low finesse (130) cavity.

commonly used device to perform this is called a piezoelectric transducer. The standard transducer material used is lead zirconate titanate, abbreviated PZT.

The final element required in the control loop is the controller itself. The controller transfer function should ideally be the inverse of the combined transfer function of the rest of the locking loop, but this is very difficult to implement in an analogue device. The locking systems were one of the major technical developments of the experiment, and so they will be discussed in detail in the next chapter. This includes a discussion of the controllers, optical detectors and generation of the error signal.

The reliability and consistency of the experiment rests on the strength of the locking loops of the optical resonators more than any other factor. If the cavities are not locked properly on resonance then the measured results will reflect this strongly both in terms of the calibrated shot noise level and the actual amount of squeezing produced. Stabilisation of optical cavities is therefore a very significant technical consideration in the experiment.
5.7 Detection of the Squeezing Spectrum

The most useful interpretation of a squeezed state for the purposes of measuring the squeezing is that the state has correlated sidebands. The role of the detector will be to measure the correlation in the sidebands of the squeezed beam at the frequencies of interest. The fundamental limiting factor of optical detectors in this process is that they cannot directly measure the electric field of the beam, only the intensity envelope. This means that phase information cannot be directly measured as it is, for example, in radio receivers.

The most basic scheme of measuring squeezed light is directing the beam onto a photodiode and looking at the detector output on a spectrum analyser. As explained above, this method is only sensitive to intensity noise and therefore can only measure amplitude squeezing. Furthermore, the detector must have a very low electronic noise floor to be properly sensitive to the shot noise of a (weak) squeezed beam. These limitations make this method very unsuitable for a general squeezing measurement device.

A detection scheme which is sensitive to the quadratures of a beam can be achieved by interfering the beam to be measured with a local oscillator. Of course this requires that the two beams be phase locked together so that phase interference takes place. If the local oscillator is strong enough relative to the squeezed beam then, to first order in the noise terms, changing the phase difference between the beams will simply rotate the quadrature axes of the squeezed beam relative to the intensity on the detector. As a result, this detection scheme can measure the full variance properties of any squeezing ellipse, whether phase or amplitude squeezed.

This scheme can be further improved through the use of two balanced detectors, instead of direct detection. The two beams can be interfered on a beamsplitter and then detected by a pair of matched photodetectors (with similar efficiencies and electrical properties). In such a scheme, if the central frequency of the local oscillator beam is the same as the central frequency of the squeezed beam then this is referred to as homodyne detection, otherwise it is called heterodyne detection. A diagram of a homodyne detector is presented in figure 5.5, and the detection properties are derived after this.

Figure 5.5: A basic setup to perform a homodyne measurement of a beam exiting some optical system. The system is placed in one arm of an interferometer while the other arm is shifted in phase by a variable amount.
We can see in figure 5.5 that the most simple method of maintaining a constant phase between the local oscillator and the beam to be analysed is by placing the experiment in a Mach-Zehnder interferometer, with a device in one path to give a well known phase shift. Based on this setup we can derive an expression for \( I(t) \) (the subtraction of the two detectors) based on the local oscillator and information beam modes. We are interested in the noise of the information beam, so we assume that the detectors are AC coupled (using a high-pass filter so only noise terms are outputted, not the mean field intensity). The following derivation follows the work in [2]. We first write down the operators for the fields after beamsplitter 2

\[
\begin{align*}
\mathbf{A}_{D1} &= \frac{1}{\sqrt{2}}((\mathbf{A}_{lo} + \delta \mathbf{A}_{lo})e^{i\phi} + (\mathbf{A}_{in} + \delta \mathbf{A}_{in})) \\
\mathbf{A}_{D2} &= \frac{1}{\sqrt{2}}((\mathbf{A}_{lo} + \delta \mathbf{A}_{lo})e^{i\phi} - (\mathbf{A}_{in} + \delta \mathbf{A}_{in}))
\end{align*}
\]

where we assume that the transmission ration of beamsplitter 2 is 0.5. Now the photocurrents generated by the detectors (assume perfect efficiency) \( I_{D1} \) and \( I_{D2} \) are given by

\[
\begin{align*}
\mathbf{A}^\dagger_{D1}\mathbf{A}_{D1} &= \frac{1}{2}((\mathbf{A}_\text{lo}^\dagger + \delta \mathbf{A}_\text{lo}^\dagger)(\mathbf{A}_\text{lo} + \delta \mathbf{A}_\text{lo}) + (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{in} + \delta \mathbf{A}_\text{in})e^{-i\phi} \\
&\quad + (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{lo} + \delta \mathbf{A}_\text{lo})e^{i\phi} + (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{in} + \delta \mathbf{A}_\text{in})) \\
\mathbf{A}^\dagger_{D2}\mathbf{A}_{D2} &= \frac{1}{2}((\mathbf{A}_\text{lo}^\dagger + \delta \mathbf{A}_\text{lo}^\dagger)(\mathbf{A}_\text{lo} + \delta \mathbf{A}_\text{lo}) - (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{in} + \delta \mathbf{A}_\text{in})e^{-i\phi} \\
&\quad - (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{lo} + \delta \mathbf{A}_\text{lo})e^{i\phi} + (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{in} + \delta \mathbf{A}_\text{in}))
\end{align*}
\]

Subtracting these photocurrents gives

\[
I(t) = I_{D1} - I_{D2} = (\mathbf{A}_\text{lo}^\dagger + \delta \mathbf{A}_\text{lo}^\dagger)(\mathbf{A}_\text{in} + \delta \mathbf{A}_\text{in})e^{-i\phi} + (\mathbf{A}_\text{in}^\dagger + \delta \mathbf{A}_\text{in}^\dagger)(\mathbf{A}_\text{lo} + \delta \mathbf{A}_\text{lo})e^{i\phi}
\]

This photocurrent clearly has terms containing information about the quantum noise of the information beam - \( \delta \mathbf{A}_{in} \). In order to look at the variance of this noise we need to take the variance of the electrical photocurrent. This is done by converting the current to a proportional voltage (using a resistor in the simplest case) then viewing the resulting voltage on a spectrum analyser. If we assume the mean field amplitude of the information beam is much smaller than the mean field amplitude of the local oscillator then we can ignore (to first order) many of the terms in equation 5.11. Then the variance of the photocurrent is

\[
\Delta^2 I(t) = \alpha_{lo}^2(\cos^2(\phi)V_{1in} + \sin^2(\phi)V_{2in})
\]

where \( \alpha_{lo} \) is the mean field amplitude of the local oscillator. Provided the information beam amplitude is small compared to the local oscillator amplitude then the homodyne
5.7 Detection of the Squeezing Spectrum

detection scheme is an excellent method of measuring the optical noise properties (the variance) of the information beam. Due to the subtraction process of the detectors none of the noise on the local oscillator contaminates the measurement. Furthermore, the variance in different quadratures is simply selected by changing the phase of the local oscillator beam. This property is essential for measuring the complete properties of a quadrature squeezed state as noise is transferred between the quadratures. Finally, equation 5.12 demonstrates that the overall variance measured by a spectrum analyser is scaled by the local oscillator intensity. The result of this is that we need to calibrate the quantum noise limit by measuring the variance of the photocurrent when the information beam is not present. This calibration gives a normalized variance measurement.

As well suited as the homodyne scheme is for measuring squeezed light, its technical requirements are not currently able to be met for measuring the squeezing spectrum in the gigahertz range. Namely, only a single detector at this frequency is available and the means of subtracting weak signals at such high frequencies with low enough noise is not accessible. Therefore, the detection scheme to be used in the experiment is a single detector - measuring the squeezed beam interfered with a local oscillator, on a spectrum analyser.

The derivation of the homodyne properties presented above highlights those desirable characteristics that are not present in the single detection scheme and also those that remain. The most critical property that is lost in a single detection scheme is the insensitivity to the local oscillator technical noise. The measurement of the squeezed beam noise properties now relies upon the local oscillator being quantum noise limited (free of technical noise) since the second detector is no longer present to subtract that noise.

Additionally, the single detection scheme has the problem that the shot noise is no longer constant as the phase of the local oscillator is changed relative to the squeezed beam. Again, without the second detector subtracting the required noise terms, the squeezed beam mean field intensity will add to and subtract from the shot noise during constructive and destructive interference respectively.

Finally, there is no lossless means of interfering two beams of the same spatial mode so that they co-propagate. This means that the squeezed beam will have a fixed loss in its path as a result of the combination process, reducing the observed squeezing. On the other hand, the scheme retains the ability to be sensitive to a chosen quadrature through the adjustment of the relative phase of the local oscillator to the squeezed beam.

The bandwidth considerations involved in this measurement process revolve around the photodetector itself and the spectrum analyser used. In order to measure the squeezing spectrum at several different free spectral ranges we need a detector that works up into the mid gigahertz range. Photodetection with a bandwidth of several gigahertz is a novel technique [23]. The prototype detector available did not have sufficient gain at radio frequencies (below 1 GHz) such that it could measure the squeezing spectrum component around DC (the zeroth FSR). Therefore, a more conventional low frequency detector is required to measure this part of the squeezing spectrum. The following convention will be used in the rest of this work - LF is an abbreviation for low frequency (below 100 MHz) and HF abbreviates high frequency (between 1 GHz and 6 GHz).

The final consideration with the detectors used is the optical power required to separate the measured shot noise from the dark noise (electronic noise) of the detector. This separation is most critical since the goal is to measure noise levels below the quantum noise. The LF detector requires only a very small optical power level to get sufficient separation (more than 5 dB) - in the order of 1 mW. The HF detector, on the other hand,
Figure 5.6: A schematic of an experiment to measure the high frequency noise spectrum of a squeezed state based on the requirements outlined in this chapter.

requires much more optical power for a smaller separation of shot noise from dark noise - a 4 dB separation with 7 mW of optical power. This consideration sets a requirement on the ratio of the beamsplitter used to combine the squeezed beam and the local oscillator and therefore the loss of squeezing at this point, as the local oscillator has only limited maximum power.

5.8 Experimental Layout

Taking into consideration the experimental requirements discussed up to this point we can develop a basic experimental layout of the key components required. This layout is displayed in figure 5.6.

In figure 5.6 the isolators are devices which allow light to pass in one direction only. These are required so that any reflected beams from the OPO cavity or other components do not affect the laser or create their own unwanted resonators. The mode transfer cavity (MTC), also referred to as a mode cleaning cavity (MCC), is ideally a high finesse cavity. The mode transfer cavities used in the experiment are as depicted in figure 5.6 - three mirrors arranged to give a ring cavity.

The novelty in realising this experiment was the combination of the following three elements

- **A high frequency detector** - This use of a new, high frequency detector was obtained through collaboration with the Australian Defense Force Academy (ADFA). The detector uses microwave frequency electronics, together with a photodiode that is two orders of magnitude smaller in area than other photodiodes used in the experiment. This provides a detection bandwidth that is much larger than detectors at this wavelength built previously. This allowed for the detection of a much wider range of the frequency spectrum of the squeezed light. The details of the detector are published in ref. [23].
• **A stable, cavity enhanced OPO squeezed light source** - The OPO used to produce the squeezed light operated with high temporal stability and reliability, while using a longer cavity length than many OPO squeezing experiments. The long cavity length was used to give a FSR which was within the detection bandwidth. This enabled the measurement of the novel feature of the OPO squeezed light - the repetition of squeezing at sequential free spectral ranges. The means by which the stability of the experiment was improved is discussed in detail in chapter 6.

• **A method of selecting the spatial mode of the squeezed light** - The method of producing squeezed higher order spatial modes pioneered in [18] lacked the stability we desired for the experiment. An improved method of generating the squeezed higher order spatial modes was therefore implemented in the experiment. The details of this improvement are also discussed in detail in chapter 6.
Chapter 6

Experimental Developments

This chapter describes in detail the important technical developments made in constructing the experiment used to measure the high frequency squeezing spectrum. The developments presented in this chapter are considered important to the experiment as they strongly influence the reliability, repeatability and stability of the experiment. First, the details of the feedback controllers implemented in the experiment are described, along with criteria for assessing their performance. Next, two schemes for locking the pump phase to the seed in the OPO will be discussed. Also, different kinds of detectors are assessed in their capacity to generate an error signal for the locking loops in the experiment to further optimise the feedback controllers. Finally, a different method of spatial mode selection compared to the old way of generating squeezed higher order spatial modes is presented and its benefits are explained.

6.1 Feedback Controllers

Precise locking of the optical cavities in the experiment is critical to making it reliable and repeatable. Therefore a significant effort was made in the experiment to develop the stability and accuracy of the feedback loops. Three new feedback controllers were implemented in the experiment, with the development of each controller taking the following path.

6.1.1 Control Element Transfer Function

The control element used to keep the optical cavities locked on resonance is a PZT element attached to a mirror. Changing the voltage applied to the PZT causes it to change its length, moving the mirror attached to it. A PZT element has a bandwidth of approximately 50 kHz. To have a sufficient dynamic range of movement of the PZT a voltage range of approximately 200 Volts is required. Therefore each PZT is driven by a high voltage amplifier, the electrical schematic for which is attached in appendix A.3.

In order to determine the transfer function for the controller we need to know the transfer function for the error signal and the control element. The error signal has a flat frequency response far beyond the bandwidth of the PZT, so we can ignore this element from the point of view of finding the transfer function shape for the controller. The only impact the error signal will have will be on the overall gain needed in the controller - which is determined experimentally later on.

The control element has a strongly frequency dependent transfer function, combining the transfer function of the PZT itself with the resonant properties of the mechanical
housing of the cavity. An example of a measured transfer function is plotted in figure 6.1, which shows the amplitude and phase response of the MTC PZT.

Figure 6.1 illustrates how strongly non-linear a PZT transfer function is - with the amplitude spanning almost four orders of magnitude. Additionally, the phase of the PZT response changes dramatically in the measured bandwidth, primarily about the resonance feature at 40 kHz.

### 6.1.2 Resonances in Feedback Loops

Ideally, the transfer function of the controller would be the precise inversion of the PZT transfer function (in both amplitude and phase). Constructing a circuit as complicated as the inversion of the transfer function depicted in figure 6.1 is impossible with current analogue technology. The next best step is to build a controller with a response that cancels out the most dominant feature of the PZT response - a resonance.

In figure 6.1 we can clearly see two strong resonance features in the PZT - one at 17 kHz and one at 40 kHz. While the 17 kHz resonance does not bring about a full $\pi$ phase change the 40 kHz resonance does, sending the total phase change through $\pi$ causing a basic proportional controller to oscillate at 40 kHz.

The simplest method to deal with resonances in feedback loops is to strongly attenuate the round trip loop gain at the resonance frequency by using a notch filter in the controller circuit. If the resonant feature does not have a corresponding $\pi$ phase change then the controller bandwidth can extend past the resonance stably with no phase adjustment in the controller. Resonant features with a phase change require the control loop to compensate for this phase shift.
6.1.3 PID Controller

A common type of feedback controller is the Proportional-Integral-Derivative controller, or PID. The PID controller provides significant noise suppression in a feedback loop for an optical cavity across a very wide bandwidth - suppressing not only long term drift noise but short term mechanical vibrations as well. This gives a low noise output of the transmitted beam, as well as a stable lock for up to several hours. The three stages of a PID are explained below:

- **Proportional** – The proportional stage of the controller outputs a value that is determined by multiplying the error signal by some fixed function. Intrinsically the proportional stage responds equally to all frequencies. This response (the multiplying function) is normally altered to give a round trip loop gain that provides a stable control system.

- **Integral** – The integral stage of a controller outputs a value that is determined based on the integral of the error signal over some previous time window. The integral stage therefore provides a means of correcting for slow moving drift in the system - as a small drift integrated over a long period makes a large correction signal. This stage allows the controller to make use of history to improve performance.

- **Derivative** – The derivative stage of a controller outputs a value calculated from the derivative of the error signal. This stage allows the controller to attempt to predict the near future and correct for it. Derivative stages are useful in systems with limited bandwidth and a slow system response.

In a control loop for an optical cavity only the first two stages (proportional and integral) are normally used, as the large bandwidth of the loop (at least four decades) makes the derivative stage redundant. The derivative stage is normally required to introduce a phase shift in the controller to drive through low frequency resonances of the PZT. All of the PZT elements used in the experiment have sufficiently high frequency on the lowest resonance so that derivative stages are not required on the controllers.

The proportional stage of a PID for use in a feedback loop for optical cavities provides the main response of the controller above approximately 10 Hz. The integrator stage provides correction for long term drift in the flexure and temperature of the cavity. The analogue circuit used to implement the PIC controllers for the new locking loops in the experiment is shown in appendix A.2. The functional components of this circuit are an elliptic filter, an integrator, a low pass filter and a total gain (amplification). The elliptic filter provides a notch response as well as a low pass roll-off. The integrator is simply a capacitor included into the feedback loop of one of the operational amplifiers. The low pass filter is comprised of a resistor-capacitor combination at the output of the circuit. Finally, amplification is performed in the circuit through four gain stages (again using operational amplifiers).

6.1.4 Total Feedback Loop

Combining the transfer functions for the PZT elements and the PID controllers we can determine the round trip loop gain of the feedback control loops for the three new locking loops developed for the experiment. The round trip loop gain spectra are plotted in figure 6.2.
Figure 6.2: Total round trip loop gain functions, plotted as a function of frequency, for three control loops in the experiment - (a) the OPO (b) the pump phase (c) the MTC. The amplitude measurements displayed indicate the relative magnitude between different frequencies. The controller has a variable gain which uniformly raises or lowers the displayed amplitude traces to obtain the optimal gain experimentally.

Figure 6.2 illustrates the control concepts of phase and amplitude of the round trip loop gain described in chapter 5. The optimal amplitude level for the loop is found experimentally by adjusting the overall gain in the controller circuit. This results in the uniform raising or lowering (on a logarithmic scale) of the amplitude of the round trip loop gain. The optimal point is to set the level such that the frequency with a phase shift of exactly $\pi$ has a gain just below unity.

The two most important criteria by which the feedback loops should be evaluated are the noise levels on the transmitted beam and the long term stability of the lock. The former is most easily assessed by directly measuring the intensity noise of the transmitted beam with a photodiode and a spectrum analyser. These measurements for the three loops are plotted in figure 6.3. In addition, each plot contains a trace which measures the noise when the gain of the loop is too high and oscillation occurs.

Each of the spectra in figure 6.3 were measured after calibrating the DC signal on the detector to 250 millivolts (1 dBm). The measured noise power displayed is the difference
between the signal of interest (the DC component of the field at 1 dBm) and the measured noise at each frequency. This is measured in units referred to as decibels relative to the carrier, or dBc. For example, when oscillating, the MTC has a peak at 84 kHz with amplitude -20 dBc. This means the 84 kHz signal on the measured on the transmitted beam is 1% in power (10% in voltage) of the DC signal. This calibration allows the different spectra in figure 6.3 to be compared to each other directly.

The noise spectra for oscillating control loops are displayed in figure 6.3 to compare the resulting oscillation peaks with figure 6.2 in relation to the stability criteria for a control loop. For example, in the mode transfer cavity loop gain the phase passes the $-\pi$ point at about 80 kHz and in the spectrum of oscillation we see a strong peak at 83 kHz.

The total noise levels (peak-to-peak) observed on the transmitted beams for the three locking loops are displayed in table 6.1. The error values displayed arise from a combination of the division uncertainty of the oscilloscope and the finite display rate of the oscilloscope, where the uncertainty results in imprecise definition of the peak values of the

Figure 6.3: The intensity noise power spectra on the transmitted beams of three cavities when locked (a) the OPO (b) the pump phase and (c) the MTC. Each control loop has two measured noise spectra, one with the optimal gain for a stable loop and one where the loop is oscillating. The measurements are calibrated so the noise amplitudes (in dBc) can be directly compared between the plots.
noise. While this method is a more crude means of comparing noise in the control loops (figure 6.3 is the correct method) it does provide a more intuitive comparison.

<table>
<thead>
<tr>
<th>Locking Loop</th>
<th>p-p Noise (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPO</td>
<td>9 ± 2</td>
</tr>
<tr>
<td>Pump Phase</td>
<td>14 ± 2</td>
</tr>
<tr>
<td>MTC</td>
<td>3 ± 2</td>
</tr>
</tbody>
</table>

Table 6.1: The peak-to-peak transmitted intensity noise measurements of three control loops in the experiment.

The final criteria for the quality of the locking loop is the long term stability. The MTC and the OPO can each remain locked for several hours at a time while the pump phase loop remains locked for 5-10 minutes on average. The latter case is caused not by a problem with the control loop but rather by the integrator of the PID circuit causing the output to rail on its voltage limits (+/−15V), due to small signal amplitude of the pump phase error signal. Nonetheless, the total stability of all three locking systems operating in tandem is sufficient to provide an excellent repeatability of the experiment.

### 6.2 Locking the Pump Phase

The next major development required in the experiment was to find a different method of generating the error signal to lock the pump phase. The previous scheme of generating squeezed higher order spatial modes relied on the transmission of modulation sidebands through the OPO cavity to be measured by the detector measuring the squeezing spectrum, and hence generate the error signal. This method of locking the pump phase cannot be used in the experiment we want to perform because the HF detector (used to measure the squeezing spectrum) is not sensitive to radio frequency modulation sidebands. Therefore we need a method of generating the error signal for the pump phase locking loop that is independent of the squeezed beam.

The two remaining options for beams that interact with the OPO cavity which we can possibly use to generate the error signal are the reflected seed beam and the reflected pump beam. The reflected pump beam is not a desirable choice because we operate the OPO in a regime where the pump depletion is negligible. Therefore, any phase shift between modulation sidebands on the pump beam will be small - giving a weak error signal.

Conversely, the seed beam undergoes the full amplification-deamplification process, so any sidebands on the incident seed beam will more strongly pick up a relative phase shift depending on the phase of the pump beam. Therefore the reflected seed beam from the OPO cavity is the best choice to detect for the purpose of generating the pump phase error signal.

The reflected seed beam will therefore be used for making the error signals for two locking loops - the OPO cavity loop and the pump phase loop. The linewidth of the OPO cavity is approximately 10 Mhz, which sets an effective minimum to the frequency of the modulation sidebands used for the OPO cavity error signal.

We have shown that generating an error signal using the Pound-Drever-Hall method requires the introduction of a phase or magnitude change between the frequency modulation triplet (the carrier and two sidebands). In the case of an OPO either amplifying or deamplifying a seed beam, simple reflection off the cavity will not change the phase or amplitude of the sidebands. The sidebands need to be transmitted into the cavity to
be amplified or deamplified themselves. Sidebands on the reflected seed beam will then interfere with the cavity modes at the sideband frequency - giving the magnitude change in the reflected sidebands needed to generate the error signal.

The choice of modulation frequency for the pump phase error signal is therefore a tradeoff between transmission of the sidebands into the cavity and the strength of the sidebands reflected, which determines the electronic strength of the error signal. The upper limit on the modulation frequency is therefore the linewidth of the cavity - 10 Mhz. From a technical standpoint this is convenient as it allows the RF signals for the two different error signals (the OPO cavity and the pump phase) to be separated by analogue filters.

Locking of the pump phase using the reflected seed beam was successfully used in the experiment with two main advantages - the lock is independent of the squeezed beam produced (the goal to start with) and the lock quality is significantly improved, giving less intensity noise on the squeezed beam. A comparison of the intensity noise on the squeezed beam for reflection locking and transmission locking of the pump phase is displayed in figure 6.4.

Figure 6.4: The intensity noise spectra on the transmitted beam from the OPO for the two locking schemes.

The main difference in the noise performance of the two error signal generation schemes is that the sidebands used for transmission locking are transmitted through at least one cavity. This process effectively writes the intensity noise of the first cavity onto the sidebands, which then enters the feedback loop for the pump phase. This is why, as proved with figure 6.4, the reflection locking scheme for the pump phase is superior to transmission locking.
6.3 Locking Detectors

The photodetection process adds its own electronic noise onto the error signal generated, which in turn manifests as noise in the locking loop. Minimizing this problem is equivalent to the process of maximizing the signal to noise ratio produced by the detector. Increasing the signal detected requires proper selection of the detector bandwidth and decreasing the noise involves redesigning the photodetector circuit to use lower noise amplifiers.

The operating bandwidth of a photodetector is the region in frequency space where the detector will give a significant signal output (strictly speaking this is the frequency range where the detector gain is appreciable). Since we want to use RF modulation frequencies then correspondingly we want the detector bandwidth to contain this RF region, approximately 1 MHz to 20 MHz. The bandwidth of the detectors is an even more salient problem for the reflection locking scheme discussed in the previous section as we want a single detector to operate strongly at two different frequencies.

The detector bandwidth is governed primarily by the bandwidth of the operational amplifiers used in the detector circuit and the size of the photodiode used. Smaller photodiodes can readout current from the detector surface, giving them a higher bandwidth. For operation up to 50 MHz, the signal response of 1 mm photodiodes was found to be sufficiently flat, meaning the operational amplifiers in the circuit most strongly affect the effective bandwidth of the detector. Furthermore, the choice of operational amplifier changes the dark noise (electronic noise) of the detector by orders of magnitude.

Three different detectors (each using a different type of amplifier) were compared based on their noise and bandwidth properties to find the optimal detectors for the locking loops in the experiment. The electronic circuit diagram for the detectors used can be found in appendix A.1. The three different amplifiers used to make the different detectors were the LMH6624, the LM7121 and the AD829.

At a basic level the bandwidth of a detector can be investigated by analysing the dark noise spectrum (electronic noise) of the detector. The combination of the amplifier bandwidth and analogue electronics in the detector creates an under-damped resonant circuit. Below the resonance in the circuit a detector will have a strong response to signals (desired for making error signals) whereas above resonance the signal response of the detector falls off rapidly. Therefore a measurement of the resonance point of the detector circuit gives an indication of the bandwidth of the detector. This resonance point can be determined simply by inspection of the dark noise spectrum of a detector, as the dark noise closely follows the resonant characteristic of the detector circuit for low noise amplifiers. The dark noise spectra for the three detectors considered are displayed in figure 6.5.

Figure 6.5 illustrates firstly that the approximate bandwidth of the detectors with the LMH6624 and LM7121 amplifiers is 25 MHz while the bandwidth of the detector with the AD829 amplifier is only 5 MHz. Furthermore, we can see that the detector with the LMH6624 amplifier has significantly reduced electronic noise (up to 10 dB) below 10 MHz over the detector with the LM7121 amplifier. This plot immediately indicates that the LMH6624 is the most suitable amplifier for the wide bandwidth, low noise detectors that we want to use for locking the control loops in the experiment. Next we want to confirm this initial result by comparing the actual error signals generated by the three different detectors through measurements of the signal to noise ratios.

The most crucial requirement of the locking detector design is that the detector measuring the field reflected from the OPO is capable of generating an error signal from both 1 MHz and 16 MHz sidebands. The three detectors are therefore compared in their capacity
to meet this requirement. The signal to noise ratio of the error signal generated from the 1 MHz and 16 MHz sidebands is measured by dividing the voltage span of the linear section of the error signal by the voltage span of the noise (peak to peak value). Traces of the error signals measured can be seen in figure 6.6. While the acquisition device used to record these error signal traces suffers from digitization issues with the small voltages present, figure 6.6 provides a clear indication of the differences between the detectors for locking purposes. A more precise measurement of the signal to noise ratios for the linear section of the error signals was made using a high speed, higher resolution oscilloscope. The results are displayed in table 6.2.

<table>
<thead>
<tr>
<th>Detector Amplifier</th>
<th>Locking Loop</th>
<th>Signal (mV)</th>
<th>p-p Noise (mV)</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMH6624</td>
<td>OPO</td>
<td>46 ± 10</td>
<td>12 ± 2</td>
<td>3.8 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>Pump Phase</td>
<td>45 ± 10</td>
<td>11 ± 1</td>
<td>4.1 ± 0.2</td>
</tr>
<tr>
<td>LM7121</td>
<td>OPO</td>
<td>13 ± 3</td>
<td>9 ± 1</td>
<td>1.4 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>Pump Phase</td>
<td>35 ± 13</td>
<td>20 ± 1</td>
<td>1.8 ± 0.4</td>
</tr>
<tr>
<td>AD829</td>
<td>OPO</td>
<td>5 ± 1</td>
<td>2 ± 1</td>
<td>3 ± 1</td>
</tr>
<tr>
<td></td>
<td>Pump Phase</td>
<td>20 ± 5</td>
<td>22 ± 2</td>
<td>0.9 ± 0.3</td>
</tr>
</tbody>
</table>

*Table 6.2:* Signal, noise and signal to noise ratio measurements of error signals for three detectors with different amplifiers. Error estimates on the signal measurements are upper bounds determined by the noise present. Error estimates on the noise arise from the division limit on the oscilloscope.

From figure 6.6 and table 6.2 it is clear that the LMH6624 amplifier produces the detector with the best bandwidth and noise performance for the experiment. Therefore, this detector is used in the three new control loops implemented in the experiment.
Figure 6.6: Error signals generated by three detectors with different operational amplifiers. The OPO error signal uses 16 MHz sidebands while the pump uses 1 MHz sidebands. Clearly the LMH6624 amplifier gives the best signal to noise ratio over the widest frequency range. The time axis is proportional to the length of the cavities (or phase of the pump) as the PZT control elements were scanned in time at 100 Hz. This frequency was required to avoid low frequency mechanical noise distorting the signal.

6.4 Improved Spatial Mode Selection

The final major technical development made in the experiment concerns the method by which squeezed higher order modes are produced. The experiment developed by Lassen et al. [18] misaligned the OPO cavity in order to produce a squeezed higher order spatial mode. This in turn required the local oscillator be produced by misaligning another, separate cavity. While the OPO locking system is stable when the OPO cavity is well aligned, misalignment degrades the signal to noise quality of the error signal - resulting in a lock which is more noisy and less stable.

A powerful improvement to this method of producing squeezed higher order modes is to misalign the mode cleaning cavity (MCC) immediately after the laser, instead of both the OPO cavity and the mode transfer cavity, in order to produce the higher order spatial mode. The major reason why this is such an improvement relies upon the simple observation that the signal to noise ratio of the MCC error signal is much higher than that of the OPO cavity. Reducing the error signal strength by a factor of three for the MCC gives an error signal that is still very well separated from the electronic noise. A similar reduction in the error signal of the OPO cavity brings it relatively much closer to the electronic noise floor, making the MCC the better choice for the mode selection device from the perspective of feedback loop noise.
The other significant advantages in this method of spatial mode selection are as follows

- **Purity of Spatial Mode** - Using a high finesse cavity, such as the mode transfer cavity, means that the overlap in frequency space of different spatial modes is minimized - thereby giving a high purity spatial mode as the transmitted beam. This is because a lossy cavity will transmit power off resonance depending on the finesse of the cavity. Even though different spatial modes are separated in frequency space in the MTC, the small transmission of the other spatial modes contaminates the mode purity. This effect is significantly worse when misaligning the OPO cavity as it has a much lower finesse than the MTC (by a factor of 8).

- **Definition of spatial mode axes** - The Hermite-Gauss modes display rectilinear symmetry, but the axes on which this is based is defined in the experiment. If the spatial modes are produced by misalignment of a cavity then these axes are defined by the cavity path. A ring cavity is machined so that the optical path is parallel to the table - meaning misalignment of the MTC produces higher order spatial modes with an x-axis that is parallel to the table. By contrast, the OPO cavity is linear which means that the output mode axes are often not parallel to the table - reducing the interference of the local oscillator with the squeezed beam.

- **Increased Local Oscillator Power** - The design of the mode transfer cavities allows significantly more power to be coupled into higher order spatial modes than the OPO cavity does. By only misaligning one MTC and not the OPO cavity, the net optical power available in the experiment is increased - allowing for a stronger local oscillator.

- **Conservation of Control Loops** - The old scheme of generating higher order spatial modes used a mode cleaning cavity to directly filter the laser output and then an additional mode transfer cavity to produce the higher order spatial mode. In the new scheme only one MTC is required - meaning one less control loop is needed to run the experiment. This makes the experiment more stable, allowing superior measurements to be collected.
Results

This chapter details the results measured of the noise spectrum of an OPO produced squeezed state, as well as the method of obtaining these results. The full specifications of the experiment used in obtaining these results are presented, as well as the method by which this experiment was used. The data obtained by the experiment is displayed and the subsequent data analysis is explained. Finally, the complete, analysed results are presented and compared to the theoretical simulations developed in chapter 4.

7.1 Experimental Method

This section describes the full details of the final state of the experiment - where it was used to take the results presented later in this chapter. The details presented can be compared against chapter 5, which described the basic experimental setup for measuring the spectrum of a squeezed state and chapter 6 which then described the technical contributions made in improving this basic setup from the existing technical methodology.

7.1.1 Experimental Setup

A diagram of the experiment is displayed in figure 7.1, providing a schematic layout of how the pump and seed beam interact with the OPO and the detector. Figure 7.1 can be compared with the corresponding figure at the end of chapter 5 to highlight the developments described in chapter 6. A complete diagram of the experiment is given in appendix A.4, which includes all optical and electronic components used.

The following is a list of the important details in the experiment:

- The laser used is a diode-pumped Nd:YAG laser with an internal frequency doubling feature (second harmonic generation) using a portion of the infrared beam to produce the 532 nm pump beam. Since the pump is produced using an SHG then no external phase locking between pump and seed needs to be performed.

- Squeezed light is produced using a cavity enhanced Optical Parametric Oscillator (OPO) below threshold. The OPO is seeded with a laser at 1064 nm and pumped at 532 nm.

- The spatial mode of the squeezed beam is selected by misaligning the seed beam into a mode transfer cavity (MTC). The MTC is locked on resonance using the Pound-Drever-Hall technique. The modulation sidebands for locking the MTC are generated internally by the laser at 12 MHz using phase modulation from an electro-optic modulator (EOM).
• The non-linear crystal used in the OPO is bulk LiNbO$_3$ which is 7% doped with MgO. Phase matching is performed by tuning the absolute temperature of the crystal around 60°C. While the absolute temperature of the crystal is not well known beyond a precision of ±1°C (due to calibration of the temperature sensor), the relative temperature of the crystal was held to within 0.01°C of a set point to maintain the phase matching condition. The phase matching temperature was different for the TEM$_{00}$ and TEM$_{10}$ squeezed beams [18].

• One surface of the crystal has a radius of curvature of 8 mm and is coated to be a high reflector for 532 nm and reflect 99.9% of 1064 nm. The other surface is anti-reflection coated for both 532 nm and 1064 nm, with a radius of curvature of 100 mm.

• The OPO cavity is linear, and is formed by the high reflecting surface of the crystal (8 mm radius of curvature) and a 96% reflecting mirror, with a 75 mm radius of curvature. Combined with the high index of refraction of the OPO crystal the effective optical path length of the OPO cavity is approximately 90 mm.

• The beam waist inside the cavity is located close to the 8 mm radius of curvature mirror, inside the crystal. This is to ensure optimal pump intensity for the non-linear effect. This is also the reason for the curvature of the anti-reflecting side of the crystal - the matching of the crystal surface to the radius of curvature of the beam at that point. Note that figure 7.1 does not display the OPO to scale, it is much longer relative to the crystal - the figure is a schematic only.

• The OPO cavity is backseeded with the mode selected by the mode transfer cavity and locked using the Pound-Drever-Hall technique. The modulation sidebands for locking the cavity at 16 MHz are generated by an electro-optic modulator (EOM). The phase of the pump is locked to the phase of the seed also with Pound-Drever-Hall locking, using modulation sidebands at 1 MHz generated by the same EOM as the 16 MHz sidebands. Detecting the reflected signal from the OPO cavity is done...
by using a 90/10 beamsplitter in the path of the seed beam, so as to transmit 10% of the reflected beam from the OPO cavity onto a detector. This detector measures the modulation sidebands at both 1 MHz and 16 MHz.

- The OPO is pumped with the second harmonic of the seed beam at 532 nm in the TEM$_{00}$ spatial mode. This pumping scheme is optimal when generating a TEM$_{10}$ squeezed beam but results in reduced squeezing in comparison for TEM$_{10}$ [18]. The phase of the pump is locked in quadrature with the phase of the seed, thereby de-amplifying the infrared field in the OPO, producing amplitude squeezed light. The squeezed beam is separated from the pump by using a dichroic mirror (DM).

- The phase of the local oscillator is adjusted by changing the voltage across PZT-4, which in turn moves the mirror it is attached to. This changes the path length difference of the local oscillator and squeezed beam, which changes the relative phase between them.

- The control element used to lock the OPO cavity on resonance was PZT-2, attached to the output coupler. The mode transfer cavity was locked using PZT-1 and the phase of the pump beam was locked using PZT-3, attached to a mirror in the pump beam path.

- The squeezed light was detected directly with a single detector after being overlapped with a strong local oscillator on a 90/10 beamsplitter.

- Two detectors were used in the measurement of the squeezing spectrum - the low frequency detector operated up to 15 MHz and measured the zeroth FSR data. This detector used a 500 $\mu$m diameter InGaAs (Indium Gallium Arsenide) photodiode with AD829 operational amplifiers. The high frequency detector measured the first, second and third free spectral range squeezing data between 1 GHz and 6 GHz. This detector is described in ref. [23].

- The detectors used for locking the mode transfer cavity, the OPO cavity and the pump phase used 1 mm InGaAs photodiodes with LMH6624 operational amplifiers.

7.1.2 Measurement Technique

The geometry of the OPO cavity resulted in a threshold power for a TEM$_{00}$ squeezed beam of 200 mW. Production of the squeezed beam required the OPO to be operated below this threshold, as discussed in chapter 3. The pump power was selected to give optimal squeezing experimentally by increasing the power until the squeezed beam became noisy, then reducing it just enough to eliminate this excess noise. The pump power used was 60 mW when producing the TEM$_{00}$ squeezed beam, which de-amplified the seed to a factor of 0.3 of its original power.

The same procedure of determining the pump power was used in the case of the TEM$_{10}$ squeezed beam. Threshold was 350 mW of pump power, and 110 mW was used. The seed was de-amplified by a factor of 0.45 of its original intensity. The OPO was seeded with 5 mW of power when producing both TEM$_{00}$ and TEM$_{10}$ squeezed beams. The local oscillator power after the 90/10 beamsplitter was 7 mW for TEM$_{00}$, and 4.2 mW for TEM$_{10}$.

The bandwidth of the high frequency detector was sufficient to measure the noise properties of the beam between 1.5 GHz and 6 GHz. The OPO cavity free spectral range
is approximately 1.7 GHz, allowing the detector to measure the presence of squeezing at
the first, second and third free spectral range. The zeroth free spectral range measurements
were made using a detector with a 500 µm photodiode and a total detector bandwidth of
15 MHz.

At each free spectral range two sets of data were collected. The first looks at a 30
MHz span centred on the multiple of the free spectral range and contains the spectra of
both squeezing and anti-squeezing. The second data set is a zero span measurement at
the frequency displaying maximal squeezing from the wide span trace. The zero span
measurement is a two second sweep that looks at the noise properties of the beam as the
phase of the local oscillator is swept at a frequency of below one Hertz.

The sweeping of the local oscillator involves scanning the phase forwards and backwards
based on a periodic sawtooth signal at 0.4 Hz. This sweeping method leads to flyback
issues in the zero span data, which is where the sawtooth signal reaches an extreme point
and changes direction. Zero span data with this issue display symmetry about this point.
The phase of the local oscillator must be swept in this manner as the travel range of the
PZT used to adjust the local oscillator phase is limited. Flyback issues were avoided as
much as possible - with the only significant case occurring in the data for the first FSR
zero span measurement of the TEM\textsubscript{00} squeezed beam.

Ultimately the aim of the experiment is to demonstrate a photon field with a detected
noise power that is below the quantum noise limit (the QNL). The measurements recorded
therefore require calibration to this QNL for a true comparison. This calibration process
can be thought of most clearly as replacing the squeezed beam with a coherent state -
thereby measuring the classical noise floor (the QNL). In the single detection scheme,
power from the squeezed beam and local oscillator is lost out of the fourth port of the
combining beamsplitter. Since the squeezed beam intensity is not negligible relative to
the local oscillator then changing the relative phase between these beams will change the
number of photons lost out this dark port. The result of this is a change in the quantum
noise limit with the local oscillator phase.

Since we want to determine the squeezing of a state relative to the QNL and the
anti-squeezing of a state (also relative to the QNL) then two calibration measurements of
this QNL are required, one for squeezing and one for anti-squeezing. Since it is difficult
to accurately replace the squeezed beam with a coherent state of the same intensity, the
QNL measurements are made by blocking the squeezed beam and simulating the correct
intensity by attenuating the local oscillator. The correct intensity is determined by the
type of calibration required, either squeezing or anti-squeezing.

All squeezing measurements were made with the local oscillator π out of phase from the
squeezed beam. In this case the QNL, referred to as shot noise (squeezing), is measured
by recording the DC component of the detector photocurrent when the squeezed beam
destructively interferes with the local oscillator. The squeezed beam is then blocked and
the local oscillator attenuated until the same DC photocurrent is obtained. Shot noise
(squeezing) is then measured. The DC photocurrent comparison is accurate to within 1%
(since an oscilloscope was used), which means the shot noise (squeezing) measurement is
accurate to within 0.05 dB of the true value.

Anti-squeezing measurements are made when the phase of the local oscillator is in
quadrature with the squeezed beam. In this case the QNL measurement, referred to as
shot noise (anti-squeezing), is made by simply blocking the squeezed beam and recording
the noise spectrum. This is accurate also to within 0.05 dB of the true value, assuming a
100:1 ratio of local oscillator power to squeezed beam power - which was easily the case
in the experiment.

7.2 Results

This section presents the main results of this thesis as a solution to the aim of measuring the high frequency noise spectrum of a squeezed state. The complete evolution of the data is presented - starting with the data immediately obtained from the experiment, then moving to the analysis of this initial data and finishes with the display of the complete set of results, together with a comparison to the theoretical simulations.

7.2.1 Initial Data

Between the LF and HF detectors, squeezing was observed around four frequency sidebands - DC and the 1st, 2nd and 3rd free spectral range multiples. Initially a spectrum spanning the entire bandwidth of the HF detector (1-6 GHz) was studied and no squeezing was observed between the multiples of the FSR. The power spectrum of the squeezing around each of these four frequencies was investigated by measuring, for each FSR, the following set of traces:

- Five wide-span traces centered on the frequency where maximum squeezing was observed. The five traces contain the following information:
  - Dark noise: This provides a measure of the electronic noise of the detector, which is later subtracted from the total noise spectrum to give the optical noise only. This trace was averaged 100 times in the spectrum analyser.
  - Shot noise (anti-squeezing): This determines the quantum noise limit which is needed to calibrate the optical noise measurements in order to measure anti-squeezing. This trace was averaged 100 times in the spectrum analyser.
  - Shot noise (squeezing): This is the shot noise calibration for when the squeezed beam is out of phase with the local oscillator, which is needed as a reference to measure squeezing. This trace was averaged 100 times in the spectrum analyser.
  - Squeezing: This measures the optical noise spectrum about the central frequency when the squeezed beam is held $\pi$ out of phase with the local oscillator, giving the squeezing spectrum.
  - Anti-Squeezing: This measures the optical noise spectrum about the central frequency when the squeezed beam is held $\pi/2$ out of phase with the local oscillator, giving the anti-squeezing spectrum.

- Four zero-span traces measuring the noise power at the selected frequency over a two second sweep. The phase of the local oscillator is swept back and forth relative to the squeezed beam at a frequency of 0.4 Hz for all zero-span traces. The traces contain the following information:
  - Dark noise: This provides a measure of the electronic noise of the detector, which is later subtracted from the total noise spectrum to give the optical noise only. This trace was averaged 15 times in the spectrum analyser.
  - Shot noise (anti-squeezing): This determines the quantum noise limit which is needed to calibrate the optical noise measurements in order to measure anti-squeezing. This trace was averaged 15 times in the spectrum analyser.
– Shot noise (squeezing): This is the shot noise calibration for when the squeezed beam is out of phase with the local oscillator, which is needed as a reference to measure squeezing. This trace was averaged 15 times in the spectrum analyser.

– 'M'-Trace: This measures the optical noise spectrum as a function of the phase of the local oscillator, graphically demonstrating squeezing and anti-squeezing.

Two examples, one of each of the above sets, of the initial data collected directly from the spectrum analyser are displayed in figure 7.2.

![Figure 7.2: Examples of the data traces directly obtained from the spectrum analyser before analysis. Left: Wide span traces about the first FSR for TEM\(_{00}\). Right: Zero span traces at 1706 MHz for TEM\(_{00}\). The zero span 'M' trace displays an example of flyback - where the periodic signal which scans local oscillator phase reached an extremum and reversed direction, mirroring the data.](image)

### 7.2.2 Data Analysis

In order to satisfy the aim of measuring the squeezing spectrum we need to transform the initial results from the spectrum analyser into a normalized variance measurement of the squeezed state, relative to the quantum noise limit.

The first step in analysing the data to this end is to decouple the optical noise information in the measurements from the extraneous information introduced by the detector. The detector introduces two terms into each of the optical measurements - the first is dark noise and the second is a transfer function, describing the frequency response of the detector. In linear space, decoupling these two terms from the optical noise requires two processes
First, every wide span trace (dark noise, shot noise, squeezing etc.) needs to be divided by the transfer function of the detector.

Finally, the transfer function corrected traces containing optical information (shot noise, squeezing, etc.) have the transfer function corrected dark noise power at each frequency subtracted from them.

The final step is then to divide the squeezing and anti-squeezing traces by the corresponding quantum noise limit measurements to obtain the normalized variances of the state. The procedure used to analyse the data, achieving the three separate steps mentioned, was

1. Subtract, in linear space, the dark noise power at each frequency from all other traces.

2. Subtract, in logarithmic space, the dark noise corrected squeezing and anti-squeezing traces from the dark noise corrected quantum noise limit measurements.

Calibration of the zero span measurements to the true quantum noise limit is more difficult, as the measurements record the noise spectrum over the full $2\pi$ phase shift of the local oscillator relative to the squeezed beam. The quantum noise limit is measured at only two points over the $2\pi$ span. Therefore the zero span measurements are calibrated relative to the mean of shot noise (squeezing), while shot noise (anti-squeezing) is similarly calibrated and displayed to provide a reference for anti-squeezing. This allows for true calibration of the squeezing and anti-squeezing measurements on the zero span figures, while the measurements at a phase between squeezing and anti-squeezing need to be calibrated to a point between shot noise (squeezing) and shot noise (anti-squeezing).

From figure 7.2 it is clear that the precision of the measurements is limited, as indicated by the sharp differences in the noise power measurements between local areas of the traces (in rough terms, the 'fuzziness' of the plots). This is an inherent property of the spectrum analyser used to record the traces [24], [25], as opposed to the variance of the squeezed state changing. Indeed, this local deviation provides an easy visual indication of the uncertainty in the measurements, as this local deviation in the squeezing and anti-squeezing measurements are much larger than other random errors present. This is also true of the shot noise measurements, which were found earlier to have an accuracy uncertainty of 0.05 dB, a term dominated by the uncertainty from the spectrum analyser.

7.2.3 Final Results

Figures 7.3, 7.4, 7.5 and 7.6 display the wide span and zero span data for spatial modes TEM$_{00}$ and TEM$_{10}$. The analysis performed on the data before presentation is as described in the previous section.

In all results the resolution bandwidth was 300 kHz for the low frequency measurements (zeroth FSR) and 1 MHz for the high frequency measurements. The video bandwidth was 300 Hz for the zeroth FSR and 1 kHz for the high frequency measurements.

The wide span measurements (figures 7.3 and 7.5) are overlaid with simulated results based on the theory in chapter 4. The simulations were fitted to the measured results using three parameters:
• $\tau$ - The OPO cavity round trip time determines the frequency spacing (the FSR) between adjacent maxima in the squeezing spectrum. Changing this parameter aligns the peak of the simulated squeezing spectrum with the peak of the measured spectrum.

• $\chi$ - The non-linearity parameter determines the magnitude of the squeezing produced by the OPO (assuming no losses). $\chi$ is expressed as a fraction of the the threshold value of the non-linearity $\chi_{\text{threshold}}$, which is equal to the total decay rate of the OPO cavity $\kappa$.

• Total Loss - Any loss affecting the squeezed beam, whether optical or detection related, will reduce the magnitude of the squeezing and anti-squeezing of the beam as per the beamsplitter equation \[2\]. Furthermore, loss will increase the uncertainty product of the quadratures - meaning the squeezing will degrade more than the anti-squeezing.

We can estimate a lower bound for the total loss in the system by multiplying the transmission factors from the known losses. Estimates for the known losses are listed below:

- $(23\pm3)\%$ for the intracavity losses of the OPO (loss from escape efficiency)
- $(6\pm1)\%$ due to optical losses between the OPO and the combining beamsplitter
- $(8\pm2)\%$ due to the quantum efficiency of the detector
- $(7\pm2)\%$ due to non-ideal overlap of the local oscillator with the squeezed beam
- $(10\pm0.1)\%$ from the loss of squeezing at the combining beamsplitter

This gives a lower bound on the total system loss of $(44\pm19)\%$.

A phenomenon observed in the experiment was the increased asymmetry between the magnitude of squeezing and anti-squeezing as the pump power was increased. This phenomenon can be modeled as a loss term that increases with pump power. An explanation of this that is currently popular is that the green pump induces infrared absorption in the non-linear crystal \[26\]. As this work is not concerned with this phenomenon itself we simply acknowledge that the loss required for fitting the theoretical results to the measured results will be larger than predicted from the known losses.

Table 7.1 displays the parameters used to fit the simulations to the measured data in figures 7.3, 7.4, 7.5 and 7.6.
### Table 7.1: Parameters used to fit the theoretical simulations to the measured results.

<table>
<thead>
<tr>
<th>Spatial Mode</th>
<th>Frequency</th>
<th>$\tau$ (GHz$^{-1}$)</th>
<th>$\chi$</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEM$_{00}$</td>
<td>0 × (FSR)</td>
<td>0.5</td>
<td>0.32</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>1 × (FSR)</td>
<td>0.586</td>
<td>0.32</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>2 × (FSR)</td>
<td>0.58337</td>
<td>0.32</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>3 × (FSR)</td>
<td>0.58111</td>
<td>0.325</td>
<td>0.57</td>
</tr>
<tr>
<td>TEM$_{10}$</td>
<td>0 × (FSR)</td>
<td>0.55</td>
<td>0.17</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>1 × (FSR)</td>
<td>0.5844</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>2 × (FSR)</td>
<td>0.58295</td>
<td>0.19</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>3 × (FSR)</td>
<td>0.58117</td>
<td>0.15</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Figure 7.3: TEM$_{00}$ widespan results of squeezing and anti-squeezing, normalised to the quantum noise limit, measured at four frequency sidebands of the squeezed beam - zero, one two and three multiples of the cavity FSR. Theoretical predictions (smooth curves) are overlaid on the measurements.
§7.2 Results

Figure 7.4: TEM$_{00}$ zerospan results, normalised to the quantum noise limit for the amplitude quadrature, measured at four frequency sidebands of the squeezed beam - zero, one two and three multiples of the cavity FSR. The plots display the noise power as the local oscillator phase is swept over a two second period. The shot noise level for the phase quadrature is also displayed in the figures.
Figure 7.5: TEM$_{10}$ widespan results of squeezing and anti-squeezing, normalised to the quantum noise limit, measured at four frequency sidebands of the squeezed beam - zero, one two and three multiples of the cavity FSR. Theoretical predictions (smooth curves) are overlaid on the measurements.
Figure 7.6: TEM$_{10}$ zerospan results, normalised to the quantum noise limit for the amplitude quadrature, measured at four frequency sidebands of the squeezed beam - zero, one two and three multiples of the cavity FSR. The plots display the noise power as the local oscillator phase is swept over a two second period. The shot noise level for the phase quadrature is also displayed in the figures.
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We have measured non-classical noise reduction on an optical beam at microwave frequency sidebands produced by an optical parametric oscillator. This is the first measurement of noise reduction in a squeezed state at such high frequencies, the highest measured being 5.1 GHz. The measurements also proved that the observed squeezing spectrum mirrors the classical intensity transmission spectrum of the cavity enhancing the OPO. This was evidenced by observations of squeezing at the zeroth, first, second and third integer multiples of the cavity free spectral range. Not only was the microwave frequency spectrum of squeezing measured for the fundamental Gaussian spatial mode, but the corresponding spectrum for a squeezed higher order spatial mode (TEM$_{10}$) was measured also.

A detailed theoretical model of the optical parametric oscillator was presented in addition to the measurements of squeezing at microwave frequency sidebands. Computer simulations of the predictions from this model were developed and compared directly with the experimental measurements, showing an excellent agreement. This confirms the suitability of the physically simple model used to obtain the theoretical predictions. The model made the single spatial mode assumption, and therefore was not able to compare the squeezing between different spatial modes, but the success of the model was in predicting the squeezing present at different frequencies for a particular spatial mode.

A comprehensive review of the quantum optics literature is performed in this work so as to draw together the important concepts required to understand squeezed light, and the reasons why such a state is indeed non-classical. Furthermore, a detailed explanation of parametric amplification is laid out, leading to the conclusion of how optical parametric amplification can produce a squeezed state.

In addition to the novel physics explored in this thesis, significant technical developments were made in successfully operating the experiment used to perform the squeezing measurements. The four major improvements were

1. Three new feedback controllers were implemented in the experiment to keep the optical cavities on resonance and the phase of the pump beam in quadrature with the seed beam in the OPO. The round trip loop gains are measured and displayed for these control loops which, in addition to transmitted noise measurements, demonstrated the success of these control loops in technical noise suppression.

2. A new method of locking the phase of the pump beam to the phase of the seed beam was developed that was independent of the squeezed beam. Not only did this method alleviate the requirement on the squeezing detector of locking the pump phase but also decreased the noise present in the control loop.
3. A new design of photodetectors used to generate the error signal for the control loops in the experiment was implemented. The design improvement was in the operational amplifiers used in the detector circuit, which were changed to give the detectors a larger signal bandwidth and a superior signal to noise ratio.

4. An improved method of selecting the spatial mode of the squeezed beam was developed, which resulted in a more stable experiment with less losses in the squeezed beam path. The latter benefit results from the superior mode overlap of the squeezed beam and the local oscillator. The results measured using this improved method display more squeezing and have increased reliability.

8.1 Discussion of Results

The conclusions arrived at in this thesis are based on experimental results, which inevitably contain sources of error. This section will discuss the accuracy and precision of the results and how this affects the validity of the conclusions drawn in this chapter. Additionally, minor anomalies in the measurements will be identified and their causes hypothesized. The three minor anomalies in the results that are discussed are the excess phase noise at the zeroth FSR, the different fitting quality of squeezing versus anti-squeezing and the difference in frequency spacing of the observed squeezing maxima.

There exist two dominant sources of experimental uncertainty in the results obtained - a systematic error due to non-linear electronic response and a random error introduced by the spectrum analyser. Non-linear electronic response results when an amplifier is forced to deal with signals of very different relative strengths. The amplifier responds linearly to the strong signal, but not for the weak signal. In this experiment the noise spectrum is the quantity of interest - which is much weaker than modulation signals encoded on the beam (used in PDH locking for example). A non-linear response of the electronics in amplifying noise can lead to an offset in the measurement from the true value. This systematic error may be an issue at low frequency, as there are strong modulation signals within the detector bandwidth. At high frequency, though, there are no strong modulation signals present within the detector bandwidth - so this systematic error is not an issue for the high frequency results.

The random error introduced by the spectrum analyser is the largest source of statistical uncertainty in the experiment. This uncertainty results, to a large extent, from the fact that the spectrum analyser used was not designed to measure noise - but rather signal. Nonetheless, the absolute optical squeezing noise measurements were calibrated to the shot noise measurements. This means that we can say with confidence that the normalized optical variance is very close to the mean of the measured traces, within the spectrum analyser uncertainty. This uncertainty is the width of the 'fuzziness' of a trace, the standard deviation of the measured noise from the mean noise in a local area of the trace. Visually, this uncertainty is easily identified on the traces - providing a natural indication of the random error size of the variance measurement.

In light of this error analysis, it is clear that the strongest correlation of the theoretical simulations with measured results comes from the anti-squeezing results of the TEM$_{00}$ spatial mode, as these have the lowest uncertainty. Even though the uncertainty on the squeezing measurements is not sufficiently low to confirm or deny the exact shape of the squeezing spectrum about resonance, we can still draw important conclusions from these results.
• Squeezing was observed at sidebands significantly larger the linewidth of the OPO cavity.

• The maximum squeezing occurs at the same frequency as maximum anti-squeezing.

The first point mentioned above in important as this is a key conclusion of this work - that we measured, beyond experimental uncertainty, a state with squeezing at sideband frequencies well outside the cavity linewidth.

It was concluded that the theoretical simulations were in excellent agreement with the measured results. While it is clear from chapter 7 that the simulations can be made to overlay the data very well, this conclusion requires stronger support than this. In particular, consistency between the fitting parameters is required. Table 7.1 illustrates how the loss used to fit the simulations to the results was constant for all three high frequency measurements with the two spatial modes. Additionally, the non-linearity parameter $\chi$ was also approximately constant, especially for the TEM$_{00}$ results. While the low frequency measurements required a different loss parameter this is not anomalous, as separate detectors, with different efficiencies were used. The consistency of the two parameters of loss and non-linearity for the simulations provides the real strength in the evidence supporting the conclusion of the agreement between the theory and the experiment.

8.1.1 Excess Noise at Low Frequency

At low frequency (the zeroth FSR) it is clear from the widespan results (7.4 and 7.6) that excess noise is present in the phase quadrature of the squeezed beam. The noise is most likely not quantum in nature as it exceeds quite liberal theoretical fitting parameters. Additionally, this frequency range is where many previous squeezing measurements have obtained states without this excess noise.

The most likely cause of this excess noise is the close proximity of the three phase modulation signals - 1 MHz, 12 MHz and 16 MHz. These modulation signals are measured to be up to 80 dB higher than shot noise. This large difference in the power between the signal frequencies and the frequencies of interest (where we want to see squeezing) can cause the operational amplifiers inside the detector to operate in some non-linear fashion. This can result in systematic error (a measurement of excess noise) as discussed previously. This is the manifestation of the finite dynamic range of the amplifiers, which is the maximum relative power difference of two signals which will both still be amplified linearly.

Even though this is anomalous with respect to the derived theory, it does not detract from the conclusion drawn that squeezing was observed at higher multiples of the FSR, where no excess noise was observed. It is a safe assumption that the high frequency detector was not affected by this systematic error as there were no strong modulation signals in the microwave frequency range.

8.1.2 Theoretical Fitting of Squeezing Versus Antisqueezing Results

The precision of the fitting of theoretical simulations to the measured widespan results determines to a large extent the experimental verification of the derivation of the theory. Since these fits were, for the high frequency results, very good then it was concluded that the experimental results provided excellent verification of the theory. One uniform discrepancy in this fitting process was that the anti-squeezing measurements fitted much more closely the simulations that the squeezing results at high frequency.
The maximum squeezing of a state is measured when the local oscillator phase is exactly in phase or $\pi$ out of phase. For a local oscillator phase close to these points, squeezing will be measured but not at the same magnitude as at the maximum squeezing point. The observed squeezing reduces rapidly when the phase deviates from the optimal squeezing point, whereas anti-squeezing reduces more slowly as the phase changes. This fact is exactly reflected in the zero-span measurements where the phase of the local oscillator is swept through a full $2\pi$.

It is clear then, that the anti-squeezing measurements represent much more closely the full anti-squeezing spectrum of the state than the squeezing measurements do the full squeezing spectrum. This is due to the human error of recording the traces at the correct local oscillator phase. It is this error in the phase of the local oscillator (not being precisely out of phase) which most likely gives rise to the differences of the squeezing traces from the theoretical simulations.

That we have more certainty in the anti-squeezing measurements is advantageous for the outcome of verifying the theoretical simulations, as the anti-squeezing has a noise profile above the shot noise. At high frequency the shot noise is close to the electronic noise of the detector. Since the anti-squeezing measurements are better separated from the electronic noise than the squeezing measurements then the uncertainty on these measurements is lower. This fact reinforces the conclusion that the theoretical simulations fit the experimental results very well.

### 8.1.3 Frequency Pulling

The theoretical model in chapter 4 predicted the maxima in the magnitude of the observed squeezing to be at exact multiples of the OPO cavity FSR. In particular, this means that the frequency difference between successive FSR multiples should be equal. The frequencies of the measured maxima in the squeezing spectra for TEM$_{00}$ and TEM$_{10}$ are displayed in table 8.1.

<table>
<thead>
<tr>
<th>Spatial Mode</th>
<th>Maxima Number</th>
<th>Frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEM$_{00}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1706$\pm$1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3428$\pm$1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5163$\pm$1</td>
</tr>
<tr>
<td>TEM$_{10}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1711$\pm$1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3431$\pm$1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5162$\pm$1</td>
</tr>
</tbody>
</table>

Table 8.1: The frequencies of the observed maxima in the magnitude of squeezing.

From table 8.1 it is straightforward to calculate the difference in frequencies of adjacent squeezing maxima. These frequency differences are then plotted in figure 8.1.

It is evident from figure 8.1 that the measured results are *frequency pulled* from the theoretical prediction, meaning the difference plot is increasing instead of being flat. This effect is a small offset - a 45 MHz shift on a 1.7 GHz FSR, but how does this small offset relate to the possible causes in the experiment?

Consider the possibility that the frequency pulling is due to the PZT of the OPO cavity locking at a different point - giving a slightly different cavity length. In order to achieve...
Figure 8.1: The difference in frequency between adjacent squeezing maxima. The horizontal axis indicates which maxima had the higher of the two frequencies being subtracted, 1 denotes the first FSR minus the zeroth FSR, 2 denotes the second FSR minus the first FSR and so on.

the 45 MHz shift at the third FSR, the OPO length would have to change by 765 μm. The largest travel length of the OPO PZT is under three microns, meaning the required travel is two orders of magnitude greater than what is possible. It is therefore very unlikely that the frequency pulling is a result of physical length change in the cavity.

Another possible cause of frequency pulling is dispersion in the path length of the cavity. It is well known that the non-linear crystal used in the experiment is a dispersive medium, meaning that there is a real possibility that this is the issue causing frequency pulling. This difficulty in this explanation, though, is that if we assume dispersion to be linear through the seed frequency then the observed squeezing at high frequency should be much lower than squeezing at the zeroth FSR. This is because the resonant mode at frequency $-N \times \text{FSR}$ would be offset differently from the mode at $+N \times \text{FSR}$. Therefore, to achieve a frequency pull outside the linewidth of the cavity (as was observed), significant loss would be introduced in one sideband relative to the other - reducing the observed squeezing.

Presented above are discussions of two possible causes of the frequency pulling observed, but there is not yet a definite solution to this question. The investigation of this issue is one of the interesting future directions for this work presented in section 8.2.

8.2 Future Directions

The most immediate future direction for this research is the further characterisation of squeezing at high frequencies from an OPO. In this work a prototype detector was used and squeezing was only measured three multiples of the FSR away from the carrier beam. While this initial characterisation proved the existence of squeezing at integer multiples of the cavity FSR and its correlation with the theoretical model, there remains significant
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scope in understanding more about OPO squeezed light.

One of the fundamental assumptions in the theory was that the frequency response of the non-linearity generating the squeezing was flat. Therefore the model predicted an infinitely wide comb of squeezing in frequency space. It is obvious that this cannot be the case experimentally, since the optical beam has a finite frequency and the non-linear crystal has a finite phase matching bandwidth. What is not presently known is how this approximation breaks down, if the phase matching bandwidth is the dominant factor or if there are competing terms limiting the squeezing bandwidth that have not been considered previously.

The first step in the further characterisation of high frequency squeezing is the construction of an experiment capable of measuring optical noise at many more integer multiples of the OPO cavity FSR. Achieving this end requires primarily the following changes from the experiment presented in this work

- **Detection** - Firstly, a second matched detector needs to be built and a low noise method of microwave frequency signal subtraction found. This allows for the use of a homodyne measurement in the experiment - improving the available local oscillator power per detector and eliminating the loss of squeezing at the combining beamsplitter. Next, the detector used needs improved clearance of shot noise from the dark noise of the detector. This enables more precise measurements of the sub-shot noise variance of the squeezed beam.

- **Cavity Length** - In order to measure squeezing at many more multiples of the cavity FSR the cavity length needs to be significantly increased. Lengthening the cavity used in this experiment to 90 cm would allow the same detector to measure squeezing at over 30 different multiples of the cavity FSR. Lengthening the OPO cavity presents the immediate difficulty of selecting a suitable cavity geometry with the available mirrors that gives a sufficiently small waist (for a strong non-linearity). While technically challenging, this problem is most certainly able to be overcome using gaussian optics and the control theory described in this work.

Another interesting characterisation of high frequency squeezing would be the comparison of simultaneous measurements of squeezing at different multiples of the FSR. This may not initially seem an interesting question but it has been noted previously that frequency pulling was observed between the different multiples of the FSR, indicating there is a process in the system not properly modeled. Simultaneous measurement is most simply performed by splitting the detector output and using two spectrum analysers. A more interesting method of simultaneous measurement would entail the use of a high-speed digitizer to record directly the detector photocurrent. This can then be Fourier transformed using a computer, providing a direct comparison of squeezing at different frequencies.

One of the truly interesting features of quantum mechanics is the ability to entangle states, a phenomenon that can be achieved with squeezed beams [27], [28]. This process requires at least two squeezed beams (for quadrature entanglement), greatly increasing the complexity of the experiment. The results of this work demonstrate that a single OPO can be used to produce a beam with many different squeezed frequency modes, which can be separated with an optical cavity yielding separate squeezed beams. The complication in using the different beams for an entanglement experiment is that the squeezing is at different frequencies on the different beams. Therefore, one exciting future direction of this work is the development of a means by which squeezing at different frequencies can
be entangled. If this was achieved then a single, stable OPO could be used to implement large squeezing networks as in [29].
Appendices

A.1 Photodiode Circuit Diagram

Figure A.1: Photodiode circuit diagram
A.2 PID Circuit Diagram

Figure A.2: PID controller circuit diagram
A.3 High Voltage Amplifier Circuit Diagram

Figure A.3: High voltage amplifier circuit diagram
A.4 Complete Experimental Setup

Figure A.4: Complete optical and electronic experimental diagram
Appendix B

Glossary

**Bandwidth** In general, bandwidth is the region in frequency space where a system operates properly. An amplifier, for example, has a bandwidth of operation - and signals outside this bandwidth are not amplified or are attenuated.

**Beamsplitter equation** The equation describing the mixing of the variances of the input modes of a beamsplitter to give the output mode variances. This equation describes why squeezed or noisy states come closer to the quantum noise limit when attenuated.

**Coherent state** A quantum model of a photon field that closely approximates lasers available in experiments.

**Dark noise** The total electronic noise of a detection system. The this is described as dark noise as it is the noise measured when a detector has no input (is dark).

**Downconversion** The process of converting one high frequency photon into two lower frequency photons.

**Error signal** An electronic signal (voltage) that is proportional to how far an optical cavity is from resonance. This is required for the feedback loop to lock the cavity on resonance.

**Feedback loop** A type of control loop which uses constant measurement of a system output to keep the system stable to a reference value.

**FSR** Free Spectral Range. The frequency separation between resonant modes of an optical resonator.

**HF** High frequency - above 1 GHz.

**Homodyne Detection** A method of detection whereby a beam of interest is interfered with a local oscillator (at the same frequency) on a beamsplitter. The resulting two beams are detected and the electronic photocurrents subtracted.

**In quadrature** Two waves (optical beams for example) are in quadrature when they are 90 degrees (π/2 radians) out of phase.

**LF** Low Frequency - below 100 MHz.

**Linewidth** The Full Width Half Maximum of a cavity transmission spectrum. This is the spectral width of the resonant lines at half of the maximum transmission in an optical cavity transmission spectrum.
Local oscillator  A strong reference beam, phase locked to some beam of interest, that can be interfered with the beam of interest for detection purposes.

Locking a cavity  The process of keeping a laser locked onto the resonance of a cavity so the transmitted beam has a stable intensity.

Number state  A quantum state of a photon field which contains a precisely defined number of photons in its mode.

OPO (below threshold)/OPA  Optical Parametric Oscillator (below threshold)/Optical Parametric Amplifier. A device which can parametrically amplify an optical field, usually a non-linear crystal coupling a seed field to its second harmonic field.

Optical cavity/Optical resonator  An arrangement of reflecting surfaces in which light can find a closed circuit. The most basic example is two flat mirrors parallel to each other.

Parametric amplification  Phase sensitive amplification. This amplifies one quadrature and de-amplifies the other quadrature.

PID Controller  Proportional-Integral-Derivative controller. A type of feedback controller that is very commonly used in systems around the world.

PZT  Lead Zirconate Titanate. This is a piezoelectric material, meaning it expands mechanically in proportional to a voltage applied to it.

QNL  Quantum Noise limit. This is the noise that occurs on measurements of a laser without any technical noise (a coherent state). Quantum noise originates from uncertainty in the quantum statistics of a laser photon field.

Quadratures  Parameters of an optical beam derived from the amplitude and phase of the beam. The transformation to quadratures is made because the amplitude quadrature and phase quadrature can be directly compared, unlike amplitude and phase.

Shot noise  This refers to the quantum noise of a photon field.

Sidebands  Signal or noise at frequencies offset from a carrier beam by a small amount, symmetrically above and below the carrier in frequency.

TEM_{nm}  The Hermite-Gauss solution too the paraxial Helmholtz equation of order n in one axis and m in the other. This describes the intensity distribution of a laser, known as the spatial mode. These modes have rectilinear symmetry (hence the two indices).

Upconversion  The process of converting two photons of low frequency into one with a high frequency.

Vacuum state  A quantum state describing a photon field with a mean field amplitude of zero. There are a continuum of vacuum modes in space, which can couple into an optical system whenever loss is present, for example with a beamsplitter.

Waist  A waist is the part of an optical beam that has the smallest diameter. This is a property of Gaussian optics.


[17] MATLAB is a registered trademark of The Mathworks, Inc.


[25] for example see hewlett packard spectrum analyser HP B6457 or similar.


