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Third-party Guarantees

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Spousal Guarantees*

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Abstract

Lenders often require of small investors that they sign a personal guarantee before forwarding funds, and if the borrower's own funds or assets are insufficient backing for a guarantee, then a third party may be asked to sign. Since strangers do not guarantee each other's debts, it is in the nature of such guarantees that they straddle the private and business worlds within any relationship. Important relationship assets (such as the family home) are often at stake, and courts struggle with the policy tradeoffs inherent in such deeds or contracts between 'freedom of contract' and a concern with the potential for 'coercion' of the one signing. This paper explores the nature of the optimal third party guarantee within the incomplete contracting environment inaugurated by Grossman and Hart (1986). The optimal contract trades off the ex post amount of relationship asset to be foreclosed by a bank against the desirability of ensuring the ex ante release of funds to promote the exploitation of viable investment opportunities. The role of ex post bargaining power, as a proxy for 'coercion' is examined and it is found that for certain parameter values in the model increased coercion, while freeing more funds ex ante, simultaneously lowers social welfare.

Journal of Economic Literature Classification: D14, D18, G21, G33, K12, K35.

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1 The problem

It is well-known that the debt structure of the majority of small to medium sized businesses is bank debt.[1] Such businesses have limited access to capital markets and are much more affected than larger firms by business cycle-related fluctuations.[2] It is known that in the event of financial distress banks rarely forgive principal.[3] Empirically, the more bank debt a company has, the more likely that asset sales will be forced upon it during bankruptcy.[4] Banks rarely lend to small businesses without the comfort of guarantees or collateral.[5] Even when such businesses operate via a corporate form that legally provides limited liability protection, banks generally insist on personal guarantees.[6]

This paper is concerned with the contractual phenomenon of third-party guarantees. If for some reason (such as the business being a start-up) the person seeking a loan does not possess business assets to act as collateral, a third party may be asked to act as guarantor in their stead. Since strangers do not act as guarantors for each other's debts, it is intrinsic to such guarantees that they are signed within the context of a longer-term, continuing, personal relationship. Concrete examples of people likely to be asked to act as a third-party guarantee include wives guaranteeing the loans of husbands (and vice-versa), parents the loans of children, grandparents the loans of grandchildren. A problem potentially arises when the asset which acts as collateral is an important relationship asset like the family home, which has a value to its occupants greater than the market value a foreclosing bank might receive. Although it might be thought that guarantors worried about the future loss of such an important relationship asset should then not sign a guarantee, such an approach

[1] See Gertler and Gilchrist (1994) and Petersen and Rajan (1994) for details and evidence.

[2] See Gertler and Gilchrist (1994).

[3] See Glassman and Struck (1982).

[4] See Dennis, Dunkelberg and van Hulle (1988).

[5] In a 1983 survey of its members, the National Federation of Independent Business found about 60 percent of firms with commercial bank loans provide collateral as security for the loan agreement. They also found that collateral secured 78 percent of the total volume of small business loans. Similarly, the Interagency Task Force on Small Business Finance found, in a survey conducted in 1982, that some form of collateral securing almost 80 percent of the dollar volume of large and small business loans from all sources. These figures are cited in Leeth and Scott (1989).

[6] See Chesterman (1982), and also Petersen and Rajan (1994) (summarizing economtric analysis of NFIB (1983)): "The owner's reputation is apparently more important than that of the business" (at page15).

ignores the potential for coercion in close domestic relationships. With perhaps an excessive regard for the ex post regret obviously felt in those instances when loans or loved ones turn sour, such third party guarantees have been dubbed in some legal scholarship a form of 'sexually transmitted debt'.[7]

During the nineties courts in the Anglo-American world grappled with the policy trade-offs involved in permitting the enforceability of third-party guarantees.[8] As an example the leading House of Lords case involved a wife suing to prevent a bank foreclosing on the matrimonial home. She had co-signed a guarantee as backing for business interests in which her husband was involved (and which did not directly involve her). In their decision the law lords were aware that any desire for paternalistic circumvention of the usual legal and economic norms of freedom to contract should be balanced against the concern that 'the wealth currently tied up in the matrimonial home does not become economically sterile'.[9] This paper analyzes the tension between freedom and regulation of third party guarantees, between the desire to permit efficiency increasing investment projects being undertaken and also to protect guarantors who for whatever reason the law might regard as more susceptable to coercive pressures within a relationship.[10] A ban on such guarantees would freeze forever all assets held in domestic use while unfettered freedom exposes a subset of guarantor's to intolerable risk of primary asset loss. The optimal guarantee contract will trade off these concerns.

Aware of legal concern about 'coercion', banking associations in the United Kingdom and United States have drawn up conventions which branch managers must take into account when presenting third-party guarantees for signing.[11] Such conventions include the

[7]See for example Fehlberg (1997).

[8]See the surveys of cases and jurisdiction by (for example) Fehlberg (1995) and Trebilcock and Ballantyne-Elliot (1998?).

[9]See Barclays Bank Plc v O'Brien [1994] 1 AC 180. The instance of signing of third party guarantees appears to be classifiable into two broad categories - those where the guarantee is for a start-up and those where the guarantee is for the refinancing of an already existing investment project. The Barclays Bank case concerned the latter, and such cases do appear to offer cause for greater concern than do the former type of cases (see Section 5 of this paper).

[10]The contractual legal doctrines protecting disadvantaged persons in common law countries fall under the rubric of equity. For a summary of equitable doctrines in contract see Hanbury and Martin: Modern Equity (2002).

[11]See for the UK BBA (1994).

requirement on lenders to provide basic information about the nature and possible consequences of signing a guarantee (like a 'health warning') and also to urge guarantees to seek independent (that is, independent of the guarantee's representatives) legal advice before signing.[12] The paradox appears to be that those aspects of a domestic relationship whose presence is likely to lead a court to suspect coercion in the signing of a guarantee are precisely those that make the guarantee valuable from a bank's perspective. Thus one bank manager reported in a survey study his belief that 'any borrower who is undergoing difficult financial times is far more likely to repay a debt which is secured on his or her home so that itself is a factor in assessing risk' while another reported that the family home in particular was an important 'motivational asset'.[13] As the author of the study concluded after her survey of lending institutions, "private commitments enhanced public enforceability":

> Lenders acknowledged the problems inherent in taking security from a person in an intimate relationship with the debtor, but they also emphasized the importance to them in commercial terms of the surety's emotional investments in both the *relationship* with the debtor and the *home* (where relevant). [emphasis original]

Economists familiar with the literature on financial contracting and the 'income diversion' stories therein (see chapter 5 of Hart (1995)) will be unsurprised by these expressions by lenders of the desirability of any factor that forces the borrower to pay out funds rather than default, both in times of financial distress and even in good times.

While data on the incidence of third party guarantees is not available (and not easily made available), the significance of internal financing for young or small businesses is known. Thus Petersen and Rajan (1994), summarizing the National Survey of Small Business Fincances (NSSBF) collected during 1988 and 1989 under the guidance of the Board of Governors of the Federal Reserve System and the Small Business Association note that (at page 8):

[12]These are the same type of measures (although perhaps strengthened) adopted by the House of Lords in its *Barclays Bank* decision.

[13]See Fehlberg (1997) at page 204.

> The smallest 10 percent of firms in our sample borrow about 50 percent of their debt from banks. Another 27 percent comes from the firm's owners and their families. The table [referring to Table II on page 9 of their article] shows that the fraction from personal (owner and family) sources declines to 10 percent for the largest 10 percent of firms in our sample.

On the same page they go on to note that "The youngest firms (age less than or equal to 2 years) rely most heavily on loans from the owner and his or her family" and again on page 10 "[F]irms follow a 'pecking order' of borrowing over time, starting with the closest sources (family) and then progressing to more arm's length sources."[14] This data would seem to suggest that, if data on the incidence of third party guarantees were available, it would most likely be concentrated among young and/or small firms. They are likely to play an important role in many start-up companies.

Obviously the first best can be achieved if comprehensive contracts could be written ex ante. But the type of relationships modelled in this paper are not in reality governed by comprehensive contracts or indeed often by any formal contract at all. Indeed, in some common law jurisdictions pre-marital contracts are not permitted as a matter of public policy, and even in those jurisdictions where they are, their content is heavily circumscribed again on public policy grounds.[15] Even if they are permitted, there may be adverse signalling reasons preventing their widespread adoption.[16] The 'incompleteness' of the guarantee contract permits ex post renegotiation and hence the possibility of that 'coercion' or hold-up that exercised the minds of common law courts during the nineties. Paradoxically, it will be seen that what is 'bad' for the guarantor (coercion) is 'good' from the point of view of

[14] The actual figures are: "The youngest 10 percent of firms in our sample borrow an amount equal to 0.32 of their book assets, while the oldest 10 percent of firms in our sample borrow only 0.15. The smallest 10 percent of firms in our sample borrow 0.22 of their book assets while the largest 10 percent of firms in our sample borrow 0.30 of their book assets." (at page 10, footnote 8).

[15] See Sanford N. Katz (Editor), Cross Currents: Family Law Policy in the US and England (2001).

[16] See for example Spier (1992). An obvious reason why complete guarantees are rarely seen (that is, contracts which not only involve clauses concerning the loan but also the domestic relationship which forms the context of the request for the guarantee) is the non-verifiable nature of many of the relationship variables, and it is known that when some are unverifiable, it may be in the contractual parties' interests to leave other, verifiable variables unspecified also (see Holmstrom and Milgrom (1992) and Bernheim and Whinston (1995)).

overall societal welfare (more projects are financed ex ante), though only up to a point.

Relationship to literature The paper contributes to that recent literature which analyzes financial decisions from the 'incomplete contracting' perspective inaugurated by Grossman and Hart (1986) and applied to the financial contracting setting by Aghion and Bolton (1992), Hart and Moore (1994) and Hart and Moore (1998).[17] In these papers (let's call it the standard framework) a financially constrained entrepreneur seeks funds from an investor in order to exploit an investment opportunity. The funds are used to buy project assets which in turn generate return streams. The theory assumes that, at the time the loan contract is written, the parties to the contract are not able enforceably to condition on all future states of the world (especially return streams), so that the contract instead must specify who gets control of the project assets in the event the entrepreneur defaults. Because loan repayment cannot be conditioned on return streams (meaning that any contractually specified repayment amount can be renegotiated), default can occur strategically and not just because returns are low. To minimize the incentives for such strategic default the investor must liquidate part of the project assets in the event of non-strategic default, even though such liquidation is ex post inefficient. The reason such liquidation has the right incentive effects is because future project return streams (which accrue only to the entrepreneur) depend on the entrepreneur controlling the project assets. It is this which gives the asset control decision in the standard framework an important 'leverage' effect between entrepreneur and investor. The current paper differs crucially from this standard framework in that the asset on which the security is taken is not a project asset but a 'relationship' asset independent of the entrepreneur's business. Consequently, signing a security shifts the leverage from the investor/entrepreneur relationship to the entrepreneur/guarantor relationship.

This indicates another aspect of the current paper that differs from the standard framework, in that the model involves three agents rather than two. Papers extending the standard framework to more than one investor include Bolton and Scharfstein (1996), De-

[17]For a summary of the standard framework and related literature see chapter 5 of Hart (1995). For another early example utilizing a similar style of incomplete contracts model, but analyzing predation in industrial organization theory instead, see Bolton and Scharfstein (1990).

watripont and Maskin (1995), Dewatripont and Tirole (1994) and Berglöf and von Thadden (1994). The first two involve multiple investors with the same asset claim while the second two explore the effects of having different investors hold different asset claims. In both types of paper the purpose of multiple investors and/or multiple claims is to harden the budget constraint of the entrepreneur and consequently ease the threshold borrowing condition for the investors, thus enabling more loans to be provided (and more investments made). Indeed, anything (such as more agents involved in the bargaining, or asymmetric information in the ex post bargaining) which makes renegotiation harder to achieve will have this effect on the entrepreneur, with the cost of course that the possibility of efficient renegotiation (that is, when default has been necessary) is lost.

Outline of paper Section 2 outlines the basic structure of the model and section 3 solves for the optimal contract. Section 4 explores the relationship between coercion and investment determines when only an outside asset is available to the parties (the paradigmatic case), while 5 assumes that both types of asset are available and explores the conditions determining when total versus partial foreclosure is preferred, as well as the conditions under which third-party guarantees are preferred to personal guarantees. Section 6 concludes.

2 The model

2.1 The ex ante contract

The agents At date 0 a bank (denoted B), an entrepreneur (denoted E) and a guarantor (denoted G) convene to sign a guaranteed loan contract to enable the entrepreneur to invest in a long-term profitable project.[18] The entrepreneur must borrow funds because he is wealth-constrained; in particular, we assume that he has zero (liquid) wealth ex ante, and we assume the same for the guarantor. All agents are risk neutral and discount factors are normalized to zero.

[18] Hereafter the entrepreneur is referred to generically as ‘he’ and the guarantor as ‘she’.

The project The project lasts two periods. There is no intertemporal interest rate. The project provides non-negative returns of $\tilde{R}_1$ (a random variable with support $\{0, x\}$) at date 1 and $R_2 = r$ at date 2. In the first instance these returns accrue to the entrepreneur.[19] The commonly known distribution on $\tilde{R}_1$ is given by:[20]

$$\tilde{R}_1 = \begin{cases} 0 & \text{with probability } 1-\theta, \\ x & \text{with probability } \theta. \end{cases}$$

The project's initial cost is $K > 0$. The project is ex ante viable or productive since $\theta x + r > K$ regardless of θ. Thus, we have biased the spousal guarantee problem in favor of financing indubitably worthwhile investments. The amounts R_1 and r are uncontractable: that is, ex ante describable but ex post unenforecable (or 'observable' but not 'verifiable' in the language of Grossman and Hart (1986)).

The loan The entrepreneur borrows K from the bank at date 0 for a promise to repay P at date 1.[21] The promised repayment amount P is also uncontractable. One way to think about this in concrete terms is to imagine the existence of a 'savings account' belonging to the entrepreneur into which the return is deposited when it accrues. Any amount in this 'savings account' is untouchable by the bank, even in the event that the entrepreneur defaults on the repayment of P. This is the 'diversion' or 'stealing' assumption of Hart and Moore (1998), a possibly extreme but nonetheless useful assumption designed to capture the more realistic phenomenon of managerial discretion in the use and disbursement

[19] These returns are specific to the entrepreneur - that is, neither the bank nor the guarantor can obtain these returns from the project without the entrepreneur. However, we do not model the process by which the entrepreneur generates these returns, assuming instead that they are exogenously given.

[20] Note that there is no loss of generality in confining attention to a two-state date 1 return, since even if R_1 is an interval (say $\mathbb{R}^+$) it is clear that it is never optimal for the entrepreneur to make a partial payment, so that default in that more general case would be defined as not paying anything at all at date 1.

[21] The date 0 contract to be signed between the bank and the entrepreneur is a *standard debt contract*, namely (B, P). That is, the bank agrees to lend B at date 0 for a promise by the entrepreneur to repay the *non-contingent* amount P at date 1. This paper does not consider the issue of whether other, more elaborate, types of debt contract would be pareto superior to the standard debt contract examined here: for example contracts utilizing options to own a là Nöldeke and Schmidt (1995), or contracts utilizing, in the tradition of Maskin (1999), ex post message games such as is examined in the appendix to Aghion and Bolton (1992) or in the latter half of Hart and Moore (1998) (where necessary and sufficient conditions for a standard debt contract to be optimal are derived).

of corporate funds. At least within the context of family businesses a reason for such untouchability lies in the ability of entrepreneurs potentially to divert business profits into family gifts and trusts.

Because the date 1 return is describable, the date 0 contract can stipulate the date 1 payment P to be conditional on $\tilde{R}_1$. Thus the contract can stipulate that at date 1 the entrepreneur should repay P_0 when $R_1 = 0$ and P_x when $R_1 = x$. However, because any contract terms conditioned on $\tilde{R}_1$ are ex post unenforceable, we need to denote the *actual* payment made by the entrepreneur at date 1 by $\hat{P}$. We restrict this date 1 action set to be the same as the date 0 contractually specified repayment schedule: $\hat{P} \in \{P_0, P_x\}$. Actual date 1 repayment is contractable.

Security asset Because of the noncontractability of the return stream, the bank requires security for the loaned funds K. There exist two types of asset which might act as security. Call the combined assets AH. A security on the assets (either or both) is contractable.

The first asset (asset A) is a business asset that will be bought with the borrowed funds. It lasts one period. This asset is essential to the production process: in combination with the entrepreneur's skill it produces the return stream over the two periods. If the asset is liquidated at date 1, then the entrepreneur is unable to earn the date 2 return r. The date 1 liquidation value to the bank is $L^A = \alpha r$, where $\alpha \in [0, 1)$.

The second asset (asset H) is a shared non-liquid relationship asset which is completely independent of the business. It has a deterministic market value of z, which can be interpreted as the value of (say) a family home to its occupants. The date 1 liquidation value to the bank is $L^H = \lambda z$, where $\lambda \in [0, 1)$. This modelling assumption captures the fact that the relationship asset is worth more when maintained as a relationship asset than when in the possession of the bank. Specifically, it captures the fact that a relationship asset like a family home provides a value to its occupants not encapsulated in liquidated sale price alone.

Note that when both assets are used as security, the date 1 liquidation value of the assets AH to the bank is $L^{AH} = \alpha r + \lambda z$. These liquidation values constitute ex post exogenously determined inefficiencies which play an important role in the ex post renegotiation to be described below. They are summarized in Panel A of Table 1.

Relationship asset share At date 2 (when the model ends) the relationship asset is sold and consumed by the entrepreneur and/or guarantor according to their exogenously determined share of the asset. Let $S^E \in [0,1]$ denote the entrepreneur's date 2 share of the relationship asset (or the date 2 sale proceeds thereof). Hence the guarantor's share is $(1 - S^E)$.

Entrepreneur's promised payment to guarantor The guarantor must be compensated for the risk of permitting (her share of) the relationship asset to act as security for the loan. We denote by y_i (where $i \in \{0, x\}$) the amount the entrepreneur promises to pay the guarantor at date 2 (conditional on the entrepreneur's date 1 actual repayment $\hat{P}$) in return for her permitting the relationship asset to be utilized as security. Note that y_i need not be interpreted as an explicit payment arising out of the guarantee contract but can be interpreted more expansively as the promise of a 'standard of living' arising out of the relationship. The y_i's are enforceable since they are conditioned on actual date 1 repayment by the entrepreneur. Such enforcement can be interpreted as divorce law in the case of spoual guarantees.

Contractual provision for default We will assume that the assets are discrete so that they cannot be partially liquidated. In the case of a family home at least this assumption is realistic. The most general type of default provision then specifies that when the entrepreneur makes a date 1 payment $\hat{P}$, the bank has the right to liquidate the secured asset(s) with probability $\beta(\hat{P}) \leq 1$.[22] The date 0 contract will therefore specify that when the entrepreneur makes the payment P_x the bank has the right to liquidate the secured asset with probability β_x, and when the entrepreneur makes the repayment P_0 the bank has the right to liquidate the secured asset with probability β_0. The β_i's (where $i \in \{0, x\}$) are enforceable since they are conditioned on actual date 1 repayment by the entrepreneur.

[22] A model of this type was first used in Bolton and Scharfstein (1990).

Payoffs Payoffs for the agents are described in the next section when the optimal guarantee is solved. They are linear in income/payments and (for the entrepreneur and guarantor) linear in asset share and (for the bank) linear in expected foreclosure value. The relative greater importance of the relationship asset to the guarantor vis-a-viz the entrepreneur is modelled via a weight function $\phi(\lambda)$ which is increasing in λ, explained further below.

Contractability It is useful to summarize the contractability assumptions in the model. The return stream and agent payoffs are not contractible, while the asset(s) and the entrepreneur's *actual* date 1 payments are contractible. Contractibility assumptions rest on the idea that it is easier to divert cash flow than physical, non-liquid assets, a distinction emphasised in Hart and Moore (1998).

The date 0 contract We assume that the entrepreneur has all the ex ante bargaining power and chooses the date 0 contract $\Gamma = \{P_0, P_x, \beta_0, \beta_x, y_0, y_x\}$. The first two terms (conditioned on $\tilde{R}_1$) are not enforceable but the remaining terms (conditioned on $\hat{P}$) are enforceable. Since the first two terms are not enforceable, at date 1 the entrepreneur might choose to 'deviate' from the loan repayment amounts specified in Γ.

The timeline of the game at date 1 is as follows. First nature moves determining whether the date 1 return is either x or 0. Then the entrepreneur decides whether to pay P_x or P_0. Depending on this vbnrepayment amount, the bank acquires the right to foreclose on the secured asset(s) with probability β_x or β_0. However, liquidation of the secured asset(s) is not automatic because such liquidation is ex post inefficient. The agents would prefer to renegotiate the ex ante contractual terms β_x and β_0, setting them to zero and dividing amongst themselves the ex post surplus thereby saved. If renegotiation suceeds then the secured asset(s) is not liquidated and if it fails then it is liquidated by the bank. The specifics of renegotation is outlined in subsection 2.2. The timeline for the complete model is summarized in Figure 1.

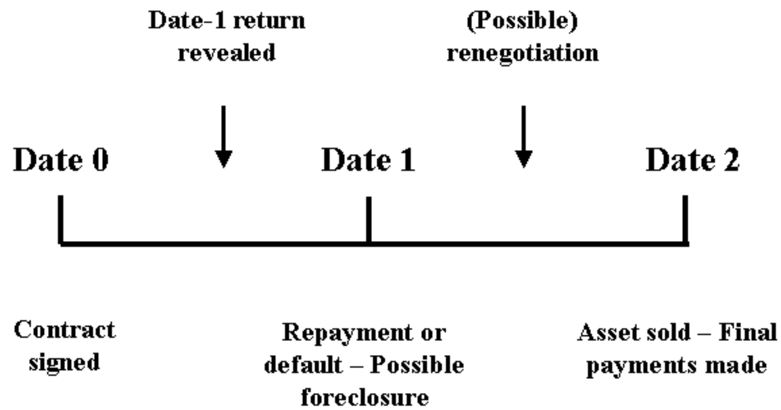

Figure 1: Spousal Guarantee Timeline

First-best If a comprehensive contract could be signed, then given the assumptions on the productivity of the project the entrepreneur would have no difficulty getting a bank to finance the project, and the first-best would be achieved. Note therefore that securing the loan would not be necessary and, if nonetheless undertaken, liquidation would never be part of a first-best outcome. However, the inability to contract on the return stream means that, without a mechanism to enforce date 1 repayment, no bank will lend to the entrepreneur in spite of the overall viability of the project. The usual mechanism in the literature is a security over the project asset A that is bought with the borrowed funds. Since the entrepreneur values continuance of the project, the possibility of liquidation of A at date 1 gives the bank leverage over the entrepreneur ensuring that the latter pays the loan out of the date 1 return stream. Securing the relationship asset rather than the project asset switches the leverage problem from the bank/entrepreneur relationship to the entrepreneur/guarantor relationship, as we now see.

Table 1: **Table of ex post values**

Panel A	
Secured Asset	Foreclosure Value
$i = A$	$L^A = \alpha r$
$i = H$	$L^H = \lambda z$
$i = AH$	$L^{AH} = \alpha r + \lambda z$
Panel B	
Secured Asset	Total Ex Post Surplus Value
$i = A$	$\Pi^A = r - \alpha r$
$i = H$	$\Pi^H = z - \lambda z$
$i = AH$	$\Pi^{AH} = r + z - (\alpha r + \lambda z)$
Panel C	
Secured Asset	Entrepreneur's Share of Ex Post Surplus
$i = A$	$g_E^A = \tau_E(r - 2\alpha r)$
$i = H$	$g_E^H = \tau_E(z - 2\lambda z)$
$i = AH$	$g_E^{AH} = \tau_E[r + z - 2(\alpha r + \lambda z)]$

2.2 The ex post renegotiation

Renegotiation takes the form of the entrepreneur bribing the bank not to exercise its right to liquidate the foreclosed asset(s). Since the entrepreneur has zero ex ante (liquid) wealth, such a bribe is only possible in the case where $R_1 = x$. If the payment of P_0 is called a *default*, then default can either be either *strategic* or *necessary* according to whether it occured when R_1 equalled x or 0 respectively. Consequently there can be no renegotiation after a necessary default while after a strategic default renegotiation is possible since the funds potentially exist to 'buy back' the seized asset(s)).

The ex post surplus over which the agents renegotiate depends on which asset(s) is used as security. Denote by Π^i the social surplus salvaged by the parties when the liquidation of asset i (where $i = A, H$ or AH) is prevented via renegotation. These different amounts are shown in Panel B of Table 1. Note that they depend on the exogenously given liquidation values so that the ex post surplus is also exogenous.

In the literature a number of bargaining conventions are used, the most common being the Nash bargaining solution where the parties equally split the surplus. Because in sections

4 and 5 we wish to explore the effects of differing bargaining power between entrepreneur and guarantor, in this paper renegotiation takes the form of Generalized Nash Bargaining. We assume that the bank is exactly compensated for the loss of liquidation value which it gives up, and that the entrepreneur and guarantor then engage in two-way bargaining over the surplus that remains. The agents are exogenously endowed with ex post bargaining power τ_i (where $i = E$ or G) which sum to one. Appendix A presents the details while Panel C of Table 1 summarizes the results.

As a final note to renegotiation, it is worth pointing out that since the relationship asset H is contractable and becomes liquid at date 2, it is possible within this model for the entrepreneur to propose at date 1 (say in exchange for forbearance on the part of the bank in foreclosing on the project asset A) a share of the date 2 relationship asset proceeds. However, for convenience in what follows we rule out this possibility of 'constant recontracting' on the security over H.

3 The optimal guarantee

In this section we set out the optimization program that the entrepreneur solves at date 0 and use it to characterize the optimal contracts for each of the three possible cases of secured asset, namely, for asset i where $i \in \{A, H, AH\}$. We state the model in its general form, and then at the end briefly give the concrete forms of the solution variables that will be needed in the rest of the paper (in fact, only β) for each of the three cases of $i = A$, H and AH. We are interested in the renegotiation-proof contract, where the possibility of future renegotiation is anticipated by the parties when they come to draw up the date 0 contract so that renegotiation does not in fact occur in equilibrium. Renegotiation-proofness manifests itself in the optimization problems in the form of an added constraint.

At date 0 the entrepreneur solves the following linear program (call it $(\bigstar^{(i)})$), choosing over $\Gamma^i = \{P_0, P_x, \beta_0, \beta_x, y_0, y_x\}$ to maximize his expected payoff[23]

$$\theta[x - P_x - y_x + (1 - \beta_x \kappa_r^i) r + (1 - \beta_x \kappa_z^i) S^E z + \beta_x g_E^i] \tag{1}$$
$$+ (1 - \theta)[-P_0 - y_0 + (1 - \beta_0 \kappa_r^i) r + (1 - \beta_0 \kappa_z^i) S^E z]$$

subject to the individual rationality constraint of the bank (IRB)

$$\theta[P_x + \beta_x L^i] + (1 - \theta)[P_0 + \beta_0 L^i] \geq K \tag{2}$$

as well as to the individual rationality constraint of the guarantor (IRG)

$$\theta[y_x + (1 - \beta_x \kappa_z^i)(1 - S^E) z + \beta_x g_G^i] \tag{3}$$
$$+ (1 - \theta)[y_0 + (1 - \beta_0 \kappa_z^i)(1 - S^E) z] \geq (1 - S^E) z$$

and, in order to ensure that the entrepreneur does not strategically default when $R_1 = x$, subject also to the following 'renegotiation constraint' (RC)

$$x - P_x - y_x + (1 - \beta_x \kappa_r^i) r + (1 - \beta_x \kappa_z^i) S^E z + \beta_x g_E^i \tag{4}$$
$$\geq x - P_0 - y_0 + (1 - \beta_0 \kappa_r^i) r + (1 - \beta_0 \kappa_z^i) S^E z + \beta_0 g_E^i$$

and subject to the following 'limited liability' constraints for the entrepreneur and guarantor owing to the assumption of ex ante zero liquid wealth

$$P_0 \leq 0 \text{ and } P_x \leq x$$

$$0 \leq y_0 \leq r \text{ and } 0 \leq y_x \leq x + r - P_x \tag{5}$$

and finally subject to feasibility constraints on the foreclosure probabilities

$$0 \leq \beta_0, \beta_x \leq 1$$

There are three models in one here, depending on whether $i = A$, H or AH.[24] The difference between the models depends only on the different values of κ_r^i and κ_z^i for each

[23]Note that each of these contractual terms should also have an i superscript, but to avoid notational clutter we omit them.

[24]There is of course a fourth model entailed in this general form, which of course we ignore, namely, the case of no security at all.

value of i. The κ's are indicator functions, taking on the values of zero or one only. In particular we have

$$\kappa_r^i = \begin{cases} 1 & \text{when} \quad i = AH, A \\ 0 & \text{when} \quad i = H \end{cases}$$

and

$$\kappa_z^i = \begin{cases} 1 & \text{when} \quad i = AH, H \\ 0 & \text{when} \quad i = A \end{cases}$$

The payoffs of each of the three agents are written assuming that the contractual terms of Γ^i are honored. Thus, equation (1) shows the entrepreneur's payoff for the two cases when $R_1 = x$ and he pays the bank P_x (and the guarantor y_x) and when $R_1 = 0$ and he pays the bank P_0 (and the guarantor y_0). When P_x is paid then with probability $1 - \beta_x$ the entrepreneur keeps the secured asset(s) while with probability β_x he needs to buy it back and then split the surplus with the guarantor, giving him g_E^i. The payoffs for the bank and guarantor are derived in the same way. Regarding equation (4), the LHS is taken from the LHS of (1) while the RHS has the same form except that now the entrepreneur has paid P_0 so the other contractual terms (β and y) conform to that payment. The second renegotiation constraint, ensuring that the entrepreneur pays P_x instead of P_0 when $R_1 = 0$, is otiose because when income is zero it is not feasible for the entrepreneur to pay P_x due to the assumption that the entrepreneur is wealth constrained. For the same reason we need not include renegotiation payoffs for either the entrepreneur or the guarantor for the case when $R_1 = 0$ since they are automatically zero. Finally, both sides of both inequalities of 5 model the fact that, for both the guarantor (the LHS of both inequalities) and the entrepreneur (the RHS of both inequalities), the relationship asset is untouchable when it comes to agreeing on the possible amounts of the side payments (the y's) between them.

The following proposition and corollary characterize the optimal contract.

Proposition 1 (Contract Characterization) *In the optimal contract*

(i) $P_0 = 0$,

(ii) $y_0 = y_0(1 - \kappa_r^i) \geq 0$,

(iii) $\beta_x = 0$,

(iv) both the bank's and guarantor's individual rationality constraints bind,

(v) the renegotiation constraint binds.

Proof See Appendix B. ■

The proofs of parts (i) and (ii) are trivial, following immediately from assumptions on the contracting technology made in subsection 2.1. As part (ii) of the propostition indicates, the main difference between the solutions to $(\bigstar^{(AH)})$ and $(\bigstar^{(A)})$ when compared with the solution to $(\bigstar^{(H)})$ is that in the latter case y_0 is not *necessarily* zero. The reason is that, when $R_1 = 0$ and the entrepreneur pays (as the contract requires) P_0, foreclosure of the secured relationship asset with probability β_0 (as the contract requires) still leaves the project asset available to generate the date 2 return r, from which a positive payment to the guarantor can be made if desired. Unsurprisingly, now that it is only the relationship asset which carries the security, the guarantor needs to be compensated even more for the greater risk of offering up her property. In fact it will be seen that in this case $y_0 = r$, the maximum possible payment. Part (iii) is proved by showing that a strictly positive β_x cannot be optimal, since in that case decreasing β_x without changing the payoffs of the bank and guarantor strictly increases the entrepreneur's payoff. The intuition for the result is that the contract needs to provide the entrepreneur with incentives not to strategically default. Foreclosing on the secured assets when $R_1 = x$ gives the entrepreneur precisely the opposite incentives from the point of view of the goal of preventing strategic default. Part (iv) is proved by utilizing the fact that the entreprenuer, who has all the ex ante bargaining power, maximizes his payoff by paying both the bank and guarantor as little as possible. The proof of part (v) is by contradiction - the relaxed program is solved and the result shown to contradict the ignored renegotiation constraint. Essential to the proof is the assumption that the y's cannot be paid out of the guarantor's (or entrepreneur's) share of the relationship asset. The intuition is that the renegotiation constraint must bind in order to provide the incentive for the entrepreneur to repay the debt in the good income state, since no other reason exists for him to repay the loan in that state. If there is no incentive for the entrepreneur to repay in the good income state, then the guarantor will not sign the contract.

Although we do not show it explicitly, it is worth pointing out one difference between the three specific models. The main difference between the solutions to $(\bigstar^{(AH)})$ and $(\bigstar^{(H)})$ when compared with the solution to $(\bigstar^{(A)})$ is that in the latter case $y_x = 0$. Since the guarantor's asset is not at risk, paying a positive y_x simply makes the guarantor an investor in the project without any quid pro quo, something that is clearly not in the entrepreneur's interest to permit. However, a negative y_x is not permitted by 5, so the optimal payment to the guarantor sets y_x equal to zero: the guarantor is irrelevant. It should also be noted that in model $(\bigstar^{(A)})$, even though there is no relationship asset so that the presence of the guarantor is irrelevant to that specific model, nonetheless, because of the ex post bargaining convention adopted, she still obtains some positive surplus. This is an artifact of all generalised Nash-type bargaining models.

Corollary 1 *the optimal β_0 is bounded away from zero.*

Proof The corollary follows immediately from the proof of part (v) of proposition 1. See Appendix B for details. ■

Even though asset foreclosure is ex post inefficient when $R_1 = 0$, nonetheless it will occur. This inefficiency arises from the twin effects of limited liability and contractual incompleteness. The model is characterized by the fact that the greater the ex post *inefficiency* when $R_1 = 0$, the greater the ex ante *efficiency*. Stated another way, there exists a trade-off between ex post costs of financial distress and ex ante efficiencies in ensuring that viable projects are undertaken. Financial distress in the case of third-party guarantees involves important relationship asset loss, which hits the guarantor in particular hard. From the point of view of a court deciding between ex ante versus ex post efficiency, it is not obvious normatively which should receive judicial preference. Ex post efficiency should be favored in the case of a one-shot decision, but in a repeated context such reasoning undercuts the valuable commitment role of the law in enabling contracting parties to use *[. . .]*. How the courts should balance these twin concerns has long been an issue in jurisprudence.

Proposition 1 enables us to simplify $(\bigstar^{i})$ and consequently to find this optimal level of contractual inefficiency.[25]

Proposition 2 (Contractual Inefficiency)

(i) In the optimal contract, the efficiency loss due to contractual incompleteness is

$$EL^i \equiv (1-\theta)\beta_0^i[\kappa_r^i r + \kappa_z^i z - L^i] \tag{6}$$

(ii) while the optimal foreclosure probability is

$$\beta_0^i \equiv \frac{K-(1-\kappa_r^i)y_0}{\theta[\kappa_r^i r + \kappa_z^i S^E z - g_E^i] + (1-\theta)[L^i - \kappa_z^i(1-S^E)z]} \tag{7}$$

(which will be a solution to ($\bigstar^i$) provided the RHS is not greater than one).

Proof See Appendix B. ■

The efficiency loss due to contractual incompleteness is the expected loss of surplus when the date 1 return is zero (with probability $1-\theta$) and the bank gets the right to foreclose on the secured asset(s) (with probability β_0^i). The term in square brackets in 6 is the exogenous ex post surplus Π^i.

Equations 6 and 7 can be written in terms of the model's exogenous parameters (from Table 1). Depending on the value of i, this table will give different parameterized forms for the two equations. For example, for the case of $i = AH$ we have

$$\beta_0^{AH} \equiv \frac{K}{\theta[r + S^E z - \tau_E(r + z - 2[\alpha r + \lambda z])] + (1-\theta)[(\alpha r + \lambda z) - (1-S^E)z]} \tag{8}$$

$$EL^{AH} \equiv \frac{(1-\theta)K[r + z - (\alpha r + \lambda z)]}{\theta[r + S^E z - \tau_E(r + z - 2[\alpha r + \lambda z])] + (1-\theta)[(\alpha r + \lambda z) - (1-S^E)z]} \tag{9}$$

and the parameterized forms of β_0 and EL can be easily found for the other two cases also. These parameterized forms will be used in the rest of the paper.

[25] Here we are re-inserting the superscript on the contract variable β. (See footnote 23)

4 Outside security assets only

Often the parties to the transaction are unlikely to have the luxury of an inside asset. This section examines the paradigmatic case of outside guarantees and the possibility of coercion when only the relationship asset is able to be secured. That is, we focus on the case of asset H alone.

Writing β_0^H and EL^H in terms of the model's exogenous parameters (from Table 1) gives

$$\beta_0^H \equiv \frac{K - r}{\theta[S^E z - \tau_E(z - 2\lambda z)] + (1-\theta)[\lambda z - (1 - S^E)z]} \tag{10}$$

$$EL^H \equiv \frac{(1-\theta)\{K - r\}[1-\lambda]}{\theta[S^E - \tau_E(1-2\lambda)] + (1-\theta)[\lambda - (1 - S^E)]} \tag{11}$$

Note that the efficiency loss does not depend on the value of the secured asset z. The numerator of 10 can be either negative or positive depending on whether the second period return is greater than or less than the project cost. The paradigmatic case might be thought to involve a residential house value orders of magnitude greater than the possible return from the project. This would certainly be true for cases of refinancing or bridging loans taken out in times of cash-flow problems (the House of Lords case mentioned in the introduction falls into this category). But since it is known that outside guarantees are also used to finance start-ups, it is not inconceivable that at least in some of those instances final period return is much greater than the value of the residential home used to support it. We have the following proposition determining the risk profile of projects which an outside security can support.

Proposition 3 (Risk Profile) *Define* $\bar{\theta}(\lambda, \tau_E, S^E) \equiv \dfrac{1 - S^E - \lambda}{1 - \lambda - \tau_E(1-2\lambda)}$ *and assume that* $1 - \lambda \geq S^E \geq \tau_E(1-2\lambda)$. *Then*

(i) the relationship asset serves as a commitment device when

(a) (for $K > r$) $\theta > \bar{\theta}$, and

(b) (for $K \leq r$) $\theta \leq \bar{\theta}$

(ii) For each of the arguments of $\bar{\theta}$ we have that $\bar{\theta}$ is decreasing in S^E and λ, and increasing (decreasing) in τ_E when $\lambda > \frac{1}{2}$ ($\lambda < \frac{1}{2}$).

Proof See Appendix B. ■

The assumption comes from the need to ensure that the parameters λ, τ_E and S^E are such that $\bar{\theta} \in (0, 1)$. It is easily verified that the denominator of $\bar{\theta}$ is always a positive fraction. Consequently the LHS of the assumption ensures that the numerator is also positive. If it is violated, then part (i) of the proposition informs us that (since $\bar{\theta}$ can't be negative and so is at its lower bound of zero) start-ups will not be financed regardless of their risk profile while all refinancing projects will be. The RHS of the assumption ensures that $\bar{\theta}$ is less than one. If it is violated then part (i) of the proposition informs us that (since $\bar{\theta}$ can't be greater than one and so is at its upper bound of one) refinancing will never occur while all start-ups (regardless of risk profile) will be financed. The assumption therefore ensures that both types of projects remain possible given the parameters.

The proof of part (i) of the proposition relies simply on the feasibility constraint on β_0 (in particular, that β_0 must be positive). As an aside, the fact that β_0 must also be a fraction places a constraint on the size of the absolute value of $K - r$, in particular $|K - r| < 2z$. As we would expect, the greater the value of the available outside asset the higher the cost of the project that can be financed for any given return profile. Note also that when $K = r$ then θ can take any value, in particular, it can take the value specified in the proposition. The proof of part (ii) involves finding the signs of the respective derivatives.

Part (i) of the proposition states that the risk profile which can be supported with an outside security asset depends on whether the funds are required for refinancing an existing project or are required for a start-up. In the case of refinancing, since $K > r$, all the risk is carried by the date 1 return. This greater risk means that only low risk projects ought to be funded (recall that θ is the probability of the good date 1 return). In the case of a start-up, since the risk of eventual project non-viability is much less, high risk projects can be supported with the outside security.

Part (ii) of the proposition shows that the comparative statics of changes in the parameters on changes in the cut-off θ depends on which case (refinancing or start-up) is examined.

In particular, increases in entrepreneur bargaining power, thought to be beneficial by banks and suspicious by courts, can be seen to affect the risk profile of the two different types of project differently. In addition it depends on the amount of closeness in the relationship. When the relationship is not close, then in the case of refinancing it leads to fewer and fewer low risk projects being financed, and in the case of start-ups to more and more low risk projects being financed. In the limit as τ_E approaches one, regardless of risk profile, no projects get refinanced while all start-ups are funded. The opposite results obtain when the relationship is close. The same results obtain when the focus is on relationship closeness rather than intra-familial bargaining power; that is, the more close the relationship (lower λ), the fewer high risk projects are refinanced and the more high risk start-ups are supported.

In this model there is no inherent tension between the views of the guarantor and bank regarding the utility of intrafamilial factors for enabling project finance. The different perspectives mentioned in the introduction might reflect different views of the nature of the project. In this model the type of project dealt with is common knowledge ex ante. Ex post, if the courts perceive that the husband possesses a high degree of influence over a wife and that the project was for refinancing (as occured in the House of Lords case mentioned in the introduction) then suspicion is rightly raised about why the guarantee was signed at all. In this model, where all information is known and agents are rational, such contracts would not be signed, so that an explanation for such an occurance would need to lie in some behavioural economic explanation outside the scope of this paper.

One of the factors that courts are right to focus on is entrepreneurial asset share. This can be seen via further examination of the constraint the assumption in the proposition places on the parameters of the model. Note that it can be summarized as a correspondence $S^E(\lambda, \tau_E)$. Consider the following five cases where λ is treated as a parameter rather than a variable:

Value of λ	Range for S^E
0	$[1, \tau_E]$
$\frac{1}{4}$	$[\frac{3}{4}, \frac{1}{2}\tau_E]$
$\frac{1}{2}$	$[\frac{1}{2}, 0]$
$\frac{3}{4}$	$[\frac{1}{4}, -\frac{1}{2}\tau_E]$
1	$[0, -\tau_E]$

The pattern can be summarized as follows. The entrepreneur's share of the relationship asset is increasing in relationship closeness and decreasing in his bargaining power. When the relationship is close the size of the set of possible asset shares depends on his bargaining power, the more he has, the smaller the range of possible asset shares. When the relationship is not close ($\lambda > \frac{1}{2}$) then his bargaining power no longer matters in determining the size of the set and the size of the feasible asset share is determined solely by the level of relationship closeness. The two extreme values of λ are worth noting. When $\lambda = 0$, so that the relationship is very close, the range of the entrepreneur's share of the relationship asset is decreasing in his bargaining power. In particular, the more powerful he is within the relationship, the greater the share of the asset he needs to ensure that the relationship can act as a commit device to support the project. In the limit when he has all the bargaining power the model mandates that he also completely own the asset. This is equivalent to saying (if the guarantor *in fact* owns the asset) that, in close marriages (say) with a dominant husband, the third-party security should *not* be signed, confirming the suspicions of the courts. When $\lambda = 1$ we have *etc*

An important consideration in public policy is maintaining the institution of marriage. If there were no relationships then there would be no efficiency loss in security guarantees. A policy of promoting closeness in relationships therefore would set $\lambda = 0$. The following proposition states the welfare effects of changes in the parameters of the model.

Proposition 4 (Welfare)

(i) *[Incomplete proposition - still to do]*

Case $K > r$**:** EL^H is decreasing in S^E, increasing in τ_E as $\lambda < \frac{1}{2}$, and ?? in λ as

Case $K < r$**:** EL^H is increasing in S^E, decreasing in τ_E as $\lambda < \frac{1}{2}$, and ?? in λ as

(ii) Define $\lim_{\lambda \to 0} EL^H \equiv \underline{EL}^H = \dfrac{(1-\theta)(K-r)}{\theta[S^E - \tau_E] - (1-\theta)[1-S^E]}$. Then

(a) (when $K > r$).

(b) (when $K \leq r$)

Proof See Appendix B. ■

[Summarize results.]

[Tension between funding more projects and promoting relationships.]

5 Outside versus inside security assets

5.1 Total versus partial foreclosure

Define *partial* foreclosure as the foreclosure of *only* asset A or *only* asset H, and *total* foreclosure as the foreclosure of *both* assets AH. In the next subsection we are interested in comparing partial liquidations only. In this subsection we determine the conditions under which partial foreclosure will be preferred. We assume that securing oneself up to the eyeballs is not something that either the entrepreneur or guarantor want. It is not the case that for all parameter values partial liquidation is preferred to total liquidation, as the following proposition shows.

Proposition 5 (Partial Liquidation) *There exists a* $\hat{\theta}(\alpha, \lambda, \tau_E, S^E, r, z, K) \in (0, 1)$ *such that partial liquidation is preferred to total liquidation when* $\theta > \hat{\theta}$*; otherwise total liquidation is preferred.*

Proof See Appendix B. ■

The lower is θ the more risky the investment project. When the project is risky, understandably the entrepreneur must choose to permit the bank to foreclose on all available assets. Put another way, the more assets one has available to act as collateral, either the greater the loan one can take out or the riskier the project one can finance. By confining ourselves to situations where only one asset at most is permitted to be foreclosed, we are limiting the riskiness of projects that can be financed by spousal guarantees.

5.2 Comparing partial foreclosure

In this subsection we assume that the conditions ensuring partial foreclosure expounded in the previous subsection hold. In order to investigate the conditions under which we might anticipate the use of either personal or third party guarantees we need to compare contractual inefficiencies in these two different scenarios. The following proposition outlines the conditions determining whether personal or third-party guarantees are likely to be used.

Proposition 6 (Guarantee Conditions)

(i) Investment Risk *There exists a* $\tilde{\theta}(\alpha, \lambda, \tau_E, S^E, r, z, K) \in (0, 1)$ *such that when* $\theta < \tilde{\theta}$ *the entrepreneur prefers foreclosing the project asset (personal guarantees) and when* $\theta > \tilde{\theta}$ *the entrepreneur prefers foreclosing the relationship asset (third party guarantees).*

(ii) Entrepreneur Bargaining Power *There exists a* $\tilde{\tau}_E(\alpha, \lambda, \theta, S^E, r, z, K) \in (0, 1)$ *such that when* $\tau < \tilde{\tau}_E$ *the entrepreneur prefers foreclosing the relationship asset (third party guarantees) and when* $\tau > \tau_E$ *the entrepreneur prefers foreclosing the project asset (personal guarantees).*

(iii) Entrepreneur Relationship Asset Share *There exists a* $\tilde{S}^E(\alpha, \lambda, \tau_E, \theta, r, z, K) \in (0, 1)$ *such that when* $\tilde{S}^E < S^E$ *the entrepreneur prefers foreclosing the project asset (personal guarantees) and when* $\tilde{S}^E > S^E$ *the entrepreneur prefers foreclosing the relationship asset (third party guarantees).*

(iv) Ex post Inefficiency

(a) There exists a $\tilde{\lambda}(\alpha, \lambda, \tau_E, S^E, r, z, K) \in (0, 1)$ *such that when* $\lambda > \tilde{\lambda}$ *the entrepreneur prefers foreclosing the relationship asset (third party guarantees) and when* $\lambda < \tilde{\lambda}$ *the entrepreneur prefers foreclosing the project asset (personal guarantees).*

(b) There exists a $\tilde{\alpha}(\alpha, \lambda, \tau_E, S^E, r, z, K) \in (0, 1)$ *such that when* $\alpha > \tilde{\alpha}$ *the entrepreneur prefers foreclosing the project asset (personal guarantees) and when*

$\alpha < \tilde{\alpha}$ the entrepreneur prefers foreclosing the relationship asset (third party guarantees).

Proof See Appendix B. ■

The proofs of each part involves comparing the expected efficiency loss from contractual incompleteness in the two cases of when the project asset is foreclosed and when the relationship asset is foreclosed. Part (i) shows that third party guarantees open up the possibility to financing of low risk projects while personal guarantees open the possibility of financing low risk projects. Parts (ii) and (iv) separate the effects of (ex post) distribution and (ex post) inefficiency. Part (ii) of proposition 6 indicates that court concern with 'coercion' per se appears not to be misplaced. The greater the entrepreneur's bargaining power viz-a-vis the guarantor, the more he prefers to use his own project assets as security rather than an outside asset. Consequently, if a court finds that there exists a large potential for coercion in a relationship it has the right to wonder why it is being asked to examine the validity of a spousal guarantee. Part (iv) shows that whether third-party guarantees are used depends on the size of the ex post inefficiency - the larger the potential ex post waste, the more efficient it becomes to use the other asset instead as a commitment device. Third party guarantees have value only when they are able to act as substitute leverage for that lost between the entrepreneur and bank when the option of securing project assets is waived. Part (iii) indicates how that substitute leverage is effected by giving the entrepreneur a greater stake in the potential loss of the relationship asset (when that is the asset secured). A lower S^E increases the capacity of the relationship asset to act as leverage, which makes foreclosing on the relationship asset more favorable.

[comparison of $\hat{\theta}$ and $\tilde{\theta}$] [comparative stats on various of these]

6 Conclusion

This paper analyzes a form of secured transactions that has received much judicial attention during the last fifteen years. The main theme of the paper is that the relationship between guarantor and guarantee, represented in this paper as coercion and relative asset share, has both socially beneficial and detrimental effects. Third party guarantees involve an inherent tension between the good and bad effects of coercion within relationships. The good effect allows otherwise wealth constrained individuals to finance projects that would not otherwise obtain financing, involving an efficiency loss to society (given the assumption in this paper that such projects are socially beneficial). The mechanism by which this is achieved is precisely via the exploitation of that relationship connection that exists between guarantor and guarantee - a connection which reassures banks that the loan will be repaid. However, that connection can also lead to parameter values in which the project is high risk and social welfare has decreased. Indeed, it was shown that in the paradigmatic third party guarantee case, where such guarantees are likely to have their most socially beneficial impact (in terms of opening financial access to low risk projects), too much coercion leads to the situation that any project is supported regardless of risk profile.

A Generalized Nash bargaining

Let g_E^i denote the share of ex post surplus obtained by the entrepreneur during renegotation (where $i = A$, H or AH). Correspondingly, let g_B^i denote the share of the ex post surplus obtained by the bank and g_G^i the share of the ex post surplus obtained by the guarantor. Obviously $g_E^i + g_B^i + g_G^i = \Pi^i$. We always assume that the bank is exactly compensated for giving up its right to liquidate asset i so that $g_B^i \equiv L^i$. Let the exogenous bargaining powers of the entrepreneur and guarantor be τ_E and τ_G respectively, where $\tau_E + \tau_G = 1$. The generalized Nash bargaining problem for two people takes the following form

$$\max_{g_E^i, g_G^i} \phi \equiv (g_E^i)^{\tau_E} (g_G^i)^{1-\tau_E}$$

subject to

$$g_E^i + g_G^i = \Pi^i - L^i$$

The first order condition is

$$-\frac{\phi(1-\tau_E)}{\Pi^i - L^i - g_E^i} + \frac{\phi \tau_E}{g_E^i} = 0$$

which after manipulation gives

$$g_E^i = \tau_E(\Pi^i - L^i)$$

The entrepreneur's share of the ex post renegotiaiton surplus increases as his power within the relationship increases.

B Mathematical proofs

Proof of Proposition 1 (Contract Characterization)

(i) From $P_0 \leq 0$ in **??** we have either $P_0 = 0$ or $P_0 < 0$. Assume the latter. This means that the bank (it can't be the guarantor, who also has zero (liquid) wealth) pays the entrepreneur something when $R_1 = 0$. But then it would be more socially efficient (since foreclosing on either the project or relationship asset is always inefficient) to increase P_0 and so reduce β_0. Hence $P_0 = 0$.

(ii) For the cases of $i = AH, A$: From **??** we know that $y_0 \geq 0$. The amount y_0 is paid at date 2 when the entrepreneur pays P_0 at date 1. Under the assumption that the contractual terms are carried out as intended ex ante, such a payment occurs only when $R_1 = 0$. Under that scenario the assets are foreclosed with probability β_0 and not foreclosed with probability $1 - \beta_0$. In the former case $y_0 = 0$ since there is no income from either date 1 or date 2 with which to make the payment. In the latter case date 2 income (r) does accrue to the entrepreneur so that a positive payment is not infeasible, but the assumption on the contracting technology made in subsection 2.1 (namely, that the parties to the contract are constrained to stipulate the same amount, y_0, in both cases) means that the agents choose the lessor amount when designing the contract at date 0. Hence $y_0 = 0$. Note that this last part of the proof only applies to the cases where the business asset is secured. When it is not secured (as in the case of $i = H$) then even when the (relationship) asset is foreclosed, the business asset still exists to provide a date 2 return of r. The indicator function κ_r^i can be used to encapsulate all three models in one term, as shown in part (ii) of the proposition.

(iii) Suppose to the contrary that β_x is strictly positive at an optimum. Now reduce β_x by some infinitesimal amount, say ϵ, without thereby changing the bank's and guarantor's payoffs (this can be effected if we simultaneously ensure that P_x is increased by ϵL^i in the bank's payoff and y_x is decreased by $\epsilon[\kappa_z^i(1-S^E)z - g_G^i]$ in the guarantor's payoff). With these changes, the entrepreneur's payoff changes by $-\theta\epsilon[L^i - \kappa_z^i(1-S^E)z + g_G^i - (\kappa_r^i r + \kappa_z^i S^E z) + g_E^i]$ and the LHS of the renegotiation constraint changes by $-\epsilon[L^i - \kappa_z^i(1-S^E)z + g_G^i - (\kappa_r^i r + \kappa_z^i S^E z) + g_E^i]$. Hence, the assumption which began this proof will be contradicted provided that $[L^i - \kappa_z^i(1-S^E)z + g_G^i - (\kappa_r^i r + \kappa_z^i S^E z) + g_E^i] < 0$. Now note from Table 1 that $g_G^i + g_E^i = \Pi^i - L^i$. Incorporating this and rearranging gives $\Pi^i - (\kappa_r^i r + \kappa_z^i z)$ which can easily be verified as negative for each case of i by consulting Table 1. These changes therefore strictly increase the entrepreneur's payoff while slackening the renegotiation constraint. Hence we have shown the contradiction in the assumption that a strictly positive β_x can be an optimum.

(iv) The bank's individual rationality constraint binds at an optimum since, if it did not, it would be possible to decrease P_x and consequently raise the entrepreneur's payoff. Such a change would not effect the guarantor's payoff and would slacken the renegotiation constraint.

The guarantor's individual rationality constraint binds at an optimum since if it did not, it would be possible to decrease y_x and consequently raise the entrepreneur's payoff. Such a change would not effect the bank's payoff and would slacken the reneogotiation constraint.

(v) Suppose to the contrary that (4) is slack. We solve for the optimal contract assuming this and show that the solution to this relaxed program violates the renegotiation constraint. Using the results of parts (i)-(iv) of proposition 1 the optimization problem ($\bigstar^{(i)}$) can be reformulated as choosing over $[P_x, \beta_0, y_x, y_0]$ to maximize

$$\theta[x - P_x + r - y_x + S^E z] \\ + (1-\theta)[-y_0(1-\kappa_r^i) + (1-\beta_0\kappa_r^i)r + (1-\beta_0\kappa_z^i)S^E z] \tag{B.1}$$

subject to:

$$\theta P_x + (1-\theta)\beta_0 L^i - K = 0 \tag{B.2}$$

$$\theta[y_x + (1-S^E)z] + (1-\theta)[y_0(1-\kappa_r^i) + (1-\beta_0\kappa_z^i)(1-S^E)z] - (1-S^E)z = 0 \quad \text{(B.3)}$$

$$x - P_x + r - y_x + S^E z \quad \text{(B.4)}$$

$$\geq x - y_0(1-\kappa_r^i) + (1-\beta_0\kappa_r^i)r + (1-\beta_0\kappa_z^i)S^E z + \beta_0 g_E^i \quad \text{(B.5)}$$

$$P_x \leq x \quad \text{(B.6)}$$

$$0 \leq y_0 \leq r \text{ and } 0 \leq y_x \leq x + r - P_x \quad \text{(B.7)}$$

$$0 \leq \beta_0 \leq 1 \quad \text{(B.8)}$$

Ignoring the renegotiation constraint (B.4), we can substitute (B.3) and (B.2) into (B.1) to obtain (after some manipulation and collecting the β_0 terms) a reformulated objective function in terms of β_0

$$\theta x - K + [r + S^E z] - (1-\theta)\beta_0[\kappa_r^i r + \kappa_z^i z - L^i]$$

This objective function is linear in β_0 so we have a corner solution. The feasibility constraint (??) on β_0 means that the entrepreneur's payoff is maximized when $\beta_0 = 0$ because $[\kappa_r^i r + \kappa_z^i z - L^i]$ is positive for each of the three cases of i, as can easily be verified by consultation with Table 1. Consequently, from ?? and ??, when $\beta_0 = 0$ we have that $P_x = \dfrac{K}{\theta}$ (which does not violate ??) and that $y_x = -\dfrac{(1-\theta)}{\theta} y_0(1-\kappa_r^i)$. Returning now to the renegotiation constraint ??, it can be rewritten as

$$P_x + y_x < \beta_0[\kappa_r^i r + \kappa_z^i S^E z - g_E^i]$$

which, after subtituting in the solutions $\beta_0 = 0$, $P_x = \dfrac{K}{\theta}$ and $y_x = -\dfrac{(1-\theta)}{\theta} y_0(1-\kappa_r^i)$, gives $\dfrac{K}{\theta} - \dfrac{(1-\theta)}{\theta} y_0(1-\kappa_r^i) < 0$. For the two cases of $i = A$ and AH, $y_0 = 0$ and this provides the required contradiction. For the case of $i = H$ we have that ?? ■

Proof of Corollary 1 From the proof of part (v) of proposition 1 it can be seen that the solution to the *relaxed* maximization problem shown in that proof, namely $\beta_0 = 0$, cannot therefore (because it led to a contradiction) be the solution to the complete maximization problem ($\bigstar^{(i)}$), which therefore must have an optimal β_0 strictly greater than zero. ■

Proof of Proposition 2 (Contractual Inefficiency) Using the results of proposition 1 the optimization problem ($\bigstar^{(i)}$) can be reformulated (as in the proof of part (v) of proposition ??) as choosing over $[P_x, \beta_0, y_x, y_0]$ to maximize

$$\begin{aligned} &\theta[x - P_x + r - y_x + S^E z] \\ &+ (1-\theta)[-y_0(1-\kappa_r^i) + (1-\beta_0\kappa_r^i)r + (1-\beta_0\kappa_z^i)S^E z] \end{aligned} \tag{B.9}$$

subject to:

$$\theta P_x + (1-\theta)\beta_0 L^i - K = 0 \tag{B.10}$$

$$\theta[y_x + (1-S^E)z] + (1-\theta)[y_0(1-\kappa_r^i) + (1-\beta_0\kappa_z^i)(1-S^E)z] - (1-S^E)z = 0 \tag{B.11}$$

$$x - P_x + r - y_x + S^E z = x - y_0(1-\kappa_r^i) + (1-\beta_0\kappa_r^i)r + (1-\beta_0\kappa_z^i)S^E z + \beta_0 g_E^i \tag{B.12}$$

$$P_x \leq x \tag{B.13}$$

$$0 \leq y_0 \leq r \text{ and } 0 \leq y_x \leq x + r - P_x \tag{B.14}$$

$$0 \leq \beta_0 \leq 1 \tag{B.15}$$

We prove each part of the proof in turn.

(i) Now both ?? and ?? can be rewritten as linear functions of β_0 as follows

$$P_x = \frac{1}{\theta}K - \frac{(1-\theta)}{\theta}\beta_0 L^i$$

$$y_x = \frac{1}{\theta}\{(1-S^E)z(1-\theta) - (1-\theta)[y_0(1-\kappa_r^i) + (1-\beta_0\kappa_z^i)(1-S^E)z]\}$$

Substituting ?? and ?? into ?? gives (after manipulation) the entrepreneur's expected payoff from the contract

$$\theta x - K + (r + S^E z) - (1-\theta)\beta_0[\kappa_r^i r + \kappa_z^i z - L^i]$$

where the first three terms are the net present value of the project in the first best case of no liquidation, while the last term is the expected efficiency loss from the incompleteness of the contract, labelled EL^i in the proposition. This completes the proof of part (i) of the proposition.

(ii) The renegotiation constraint (??) can also be rewritten as a linear function of β_0 to give

$$P_x = y_0(1-\kappa_r^i) - y_x + \beta_0[\kappa_r^i r + \kappa_z^i S^E z - g_E^i]$$

Substituting () and () into () gives (after manipulation) the following reformulated renegotiation constraint

$$\beta_0 = \frac{K - y_0(1-\kappa_r^i)}{\theta[\kappa_r^i r + \kappa_z^i S^E z - g_E^i] + (1-\theta)[L^i - \kappa_z^i(1-S^E)z]}$$

The new linear program is to choose β_0 and y_0 to maximise ?? subject to ?? and ??. There are two cases. In the first case, when $i = AH$ or A, $\kappa_r^i = 1$ and so the program is equivalent to choosing the minimum β_0 compatible with ?? and ??. In the second case when $i = H$, $\kappa_r^i = 0$ and so any positive y_0 is a possible solution. The y_0 which minimises β_0 is $y_0 = r$ (the maximum possible y_0 in its range). In either case ?? is the solution provided that it falls between zero and one, which proves part (ii) of the proposition. ■

Proof of Proposition 3 (Risk Profile)

(i) $\bar{\theta}$ is obtained via manipulation of the denominator of ?? (setting it equal to zero and gathering θ terms on the LHS). Since ?? must be positive, the sign of the denominator will depend on the sign on the numerator. This gives the two cases, (a) and (b) mentioned in the proposition.

(ii) We examine each case in turn.

(a) **Case S^E:** Taking the derivative we have that

$$\frac{\partial\bar{\theta}}{\partial S^E} = \frac{-1}{1-\lambda-\tau_E(1-2\lambda)}$$

which is everywhere negative by the fact that the denominator of $\bar{\theta}$ is positive. Hence $\bar{\theta}$ is decreasing in S^E.

Case λ: The derivative gives (after manipulation of the numerator)

$$\frac{\partial\bar{\theta}}{\partial\lambda} = \frac{\tau_E(2S^E-1)-S_E}{[1-\lambda-\tau_E(1-2\lambda)]^2}$$

The denominator is obviously positive so that the sign depends only on the numerator. Whether $\bar{\theta}$ is increasing or decreasing in λ depends on which quadrant of the partitioned $\tau_E - S^E$ space we are in. Set $\bar{\tau}_E = \frac{1}{2}$ and $\tau_E^* \equiv \frac{\tau_E}{2\tau_E-1}$. Then whether τ_E is greater than or less than $\bar{\tau}_E$ and whether S^E is greater than or less then τ_E^* determine the four quadrants of $\tau_E - S^E$ space depicted as follows:

	$\tau_E > \bar{\tau}_E$	$\tau_E < \bar{\tau}_E$
$S^E < \tau_E^*$	decreasing	increasing
$S^E > \tau_E^*$	increasing	decreasing

However, since when $\tau_E > \bar{\tau}_E$, then τ_E^* (which is a hyperbola centred on $\frac{1}{2}$) is always greater than one (a simple check with some numbers plugged in verfies this), and when $\tau_E < \bar{\tau}_E$, then τ_E^* is always less than zero, it follows that the lower LHS and upper RHS quadrants are not feasible for S^E which is restricted to fall within the range $[0, 1]$. It follows therefore that only the upper LHS and lower RHS quadrants are applicable and so $\bar{\theta}$ is decreasing in λ.

Case τ_E: The derivative gives

$$\frac{\partial \bar{\theta}}{\partial \tau_E} = \frac{(1 - S^E - \lambda)(1 - 2\lambda)}{[1 - \lambda - \tau_E(1 - 2\lambda)]^2}$$

Since by fact or assumption the other terms are positive, the sign of the derivative depends on $(1 - 2\lambda)$. This gives $\lambda = \frac{1}{2}$ as the cut-off and accordingly gives the step function of the proposition. ■

Proof of Proposition 4 (Welfare)

(i) The cases of S^E and τ_E are proved in an identical manner to the proofs of the previous proposition (part (ii)). For the case of λ we have

$$\frac{\partial EL^H}{\partial \lambda} = \frac{(1-\theta)(K-r)(1-\lambda)[2\tau_E\theta + (1-\theta)] - \{\theta[S^E - \tau_E(1-2\lambda)] + (1-\theta)[\lambda - (1-S^E)]\}(1-}{\{\theta[S^E - \tau_E(1-2\lambda)] + (1-\theta)[\lambda - (1-S^E)]\}^2}$$

The denominator is obviously positive, so the sign of the derivative depends on the sign of the numerator. Setting the numerator to zero gives

$$(1-\lambda)[2\tau_E\theta + (1-\theta)] - \{\theta[S^E - \tau_E(1-2\lambda)] + (1-\theta)[\lambda - (1-S^E)]\} = 0$$

$$[2\tau_E\theta + (1-\theta)] - \lambda[2\tau_E\theta + (1-\theta)] - \theta S^E + \theta\tau_E - \theta\tau_E 2\lambda - (1-\theta)\lambda + (1-\theta)(1-S^E) = 0$$

$$-\lambda\{[2\tau_E\theta + (1-\theta)] + \theta\tau_E 2 + (1-\theta)\} - \theta S^E + \theta\tau_E + (1-\theta)(1-S^E) - [2\tau_E\theta + (1-\theta)] = 0$$

$$\lambda = \frac{-\theta S^E + \theta\tau_E + (1-\theta)(1-S^E) - [2\tau_E\theta + (1-\theta)]}{2\tau_E\theta + (1-\theta) + \theta\tau_E 2 + (1-\theta)}$$

$$\lambda = \frac{-\{S^E(2\theta + 1) + \theta\tau_E\}}{4\tau_E\theta + 2(1-\theta)}$$

1. ■

Proof of Proposition 5 (Partial Liquidation) Partial foreclosure is preferred to total foreclosure when both $EL^A < EL^{AH}$ and $EL^H < EL^{AH}$. The proof invovles three steps. We consider first the case of $EL^A < EL^{AH}$, then the case of $EL^H < EL^{AH}$, and finally we compare the two.

Step 1: $EL^A < EL^{AH}$ In this case we wish to show (using equations ?? and ??)

$$\frac{(1-\theta)K(1-\alpha)r}{\theta r[1-\tau_E(1-2\alpha)]+(1-\theta)r\alpha} < \frac{(1-\theta)\{K+(1-S^E)z[1-\phi(\lambda)]\}[r+S^E z+\phi(\lambda)(1-S^E)z-(\alpha r+\lambda z)]}{\theta[r+S^E z-\tau_E(r+z-2(\alpha r+\lambda z))]+(1-\theta)[\alpha r+\lambda z-\phi(\lambda)(1-S^E)z]}$$

Focusing on the denominator of the RHS, we have

$$\theta[r+S^E z-\tau_E(r+z-2(\alpha r+\lambda z))]+(1-\theta)[\alpha r+\lambda z-\phi(\lambda)(1-S^E)z]=0$$

$$\theta\{[r+S^E z-\tau_E(r+z-2(\alpha r+\lambda z))]-[\alpha r+\lambda z-\phi(\lambda)(1-S^E)z]\}=-[\alpha r+\lambda z-\phi(\lambda)(1-S^E)z]$$

$$\theta=\frac{-[\alpha r+\lambda z-\phi(\lambda)(1-S^E)z]}{[r+S^E z-\tau_E(r+z-2(\alpha r+\lambda z))]-[\alpha r+\lambda z-\phi(\lambda)(1-S^E)z]}$$

The RHS of (**??**) can be rearranged as follows (after much manipulation)

$$\frac{(1-\theta)K(1-\alpha)r+(1-\theta)K[S^E z+\phi(\lambda)(1-S^E)z-\lambda z]+(1-\theta)(1-S^E)z[1-\phi(\lambda)][r+S^E z+\phi(}{\theta r[1-\tau_E(1-2\alpha)]+(1-\theta)r\alpha+\theta[S^E z-\tau_E(1-2\lambda)z]+(1-\theta)[\lambda z-\phi(\lambda)(1-}$$

If for notational convenience we denote the numerator of the LHS of (**??**) by Σ^0 and the denominator of the LHS by Σ_0, then (**??**) can be written as

$$\frac{\Sigma^0+(1-\theta)K[S^E z+\phi(\lambda)(1-S^E)z-\lambda z]+(1-\theta)(1-S^E)z[1-\phi(\lambda)][r+S^E z+\phi(\lambda)(1-S^E)z-}{\Sigma_0+\theta[S^E z-\tau_E(1-2\lambda)z]+(1-\theta)[\lambda z-\phi(\lambda)(1-S^E)z]}$$

Inequality (**??**) can now be written as (substituting (**??**) into the RHS)

$$\frac{\Sigma^0}{\Sigma_0}<\frac{\Sigma^0+(1-\theta)K[S^E z+\phi(\lambda)(1-S^E)z-\lambda z]+(1-\theta)(1-S^E)z[1-\phi(\lambda)][r+S^E z+\phi(\lambda)(1-S}{\Sigma_0+\theta[S^E z-\tau_E(1-2\lambda)z]+(1-\theta)[\lambda z-\phi(\lambda)(1-S^E)z]}$$

Again for notational convenience define $X\equiv[S^E z+\phi(\lambda)(1-S^E)z-\lambda z]$, $Y\equiv r-\alpha r$ and $Z\equiv[r+S^E z+\phi(\lambda)(1-S^E)z-(\alpha r+\lambda z)]$. Note that $Z=Y+X$. We know that X, Y and Z are all positive. *[more on this]* Also define $A\equiv(1-S^E)z[1-\phi(\lambda)]$ which we know is negative for all $\lambda>0$. Finally, define $\Phi\equiv(1-\theta)KX+(1-\theta)A(X+Y)$ and $\Psi\equiv\theta[S^E z-\tau_E(1-2\lambda)z]+(1-\theta)[\lambda z-\phi(\lambda)(1-S^E)z]$. Then ?? can be written more simply as

$$\frac{\Sigma^0}{\Sigma_0}<\frac{\Sigma^0+\Phi}{\Sigma_0+\Psi}$$

where $\Phi \equiv (1-\theta)KX + (1-\theta)A(X+Y)$. There are 2 cases in which the inequality can hold.[26]

Case 1: $(\Sigma_0 + \Psi > 0)$ In this case the inequality holds if and only if $\Sigma^0\Psi < \Phi\Sigma_0$. This means that

$$[(1-\theta)K(1-\alpha)r]\theta[S^E z - \tau_E(1-2\lambda)z] + (1-\theta)[\lambda z - \phi(\lambda)(1-S^E)z]$$
$$< \theta[S^E z - \tau_E(1-2\lambda)z] + (1-\theta)[\lambda z - \phi(\lambda)(1-S^E)z][\theta r[1-\tau_E(1-2\alpha)] + (1-\theta)r\alpha]$$

$$[(1-\theta)K(1-\alpha)r]\theta[S^E z - \tau_E(1-2\lambda)z] - \theta[S^E z - \tau_E(1-2\lambda)z]$$
$$+(1-\theta)[\lambda z - \phi(\lambda)(1-S^E)z]$$
$$-(1-\theta)[\lambda z - \phi(\lambda)(1-S^E)z][\theta r[1-\tau_E(1-2\alpha)] + (1-\theta)r\alpha] < 0$$

$$[S^E z - \tau_E(1-2\lambda)z]\{\theta(1-\theta)K(1-\alpha)r - \theta\}$$
$$+[\lambda z - \phi(\lambda)(1-S^E)z]\{(1-\theta) - (1-\theta)\theta r[1-\tau_E(1-2\alpha)] - (1-\theta)r\alpha + (1-\theta)\theta r\alpha\} < 0$$

$$[S^E z - \tau_E(1-2\lambda)z]\{\theta[K(1-\alpha)r - 1] - \theta^2 K(1-\alpha)r\}$$
$$+[\lambda z - \phi(\lambda)(1-S^E)z]\{1 - \theta - (1-\theta)\theta r + (1-\theta)\theta r\tau_E(1-2\alpha) - r\alpha + 2\theta r\alpha - \theta^2 r\alpha\} < 0$$

Now $(1-\lambda)$ is always strictly positive. Consequently, showing that the inequality holds depends on the sign and/or magnitude of $(1-\theta)\lambda - \theta\tau(1-\lambda)$. If it is zero or negative then the inequality holds immediately. This is true if

$$\theta \geq \frac{1}{1+\tau\left(\frac{1}{\lambda}-1\right)} \tag{B.16}$$

If it is positive then the inequality only holds if the numerator is greater than the denominator, or

$$(1-\lambda) - [(1-\theta)\lambda - \theta\tau(1-\lambda)] > 0$$

[26] To see this, note that, as a matter of simple algebra, $\frac{a}{b} < \frac{a+c}{b+d}$ can be rearranged as (provided $b+d>0$) $a(b+d) < (a+c)b$ which reduces to $ad < cb$, and can also be rearranged as (provided $b+d<0$) $ad > cb$.

which implies

$$\theta > \frac{1 - \left(\dfrac{1}{\lambda} - 1\right)}{1 + \tau\left(\dfrac{1}{\lambda} - 1\right)} \equiv \hat{\theta} \tag{B.17}$$

where the last equivalence follows from definition **??** in section 3. Since λ by definition is never strictly one, it follows that the RHS of (B.17) is always strictly less than the RHS of (**??**). Going through analagous steps will show that $EL^B < EL^{AB}$ whenever

$$\theta > \frac{1 - \left(\dfrac{1}{\alpha} - 1\right)}{1 + \tau\left(\dfrac{1}{\alpha} - 1\right)}$$

Hence the final part of the proof involves showing that the cut-off θ when asset A is foreclosed is less than the cut-off θ when asset B is foreclosed. We know that $c'(x) < 0$. Since $\theta'(c) = -\dfrac{(1+\tau)}{[1+\tau c]^2} < 0$, then the fact that $\hat{\theta}$ is the cut-off point follows immediately from assumption **??**. ■

Proof of Proposition ?? (Guarantee Conditions) Define $\psi \equiv EL^A - EL^H$. The entrepreneur is indifferent between using the project asset or the relationship asset for foreclosure when $\psi = 0$. That is (from (??) and (??) in section 3)

$$\psi \equiv \frac{K[r - \alpha r]}{\theta[r - \tau_E(r - 2\alpha r)] + (1-\theta)\alpha r}(1-\theta) \\ - \frac{\{K + (1 - S^E)z[1 - \phi(\lambda)] - r\}[S^E z + \phi(\lambda)(1 - S^E)z - \lambda z]}{\theta[S^E z - \tau_E(z - 2\lambda z)] + (1-\theta)[\lambda z - \phi(\lambda)(1 - S^E)z]}(1-\theta) = 0$$

This implies that

$$K[r - \alpha r]\{\theta[S^E z - \tau_E(z - 2\lambda z)] + (1-\theta)[\lambda z - \phi(\lambda)(1 - S^E)z]\}$$

$$-\{K + (1 - S^E)z[1 - \phi(\lambda)] - r\}[S^E z + \phi(\lambda)(1 - S^E)z - \lambda z][\theta[r - \tau_E(r - 2\alpha r)] + (1-\theta)\alpha r] = 0$$

which can be written as (gathering θ terms)

$$\theta\{K[r-\alpha r]\{S^E z-(\lambda z-\phi(\lambda)(1-S^E)z)-\tau_E(z-2\lambda z)\}$$

$$-\{[r-\tau_E(r-2\alpha r)]-\alpha r\}\{K+(1-S^E)z[1-\phi(\lambda)]-r\}[S^E z-(\lambda z-\phi(\lambda)(1-S^E)z)]\}$$

$$-\alpha r\{K+(1-S^E)z[1-\phi(\lambda)]-r\}[S^E z-(\lambda z-\phi(\lambda)(1-S^E)z)]+K[r-\alpha r][\lambda z-\phi(\lambda)(1-S^E)z]=0$$

To avoid notational overload we make the following definition. ■

Definition 1 *$\Theta \equiv \lambda z-\phi(\lambda)(1-S^E)z$, $\Lambda \equiv K+(1-S^E)z[1-\phi(\lambda)]-r$ and $\Omega \equiv r-\alpha r$ and $\Xi \equiv z-\lambda z$.*

The assumptions of section ?? mean that Θ and Λ are negative. Of course Ω and Ξ are both positive. Then using definition ??, ? can be written more simply as

$$\theta\{K\Omega[S^E z-\Theta-\tau_E(1-2\lambda)z]-[\Omega-\tau_E(1-2\alpha)r]\Lambda(S^E z-\Theta)\}$$
$$-\alpha r\Lambda(S^E z-\Theta)+K\Omega\Theta=0$$

and when $\phi(\lambda)=1$ then ?? becomes

$$\theta\{\Omega r\Xi+\tau_E(1-2\alpha)r\Lambda\Xi-\tau_E(1-2\lambda)zK\Omega\}+K\Omega[S^E z-\Xi]-\alpha r\Lambda\Xi=0 \qquad \text{(B.18)}$$

Proof

(i) Investment Risk From (??) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$\theta=\frac{\alpha r\Lambda(S^E z-\Theta)-K\Omega\Theta}{K\Omega[S^E z-\Theta-\tau_E(1-2\lambda)z]-[\Omega-\tau_E(1-2\alpha)r]\Lambda(S^E z-\Theta)}\equiv\tilde{\theta}(\alpha,\lambda,\tau_E,S^E,r,z)$$

provided that $\tilde{\theta}(\alpha,\lambda,\tau_E,S^E,r,z)\in[0,1]$. *[existence?]* Taking the derivative of ψ with respect to θ we get

$$\frac{d\psi}{d\theta}=\{K\Omega[S^E z-\Theta-\tau_E(1-2\lambda)z]-[\Omega-\tau_E(1-2\alpha)r]\Lambda(S^E z-\Theta)\}>0$$

so that ψ is increasing in θ for all values of θ. It follows that the entrepreneur forecloses on the project asset when $\theta<\tilde{\theta}$ and forecloses on the relationship asset when $\theta>\tilde{\theta}$.

(ii) Entrepreneur Bargaining Power From (??) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$\tau_E = \frac{(S^E z - \Theta)\{\theta\Omega(\Lambda - K) + \alpha r\Lambda\} - K\Omega\Theta}{\theta(1-2\alpha)r\Lambda(S^E z - \Theta) - \theta K\Omega(1-2\lambda)z} \equiv \tilde{\tau}_E(\alpha, \lambda, \theta, S^E, r, z)$$

provided that $\tilde{\tau}_E(\alpha, \lambda, \theta, S^E, r, z) \in [\frac{1}{2}, 1]$. *[existence?]* Taking the derivative of ψ with respect to τ_E we get

$$\frac{d\psi}{d\tau_E} = \theta[(1-2\alpha)r\Lambda(S^E z - \Theta) - K\Omega(1-2\lambda)z] < 0$$

so that ψ is decreasing in τ_E for all values of τ_E. It follows that the entrepreneur forecloses on the relationship asset when $\tau_E < \tilde{\tau}_E$ and forecloses on the project asset when $\tau_E > \tilde{\tau}_E$.

(iii) Entrepreneur Relationship Asset Share From (??) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$S^E = \frac{1}{K\Omega z}\{K\Omega\Xi + \alpha r\Lambda\Xi - \theta[\Omega r\Xi + \tau_E(1-2\alpha)r\Lambda\Xi - \tau_E(1-2\lambda)zK\Omega]\} \equiv \tilde{S}^E(\alpha, \lambda, \theta, \tau_E, r, z)$$

provided that $\tilde{S}^E(\alpha, \lambda, \theta, \tau_E, r, z) \in [0, \frac{1}{2}]$. *[existence?]* Taking the derivative of ψ in (??) with respect to S^E we get

$$\frac{d\psi}{dS^E} = K\Omega z > 0$$

so that ψ is increasing in S^E for all values of S^E. It follows that the entrepreneur forecloses on the project asset when $S^E < \tilde{S}^E$ and forecloses on the relationship asset when $S^E > \tilde{S}^E$.

(iv) Relative Inefficiency

(a) From (??) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$\lambda = \frac{[\theta\Omega + \theta\tau_E(1-2\alpha)\Lambda - K(1-\alpha) - \alpha\Lambda] + K(1-\alpha)[S^E - \theta\tau_E]}{[\theta\Omega + \theta\tau_E(1-2\alpha)\Lambda - K(1-\alpha) - \alpha\Lambda] + \theta\tau_E 2K(1-\alpha)} \equiv \tilde{\lambda}(\alpha, S^E, \theta, \tau_E, r, z)$$

provided that $\tilde{\lambda} \in [0, \frac{1}{2})$. *[existence?]* Taking the derivative of ψ with respect to λ we get

$$\frac{d\psi}{d\lambda} = r[1 - \tau_E]\theta + [3\theta\tau_E - 1]K + \alpha[(1 - \theta + 2\theta\tau_E)r - 4\theta\tau_E K] > 0$$

so that ψ is increasing in λ for all values of λ. It follows that the entrepreneur forecloses on the project asset when $\lambda < \tilde{\lambda}$ and forecloses on the relationship asset when $\lambda > \tilde{\lambda}$.

(b) From (??) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$\alpha = ?? \equiv \tilde{\alpha}(\lambda, S^E, \theta, \tau_E, r, z)$$

provided that $\tilde{\alpha} \in [0, \frac{1}{2})$. *[existence?]* Taking the derivative of ψ with respect to α we get

$$\frac{d\psi}{d\alpha} = ?? > ??$$

so that ψ is increasing in α for all values of α. It follows that the entrepreneur forecloses on the relationship asset when $\alpha < \tilde{\alpha}$ and forecloses on the project asset when $\alpha > \tilde{\alpha}$. ■

Proof of Proposition ?? (Asset Share, Coercion and Project Risk Reversal) *[Need to rewrite this]* Equation (**??**) implies

$$\theta = \frac{rz\Lambda}{\rho\nabla - rz\lambda[1 - \alpha\kappa(\tau, c)]} \equiv \tilde{\theta} \tag{B.19}$$

Taking the derivate of ψ with respect to θ we get

$$\frac{d\psi}{d\theta} = \rho\nabla - rz\lambda[1 - \alpha\kappa(\tau, c)]$$

which (since $\nabla < 0$ by assumption) is positive or negative depending solely on the magnitude of κ. When $\kappa = 0$ so that there is no coercion, $\frac{d\psi}{d\theta} < 0$, which implies that for $\theta > \hat{\theta}$

the entrepreneur forecloses on the project asset (asset A) and for $\theta < \hat{\theta}$ the entrepreneur forecloses on the relationship asset (asset B). However

$$\frac{d^2\psi}{d\kappa d\theta} = rz\lambda\alpha \geq 0$$

implying that when coercion is large enough, $\frac{d\psi}{d\theta} > 0$, implying that the situation reverses where for $\theta > \hat{\theta}$ the entrepreneur forecloses on the relationship asset (asset B) and for $\theta < \hat{\theta}$ the entrepreneur forecloses on the project asset (asset A). The turning point occurs where

$$\kappa(\tau, \lambda) = \frac{rz\lambda - \rho\nabla}{\alpha rz\lambda} \equiv \kappa^* \tag{B.20}$$

■

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