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The Dynamics of Fertility and Growth: a Baby Bounce-Back?*

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ABSTRACT

This paper examines the interplay between economic growth and fertility as an economy moves through two distinct phases: women raising children full time; women entering the work force and raising children part time.

Women's relative wages rise with economic growth, as per Galor and Weil (96). Higher wages make children more affordable. On the other hand, children are more costly when maternal time, used in child rearing, could be supplied to the labor market.

I extend Galor and Weil (96) by introducing goods and services as a child rearing input. A Constant Elasticity of Substitution (CES) production function for child rearing allows for varying degrees of substitutability between goods and time.

The existence of an alternative input to maternal time generates a baby boom-bust cycle: fertility rises in the first phase and falls in the second. Whilst fertility declines unambiguously at the beginning of the second phase, as women enter the labor force, it may bounce-back before reaching steady state as income effects start dominate.

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1 Introduction

There has been a recent surge of theoretical literature linking changes in fertility to economic growth. Despite often acknowledging a non-monotonic interplay as empirically valid, the existing literature predicts a strictly negative relationship between fertility and income *per capita* in developed economies that have progressed beyond the Malthusian poverty trap where subsistence incomes limit fertility.

This literature is motivated by the much-publicized decline in fertility in most OECD countries in the latter half of the 20th century, following the post-war 'baby-boom'. This paper explores reasons why we might expect the 'baby-bust' to be followed by a rise in fertility rates – a 'baby bounce-back' – principally because of the increasing use of child-care goods and services that reduce demands on parental time. This paper also explores the implications of child-care subsidies for the evolution of fertility.

Common to recent endogenous fertility models is the idea that growth in income per capita raises the opportunity cost of time spent raising children. In a developed economy, this effect dominates the increased affordability of children, so that fertility declines. As a slight variation on this theme, Becker, Murphy et al. (1990) and Ehrlich and Lui (1991) introduce a child quantity – quality tradeoff. In this case, fertility declines with economic growth because the opportunity cost of time spent raising children increases relative to the opportunity cost of time spent educating children.

Models endogenising fertility may be differentiated according to how children enter into parental utility. Self-interested parents may view children as a consumption item (Galor and Weil (1996)) or as a source of income support in old age (Ehrlich and Lui (1991; Cigno and Rosati (1996)). Alternatively, parents may be altruistic (Barro and Becker (1988), Becker, Murphy et al. (1990)).

The model of Galor and Weil (1996) has several advantages over its counterparts. Firstly, parents derive direct utility from the number of children, simplifying analysis without altering any empirically valid predictions.[1] Secondly, a rise in female relative wages drives the fertility decline.[2] Intuitively, when children are a consumption item, higher wages have both an income effect and a substitution effect, when labor is used to rear children. These effects work in opposite directions. By construct, the substitution effect dominates. Specifically, if maternal time is the only child rearing input and female wages are a fraction of male wages, fertility unambiguously declines as female relative wages rise. Finally, rising female relative wages is a consequence of economic growth. Women supply only one type of labor that is complementary to capital. Both male and female wages rise over time with the accumulation of capital per worker, but female wages rise relatively more. Combined with the neoclassical

[1] After making the necessary extensions, Day (2001) finds economic growth, via rising female relative wages, generates a fertility decline in all three models of parental utility.

[2] In most developed countries the decline in fertility since 1960 corresponded to a steady rise in the female-male wage ratio. Cigno and Rosati (1996) present empirical evidence for US, UK, Germany and Italy.

capital intensity effect, this final feature generates a feedback loop between growth in output per worker and declining fertility. Growth in capital per worker raises female relative wages, inducing a decline in fertility that, in turn, boosts capital per worker. Thus, Galor and Weil (1996) encapsulate in one model fertility decline as both a symptom and cause of economic growth.

In addition, Galor and Weil (1996) define two phases of demographic development: initially, women are at home full time; in the second phase, women participate in the paid labor force. Fertility declines with growth in income per capita only once women have entered the labor force. Throughout the initial phase, fertility is constant.

Does this predicted negative monotonic relationship between fertility and growth accord with data for a developed economy? Referring to Figure 1, two anomalies present:

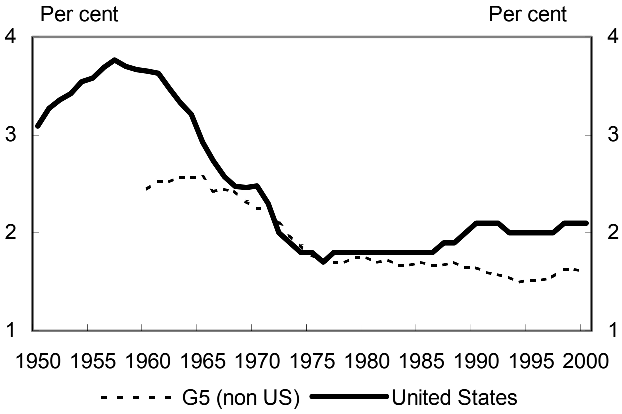
Firstly, fertility decline in leading developed economies (the G5 group) has become less pronounced in the last few decades. Fertility levels have virtually leveled off. In the US, there is evidence of a slight upturn in fertility since 1985, despite the gender wage gap continuing to fall (see Figure 2).

A footnote in Galor and Weil (1996) offers a way of reconciling the model with recent data for developed economies. If goods are also required to raise children, fertility decline due to rising female relative wages may be eradicated if the goods input is sufficiently large or, as restated in Apps and Rees (2001), the time input is sufficiently small. This raises the question: what factors determine the goods-time input mix? The obvious implication is that if goods and services are increasingly substituted for maternal time then, over time, fertility will become less sensitive to rising female relative wages and may even increase. Despite introducing goods as a child rearing input, neither paper is able to investigate the implications of the degree of complementarity between goods and maternal time. Galor and Weil (1996) assume a fixed quantity of goods. Apps and Rees (2001) introduce a generic production function for child rearing. But neither paper analyses the dynamics of fertility in the context of a child-rearing production function.

Secondly, fertility rose over the 1950's – the so-called "baby boom" which is evident in Figure 1. Galor and Weil (1996) predict *constant* fertility throughout a phase when married women allocate all of their time endowment to child rearing. This prediction follows directly from the assumption that a fixed fraction of maternal time is the only input into child rearing. This raises the question, would flexibility over the child rearing input mix alone enable fertility to rise with household income in the theoretical model?

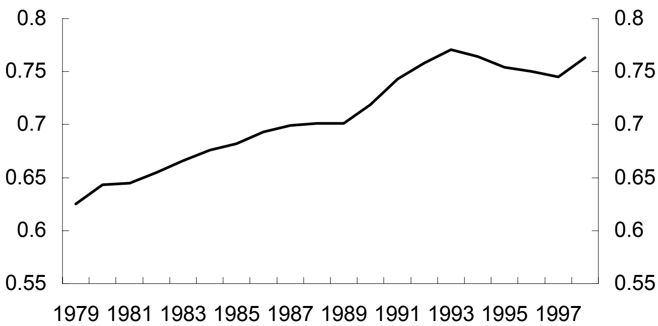
That below replacement fertility is currently observed in all G5 countries bar the United States, has perhaps biased developments in theory in favor of explanations for fertility decline. Just as we would not want theory of the 1960's to have been unduly influenced by the preceding baby boom, we do not want recent experience to blind us to the possibility of a resurgence of fertility in developed economies.

Figure 1: Total Fertility Rate (births per woman)



Source: World Bank Tables; Historical Statistics of the US

Figure 2: Female/Male Earnings Ratio (US)[3]



Source: US Bureau of Labor Statistics (Usual weekly earnings collected in the Current Population Survey)

Previous efforts to model a non-monotonic relationship between fertility and income per capita track an economy through various stages of development. Becker, Murphy et al. (1990), Ehrlich and Lui (1991) and Barro and Sala-i-Martin (1995) predict a rise in fertility at very low levels of capital per person. Kremer (1993) introduces a non-monotonic relationship by assumption. He postulates, but stops short of introducing, extensions to his Malthusian model that would generate the assumed relationship.

Relevant to Figure 1 is a model that tracks an advanced economy's transition through two demographic phases distinguished by female labor force participation, as in Galor and Weil (1996), rather than stages of purely economic development.

This paper presents a model that generates the non-monotonic relationship between fertility and GDP per capita, observed in developed economies over the last few decades. It does so without drawing on outdated Malthusian assumptions. I extend

[3]Since 1970, the gender wage gap has fallen in other G5 countries: see Blau and Kahn (2000). Harkness and Waldfogel (1999) report female-male *hourly* earnings ratios for US and Europe, 1967-1995. Blau and Kahn (2000) also report earnings corrected for differences in weekly hours worked, albeit for a shorter period. The gender gap in wages mimics that in earnings.

Galor and Weil (1996) by introducing goods and services as a child rearing input and allowing the household to choose the optimal goods-time input mix. A Constant Elasticity of Substitution (CES) production function for child rearing allows analysis of the degree of complementarity between goods and time.

2 Basic structure of the model

An overlapping generations model in which people live for three periods describes the economy. People spend the first period of life as children, consuming time, as well as goods and services, from their parents; the second period of life supplying labor to the market and raising children; and the third period in retirement, when they consume the proceeds of their savings from the previous period. The closed economy identity of savings and investment provides the link with growth in productivity and wages.

Production of final output

To generate a gender wage differential, I employ the assumption of Galor and Weil (1996): men and women differ in their wage earning ability because of different labor endowments.

Physical capital (K), physical labor L (L_t^p) and mental labor (L_t^m) are factors of production, all with non-increasing marginal products. The greater the capital-labor ratio in the economy, the more highly rewarded mental labor relative to physical labor. This is consistent with relative rise in rewards to mental labor in developed countries. Intuitively, capital does a better job of replacing human strength than human thinking.

The production function is

$$Y_t = A\left[\alpha K_t^{\rho} + (1-\alpha)(L_t^m)^{\rho}\right]^{1/\rho} + bL_t^p \quad \rho \neq 0 \tag{1}$$

where A>0, b>0, $\alpha \in (0,1)$ and $\rho \in (-\infty,1)$. The separability of the production function captures the assumption that, whereas, capital complements mental labor, physical labor is neither a complement nor a substitute for capital or mental labor.

Since each household supplies one unit of physical labor, the production function, homogeneous of degree 1, in terms of per working age household is given by:

$$y_t = Y_t / L_t^p = A\left[\alpha k_t^{\rho} + (1-\alpha)m_t^{\ \rho}\right]^{1/\rho} + b \tag{2}$$

where k_t and m_t are the per household supplies of capital and mental labor, respectively. Men supply inelastically 1 unit of physical labor and 1 unit of mental labor together. Women supply between 0 and 1 units of mental labor, so mental labor per working age household takes values: $1<m_t<2$., where (m_t-1) measures female labor force participation.[4]

Profit maximization and competitive markets imply:

[4] The crucial assumption here is that men and women are equally endowed with "brains" but only men have "brawn".

$$w_t^m = A(1-\alpha)m_t^{\rho-1}\left[\alpha k_t^{\rho} + (1-\alpha)m_t^{\rho}\right]^{1-\rho/\rho} \tag{3}$$

$$w_t^p = b \tag{4}$$

An increase in capital intensity will therefore raise the wage[5] for mental labor (w_t^m) while the wage for physical labor (w_t^p) is constant. Men earn a wage of $w_t^m + w_t^p$; women earn a wage of w_t^m. It follows that growth in the capital stock over time will increase female wages proportionately more than male wages, thus reducing the gender wage gap.

Household optimization

Households derive utility directly from the number of children. Children are essentially a durable good. There is no bequest motive.

The household utility function is:

$$u_t = \gamma \ln(n_t) + (1-\gamma)\ln(c_{t+1}) \tag{5}$$

[6]

where c_{t+1} is consumption in retirement and n_t denotes pairs of children (since the couples is the basic unit of analysis), both chosen by the household at time *t*.[7]

To raise each pair of children, households expend both goods and services[8] and a fraction of maternal time, denoted $\hat{x}$ and $\hat{z}$, respectively.

Only the wife raises children, $\hat{z}n_t \leq 1$, because the opportunity cost of female labor is lower. Men allocate their labor endowment to paid work only. This paper models the economy's transition through the Baby Boom to Bust of the latter half of the twentieth century. Neither era saw men withdrawing from the labor force to supplement women at home raising children full time. Even the arrival of "Mr. Mom" in the modern era involved substitution for, rather than supplementation of, maternal time. For the purposes of this paper, therefore, it is assumed[9] the following condition is met:

$$\hat{z}n_t \leq 1 \tag{6}$$

The production function for child-care is of CES form:

$$n_t = \left[(\alpha_1 z_t)^a + (\alpha_2 x_t)^a\right]^{\frac{1}{a}} \quad ; \quad a \neq 0 \tag{7}$$

[5] This is the real wage, with the price of the aggregate good normalized to 1.

[6] Because of the log linear specification, demand for children will be independent of the price for retirement consumption and vice versa. Consumption in the second period of life is assumed to be zero. Galor and Weil (1996) note that if couples had log utility from c_t, the equation of motion would be altered only by a multiplicative constant.

[7] The model structure up to this point corresponds to Galor and Weil (96). The analysis from this point is the original work of the author.

[8] Such goods and services could range from disposable nappies and convenience food to household appliances and child-care services.

[9] To ensure that women enter the workforce, Galor and Weil (1996) assume $\gamma < 1/2$. By implication, the household's optimal choice of fertility satisfies condition (6). The approach taken in this paper differs in that parameter values in the household optimization problem are not restricted so as to rule out the scenario where optimizing households would use child-rearing time in excess of the maternal time endowment if they could.

where a determines the constant elasticity of substitution between time and goods, given by $\varepsilon = 1/1{-}a$. Define $\alpha_1 = \delta^{1/a}$ and $\alpha_2 = (1-\delta)^{1/a}$. δ is the distribution parameter that measures the relative factor shares in production. The limits of the CES child-rearing production function are:

- as $\delta \to 1$, the assumption of Galor and Weil (1996) that child rearing requires only time input;[10]
- as $\varepsilon \to 0$, a Leontief technology, where child-care goods and services and maternal time are used in fixed proportions.

Because the child-rearing production function is homogeneous of degree one, the household optimization problem can be solved in two stages. The household first chooses, for a given n_t, the cost minimizing input mix and then chooses n_t, given the efficient input mix, so as to maximize utility subject to a budget constraint.

Cost minimization

Allowing for the possibility of a government subsidy ($0 \le \beta_t < 1$) per unit of market goods and services used[11], the total cost of rearing children is:

$$C_t = w_t^m z_t + (1-\beta_t)x_t \tag{8}$$

where x and z denote *total* goods/services and time input, respectively.

There are two cases, constrained (c) and unconstrained (u), depending on whether the maternal time constraint, (6), is binding,

The household first chooses the input mix, for a given n_t, so as to minimize (8) subject to (7), assuming, for the moment, the maternal time constraint (6) is not binding. Input demands for time and goods are, respectively,

$$z_t^{*u} = \hat{z}n_t = \left[\left(w_t^m/\alpha_1\right)^{a/a-1} + \left(1-\beta_t/\alpha_2\right)^{a/a-1}\right]^{-1/a} (\alpha_1)^{-a/a-1}(w_t^m)^{1/a-1} n_t \tag{9}$$

$$x_t^{*u} = \hat{x}n_t = \left[\left(w_t^m/\alpha_1\right)^{a/a-1} + \left(1-\beta_t/\alpha_2\right)^{a/a-1}\right]^{-1/a} (\alpha_2)^{-a/a-1}(1-\beta_t)^{1/a-1} n_t \tag{10}$$

The unconstrained per unit cost function is

$$p(w_t^m, \beta_t) = \left[\left(w_t^m/\alpha_1\right)^{a/a-1} + \left(1-\beta_t/\alpha_2\right)^{a/a-1}\right]^{a-1/a} \tag{11}$$

When (6) is binding, $z_t^{*c} = \hat{z}n_t = 1$ and the required goods input, x_t, is derived by inverting equation (7).

Utility maximization

When (6) is not binding, the household's first period and second period budget constraints are, respectively:

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[10] In the limit, $\hat{z} = 1/E$, where E is the level of production technology, analogous to A in (1). Replacing the restriction $E=1$ with $E>1$ ensures $0 < \hat{z} < 1$.

[11] The price of goods and services is normalized to 1. It is assumed the subsidy is financed by a tax on old age consumption. See second period budget constraint below.

$$p(w_t^m, \beta_t)n_t + s_t \le w_t^p + 2w_t^m \tag{12}$$

$$c_{t+1} = s_t(1+r_{t+1})(1-\tau_{t+1}) \tag{13}$$[12]

where r_{t+1} denotes the rate of return on savings, s_t , τ_{t+1} denotes the rate of taxation on old age consumption and $w_t^m(2-\hat{z}n_t)+w_t^p$ is the couple's income.

The household then chooses n_t and c_{t+1} to maximize (5) subject to (12), (13), yielding:

$$n_t^{*u} = \frac{\gamma(w_t^p + 2w_t^m)}{p(w_t^m, \beta)} \tag{14}$$

$$s_t^{*u} = (1-\gamma)(w_t^p + 2w_t^m) \tag{15}$$

When (6) is binding, the household's first period budget constraint is:

$$(1-\beta_t)\hat{x}n_t + s_t \le w_t^m + w_t^p \tag{16}$$

Intuitively, when the wife is restricted to using her entire labor endowment to raise children, the husband is the sole income earner. His earnings are then allocated between goods used in child rearing and savings.

Maximizing (5) subject to (13) and (16) yields:

$$x_t^{*c} = \hat{x}n_t = \frac{\gamma(w_t^m + w_t^p)}{(1-\beta_t)} \tag{17}$$

$$s_t^{*c} = (1-\gamma)(w_t^m + w_t^p) = w_t^m + w_t^p - (1-\beta_t)\frac{\hat{x}}{\hat{z}} \tag{18}$$

When $\hat{z}n_t = 1$, substituting from (17) into the CES production function for child rearing, each household produces:

$$n_t^{*c} = \left[\alpha_1^{\,a} + \left(\alpha_2 \frac{\gamma(w_t^m + w_t^p)}{(1-\beta_t)}\right)^a\right]^{\frac{1}{a}} \tag{19}$$

Dynamic system

Capital stock per working age couple fuels growth in this model. The capital stock in each period is determined by the saving of the working age households in the previous period, so that

$$K_{t+1} = L_t s_t^* \tag{20}$$

The number of working age households at time $t+1$ is

$$L_{t+1}^p = L_{t+1} = n_t^* L_t \tag{21}$$

Capital stock per household is therefore given by:

[12] The rate of subsidy and taxation are set so as to satisfy the balanaced-budget constraint: $\beta_t \hat{x} n_t \left[n_{t-1}L_{t-1}\right] = s_{t-1}(1+r_t)\tau_t L_{t-1}$. Although endogenous at the aggregate level, the rate of subsidy and taxation is treated as exogenous by each individual household.

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{s_t^*}{n_t^*} \tag{22}$$

From (22), an equation of motion $k_{t+1} = \phi(k_t)$ is obtained, since both household savings and fertility choices are determined by w_t^m, which in turn is a function of k_t.

In Galor and Weil (1996) capital per household evolves through two phases distinguished by female labor force participation. In the initial phase, women allocate their entire labor endowment to child rearing. Once capital per household reaches a sufficiently high level, women enter the paid labor force, marking the transition to the second phase. This paper has similar phases in respect of female labor force participation, but a different predicted path of fertility as the economy moves through the two phases.

The evolution of capital stock per household is governed by two distinct equations of motion, as the economy moves first through

- Phase1: women are at home, raising children full time $(z_t^{*c} = 1)$; and, then
- Phase 2: women participate in the labor force, raising children part time $(z_t^{*u} \leq 1)$.

Interdependence of wage for mental labor and capital per couple in Phase 2

The derivation of the equation of motion for Phase 2 is complicated by the fact that the wage for mental labor and time spent raising children are interdependent. By (3), w_t^m is a function of m_t, which in turn is a function of $\hat{z}n_t$. That is, the wage paid to mental labor reflects it marginal product, affected by household supply of mental labor, which in turn depends on the time spent raising children. As previously demonstrated, $\hat{z}n_t$ is a function of w_t^m: time spent raising children falls with the wage for mental labor when women are in the labor force. Thus, we need obtain an implicit function for $\hat{z}n_t$ when $\hat{z}n_t < 1$.

When $\hat{z}n_t < 1$, noting that

$$m_t = \frac{L_t^m}{L_t^p} = \frac{L_t(2-\hat{z}n_t)}{L_t} = 2 - \hat{z}n_t \tag{23}$$

and substituting from (3), (14) and (23) into (9),

$$\hat{z}n_t = f(\hat{z}n_t, k_t) \tag{24}$$

Let $G(\hat{z}n_t, k_t) = \hat{z}n_t - f(\hat{z}n_t, k_t) = 0$. Since $G(\hat{z}n_t, k_t) = 0$ has continuous derivatives, by the Implicit Function Theorem, if $G_{\hat{z}n_t} \neq 0$ then there is a differentiable and invertible function $\varphi(k_t)$ such that $\hat{z}n_t = \min(1, \varphi(k_t))$ where

$$\varphi'(k_t) = \frac{-G_{k_t}}{G_{\hat{z}n_t}} = \frac{\partial f / \partial w_t^m . \partial w_t^m / \partial k_t}{\left[1 - \partial f / \partial w_t^m . \partial w_t^m / \partial m_t . \partial m_t / \partial \hat{z}n_t\right]} < 0 \quad \text{if} \quad \partial f / \partial w_t^m < 0 \tag{25}$$

With the exception of $\partial f / \partial w_t^m$, signs of the partial derivatives in (25) are unambiguous. Assigning a negative value to $\partial f / \partial w_t^m$ is tantamount to assuming that the female labor supply curve is never backward bending. The possibility of a backward bending supply curve arises in this model, since, as we shall see, fertility may rise with female relative wages in Phase 2. A backward bending labor supply curve would occur if the proportionate rise in number of children exceeds the proportionate fall in time input per child. For the purposes of this paper, $\partial f / \partial w_t^m < 0$ is assumed.[13]

Thus, by (25), as the economy grows in transition to steady state, time spent raising children falls even when an input substitutable for maternal time exists.[14]

The economy enters Phase 2 once a sufficiently high level of capital per couple has been accumulated. Let k^* denote the highest level of capital per working age couple for which women raise children full time. That is,

$$\hat{z}n_t = \begin{cases} \varphi(k_t) \in (0,1] & \text{for} \quad k_t \geq k^* \\ 1 & \text{for} \quad k_t \leq k^* \end{cases} \tag{26}$$

3 Equation of motion

Substituting from (14), (15) and (18), (19) into (22) and using the definition of k^*, the equation of motion for the system is, therefore,

$$k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{s_t^*}{n_t^*} = \begin{cases} \dfrac{1-\gamma}{\gamma} p(w_t^m, \beta) & \text{if} \quad k_t \geq k^* \\ \hat{z}\left(w_t^m + w_t^p - (1-\beta_t)\dfrac{\hat{x}}{\hat{z}} \right) = \dfrac{1-\gamma}{\gamma}(1-\beta_t)\hat{x} & \text{if} \quad k_t < k^* \end{cases} \tag{27}$$

Capital stock per couple evolves from a historically given initial stock according to $k_{t+1} = \phi(k_t)$, which is readily derived from (27) by substituting for:

$$w_t^m = \begin{cases} A(1-\alpha)\left(2-\varphi(k_t)\right)^{\rho-1}\left[\alpha k_t^\rho + (1-\alpha)\left(2-\varphi(k_t)\right)^\rho \right]^{1-\rho/\rho} & \text{if} \quad k_t \geq k^* \\ A(1-\alpha)\left[\alpha k_t^\rho + (1-\alpha) \right]^{1-\rho/\rho} & \text{if} \quad k_t < k^* \end{cases} \tag{28}$$

By (28), w_t^m is a non-linear function of k_t. It follows that the equation of motion is a first order non-linear difference equation, $k_{t+1} = \phi(k_t)$.[15]

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[13] A backward bending female labor supply, although an interesting proposition in itself, adds an unnecessary layer of complication to the model.

[14] When a fixed fraction of time per child is the only input, total time spent raising children necessarily falls as the economy grows. See Galor and Weil (96).

[15] The log linear specification for household utility function ensures the equation is first order since fertility is a function of own price. Should fertility be a function of the real interest rate, a higher order equation would be obtained.

3.1 Properties

Curvature

By enabling n_t to change, the introduction of a CES production function for child rearing complicates the analysis of the curvature of the equation of motion.

Recall that capital per household at $t+1$ equals savings per household (s_t) divided by household fertility (n_t). If time is the only input or fixed quantities of time and goods are used in child rearing, n_t is necessarily constant in Phase 1. Consequently, capital per household follows the path of household savings. s_t increases one for one with the wage for mental labor. Hence, $\phi(k_t)$ is concave (convex) if the wage for mental labor increases at a decreasing (increasing) rate as capital per household accumulates. Referring to the Appendix, if the degree of complementarity between capital and mental labor is relatively low, then w_t^m is increasing and concave in k_t over the interval $(0, k^*)$.

When goods are substitutable for time, both n_t and s_t increase with the wage for mental labor. n_t rises less than s_t , proportionate to the increase in the efficient goods input. Capital per household therefore follows the path of the goods input per pair of children. As a result, the concavity (convexity) of $\phi(k_t)$ also depends on how goods input per pair of children changes with the wage for mental labor.

Proposition 1: *Given a low degree of complementarity between mental labor and capital, a low degree of complementarity between child-rearing goods and time is a sufficient condition for concavity of $\phi(k_t)$ over the interval $(0,k^*)$.*

<u>Proof</u> See Appendix

Regardless of the elasticitiy of substitution between goods and time, goods input per pair of children rises with the total quantity of goods (see Appendix), which in turn rises with the wage for mental labor. Thus, $\phi'(k_t) = (1-\gamma)\partial\hat{x}/\partial x.\partial x/\partial w_t^m.\partial w_t^m/\partial k_t > 0$. Is the sign of the second order derivative also unambiguous? Referring to Appendix, given an elasticity of substitution between mental labor and capital in excess of 1, an elasticity of substitution between child-care goods and time exceeding 0.5 (corresponding to $a > -1$) is a sufficient condition for $\phi''(k_t) < 0$.

The intuition for this result lies in diminishing marginal returns. Growth in k_t raises the wage for mental labor, which in turn boosts the quantity of goods input per child. Input of mental labor is fixed to 1 in Phase1. The wage (or marginal product) for mental labor increases at a decreasing rate when the degree of complementarity between mental labor and physical capital is relatively low. Similarly, goods input per pair of children increases at a decreasing rate when the degree of complementarity between goods and time is relatively low.

Proposition 2: *$\phi(k_t)$ is increasing and concave in k_t over the interval (k^*,∞) regardless of the degree of complementarity between goods and time in child rearing.*

Proof

Referring to (27), k_{t+1} is proportional to child rearing costs per pair of children, $p(w_t^m,\beta)$. By Euler's Theorem, $\partial p/\partial w_t^m = \hat{z} > 0$. Demand for maternal time input, given by (9) is downward sloping: $\partial \hat{z}/\partial w_t^m = \partial^2 p/\partial w_t^{m\,2} < 0 \quad \forall \quad a \in (-\infty,1)$. Thus, the per unit child rearing cost function is increasing and strictly concave in the wage for mental labor, regardless of the degree of complementarity between goods and time. Intuitively, an increase in the price of time input raises the per unit cost of child rearing, albeit at a decreasing rate as the household uses less and less time input.

As in Galor and Weil (1996), the wage for mental labor is increasing and concave in capital per household. Given (25), their proof is applicable.

Discontinuity

The equation of motion is discontinuous at k^*. Substituting (14) into (9), $\hat{z}n_t = 1 \quad \Rightarrow \quad w_t^p = \dfrac{1-2\gamma}{\gamma} w_t^m + \dfrac{1}{\gamma}(1-\beta_t)\left[\dfrac{(1-\delta)w_t^m}{\delta(1-\beta_t)}\right]^{\varepsilon}$. k^* is readily obtained, after solving out for the wage for mental labor. Equality of the two equations in (27) is not satisfied for $k_t = k^*$. Specifically, equality of the two equations of motion occurs at a lower wage for mental labor: $w_t^p = \dfrac{1-\gamma}{\gamma} w_t^m + \dfrac{1}{\gamma}(1-\beta_t)\left[\dfrac{(1-\delta)w_t^m}{\delta(1-\beta_t)}\right]^{\varepsilon}$, suggesting $\phi(k_t)_{k_t \to k^*+} > \phi(k_t)_{k_t \to k^*-}$ given concavity of $\phi(k_t)$ throughout both Phase 1 and Phase 2.

For the intuition, first consider why the equation of motion is continuous in the limiting cases, $\delta \to 1$ and $\varepsilon \to 0$. In the case of fixed proportions, $\hat{z}n_t = 1 \quad \Rightarrow \quad w_t^p = \dfrac{1-2\gamma}{\gamma} w_t^m + \dfrac{1}{\gamma}(1-\beta_t)\dfrac{\hat{x}}{\hat{z}}$. In the case of time input only, the last term in the implied equality vanishes. In either case, equality of the two equations analogous to the system in (27) is satisfied for $k_t = k^*$. Intuitively, an optimizing household takes child-rearing input(s) as given and chooses fertility so as to maximize utility. On the cusp of Phase 2, the wage for mental labor (corresponding to k^*) induces a utility maximizing household to choose fertility such that $z^*=1$. Continuity of the equation of motion at $k_t=k^*$ is assured by the fact that the time input per child or the goods-time input ratio is, by assumption, fixed and therefore the same in Phase 1 as in Phase 2.

Allowing households to substitute child-care goods and services for time introduces another dimension to household decision-making. In addition to choosing fertility so as to maximize utility, an optimizing household chooses the child-rearing input mix so as to minimize cost. Once again, on the cusp of Phase 2, a utility maximizing household chooses fertility implying $z^*=1$. However, discontinuity in the equation of motion at k_t=k* arises because the goods-time input ratio throughout Phase 1 differs from the goods-input ratio upon entering Phase 2. If the household cost minimizes throughout, why does this disparity arise? Throughout Phase 1, cost minimization corresponds to a non-tangent corner solution. Growth in the wage for mental labor does not affect the goods-time input mix required for a given number of children. However, total goods used (equivalent to goods-time input ratio when z*=1) increases with fertility due to a pure (male wage) income effect. In Phase 2, cost minimization corresponds to a tangency point. Growth in the wage for mental labor raises the goods-time input ratio required for any number of children. The effect on fertility depends on competing substitution and (male and female) income effects.

3.2 Steady state equilibria

In steady state equilibrium, capital stock per household is stationary: $\bar{k} = \phi\left(\bar{k}\right)$.

Existence

A steady state exists if:

- $\phi(0) > 0$; and
- there exists some k_t, such that $\phi(k_t) < k_t$.

The first condition is indeed satisfied. Substituting from (17) into the equations for $k_t \in (0, k^*)$ of (27) and (28) implies $\phi(0) = \hat{z}(1-\gamma)\left\{a(1-\alpha)^{1/\rho} + b\right\} > 0$.[16]

Ensuring $\phi(k_t) < k_t$ for some k_t, $\lim_{k_t \to \infty} \phi'(k_t) = \frac{(1-\gamma)}{\gamma} \lim_{k_t \to \infty} \frac{\partial p}{\partial w_t^m} \frac{\partial w_t^m}{\partial k_t} = 0$, since $\frac{\partial p}{\partial w_t^m} = \hat{z}$ is bounded below by zero and the equation for $k_t \in (k^*, \infty)$ of (28), together with (25), imply $\lim_{k_t \to \infty} \frac{\partial w_t^m}{\partial k_t} = 0$.[17]

$\phi(k_t) > k_t \quad \forall \quad k_t \in [0, k^*)$ implies a steady state exists in Phase 2 rather than Phase 1. As in Galor and Weil (1996), we confine attention to this case. Thus, discontinuity of $\phi(k_t)$ poses no problem for existence provided $\phi(k_t)_{k_t \to k^*+} > \phi(k_t)_{k_t \to k^*-}$, which is satisfied, as required.

---

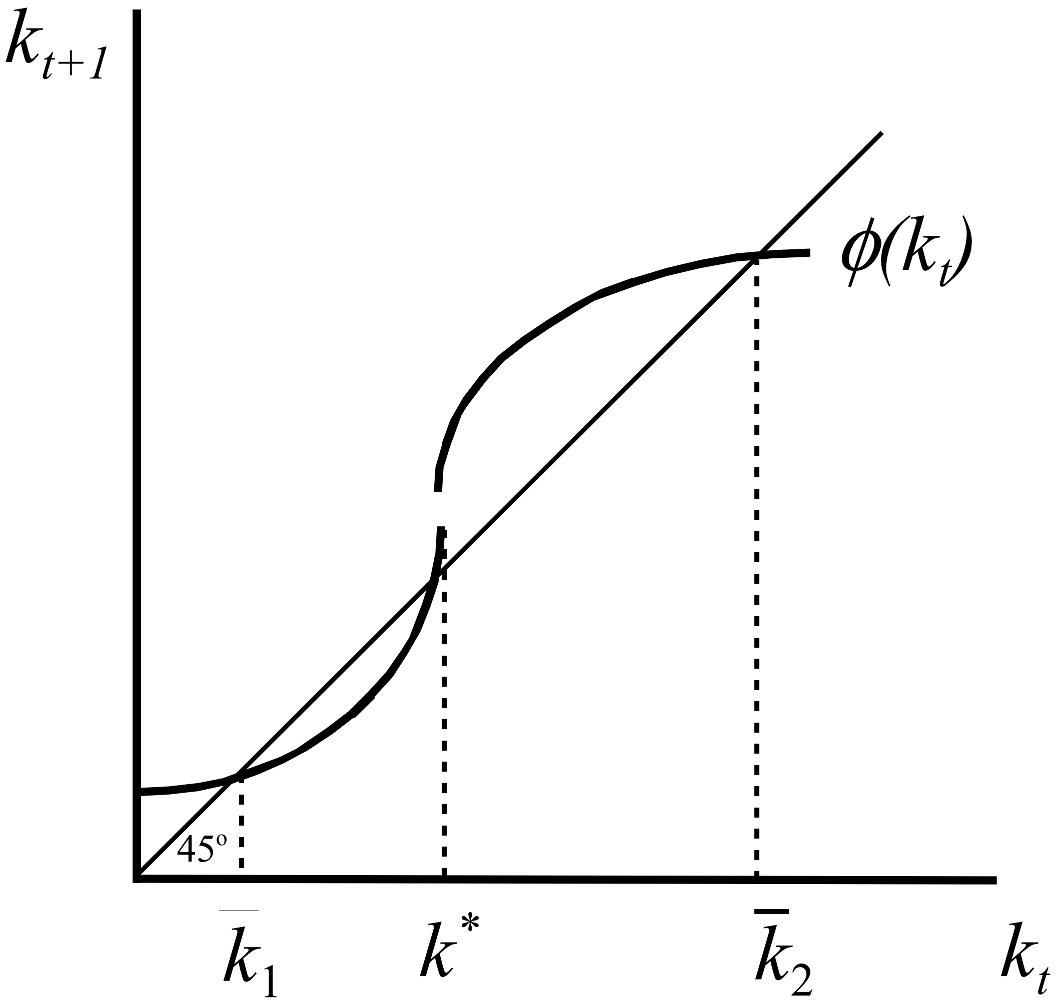
[16] Apps and Rees (2002) instead show ϕ (0) >0 for the Phase 2 equation of motion. Since the economy enters Phase 2 at a *positive* and sufficiently high level of k_t (i.e. k*), the condition for existence is correctly obtained from the Phase1 equation of motion. Their approach is correct if: 1. Phase 1 does not exist (amending their restriction $z \in [0,1]$ to $z \in [0,1)$; or 2. $k^*=0$: Phase 1 does not exist over an interval (in which case ϕ (0) >0 is obtained from the Phase 1 equation of motion, applicable at $k^*=0$).

[17] For a formal proof of $\lim_{k_t \to \infty} \frac{\partial w_t^m}{\partial k_t} = 0$, see Galor and Weil (96).

Uniqueness / Multiplicity

Given the conditions for existence are satisfied, multiple steady state equilibria may exist if the Phase 1 equation of motion is convex. Figure 3 illustrates such a case. As previously discussed, the convexity of $\phi(k_t)$ over the interval (0,k*) depends not only on the degree of complementarity between capital for mental labor, as found by Galor and Weil (1996), but also the degree of complementarity between child-care goods/services and maternal time. Should $\phi(k_t)$ be convex over the interval (0,k*), a poverty trap (represented by the stable equilibrium $\overline{k}_1$) may emerge. For the subsequent analysis, I deal with the case where there is no such low level steady state.

Figure 3: Multiple equilibria



4 Fertility dynamics

Having established that capital per household (and the wage for mental labor) rise throughout Phase 1 and Phase 2 until eventually converging to steady state in Phase 2, we can explore the dynamics of fertility.

Proposition 3: *Throughout Phase 1, fertility necessarily rises with income per household.*

It follows from (19) that rising wages for mental labor boost household fertility, regardless of the degree of complementariy between goods and time. Thus, growth in income per capita generates a baby boom during an era in which women do not participate in the paid labor force.

Recall the household's fertility decision comprises 2 stages. The household

1. chooses the cost minimizing or efficient time-goods input mix given the production possibilities; and then
2. Apportions the husband's income to children and savings given the efficient cost of child rearing.

Figure 4 depicts cost minimization for Phase 1 at the point where the isoquant n_1 intersects the vertical constraint, $z = 1$. The slope of the isocost line is $-w_t^m/(1-\beta_t)$. The wage for mental labor is sufficiently small that $z^* = 1$. Specifically $w_t^m < x^{1-a}\left(\delta/1-\delta\right)\left(1-\beta_t\right)$. The isocost is flatter than the isoquant at the initial corner solution.[18] That is, the household would lower the cost of child rearing if it were able to employ time input in excess of the women's total time endowment.

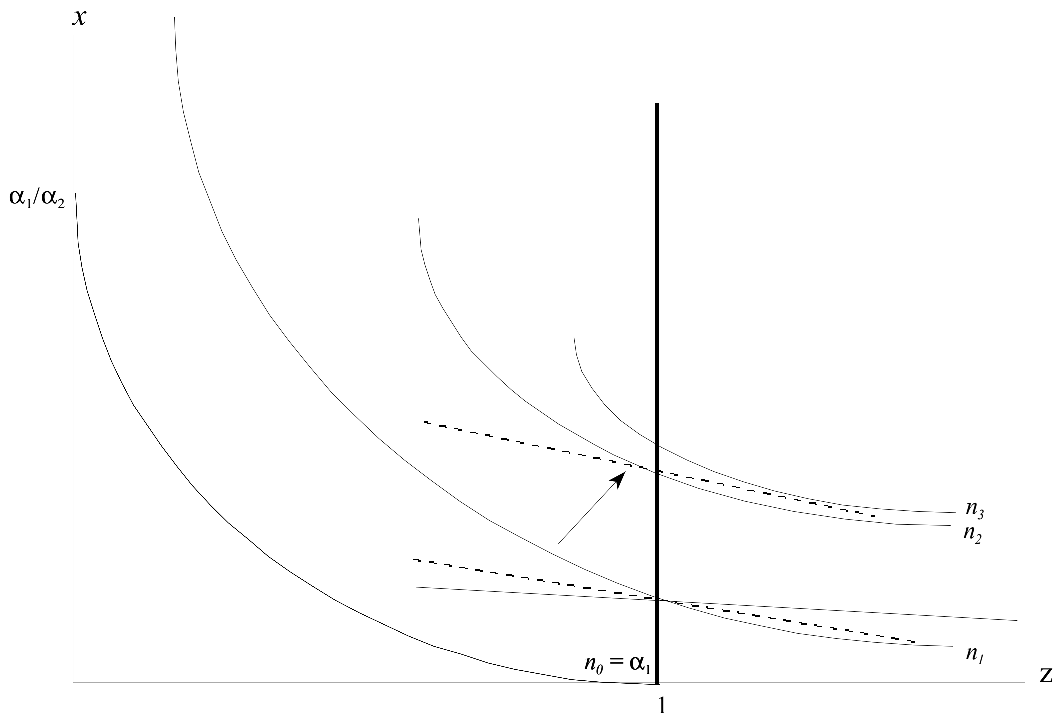
Consider a rise in the wage for mental labor that is not sufficient to induce women to enter the paid workforce. In terms of the diagram, the rotation of the isocost from the relatively flat solid line to the slightly steeper broken line does not yield an interior solution. The efficient goods-time input mix required to rear n_1 pairs of children is unaltered.

At the same time, an increase in the wage for mental labor eases the household's budget constraint. Due to a pure income effect, the demand for children (or, equivalently spending on child rearing goods) increases. An upward shift in the isoquant to n_2 depicts the corresponding scale effect in Figure 4.

Recall that for capital per household to accumulate over time, $\hat{x}$ must rise.
In addition to illustrating the baby boom in Phase 1, Figure 4 provides a diagrammatic proof that $\hat{x}$ rises with the wage for mental labor. When time input is bounded above by 1, an increase in scale can only be met by raising goods input more than proportionate to the rise in n. Hence, $\hat{x}$ necessarily rises.

[18] If the corner solution were a point of tangency, a rise in the wage for mental labour would imply a move to an interior solution and, therefore, a transition to Phase 2. We want to analyse the implications of a rise in the wage for mental labour while the economy is in Phase 1.

Figure 4: The household's child rearing production decision in Phase 1



Proposition 4 *As income per household rises throughout Phase 2, fertility may either:*

1) decline monotonically;

2) rise monotonically; or

3) *initially decline and then rise.*

Proof

Referring to the Appendix, there exists a critical value of the wage for mental labor,

$$w^m ** = \left(\frac{w_t^p}{2}\right)^{1/\varepsilon}\left(\frac{\delta}{1-\delta}\right)\left(1-\beta_t\right)^{(\varepsilon-1)/\varepsilon} \tag{29}$$

such that $\partial n_t^u / \partial w_t^m < 0 \quad \forall \quad w_t^m < w^m **$; $\partial n_t^u / \partial w_t^m > 0 \quad \forall \quad w_t^m > w^m **$.

Let $k**$ denote the level of capital per household, corresponding directly to w^m** (as per (28)), sufficient to induce a baby bounce-back in Phase 2.

Three possible cases arise:

1) $k* < \bar{k} < k** \quad : \quad \partial n_t^u / \partial w_t^m < 0 \quad \forall \quad k_t \in \left(k*, \bar{k}\right)$

Thus, for a sufficiently large w^m**, fertility declines monotonically throughout Phase 2. Although the following section examines the determinants of w^m**, consider, for the moment, the influence of the production share of maternal time in child-rearing. As $\delta \to 1$, $w^m ** \to \infty$, ensuring that in the limit the above inequality holds, since the steady state value of capital per household is finite. That is, when time is the only input, fertility unambiguously falls with the wage for mental labor throughout Phase 2, confirming Galor and Weil (1996).

2) $k* = k** < \bar{k} \quad : \quad \partial n_t^u / \partial w_t^m > 0 \quad \forall \quad k_t \in \left(k*, \bar{k}\right)$

Should the wage for mental labor needed to induce a baby bounce-back equal the wage needed to induce women to enter the paid labor force, fertility rises monotonically throughout Phase 2. An instantaneous baby bounce-back, although possible, is unlikely. We can infer from the time input only case that substitution to child-care goods and services is integral to a baby bounce-back eventuating. Market substitutes for maternal time *emerge* in response to continuing cost pressures.

3) $k* < k** < \bar{k}$

In this case, growth in capital (income) per household generates a non-monotonic path in fertility throughout Phase 2:

$$\partial n_t^u / \partial w_t^m < 0 \quad \forall \quad k_t \in \left(k*, k**\right) \quad ; \quad \partial n_t^u / \partial w_t^m > 0 \quad \forall \quad k_t \in \left(k**, \bar{k}\right)$$

Although a steep rise in the opportunity cost of maternal time causes an initial decline in fertility, further rises induce households to substitute out of maternal time, removing the pressure to reduce fertility and allowing the income effect to dominate. This pattern of gradual rather than instantaneous substitution fits with observed trends. For instance, early childhood care centers, non-existent a few decades ago, are now prevalent.

Before investigating the factors underpinning w^m**, we restate (29) so as to explore the intuition behind Proposition 4.

Corollary *As income per household rises throughout Phase 2, fertility declines if and only if the child rearing goods/time input ratio is sufficiently high.*

Proof Referring to the Appendix, $\partial n_t^u / \partial w_t^m < 0$ if and only if

$$\frac{\hat{x}}{\hat{z}} = \left[\frac{w_t^m}{1-\beta_t} \frac{1-\delta}{\delta}\right]^{\varepsilon} < \frac{w_t^p}{2\left[1-\beta_t\right]} \qquad (30)^{19}$$

[19] Alternatively, this condition can be expressed as $2\gamma < \hat{z} n_t$. Hence the proposition: "If time spent in domestic child care is sufficiently small … fertility increases with the female wage" (p.7 Apps and Rees (2001)).

To abstract from the role of wage growth, for the moment, consider the limiting case, $\varepsilon \to 0$. By assumption of fixed proportions the input ratio (left hand side of (30)) remains fixed.

This is the assumption made by Galor and Weil (1996) who note briefly that in this case, a rise in female relative wages "reduces fertility if $\hat{x}$ is not too large" (footnote 12, Galor and Weil (1996)).[20] Thus, the introduction of a second child rearing input, albeit in fixed proportions, is sufficient to demonstrate that fertility decline is not an inevitable consequence of rising female relative wages with economic growth.

Nonetheless, the assumption of fixed proportions seems unnecessarily restrictive. Goods and time are unlikely to be perfect complements in the rearing of children.

Moreover, only by restoring wages as a determinant of the goods/time ratio can we see why the negative relationship between household income and fertility will unravel over time, as in the third case of Proposition 4.

The intuition lies in a comparison of substitution and income effects. When both husband and wife work, female wages constitute a portion of household income. A proportionate increase in the wage for mental labor results in a less than proportionate rise in household income. To illustrate, if female wages are two thirds of male wages, a 10% rise in the wage for mental labor will increase household income by 8%.

In the limiting case, $\delta \to 1$, when maternal time is the only child rearing input, a 10% rise in mental wages will increase the cost of raising children by 10%. Hence, given our functional forms for parental utility and for child rearing, the substitution effect dominates the income effect, and fertility necessarily declines. However, when a second input is introduced, the cost of the wife's time is only a portion of the total cost of child rearing, so that the income effect may now dominate.[21]

In the limiting case of fixed input proportions, $\varepsilon \to 0$, both potential household income and the per unit cost of child rearing increase at a constant rate with wages growth. Thus, if the substitution effect dominates the income effect at the beginning of Phase 2, further wages growth will not reverse the dominance. However, when $\varepsilon > 0$, the per unit cost of child rearing increases at a *decreasing* rate, whereas potential household income increases at a constant rate. It follows that the income effect of rising female relative wages may eventually dominate the substitution effect so that fertility rises instead of declining.

Thus, when $\varepsilon > 0$, the very aspect of economic growth that discourages fertility, rising female relative wages, ultimately remedies fertility decline by raising the goods-time input ratio.

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[20] Clearly, by the corollary to Proposition 4, the conditions of Apps and Rees (2001) and Galor and Weil (1996) are equivalent.

[21] We can infer the likely response of fertility to a rise in female relative wages at a point in time from the composition of child rearing costs and household income. To illustrate, Haveman and Wolfe (1995) provide lower and upper bound estimates of time's share of the total cost of child rearing for the US in 1992: 18% and 73%, respectively. Since the upper bound estimate approximates female earnings as a percentage of male earnings in the same year, we would expect fertility in the US to rise with female relative wages at that time.

We could simply append the previous discussion with the surmise that the goods-time input mix increases over time as higher wages induce households to switch goods and services for maternal time. However, factors other than higher wages are also at play: households' ability to substitute goods for time; the price of goods and services used in child rearing.

Overlooked in existing literature, a CES production function for child rearing allows us to clearly identify the factors underpinning the child rearing input mix. From this, we can demonstrate how changes in parameter values may either hasten or postpone the onset of a baby bounce-back.

Proposition 5 *A high rate of subsidy to child-care goods and services prolongs fertility decline in Phase 2, if the degree of complementarity between child-care goods/services and time is relatively high.*

Proof

Consider the case $k^* < k^{**} < \bar{k}$.

From (29), $\partial w^m{**} / \partial \beta_t > 0 \quad \Leftrightarrow \quad \varepsilon < 1$. Since $w^m{**}$ is increasing over time, the higher $w^m{**}$, the later the onset of a baby bounce-back.

For the intuition on this result, note that the net price of goods and services used in child rearing appears on both sides of the inequality in (30). Accordingly, a subsidy to child-care goods and services affects the fertility response to higher female relative wages in two ways:

1. For a given goods-time input mix, a subsidy raises maternal time's portion of the total cost of child rearing, accentuating the substitution effect that induces fertility decline;
2. A subsidy lowers the net price of goods and services, prompting households to raise the goods-time input ratio (left hand side of (30)). A higher goods-time input ratio weakens the substitution effect.

Unless the input mix is highly responsive to changes in relative input prices, the first effect will dominate.

Intuitively, when the degree of complementarity between child-care goods and services and maternal time is relatively high, subsidizing child-care goods and services has a negligible effect on the goods-time input mix. Under these circumstances, a subsidy serves only to boost time's share of the total cost of child rearing. Accordingly, subsidization of child-care goods and services prolongs the decline in fertility due to rising female relative wages.

A priori, we might expect subsidizing child-care goods and services would itself foster a baby bounce-back. Note that Proposition 5 deals specifically with an indirect effect of subsidization. A high rate of subsidy implies a high $w^m{**}$, thereby postponing the onset of a naturally occurring baby bounce-back induced by rising wages.

The partial derivatives of w^m** with respect to other parameters are straightforward. In brief, the lower maternal time's share in production (δ) or the lower the wage for physical labor, the earlier the onset of a baby bounce-back.

For the intuition, recall that rising female relative wages boost fertility when the income effect dominates the substitution effect. The lower δ, the smaller maternal time's share of child rearing costs and hence, the weaker the substitution effect. The lower the wage for physical labor, the greater female wages' contribution to household income, amplifying the income effect.

Referring to the Appendix, for given rates of subsidy and a given wage for physical labor, a higher elasticity of substitution hastens the onset of a baby bounce-back.

Proposition 6 *As income per household rises throughout Phase 2, rising female labor force participation is necessary but not sufficient for declining fertility.*

Proof

Recall that female labor supply is given by $1-\hat{z}n_t$.

Necessity

$\partial n_t^u / \partial w_t^m < 0 \quad \forall \quad k_t \in (k^*, k^{**})$, together with $\partial \hat{z} / \partial w_t^m < 0$ imply $\partial(1-\hat{z}n_t)/\partial w_t^m < 0$

Sufficiency

By (25), $\partial(1-\hat{z}n_t)/\partial w_t^m < 0$, which need not imply declining fertility. Provided $k^{**} < \bar{k}$, $\partial n_t^u / \partial w_t^m > 0 \quad \forall \quad k_t \in (k^{**}, \bar{k})$.

In the limit as $\delta \to 1$, $k^{**} = \infty \quad > \quad \bar{k}$ and we obtain the Galor and Weil (1996) result that rising female labor force participation is both necessary and sufficient for declining fertility. Since time input per pair of children is fixed, the proof that rising female labor supply implies declining fertility is incidental.

5 Discussion

5.1 Joint evolution of population growth and income per household

Household fertility and income evolve jointly according to the dynamic system explored in Section 3. Figure 5 depicts the path of fertility in transition to a unique globally stable steady state, corresponding to $\bar{k}$, in Phase 2. t^*, t^{**} and $\bar{t}$ denote the time periods after which, respectively, women enter the paid labor force, a baby bounce-back commences and the economy reaches steady state.

Figure 5(b) depicts the general results. For contrast, Figure 5(a) depicts the limiting cases, $\delta \to 1$ and $\varepsilon \to 0$, when maternal time cannot be substituted for child-care goods and services.

Whilst allowing for the substitution of child-care goods and services for maternal time has striking implications for the path of fertility, a feature common to both Figure 5(a) and (b) is the dramatic change in pace of capital accumulation as the economy enters Phase 2. The pace of capital accumulation slows as the economy nears the end of Phase 1. On entering Phase 2, a boost to the supply of mental labor fuels an immediate take-off in growth. There is a corresponding steep decline in fertility. The pace of capital accumulation slows again as the economy approaches steady state. This pattern is a consequence of concavity of $\phi(k_t)$ in both phases.

In Figure 5(a), the time path for fertility derived for a given goods-time input ratio *(ε =0)* follows a similar path to that derived under the assumption of time input only *(δ =0)*. Overall, allowing for a fixed quantity of goods to be used in child rearing has no dramatic implications for the joint evolution of income per household and fertility.

Compared with the time input only case, household capital and income is lower at any given time period in Phase 1, since spending on child rearing goods and services absorbs some household savings. Accordingly, k^* is lower. That is, women enter the paid workforce at a lower income per capita.[22]

In Phase 2, household savings is the same under either assumption, but the per unit cost of child rearing is higher when goods are an input. Consequently, fertility is lower at any given time period. Less capital dilution implies higher capital (income) per household. Compared with the time input only case, the economy converges to a higher income per capita and lower fertility rate.

Figure 5(b) depicts our richer, more general result of a non-monotonic path in fertility.

As per Proposition 3, growth in income per household generates a baby boom in Phase 1. Throughout Phase 1, the maternal time constraint is binding. Because household time allocation is at a corner solution, a marginal change in household income has only an income effect and demand for children increases. The rise in fertility becomes less pronounced as the economy nears the end of Phase 1, due to the concavity of the equation of motion.

This contrasts Figure 5(a), where fertility is constant throughout Phase 1. In either limiting case, because the household cannot substitute goods for maternal time, increases in household income are entirely absorbed in additional savings. Since the maternal time constraint is binding, the assumption that maternal time input per pair of children is fixed constrains fertility to a constant, $n_t = 1/\hat{z}$. Thus, the introduction of child rearing goods is, in itself, not sufficient to generate a baby boom in Phase 1. Mothers need to be able to substitute goods for time.

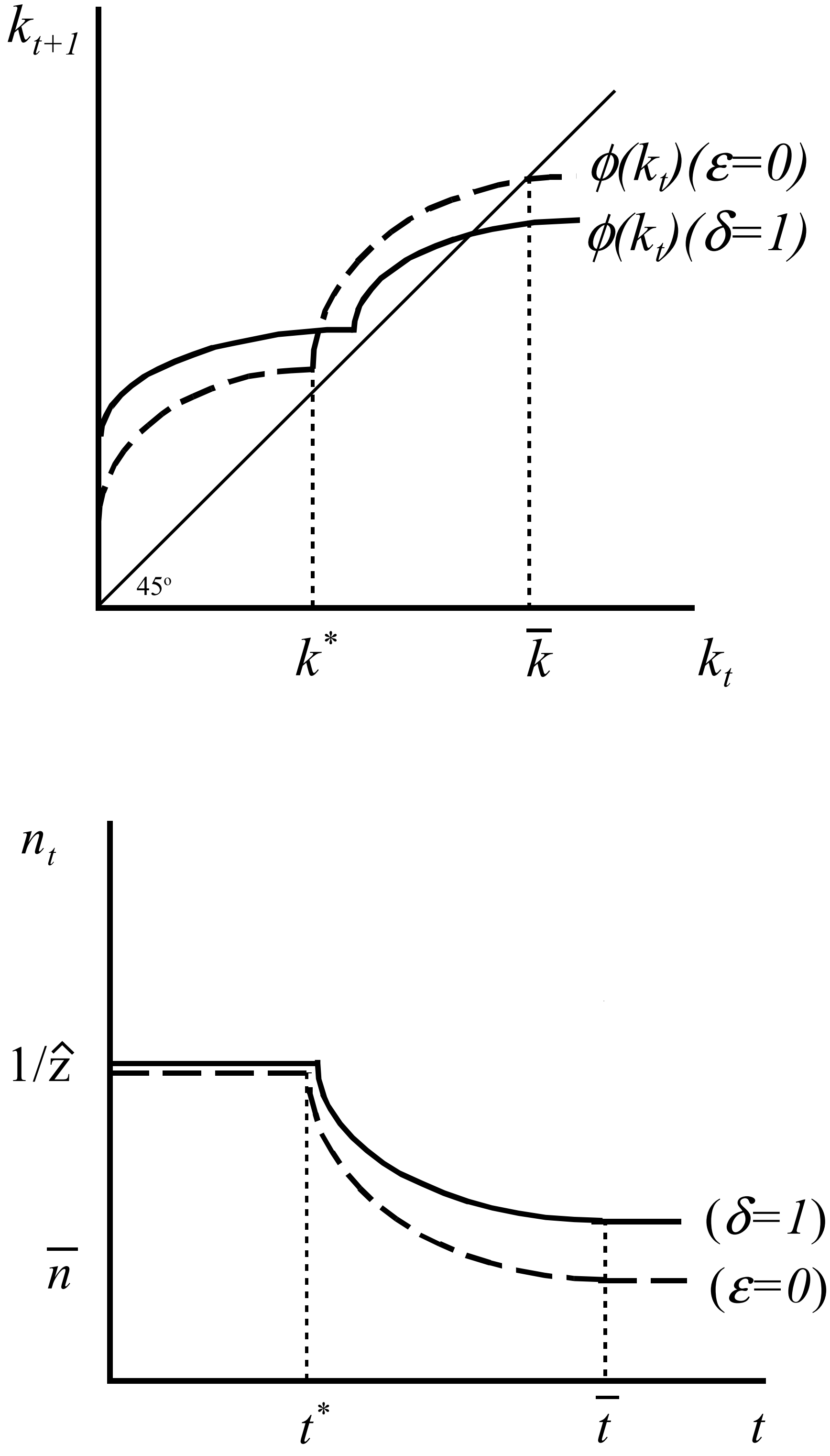
[22]See Section 3 for verification.

Phase 2, in Figure 5(b), begins with accelerated growth in capital (income) per household and a corresponding steep decline in fertility. As per Proposition 4, the inverse relationship between income per household and fertility may break down before the economy converges to a steady state. Rising female relative wages induce households to substitute child-care goods and services for maternal time. Once the maternal time input in child rearing is sufficiently low, the effect of rising wages on the cost of children is offset by the income effect. Thus, at time t^{**}, fertility ceases to decline with growth in income per capita. Any further rise in the wage for mental labor generates a baby bounce-back until the economy reaches steady state and fertility levels off because capital and income per household are no longer rising.

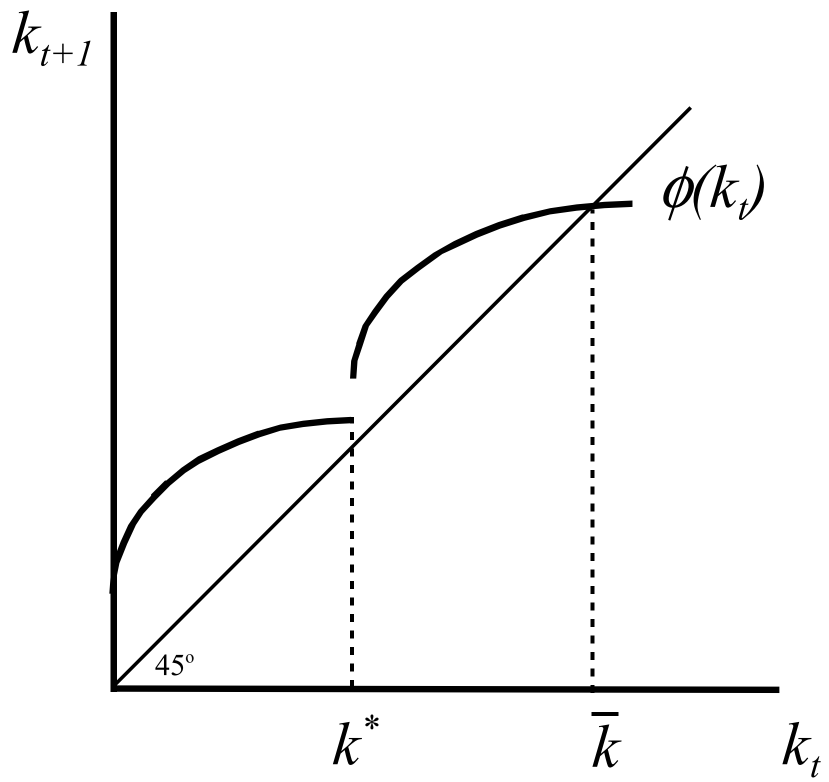
This contrasts Figure 5 (a) where, so long as income per capita is rising, fertility must decline. Of course, the path in Figure 5(a) is derived for a given goods-time input ratio. By the Corollary to Proposition 4, fertility will cease to decline with growth in income per household if goods and services come to dominate child-care inputs. However, in the limiting case $\varepsilon \to 0$, the likely path of the goods-time ratio over time remains a conjecture insofar as the input mix is an exogenous variable.

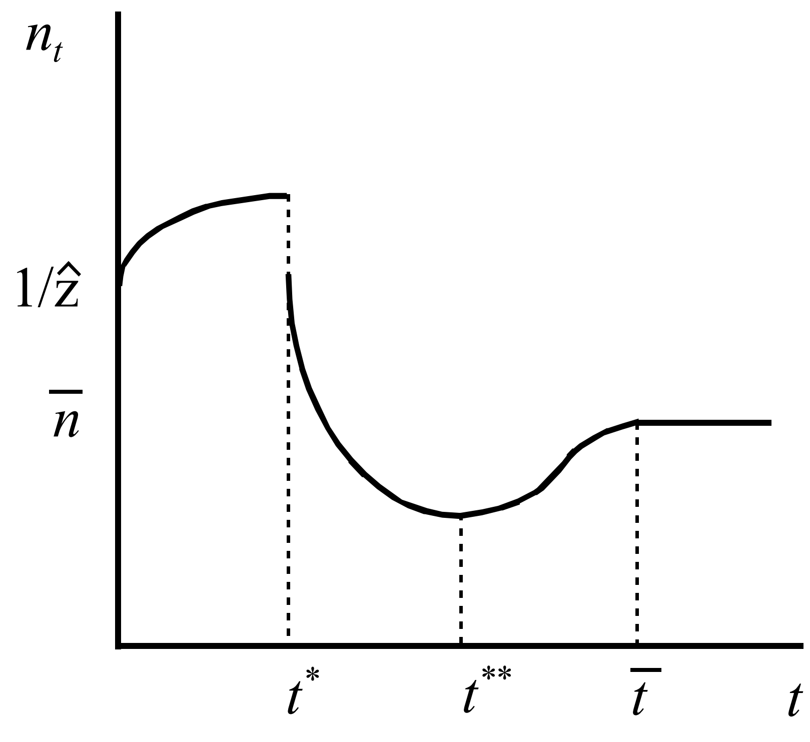
Figure 5: Evolution of capital per household and fertility

(a) Fixed quantities of time and goods



(b) CES production function for child rearing





5.2 Implications

Given that advanced economies are currently moving through Phase 2, with increasing labor force participation by women, what inferences can we draw for current policy debates?

As income per capita rises, what would the model presented in this paper predict?

1. Female labor force participation rises unambiguously, albeit at a decreasing rate.

2. Rising female labor force participation need not imply declining fertility.

3. Fertility may cease to decline before the economy reaches zero growth in per capita income in steady state.

4. Once fertility ceases to decline, any further wages growth generates a baby bounce-back.

5. A high rate of subsidy to child-care goods and services may postpone the onset of a naturally occurring baby bounce-back.

Policy

Proposition 5 has interesting implications for policy if governments are alarmed at the implications of fertility declining below replacement rates. Apps and Rees (2001) focus on the setting of child care subsidies and tax rates[23] to boost fertility. By confining their analysis to the comparative statics of steady state, in which income per capita is stationary, they forego a richer analysis of subsidies in light of rising female relative wages. Specifically, reducing the cost of bought in child care postpones an upturn in fertility induced by wages growth if the degree of complementarity between such goods and services and maternal time is relatively high. This suggests policy refocus on easing the substitutability of goods and services for time with, for example, more flexible child care / work arrangements.

The proof accompanying the corollary to Proposition 4 identifies maternal time's share of child rearing as equally important as the relative "price" of maternal time in determining the optimal child rearing input mix. The higher the share of maternal time in child rearing, the more protracted is the fertility decline. Policy effectiveness is limited to the extent that child rearing is inherently intensive in maternal time.

Cross-country differences

In addition to growth in female relative wages, the net price of child-care goods and services, maternal time's share in child rearing and the degree of complementarity between child-care goods and time all influence the path of fertility.

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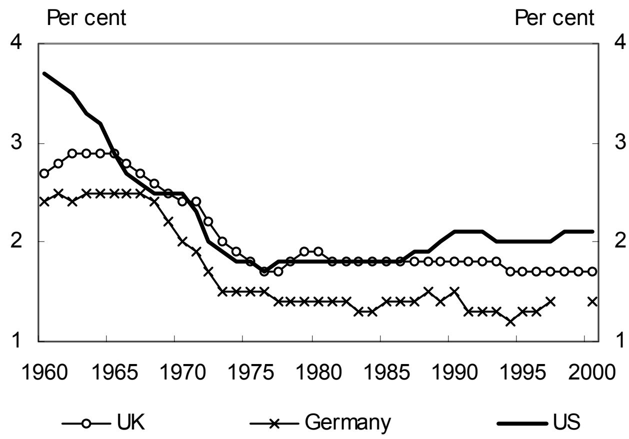
[23] The wage variable is easily amended to an after tax wage rate.

Differences in some of these parameter values could explain the purported cross-country differences in fertility decline. Across developed countries, the negative correlation between female labor supply and fertility does not seem to hold. For example, in 2000, the US is the only country with above-replacement fertility rate despite having a high percentage of women in the labor force; Germany has one of the lowest fertility rates as well as the lowest percentage of women in the labor force.[24]

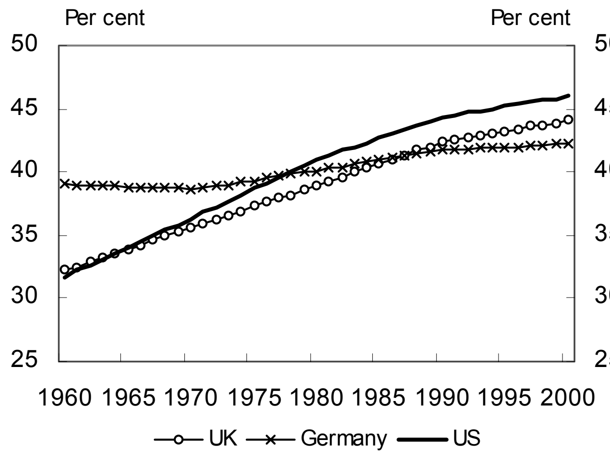
However, rather than draw inferences from a snapshot in time, I examine the dynamic interplay between fertility decline and the percentage of women in the labor force.

Figure 6: Selected G5 countries[25]

(a) Total Fertility Rate



(b) Females in Labor Force (% of)



Source: World Bank Tables

At first blush, Figure 6 supports a negative correlation between percentage of women in the labor force and fertility. For instance, Germany started in 1960 with the highest percentage of women in the labor force and the lowest fertility rate. Since then, a gradual rise in female labor force participation was met by a gradual decline in fertility.

Supporting the breakdown of the negative correlation due to substitution of child-care goods and services for maternal time, since 1975 the percentage of women in the labor force rose whilst fertility leveled off in most countries. In contrast to the UK, fertility experiences a final upturn in the US, despite both countries experiencing a similar rise in the percentage of women in the labor force.

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[24]Central to the negative fertility-female relative wage correlation is the reallocation of *time* from market work to child rearing. Participation rates may therefore be an inadequate correlate. For example, a high participation rate could be consistent with a small aggregate labor supply if most women worked only several hours a week.

[25]The selected countries depict the main cross-country differences. The fertility and labor force trends of France and Japan mimic those of UK and Germany, respectively.

Forward projections

Finally, forward projections of fertility decline should consider two questions: Will female wages rise relative to male wages to the same extent in the future? Has the wages growth to date caused households to substitute child-care goods and services for maternal time? Galor and Weil (1996) consider the first question in isolation. They present a special case of our model: as $\delta \to 1$, fertility must decline monotonically as income per capita rises in transition to steady state. Once we consider the second question, predicted fertility may assume a non-monotonic path in transition to steady state. This paper identifies the parameters central to answering the second question. From this, we may predict different paths in fertility for different advanced countries. We may also anticipate a baby bounce-back, to the extent that fertility ceased declining in most G5 countries, despite positive growth in per capita income.

6 Conclusion

In conclusion, the model presented in this paper generates a non-monotonic path in fertility for a developed economy moving through two phases, distinguished by a dramatic rise in female labor force participation.

As income per household grows, fertility:

- initially *rises* during the first phase when women allocate their total time endowment to child rearing;
- then *declines* as women enter the paid work force; and
- then may *bounce-back*.

In contrast to previous models, the inverse relationship between fertility and per capita income over the latter phase unravels over time. Rising female relative wages induce households to substitute child-care goods and services for maternal time, alleviating the cost pressure to reduce fertility. Once the wage for mental labor reaches a critical level, fertility ceases to decline with growth in income per capita. Any further rise in female relative wages causes an upturn in fertility.

Whilst the resemblance of the generated path in fertility to the G5 data illustrated in Figure 1 is compelling, I do not deny the importance of social and political changes in developed economies over this period. Arguably, some of these changes, such as the emancipation of women, are linked to growth in income per capita. Bongaarts (1999) offers a novel non-economic explanation for fluctuations in fertility. He argues that sudden changes in the mean age of child bearing distort measured fertility rates. A fall in the mean age of child bearing following World War II meant that births of successive cohorts overlapped in the same period, boosting observed fertility. Conversely, a rise in the mean age of child bearing over recent decades deflated observed fertility. However, after adjusting fertility rates for this distortion, van Imhoff and Keilman (2000) still found a remarkable baby boom-bust sequence.

Some interesting policy implications arise. A concomitant rise in fertility and female relative wages when women are in the labor force challenges the belief that reversing current fertility trends necessitates a reduction in female labor force participation. Of course, child-care goods and services must be sufficiently substitutable for maternal time for the rise in fertility to occur without a drop in female labor force participation. Traditional policies designed to eradicate declining fertility may have perverse effects. High rates of subsidy to child-care goods and services may postpone the onset of a fertility upturn induced by rising female relative wages.

Overall, introducing a CES production function for child rearing to Galor and Weil (1996) yields a rich dynamic interplay between fertility and income per capita. This paper makes an important contribution to endogenous fertility literature by both generating a baby boom without resorting to an outdated Malthusian assumption and challenging the projection of the recent baby bust into the future.

It would be useful to extend the analysis in several directions, such as recognizing human capital accumulation as central to the wages story as well as parental choice over fertility. Considerable richness and realism would be gained by drawing on the R&D based models of endogenous growth of Romer (1990) and Jones (1995). Both identify population, in level and growth rates, respectively, as the key determinant of technological progress. Galor and Weil (2000) incorporate endogenous fertility into a descriptive model of endogenous growth inspired by Romer (1990). Nesting endogenous fertility within a well-specified model of R&D is a feasible and challenging direction for future research.

APPENDIX

Proof of Proposition 1

$$\phi'(k_t) = \frac{1-\gamma}{\gamma}(1-\beta_t)\frac{\partial \hat{x}}{\partial x}\frac{\partial x}{\partial w_t^m}\frac{\partial w_t^m}{\partial k_t} > 0 \quad \forall \quad k_t \in (0, k^*)$$

Using $x_t = \hat{x}n_t$ and rearranging from (33),

$$\frac{\partial \hat{x}}{\partial x} = \frac{\partial(x/n)}{\partial x} = \alpha_1{}^a\left[\alpha_1{}^a + (\alpha_2 x_t)^a\right]^{\frac{-1-a}{a}} > 0$$

$$\frac{\partial x}{\partial w_t^m} = \frac{\gamma}{(1-\beta_t)} > 0$$

$$\frac{\partial w_t^m}{\partial k_t} = \alpha A(1-\alpha)(1-\rho)\left[\alpha k_t^\rho + (1-\alpha)\right]^{1/\rho-2} k_t^{\rho-1} > 0$$

$$\phi''(k_t) = (1-\gamma)\alpha_1{}^a\left[\alpha_1{}^a + (\alpha_2 x_t)^a\right]^{\frac{-1-2a}{a}}\left\{(-1-a)\alpha_2{}^a x^{a-1}\frac{\gamma}{1-\beta_t}\frac{\partial w_t^m}{\partial k_t} + \left[\alpha_1{}^a + (\alpha_2 x_t)^a\right]\frac{\partial^2 w_t^m}{\partial k_t{}^2}\right\}$$

Noting that

$$\frac{\partial^2 w_t^m}{\partial k_t{}^2} = \alpha A(1-\alpha)(1-\rho)k_t^{\rho-2}\left[\alpha k_t^\rho + (1-\alpha)\right]^{1/\rho-3}\left\{(1-\alpha)(\rho-1) - \alpha\rho k_t^\rho\right\}$$

where

$$\varepsilon_{L^m,K} > 1 \quad \Leftrightarrow \quad \rho \in [0,1) \quad \Rightarrow \quad \partial^2 w_t^m \big/ \partial k_t{}^2 < 0$$

$$(-1-a) < 0 \quad \Leftrightarrow \quad \varepsilon_{x,z} > 0.5$$

Given $\varepsilon_{L^m,K} > 1$, $\varepsilon_{x,z} > 0.5 \quad \Rightarrow \quad \phi''(k_t) < 0$

Proof of Proposition 4

Noting that (11) is homogeneous of degree 1, $z_t^* = \partial p / \partial w_t^m . n_t$ and $x_t^* = -\partial p / \partial \beta_t . n_t$.

$$\partial n_t * / \partial w_t^m = \frac{\gamma 2 \left[w_t^m . \partial p / \partial w_t^m + (1-\beta_t). - \partial p / \partial \beta_t \right] - \gamma (w_t^p + 2 w_t^m) \partial p / \partial w_t^m}{\left[p(w_t^m, \beta_t) \right]^2}$$

$$\partial n_t * / \partial w_t^m = \frac{\gamma 2 \left[(1-\beta_t). - \partial p / \partial \beta_t \right] - \gamma w_t^p \partial p / \partial w_t^m}{\left[p(w_t^m, \beta_t) \right]^2} < 0$$

$$\partial n_t \big/ \partial w_t^m < 0 \Leftrightarrow \frac{-\partial p / \partial \beta_t}{\partial p / \partial w_t^m} < \frac{w_t^p}{2[1-\beta_t]}$$

Substituting from (9) and (10),

$$\partial n_t \big/ \partial w_t^m < 0 \Leftrightarrow \frac{(\alpha_2)^{-a/a-1} (1-\beta_t)^{1/a-1}}{(\alpha_1)^{-a/a-1} \left(w_t^m \right)^{1/a-1}} < \frac{w_t^p}{2[1-\beta_t]}$$

$$\partial n_t \big/ \partial w_t^m < 0 \Leftrightarrow \left(\frac{\alpha_1}{\alpha_2} \right)^{a/a-1} \left(\frac{1-\beta_t}{w_t^m} \right)^{1/a-1} < \frac{w_t^p}{2[1-\beta_t]}$$

Given the definition of α_1, α_2 and ε,

$$\partial n_t \big/ \partial w_t^m < 0 \quad \Leftrightarrow \quad \left[\frac{w_t^m}{1-\beta_t} \frac{1-\delta}{\delta} \right]^{\varepsilon} < \frac{w_t^p}{2[1-\beta_t]}$$

$$\partial n_t \big/ \partial w_t^m = 0 \quad \Leftrightarrow \quad w_t^m = w^m ** = \left(\frac{w_t^p}{2} \right)^{1/\varepsilon} \left(\frac{\delta}{1-\delta} \right) \left(1-\beta_t \right)^{(\varepsilon-1)/\varepsilon}$$

Proof accompanying discussion of Proposition 5

$$w^m ** = \frac{\delta}{1-\delta}\left[e^{\ln(1-\beta_t)}\right]^{(\varepsilon-1)/\varepsilon}\left[e^{\ln\left(w_t^p/2\right)}\right]^{1/\varepsilon}$$

$$\frac{\partial w^m **}{\partial \varepsilon} = \frac{\delta}{1-\delta}\frac{1}{\varepsilon^2}(1-\beta_t)^{(\varepsilon-1)/\varepsilon}\left(\frac{w_t^p}{2}\right)^{1/\varepsilon}\left\{\ln(1-\beta_t) - \ln\left(\frac{w_t^p}{2}\right)\right\}$$

$$\frac{\partial w^m **}{\partial \varepsilon} < 0 \quad \Leftrightarrow \quad \frac{w_t^p}{2} > (1-\beta_t)$$

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