# Property Markets and Policies

# in an

# Intertemporal General Equilibrium Model

(Revised Version)

Paper prepared for the PhD Conference,

the Australian National University, 5-6 Nov., 1998

**KK Tang**

Economic Department

Research School of Pacific and Asian Studies

Australian National University

# Abstract

This paper presents a theoretical simulation model to examine the interaction between property markets and the rest of the economy. The model is a rational expectation, intertemporal general equilibrium model, with multiple sectors and real estate markets. The model emphasizes the interdependence between sectors as well as the government's role in the property markets. It is demonstrated that a resource boom in a durable non-tradable sector can improve the current account balance of the home economy in the short run. Secondly, when there is production interdependency between sectors, resource booms in one sector does not necessarily cause contraction in other sectors.

KK Tang

Economic Department, Research School of Pacific and Asian Studies
Australian National University
Telephone: 612 6249 3653
Fax: 612 6249 3700
Email: kktang@economics.anu.edu.au

# 1 INTRODUCTION

The study of property markets is already a well-developed and 'self-contained' area, lying between the disciplines of economics and urban studies. An overwhelming proportion of the studies in this area focus on microeconomic issues of real estate markets, such as inter-regional housing price and rental rate differentials, labor migration and housing demand, property taxes, and housing finance. The macroeconomic aspects of property markets, such as the supply side impact of land policies on business cycles, and the effectiveness of property taxes in absorbing demand shocks, are largely overlooked. Whereas the interrelation between property markets and the rest of the economy has already been evidenced quantitatively (Meen 1990). During the recent Asian economic crisis, it was observed that several troubled economies had experienced significant property booms before their currency problems surfaced. One hypothesis is that rocketing property prices raised production costs in those economies and, hence, severely undermined their export competitiveness. Nevertheless, very little has been done to disentangle the correlation between property boom and balance of payment problems.

This study constructs a theoretical stimulation model to capture the interrelation between property markets and the rest of the economy, and further to use the model to examine the general equilibrium effect of property market policies. The model differentiates three industries as well as three real estate markets on the one hand, emphasizes the interdependence between industries and markets on the other. It also acknowledges the important role of the government in the property markets.

While the exercise is primarily for theoretical purposes, the structure of the model roughly reflects the economic situation of a small open economy––Hong Kong (HK). In 1996, the construction sector contributed 5.8 percent to gross domestic product (GDP) in HK. Real estate services, together with financing, insurance and business services, contributed another 24.9 percent. A rough estimation shows that, in 1996, the mortgage payment for an average residential flat absorbed 70 percent of the income of a family with median level income (Lui 1997). Before the Asian currency crisis, property prices in HK once bypassed that in Tokyo, becoming the most expensive place for doing business in the world.[1]

A commonly suggested explanation for the high property prices in HK is the scarcity of land. Before the hand-over of HK to China in July 1997, according to the Sino-British Joint Declaration, land supply was formally constrained to 50 hectares a year (excluding land to be granted for public rental housing).[2] Additional amounts of land could be leased only if the British/HK authorities had sought agreement of the Chinese government.[3] While there was no conclusion about the impact of these supply restrictions on the property market, HK did experience several property booms during the 1980s and 1990s. One of these episodes occurred in 1991-94, when China was rapidly recovering from post-Tiananmen economic depression. Due to the deep economic integration between China and HK, this led to an economic boom in HK. At the same time, the US lowered domestic interest rates. In two years time, these two factors had dramatically pushed up property prices in HK. Under the present currency board arrangement in HK, interest rate policy is unavailable to the authorities. Consequently, most of the time the authorities have resorted to administrative measures like property transaction tax and mortgage limit.[4] Though the negotiating arrangement has faded out with the 1997 hand-over, land supply is still a problem for the island economy due to its geographical limitation.

The following section of the paper provides a brief review of the literature and a non-technical preview of the model. Section 3 explains the mathematical details of the model. Section 4 discusses the result of a trial simulation. The last section concludes.

---

[1] For comprehensive economic views about the property market in HK, see Renaud, et al. (1997) and Tse (1994).

[2] The reason for that transitional arrangement is political. The Chinese government tried to avoid the British government over-selling land before 1997, and thus reducing the land sale revenues for the post-hand-over HK Special Administrative Region government.

[3] The two governments did negotiate successfully to lease more than 50 hectares every year before the hand-over. However, there was still a lot of debate that the additional amounts were insufficient to meet the ever growing demand.

[4] This episode highlights a number of important elements, particularly foreign nominal shocks and the exchange rate regime. However, these issues will be left to the next monetary version of the model. This paper concentrates on the real impacts of land and public housing policies.

## 2 REVIEW AND PREVIEW

Early studies of housing and real estate are rooted deeply in neoclassical consumer theory. Housing is considered a durable good. Households and real estate developers are assumed to maximize their one period utility and profits, respectively. Examples include McDonald (1979), Friedman and Weinberg (1982) and Maclennan (1982). Later studies, following the mainstream development in economic theories, emphasize the intertemporal feature of economic decisions. One of the earliest attempts of this kind is Hammond (1987). Contemporary studies of this topic further integrate with capital theory, capturing the nature of real estate as both a durable good and a physical asset, for example Okumura (1997) and Miles (1994). Incorporating the asset nature of real estate significantly enriches the dynamics of the property market. Similar to its role in other asset markets, expectation becomes crucial in the property market.

However, studies like Okumura (1997) and Miles (1994) are still basically limited within the microeconomic framework, in the sense that a number of crucial macroeconomic variables are set to be exogenous, such as wages, interest rates, and inflation. As a result, the equilibrium of the housing market is determined separately from, rather than in conjunction with, other markets. Nonetheless, this general equilibrium framework seems to be increasingly embraced. Recent examples include Turnovsky and Okuyama (1994) and Nielsen and Sorensen (1994). However, both studies ignore the supply side constraint of land, as well as a number of macroeconomic factors such as capital adjustment costs, the external balance, and the role of money. A multi-country model constructed by McKibbin and Wilcoxen (1995), the G-Cubed model, has incorporated consumer durable goods and residential housing. Nevertheless, in the current version of the model the supplies of natural resources, including land, are assumed perfectly elastic. The model also abstracts from commercial building.

On the other side, if one widens the scope of view from specifically housing to more generally non-tradable (durable or non-durable) goods, there will be no supply shortage of general equilibrium analyzes. For example, see Rebelo (1997), van Winccop (1993), and the citations in their papers. In terms of model structure, these two studies shares some similarities with this paper. However, this paper is differentiated from them, and all other papers cited previously, by its much more detailed treatment of property markets.

This study attempts to integrate property markets into an intertemporal general equilibrium model, with a rational expectation representative agent for each sector of the economy.[5] The basic framework of the model is a combination of two 'workhorses' in economics: the Salter (1959) and the Ramsey (1928) models. As summarized in Table 2.1, the model is constructed with five agents: three production sectors, a representative household, and a government. The three industries correspond to tradable non-durable manufacturing goods, non-tradable non-durable services, and, lastly, non-tradable durable housing, respectively. Highlighting the coexistence of various real estate markets in an economy, the model distinguishes between commercial building, private-funded residential housing, and public-subsidized residential housing (the last two are abbreviated as private housing and public housing, respectively, hereafter). The purpose of incorporating public housing in the model is to capture the fact that half the population in HK inhabits various forms of public-subsidized dwellings. It also provides a platform to examine how public housing policies influence the market of private housing.

Each industry uses primary factors, intermediate goods, commercial building, and a sector-specific factor for production.[6] In the construction sector, the specific factor is land. The inclusion of commercial building into the production implies the potential influence of property prices on export competitiveness. Physical capital is wholly imported from overseas, reflecting the situation that HK has virtually no heavy manufacturing industries. Intermediate goods include manufacturing goods and services. The purpose of incorporating these two intermediates into the production is to enhance the interdependence between industries, which is crucial in the propagation of shocks.

---

[5] See Hartley (1997) for an extensive and strong criticism of this type of representative agent model, along with rational expectation.

[6] One of the functions of incorporating sector-specific factors is to stabilize the model and thus help the numerical iterations.

**Table 2.1: Structure of the Model**

| | |
|---|---|
| **Country:** | **Factors:** |
| Single small open economy | Physical capital (imported) |
| **Agents:** | Labor (mobile across sectors) |
| Household | Sector-specific factors (including land) |
| Government | **Housings:** |
| Industries: | Commercial building |
| Manufacturing (tradable, non-durable) | Residential housing: |
| Servicing (non-tradable, non-durable) | Public-subsidized housing |
| Construction (non-tradable, durable) | Private-funded housing |

The household provides labor and capital services to the three industries in return for wages and interest. The household consumes residential housing, non-durable goods, as well as leisure. Residential housing comprises of private housing and public housing. For simplicity, the distinction between renting and purchasing is ignored. All housing is assumed to be sold, but public housing is sold at a lower than market price due to government subsidy. In fact, in recent years, the HK government has strongly encouraged public housing residents to purchase their rented houses from the authorities.

The model abstracts from intertemporal issues of public finance. The government expenditure is financed by a lump sum tax on the household. The government budget is balanced every period. Nonetheless, the government still has a crucial role. It controls the supply of land, the distribution of land disposal amongst the three types of real estate, as well as the level of subsidy on public housing. It is worth to point out that, being a monopolist of the land resource, the government can restrict land sales to extract rent from the public. Thus, land supply can be used as an indirect means of taxation. In fact, this is a reason why the HK government could accumulate the third largest foreign reserves in the world, only behind Japan and China.

The model consists of only one country; the rest of the world is treated exogenous. Capital is perfectly mobile internationally. The home country is considered a small trader in the world market, so the prices of capital investment goods and manufacturing goods are set exogenously. This implies that purchasing power parity holds continuously in the model. To avoid either infinite borrowing from or infinite lending to the rest of the world, two constraints are imposed. Firstly, the domestic subjective discount rate is set equal to the world interest rate. Secondly, the domestic interest rate is equal to the world interest rate plus a risk premium, which is a function of the foreign debts to GDP ratio. These two constraints together imply that, at the steady state, the amount of foreign debts must be equal to zero.[7]

The model is a real model as the role of money is completely suppressed, suggesting that money is completely neutral. Lastly, no deterministic growth trend is imposed on the economy. In other words, if there is no disturbance, the economy will be at a stationary equilibrium.

The model affiliates with real business cycle models, in the sense that all markets clear continuously. Labor is perfectly mobile across sectors. The sources of rigidity are the adjustment costs of physical capital and housing accumulations. In real business cycle models, technology shocks constitute a major source of business cycles. As technological progress is a random event, the role of the government is limited to the demand side.[8] In contrast, in this property market model, the government can actually generate supply shocks directly through changing land policies. It can also generate demand shocks through changing public housing policies. The next section describes the structure of the model formally. The notation is listed in Table A1 in the Appendix.

[7] One can change the amount of foreign debts at the steady state by allowing a discrepancy between the domestic subjective discount rate and the world interest rate.

[8] Obviously, fiscal policies could have supply side impact through influencing the labor supply decision of households.

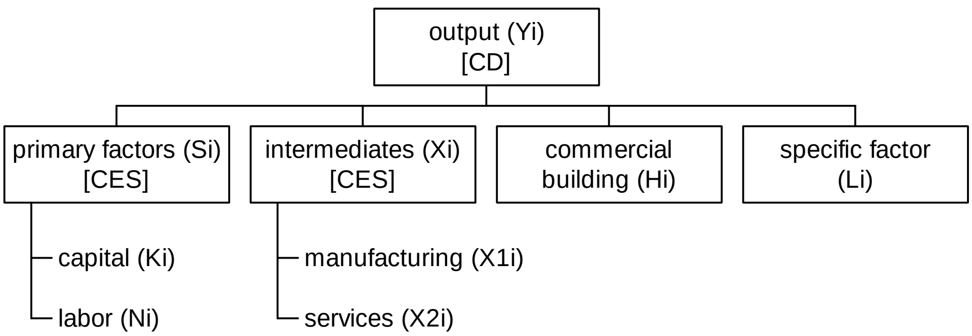
# 3 MODEL SPECIFICATIONS

## 3.1 Industries

The objective of firms is to maximize a steam of discounted current and future profits. Production in each sector involves two primary factors, two intermediate inputs, commercial building, as well as a sector-specific factor. The production nesting is illustrated in Figure 3.1.

The upper level of the production nesting is in the Cobb-Douglas (CD) form. In the lower level, the primary factor $S_i$ is composed of physical capital and labor through a constant-elasticity-of-substitution (CES) function. Indeed, this is the basic structure of the model: commodities without cross price substitutability are nested by a CD function, and commodities with cross price substitutability are nested by a CES function. For simplicity, the factor intensities of the three industries are set to be the same.

**Figure 3.1: Production Nesting**




Formally, the objective of firm *i* is to:

maximize $\int_0^\infty \Pi_i e^{-Rt} dt$

subject to:

(1) $$\Pi_i = P_i Y_i - I_{Ki} - P_H I_{Hi} - WN_i - P_{Xi} X_i - P_{Li} L_i$$

(2) $$I_{Ki} = J_{Ki}[1 + (\phi_K / 2)(J_{Ki} / K_i)]$$

(3) $$\dot{K}_i = J_{Ki} - \delta_K K_{Ki}$$

(4) $$I_{Hi} = J_{Hi}[1 + (\phi_H / 2)(J_{Hi} / H_i)]$$

(5) $$\dot{H}_i = J_{Hi} - \delta_H H_i;$$

where:

(6) $$Yi = Q^i(S_i, X_i, H_i, L_i)$$

(7) $$Q^i = \beta_{31}(S_i)^{\beta 32}(X_i)^{\beta 33}(H_i)^{\beta 34}(L_i)^{\beta 35};\ \beta_{32} + \beta_{33} + \beta_{34} + \beta_{35} = 1$$

(8) $$(S_i)^{\alpha 10} = \alpha_9(K_i)^{\alpha 10} + (1 - \alpha_9)(N_i)^{\alpha 10};\ \alpha_{10} < 1.$$

Equation (1) indicates that the profit of firm *i* is equal to the value of output minus the costs of factors and intermediates. Sector-specific factors can be considered as natural resources. Their supplies are controlled by the government. Profit tax is ignored, as it is irrelevant to the study. As labor is perfectly mobile, wages are equalized across sectors. The foreign price of capital investment goods is set to be one. Therefore, the domestic price of capital investment goods becomes the numeraire. Thus, all other prices are relative prices.

Equations (2) and (3) describe the formation of physical capital. The formation of capital is subject to an adjustment cost as a ‘standard’ setting since Lucas (1967), Uzawa (1968), and Treadway (1969). Equations (4) and (5) determine the formation of housing capital, which is used as an input in each industry. The formation of housing capital is also subject to an adjustment cost, similar to physical capital. More about housing capital formation will be discussed in the next sub-section. Equations (6)-(8) describe the production technology as illustrated in Figure 3.1.

*R(s)* is the average of the short-term interest rate, *r(t)*, over time *s* and *t*, defined as:

(9) $$R(s) = \frac{1}{(s-t)} \int_t^s r(v)dv .$$

The current value Hamiltonian of the maximization problem for the firm *i* is given as:

(10) $$h_i = P_i Q^i (K_i, N_i, X_i, H_i, L_i) - I_{Ki} - P_H I_{Hi} - WN_i - P_{Xi} X_i - P_{Li} L_i$$

$$+\lambda_{JKi}\{I_{Ki} - J_{Ki}[1+(\phi_K / 2)(J_{Ki} / K_i)]\}$$

$$+\lambda_{JHi}\{I_{Hi} - J_{Hi}[1+(\phi_H / 2)(J_{Hi} / H_i)]\}$$

$$+\lambda_{Ki}(J_{Ki} - \delta_K K_i)$$

$$+\lambda_{Hi}(J_{Hi} - \delta_H H_i).$$

Besides the original constraints, other first order conditions of the maximizing problem are given by:

(11) $$J_{Ki} = (q_{Ki} - 1)(K_i / \phi_K)$$

(12) $$J_{Hi} = (q_{Hi} - 1)(H_i / \phi_H)$$

(13) $$q_{Ki} = \lambda_{Ki}$$

(14) $$q_{Hi} = \lambda_{Hi} / P_H$$

(15) $$W = P_i Q^i_{Ni}$$

(16) $$P_{Xi} = P_i Q^i_{Xi}$$

(17) $$P_{Li} = P_i Q^i_{Li}$$

(18) $$\dot{\lambda}_{Ki} = (r^* + \delta_K)\lambda_{Ki} - P_i Q^i_{Ki} - (\phi_K / 2)(J_{Ki} / K_{Ki})^2$$

(19) $$\dot{\lambda}_{Hi} = (r^* + \delta_H)\lambda_{Hi} - P_i Q^i_{Hi} - P_H(\phi_H / 2)(J_{Hi} / H_i)^2 .$$

Equations (11) to (14) govern the formation of physical capital and commercial building. $\lambda_{Ki}$ and $\lambda_{Hi}$ are the increment to the value of firm *i* from accumulating one more unit of physical capital and commercial building stocks, respectively. So, equations (13) and (14) define the marginal Tobin's q of physical capital and commercial building, respectively. Tobin's q conveys the information about the profitability of investment. As pinned down by equations (11) and (12), it is profitable to further invest as long as Tobin's q is greater than one. Equations (15) to (17) state that the deployment of any factor is up to the point that its marginal benefit is equal to its marginal cost. Dynamic equations (18) and (19) remark that, each capital is accumulated up to where the marginal cost equates to the marginal benefit. The former is equal to the interest rate plus the depreciation rate, while the latter is equal to the rise in the output plus the fall in the adjustment cost due to a larger capital stock.

$X_i$ is an aggregate intermediate good, composed of manufacturing goods and services. The proportion of the two goods in $X_i$ is determined by a second-tier optimization process as follows:

maximize $X_i$

subject to:

(20) $$(X_i)^{\alpha 12} = \alpha_{11}(X_{1i})^{\alpha 12} + (1-\alpha_{11})(X_{2i})^{\alpha 12} \ ; \ \alpha_{12} \leq 1$$

(21) $$P_{Xi}X_i = P_1 X_{1i} + P_2 X_{2i}$$

where:

(22) $$(P_{Xi})^{1-\sigma 12} = \alpha_{11}{}^{\sigma 12}(P_1)^{1-\sigma 12} + (1-\alpha_{11})^{\sigma 12}(P_2)^{1-\sigma 12} \ ; \ \sigma_{12} = 1/(1-\alpha_{12}).$$

Solving the problem, it can be obtained:

(23) $$X_{1i} = X_i[\alpha_{11}(P_{Xi} / P_1)]^{\sigma 12}$$

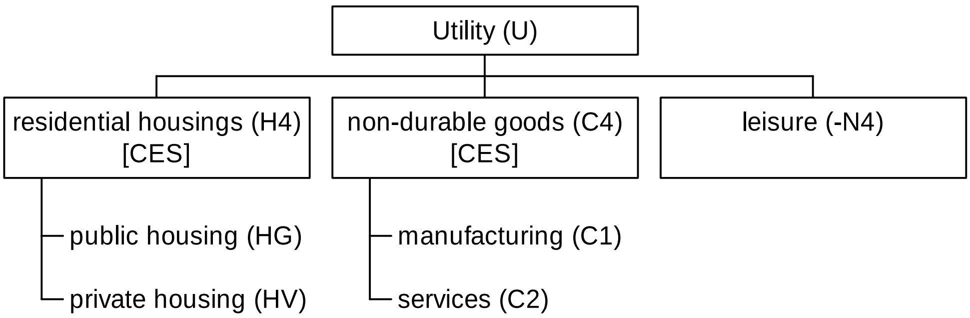
(24) $$X_{2i} = X_i[(1-\alpha_{11})(P_{Xi} / P_2)]^{\sigma 12}.$$

Equations (23) and (24) tell that the demand for goods $j$, $j = 1 \text{ or } 2$, is negatively proportional to the ratio between its price $P_j$ and the intermediate input price index $P_{Xi}$.

### 3.2 Household

The household is treated as the fourth sector in this model. The household maximizes utility from consuming the services from residential housing, non-durable consumer goods, and leisure. The consumption nesting is illustrated in Figure 3.2. Again, the first level is in the CD form, and the second level in the CES form. Residential housing comprises of private and public housing. Similar to the intermediate input $X_i$ in production, the non-durable consumer good $C_4$ comprises of manufacturing goods and services.

**Figure 3.2: Consumption Nesting**




Formally, the optimization problem of the household is stated as:

maximizing $\int_0^\infty U(H_4, C_4, N_4)\, e^{-\mu t}\, dt$

subject to:

(25) $$WN_4 + \bar{r}A = P_V I_V + \theta_1 P_G I_G + P_4 C_4 + \dot{A} + TX$$

(26) $$I_V = J_{HV}[1 + (\phi_{HV} / 2)(J_{HV} / H_V)]$$

(27) $$\dot{H}_V = J_{HV} - \delta_{HV} H_V$$

(28) $$I_G = J_{HG}[1+(\phi_{HG} / 2)(J_{HG} / H_G)]$$

(29) $$\dot{H}_G = J_{HG} - \delta_{HG} H_G$$

(30) $$(H_4)^{\alpha 2} = \alpha_1 (H_V)^{\alpha 2} + (1-\alpha_1)(H_G)^{\alpha 2}$$

(31) $$N_4 = \sum_{i=1}^{3} N_i \, ;$$

where:

(32) $$U(H_4, C_4, N_4) = \beta_1 (H_4)^{\beta 2} (C_4)^{1-\beta 2} - \beta_3 (N_4)^{\beta 4}$$

(33) $$A = \sum_{i=1}^{3} (q_{Ki} K_i + q_{Hi} H_i) + A^*$$

(34) $$\bar{r}A = r(A - A^*) + r^* A^* .$$

Equation (25) is the budget constraint of the household. The household earns labor income as well as interest from asset holdings, and allocates them amongst housing investment, non-durable goods, savings, and tax duty. While the household is consuming housing services, it is the housing investment goods, $I_V$ and $I_G$, that enter the budget constraint. The subsidy per unit cost of public housing is equal to $(1-\theta_1)$. Equations (26) to (30) specify the formation of residential housing. As expressed in equations (26), (28) as well as (5), housing investments are partially absorbed by the replacement of depreciated housing. For simplicity, the possibility of redevelopment on the land of existing dwellings by private developers is excluded. In other words, depreciated buildings are demolished, and the recovered land is returned to the government rather than to the property owners.

An additive separable utility function is used to simplify the interaction between the consumption of commodities and leisure (equation 32). Following McKibbin and Sachs (1991), it is assumed the household is holding claims against the physical capital and housing capital of firms (equation 33), rather than holding claims against firms' financial liabilities, that is, securities. However, with a perfect capital market, the two treatments are equivalent.[9] Furthermore, it is assumed that foreign assets are dominated in foreign goods, so they are not associated with risk. Consequently, the interest rate of the total financial assets is an average of the domestic and the foreign interest rates (equation 34).[10]

As described above, what the agents purchase are housing investment goods rather than 'ready-to-use' housing capital. The investment goods then accumulate into housing stocks subject to adjustment costs. Therefore, the 'adjustment process' is virtually the final part of the production process, but carried out by the users rather than by the construction sector. The same specification is also used in the accumulation of physical capital. This specification requires further justification.

In the case of physical capital, the adjustment cost is commonly explained as 'installation cost'. In the case of housing, the nature of the adjustment cost is a little more complicated. From the issue of land to the completion of a building, there is a prolonged construction period. Secondly, there is a time gap between the completion of a building and actual utilization, due to the transaction costs involved in mortgage arrangements, moving, and decorating (Harmon and Potepan 1988). While the former type of 'delay' comes from the production side, and the latter from the consumption side, both can be modeled by adjustment costs. To simplify the model, the two adjustment costs are grouped into one on the consumption side. The major benefit in doing so is to simplify the simulation program substantially by maintaining the symmetry between the settings of the three industries.

One would probably expect the adjustment cost on the production side to be much larger than that on the consumption side. As a result, the price of housing investment goods is not a good indicator of the 'actual' or effective property price. A simple, though imperfect, way to remedy this problem is to include adjustment costs in the measures of property prices. Formally, the effective property prices are defined as:

---

[9] For an example of explicit treatment of commercial securities and bonds in an intertemporal general equilibrium model, see Turnovsky (1996).

(35) $$P_{Hi} = P_H[1+(\phi_H \ / \ 2)(J_{Hi} \ / \ H_i)]$$

(36) $$P_{HG} = P_G[1+(\phi_{HG} \ / \ 2)(J_{HG} \ / \ H_G)]$$

(37) $$P_{HV} = P_V[1+(\phi_{HV} \ / \ 2)(J_{HV} \ / \ H_V)].$$

A drawback of this simplifying setting is that, it does not allow property developers to behave strategically to influence housing prices by accumulating an inventory of newly finished dwellings when prevailing market prices are low. This means the model can not capture the movement of building vacancy rates, which is one of the real estate market indicators. This strategic behavior could be important when there are unanticipated shocks, especially in stochastic models.

The current value Hamiltonian of the household's maximization problem is given by:

(38) $$\begin{aligned} h_4 &= U(H_4, C_4, N_4) \\ &+\lambda_A(WN_4 + \bar{r}A - P_V I_V - \theta_1 P_G I_G - P_4 C_4 - TX) \\ &+\sum_{i=G}^{V}\lambda_{JHi}\{I_V - J_{Hi}[1+(\phi_{Hi} \ / \ 2)(J_{Hi} \ / \ H_i)\} \\ &+\sum_{i=G}^{V}\lambda_{Hi}(J_{Hi} - \delta_{Hi}H_i) \\ &+\lambda_{H4}[\alpha_1(H_V)^{\alpha 2} + (1-\alpha_1)(H_G)^{\alpha 2} - (H_4)^{\alpha 2}] \\ &+\lambda_{N4}(N_4 - \sum_{i=1}^{3} N_i). \end{aligned}$$

Besides the original constraints, other first order conditions are given by:

(39) $$J_{Hi} = (q_{Hi} - 1)(H_i \ / \ \phi_{Hi}); \ i = G, V$$

---

[10] Replacing the world interest rate in equation (34) by the domestic interest rate does not alter the simulation result.

(40) $$q_{HG} = \lambda_{HG} / (\lambda_A \theta_1 P_G)$$

(41) $$q_{HV} = \lambda_{HV} / (\lambda_A P_V)$$

(42) $$U_{H4} = \lambda_{H4} \alpha_2 (H_4)^{\alpha 2-1}$$

(43) $$U_{C4} = \lambda_A P_4$$

(44) $$U_{N4} = -\lambda_A W$$

(45) $$\dot{\lambda}_A = (\mu - \bar{r})\lambda_A$$

(46) $$\dot{\lambda}_{HG} = (r + \delta_{Hi})\lambda_{HG} - \lambda_{H4}(1-\alpha_1)\alpha_2 (H_G)^{\alpha 2-1}$$
$$-\lambda_A \theta_1 P_G (\phi_{Hi} / 2)(J_{HG} / H_G)^2$$

(47) $$\dot{\lambda}_{HV} = (r + \delta_{HV})\lambda_{HV} - \lambda_{H4}\alpha_1 \alpha_2 (H_V)^{\alpha 2-1} - \lambda_A P_V (\phi_{HV} / 2)(J_{HV} / H_V)^2.$$

Equations (39) to (41) govern the formation of residential housing in a way similar to commercial building. Equation (42) defines the value of the marginal benefit of housing services. Equation (43) equates the marginal benefit of consuming one more unit of aggregate durable goods to its marginal cost. Equation (44) implies that the household will supply labor services up to the point that the marginal disutility of working is just compensated by the marginal value of labor incomes. Equation (45) states the dynamic of the shadow price of financial assets. When the domestic interest rate goes up, investment falls. This must be matched by an increase in consumption, other things equal. When consumption rises, the marginal utility of consumption, which is the shadow price of financial assets, declines (equation 43). Equations (46) and (47) have similar interpretations as equation (19).

Similar to the treatment of intermediate inputs in the production side, the composition of the aggregate good $C_4$ is determined by a second-tier optimization process:

maximize $C_4$

subject to:

(48) $$(C_4)^{\alpha 4} = \alpha_3 (C_1)^{\alpha 4} + (1-\alpha_3)(C_2)^{\alpha 4} \; ; \; \alpha_4 \leq 1$$

(49) $$P_4 C_4 = P_1 C_1 + P_2 C_2$$

where:

(50) $$(P_4)^{1-\sigma 4} = \alpha_3{}^{\sigma 4} (P_1)^{1-\sigma 4} + (1-\alpha_3)^{\sigma 4} (P_2)^{1-\sigma 4} \; ; \; \sigma_4 = 1/(1-\alpha_4).$$

Solving the maximization problem, it can be obtained:

(51) $$C_1 = C_4 [\alpha_3 (P_4 / P_1)]^{\sigma 4}$$

(52) $$C_2 = C_4 [(1-\alpha_3)(P_4 / P_2)]^{\sigma 4}.$$

Equations (51) and (52) state that for a given expenditure on non-durable goods, the budget share of, say, $C_1$ falls when its price rises relative to $C_2$.

### 3.3 Government

Intertemporal issues of public finance are largely ignored in the model, as they are not related to the purpose of the paper. The government is supposed to balance the budget every period. The budget constraint of the government is given by:

(53) $$\sum_{i=1}^{3} P_{Li} L_i + TX = (1-\theta_1) P_G I_G .$$

The right hand side of equation (53) is the sum of the revenues from selling sector-specific factors to the industries and from the lump sum tax on the household. For simplicity, all the specific factors are supposed to be natural resources, so no production is involved. The left hand side of equation (53) is the spending on public housing.

The remaining role of government is perhaps more important. Firstly, it controls the rate of land supply. Secondly, it controls the distribution of land between residential and commercial purposes, as well as the distribution of residential land between private and public housing. Lastly, it determines the level of subsidy on public housing.

When the government interferes the distribution of land usage, it is separating the property markets and, hence, creating price discrepancies between the three types of real estate. Nonetheless, "controlling the distribution of land usage" is only a conceptual notion. In the model, it is assumed the government controls the distribution of housing investment goods, which are something like semi-products. An important merit of this shortcut specification is that it can highly simplify the model. An essential question about the difference between controlling the distribution of land, and that of housing investment goods, is who bears the risk of property price fluctuation. In the former, it is the developers who bear the risk, as there is a prolonged construction period, while, in the latter, it is the final users who bear the risk. With prefect foresight, the two are equivalent. Nonetheless, in stochastic models such risk could be crucial in influencing agents' behaviors. This specification is actually related to the adjustment cost issue discussed in the last sub-section.

The three functions of the government can be modeled either by exogenous variables or by endogenous policy reaction functions. For simplicity, the rate of land supply and the level of subsidy are both set exogenously.[11] In contrast, in determining the distribution of housing investment goods, it is assumed the government adopts a policy reaction function to minimize certain price indexes of the goods. By minimizing the price indexes, the government indeed constrains price discrepancies between the three real estate markets. Therefore, as a whole, the government is carrying out a restrained (or balanced) market separation policy: dividing the three property markets on the one hand, and limiting the price discrepancies between the three markets on the other.

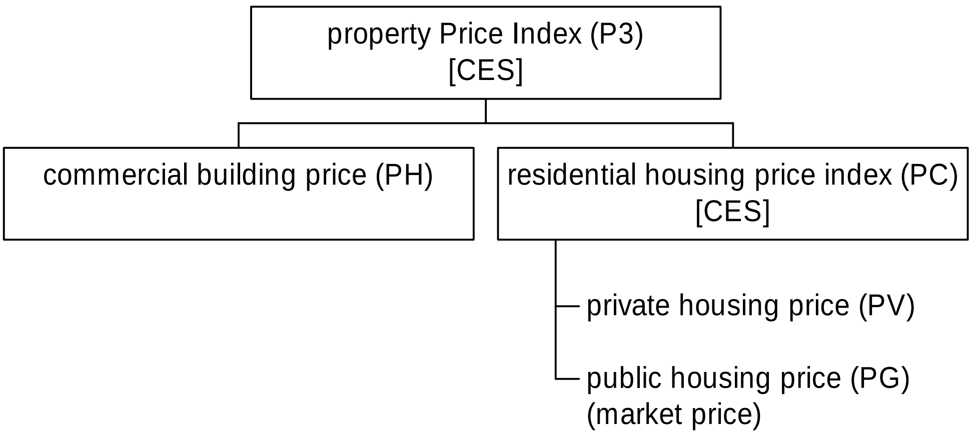
At the outset it should be pointed out that minimizing the property price index does not necessarily imply maximizing the utility of the household. The price minimization

[11] A possible way to endogenize the rate of land supply and the level of subsidy simultaneously is to impose that, the revenue collected from the former must be balanced by the expenditure on the latter. That is, the lump sum tax on the household is set to be zero.

policy is preferred mainly for its transparency and tractability. Secondly, as discussed above, effective property prices are equal to the prices of housing investment goods corrected by the adjustment costs. Therefore, minimizing the 'property price' index of housing investment goods is merely a convenient approximation of minimizing the index of effective property prices.

There are two property price indexes to be minimized. The nesting of the property price indexes is illustrated in Figure 3.3.

**Figure 3.3: Property Price Index Nesting**




The upper level minimization problem is given by:

minimize $P_3$

subject to:

(54) $(P_3)^{\alpha 10} = \alpha_9 (P_C)^{\alpha 10} + (1-\alpha_9)(P_H)^{\alpha 10}$ ; $\alpha_{10} \geq 1$

(55) $P_3 Y_3 = P_C I_C + P_H I_H$

where:

(56) $Y_3 = I_H + I_C$.

Solving the problem, it can be obtained:

(57) $$[\alpha_9 / (1-\alpha_9)](P_C / P_H)^{\alpha 10-1} = I_C / I_H.$$

Note that equation (56) implies that industrial and residential housing investment goods are prefect substitutes, as they are produced by the same production function. Therefore, the price differentiation between these two commodities is purely a result of market separation. The same point holds for private and public housing investment goods. Equation (57) indicates that to minimize the general property price index $P_3$, the share of residential housing investment goods will be increased when its price rises versus that of commercial building.

The share between private and public housing investment goods is determined by a second-tier price minimization process similar to the previous one:

minimize $P_C$

subject to:

(58) $$(P_C)^{\alpha 12} = \alpha_{11}(P_V)^{\alpha 12} + (1-\alpha_{11})(P_G)^{\alpha 12} \ ; \ \alpha_{12} \geq 1$$

(59) $$P_C I_C = P_V I_V + P_G I_G$$

where:

(60) $$I_C = I_V + I_G.$$

It should be noted that, the market price of public housing rather than the subsidized price is used in the index. Solving the problem, it can be obtained:

(61) $$[\alpha_{11} / (1-\alpha_{11})](P_V / P_G)^{\alpha 12-1} = I_V / I_G.$$

### 3.4 Model Closure

To close the model, it is needed to specify the market clearing conditions as well as the external account balances. In the above two sub-sections, the labor market is already assumed to clear, as there is no explicit distinction between labor supply and labor demand. The market for the construction sector is also assumed to clear by equation (56). The remaining markets are the manufacturing and servicing markets:

(62) $$Y_1 = \sum_{i=1}^{3} X_{1i} + C_1 + EX$$

(63) $$Y_2 = \sum_{i=1}^{3} X_{2i} + C_2.$$

Equations (62) and (63) state that the outputs of these two sectors are disposed among intermediate inputs, domestic consumption, and, in the case of the manufacturing industry, export.

The external accounts are specified as follows:

(64) $$TB = P_1 EX - P_K \sum_{i=1}^{3} I_{Ki}$$

(65) $$CA = TB + rA^*$$

(66) $$\dot{A}^* = CA$$

(67) $$\mu = r^*$$

(68) $$r = r^* + \xi$$

(69) $$\xi = -\eta A^* / GDP$$

(70) $$P_1 = P_1^*$$

(71) $$P_K = P_K^* = 1;$$

where:

(72) $$GDP = P_1 Y_1 + P_2 Y_2 + P_3 Y_3.$$

Equations (64) and (65) are merely the definitions of trade balance and current account balance, respectively. Equation (66) states that a current account surplus is matched by a capital account deficit of equal size. A capital account deficit means the domestic household accumulates a larger financial claim against the rest of the world. Equation (67) restates that the domestic subjective discount rate is set equal to the world interest rate. Equation (68) defines the uncovered interest parity condition: the domestic interest rate is equal to the world interest rate plus a risk premium. As the prices of tradable goods are exogenous, there is no expected depreciation term in the parity condition. Equation (69) specifies the risk premium as a function of the foreign debts to GDP ratio. It means that, the home economy can not accumulate foreign debts or assets infinitely. Equations (67) to (69) are crucial in the sense that they tie down the model in the long run, but allow the domestic interest rate to deviate from the world interest rate and the discount rate in the short run. Otherwise, the model will encounter the "disturbing implications from the open-economy Rasmey model" (Barro and Sala-i-Martin 1995:108) that, all the dynamics are ruled out by forcing the domestic interest rate to be equal to the world interest rate and the discount rate at every period.

Equations (69) and (70) define the terms of trade, through specifying the values of imported capital investment goods and exportable manufacturing goods, respectively. The price of capital investment goods is set to be one as the numeraire.

# 4 SIMULATION EXPERIMENTS

## 4.1 Baseline

As the modeling exercise is for theoretical rather than for forecasting purposes, the parameters and exogenous variables of the model are imposed rather than estimated. However, the values of the world interest rate, the capital adjustment cost, and the capital depreciation rate are based on McKibbin and Sachs (1991). The world interest rate is set 5 percent, indicating that one period corresponds roughly to one year. The share of government subsidy in public housing is assumed to be moderate, only 30 percent. The lump sum tax on the household turns out to be a subsidy, because the revenues collected from selling specific factors is more than enough to finance the expenditure on public housing. This reiterates the point that, monopolizing land sale provides the government an indirect but effective means of taxation. The national account and other details of the modeled economy in the baseline scenario are summarized in Tables A2 to A6, and the values of the parameters and exogenous variables are listed in Table A7 in the Appendix.

The model has about 100 equations totally. It is solved numerically using a software called Fair-Taylor which is based on the algorithm suggested by Fair and Taylor (1983). For a review of the numerical techniques used to solve this type of nonlinear rational expectation model, see McKibbin (1987), who also developed the Fair-Taylor software.

### 4.2 Land Policy Shock

Only one simulation about the government's land policy is conducted in this paper. The shock is a 10 percent permanent increase in the rate of land supply. The result is illustrated in Figures 4.1 to 4.10.

The impact of the positive land supply shock on industrial outputs, physical capital stocks as well as housing stocks is expectable. The shock is a resource boom or virtually an increase in productivity. The rise in the deployment of land resource increases the output of the construction sector immediately (Figure 3.1). The outputs of the manufacturing and services industries go up gradually too, though more moderate than the construction industry.

Although the output of the construction sector jumps up immediately after the shock, housing stocks increase only gradually. This is because the construction sector produces housing investment goods only, and the accumulation of housing investment goods into housing stocks is subject to adjustment costs. The stocks of commercial building and residential housing rise by similar proportions (Figure 4.3). Echoing this is a common falling trend of all property prices (Figure 4.9). This is because the price minimization processes guarantee any increment in housing investment goods to be distributed to all the property markets according to their weightings in the price indexes.

The slight differences between various property prices come from asymmetric weightings in the price indexes and adjustment costs.

The consumption profile of the household changes in a pattern very similar to the industrial outputs (Figure 4), though the consumption of non-durable goods goes down slightly at the beginning. It is found that the increase in land resource encourages investment and savings. Larger stocks of both domestic and foreign assets are accumulated as a result (Figure 5). Notwithstanding, in the long run, the stock of foreign assets reduces to zero, as pined down by the interest parity condition. Matching this is that both the trade account and the current account turn from balanced into credit initially and, then, into deficit in the later period (Figure 6). The two accounts resume balanced, again, in the long run.

As the home economy is a net creditor to the rest of the world, the manufacturing export must go up by more than to finance the rising import of capital investment goods. Nonetheless, all outputs are contemporaneously static. Therefore, the initial surge in the manufacturing export has to be matched by a fall in the private consumption (Figure 4.4). The requirement of higher saving is met further by an increase in the labor supply. Whereas more labor is attracted toward the expanding construction industry at the expanses of the service industry (Figure 4.7).

The most ‘usual’ result is perhaps the dynamic of the external balance. To understand this result, it is helpful to compare the impact of a resource boom in the construction industry with a similar shock in another industry. Figure 4.13-4.16 show the impact of a 10 percent permanent rise in the specific factor in the manufacturing industry on some variables. The result is a typical one for resource booms or productivity shocks. With a larger input specific factor, the output of the manufacturing industry jumps immediately after the shock, and gradually increases after. Expecting a higher future income, the home economy will try to increase the consumption of tradable manufacturing goods immediately.[12] The result is a fall in the manufacturing export. To smooth

[12] The consumption of housing commodities and services can not go up immediately because their outputs are contemporaneously static and they are non-tradable.

intertemporal consumption, the home economy borrows from the rest of the world in the initial period, and replies the debts later, when the manufacturing output goes up. This is a typical result in resource booms.

Why the resource boom in the construction sector gives an opposite result, as far as the external balance is concerned? Retrospectively, we realize that, in the long run, the effect of a larger land supply can be materialized only through higher domestic investment. This is simply because neither land nor housing investment goods are directly consumable or exportable. Secondly, both commodities are non-tradable, so the home economy can no export them in exchange for foreign consumer goods. Thirdly, since the accumulation of housing and physical capital is subject to adjustment costs, the materialization of the boom has to be delayed mostly to the future. The simulation result shows that, the rise in the domestic investment is wholly financed by a higher domestic saving. An essential equation is that, why the home economy does not borrow from overseas to finance the growing investment. This can be explained by the fact that, housing commodity and non-tradable goods enter the utility function following a Cobb-Douglas format. The utility maximization process requires the marginal utilities from consuming housing and non-durable goods to be equated. Due to the adjustment cost, the housing stock is contemporaneously static. Therefore, the household prefers to work and save more in the short run to accelerate the process of housing investment, such that it can consume more of both commodities simultaneously in the later period. On the other side, the domestic interest rate is equal to the world interest rate plus a risk premium. The risk premium is set to be a function of the foreign debts to GDP ratio. This implies that the accumulation of foreign assets must be accompanied by a fall in the domestic interest rate.

The figures suggest that the changes of the trade balance and the current account balance are rather small. This is due to the fact that, to guarantee the non-linear program to converge, it is necessary to assign the risk premium parameter $\eta$ a fairly large value--0.05. In principle, the larger the change of the external balances, the smaller the value of $\eta$. Therefore, the increase in the domestic saving is almost wholly used to finance rising domestic investment. In the medium run, the trade account and the current account turn into deficit. At the same time, the amount of foreign assets is declining towards zero. This is because the domestic subjective discount rate is set to be the same as the world interest rate, so in the long run, the home economy has to dispose any accumulated foreign assets.

It is worth to point out that, the service industry output reduces very marginally in the first few periods. This is paralleled by a fall in the output price and the labor input in the service industry in the initial period (Figures 7-8). That is, there is a slight tendency for the service industry to be 'de-industrialized'. Such a de-industrialization effect is a typical result of resource booms in Dutch disease models. Expansion in one sector can cause contraction in others because of two factors. Firstly, the booming sector attracts labor and other resources from other sectors, pushing up the production costs in the latter. This is referred as the resource movement effect by Corden and Neary (1982). Secondly, changes in relative prices shift the consumption demand towards the cheaper output of the booming sector. This is referred as the spending effect in their study. In our model, the effect of de-industrialization is small. This is because industrial productions are linked closely by intermediate inputs and commercial building. The positive land supply shock reduces the price of commercial building, and, therefore, the cost of production in the service industry. Secondly, the production of housing also requires services as an intermediate input. It thus helps to maintain the output price of the service industry. Thirdly, the land supply shock increases the total labor supply. These explain why the price of services only falls very marginally, and the output of the service industry actually raises except during the very early period. The same story can be applied to the manufacturing industry too. Nonetheless, the manufacturing industry is further sheltered from de-industrialization by the fact that it is the sole export sector and its output price is fixed at the world level.

It is useful to further point out that, the conclusion about de-industrialization becomes conditional on factor intensity, once capital is mobile across sectors in the long run. Behind this ambiguity is the Rybczynski effect: at constant output prices, when labor is extracted away (by a third booming sector), the output of the labor-intensive sector declines and that of the capital-intensive sector rises. In our model, the factor intensities of all the industries are set to be the same, and capital is importable. Therefore, the Rybczynski effect is eliminated.

# 5 CONCLUSIONS

This paper presents a rational expectation, intertemporal general equilibrium model to study the effect of property markets and policies in a macroeconomic context. The model distinguishes three industries as well as three real estate markets. It emphasizes the interdependence between industries, and the government's role in the property markets. A simulation of a permanent land supply shock is conducted. It is found that property markets have impact not only on the internal balance, but also on the external balance. The result shows that, in this model, resource booms in a non-tradable durable good sector leads to an improvement in its external balance in the initial period. The finding contrasts the result of resource booms in a tradable non-durable sector. Secondly, with production interdependence, resource boom in one sector does not necessarily result in contraction of some other sectors.

Lastly, the model used in this paper actually is still under development. The next version of the model, which is under construction, will try to incorporate money and the nominal exchange rate.

## REFERENCE


Barro, R. J. and X. Sala-i-Martin (1995). *Economic Growth*, McGraw-Hill.

Corden, W. M. and J. P. Neary (1982). "Booming Sector and De-industrialisation in a Small Open Economy." *The Economic Journal* 92: 825-48.

Fair, R. C. and J. B. Taylor (1983). "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models." *Econometrica* 51(4): 1169-86.

Friedman, J. and D. H. Weinberg (1982). *The Economics of Housing Vouchers*. New York, Academic Press.

Hammond, C. H. (1987). *The Benefits of Subsidized Housing Programs: An Intertemporal Approach*. Cambridge, Cambridge University Press.

Harmon, O. R. and M. J. Potepan (1988). "Housing Adjustment Costs: Their Impact on Mobility and Housing Demand Elasticities." *Journal of the American Real Estate and Urban Economics Association* 16(4): 459-78.

Hartley, J. E. (1997). *The Representative Agent in Macroeconomics*. New York, Routledge.

Lucas, R. (1967). "Adjustment Costs and the Theory of Supply." *Journal of Political Economy* 75: 321-34.

Lui, F. T. (1997). Hong Kong's Residential Housing Market and Policies. *The Other Hong Kong Report*. J. Y. S. Cheng. Hong Kong, The Chinese University Press**:** 357-376.

Maclennan, D. (1982). *Housing Economics: An Applied Approach*. London, Longman.

McDonald, J. F. (1979). *Economic Analysis of an Urban Housing Market*. New York, Academic Press.

McKibbin, W. J. (1987). "Numerical Solution of Rational Expectations Models With and Without Strategic Behaviour." *Research Discussion Paper of the Reserve Bank of Australia*(8706).

McKibbin, W. J. and J. D. Sachs (1991). *Global Linkages: Macroeconomic Interdependence and Cooperation in the World Economy*. Washington, D.C., The Brookings Institute.

McKibbin, W. J. and P. J. Wilcoxen (1995). "The Theoretical and Empirical Structure of the G-Cubed Model." *Brookings Working Paper*.

Meen, G. (1990). The Macroeconomic Effects of Housing Market Policies under Alternative Mortgage Conditions. *Housing and the National Economy*. J. Ermisch. Aldershot, Avebury**:** 28-42.

Miles, D. (1994). *Housing, Financial Markets and the Wider Economy*. Chichester, John Wiley & Sons.

Nielsen, S. B. and P. B. Sorensen (1994). “Inflation, Capital Taxation, and Housing: the Long Run in a Small Open Economy.” *Canadian Journal of Economics* 27(1): 198-217.

Okumura, T. (1997). “Housing Investment and Residential Land Supply in Japan: An Asset Market Approach.” *Journal of the Japanese and International Economies* 11: 27-54.

Ramsey, F. P. (1928). “A Mathematical Theory of Saving.” *Economic Journal* 38: 543-59.

Rebelo, S. (1997). “What Happens When Countries Peg Their Exchange Rates? (The Real Side of Monetary Reforms).” *NBER Working Paper*(6168).

Renaud, B., F. Pretorious, et al. (1997). *Markets at Work: Dynamics of the Residential Real Estate Market in Hong Kong*. Hong Kong, Hong Kong University Press.

Salter, W. E. G. (1959). “Internal and External Balance: The Role of Price and Expenditure Effects.” *Economic Record* 35: 226-38.

Treadway, A. B. (1969). “On Rational Entrepreneurial Behaviour and the Demand for Investment.” *Review of Economic Studies* 36: 227-39.

Tse, R. Y.-C. (1994). *Real Estate Economics: Theory and Policy---with Reference to Hong Kong, Singapore and Taiwan*. Hong Kong, EIA Publishing Ltd.

Turnovsky, S. J. (1996). *Methods of Macroeconomic Dynamics*. Cambridge, Massachusetts, MIT.

Turnovsky, S. J. and T. Okuyama (1994). “Taxes, Housing, and Capital Accumulation in a Two-Sector Growing Economy.” *Journal of Public Economics* 53(2): 245-69.

Uzawa, H. (1968). Time Preference, the Consumption Function and Optimum Asset Holdings. *Value, Capital and Growth: Papers in Honour of Sir John Hicks*. J. N. Wolfe. Chicago, Aldine.

van Winccop, E. (1993). “Structural Adjustment and the Construction Sector.” *European Economic Review* 37: 177-201.

## APPENDIX

### Table A1: Summary of Notation (in alphabetical order)

| | | |
|---|---|---|
| **Sector Subscript (*i*):** | | |
| *1* | = | manufacturing industry |
| *2* | = | service industry |
| *3* | = | construction industry |
| *4* | = | household |
| **Symbol:** | | |
| $A$ | = | financial assets |
| $A^*$ | = | net holding of foreign assets |
| $C_4$ | = | consumption of non-durable goods, composed of good *1* and *2* |
| $C_i$ | = | consumption of good $i$, $i = 1, 2$ |
| $CA$ | = | current account balance |
| $EX$ | = | net export of good *1* |
| $H_4$ | = | composite residential housing stock |
| $H_i$ | = | commercial building stock in sector $i$ |
| $H_G$ | = | public-subsidized residential housing stock |
| $H_V$ | = | private-funded residential housing stock |
| $I_C$ | = | total investment on residential housing, composed of $I_V$ and $I_G$ |
| $I_G$ | = | investment on public-subsidized residential housing |
| $I_H$ | = | total investment on commercial building, composed of $I_{Hi}$ |
| $I_{Hi}$ | = | investment on commercial building in sector $i$ |
| $I_{Ki}$ | = | investment on physical capital in sector $i$ |
| $I_V$ | = | investment on private-funded residential housing |
| $J_{Hi}$ | = | formation of commercial building in sector $i$ |
| $J_{HG}$ | = | formation of public-subsidized residential housing |
| $J_{HV}$ | = | formation of private-funded residential housing |

| | | |
|---|---|---|
| $J_{Ki}$ | = | formation of physical capital in sector *i* |
| $K_i$ | = | physical capital stock in sector *i* |
| $L_i$ | = | sector-specific factor input in sector *i* |
| $N_4$ | = | total labor supply |
| $N_i$ | = | labor input in sector *i* |
| $P_4$ | = | price of composite good $C_4$ |
| $P_C$ | = | price of residential housing investment good $I_C$ |
| $P_G$ | = | price of public-subsidized residential housing investment good $I_G$ |
| $P_H$ | = | price of commercial building investment good $I_H$ |
| $P_i$ | = | domestic price of good *i*, $i = 1, 2, 3$ |
| $P_1^*$ | = | foreign price of good *1* |
| $P_K$ | = | domestic price of imported capital |
| $P_K^*$ | = | foreign price of imported capital |
| $P_{Li}$ | = | price of sector-specific factor $L_i$ |
| $P_V$ | = | price of private-funded residential housing investment good $I_V$ |
| $P_{Xi}$ | = | price of composite intermediate factor $X_i$ |
| $Q^i$ | = | production function of sector *i* |
| $q_{Hi}$ | = | Tobin's q of commercial building in sector *i* |
| $q_{HG}$ | = | Tobin's q of public-subsidized residential housing |
| $q_{HV}$ | = | Tobin's q of private-funded residential housing |
| $q_{Ki}$ | = | Tobin's q of capital in sector *i* |
| $R(s)$ | = | the average of the short-term interest rate $r(t)$ from time *s* to *t* |
| $r(t)$ | = | short-term interest rate at time *t* |
| $r^*$ | = | world interest rate |
| $S_i$ | = | primary factor input in sector *i*, composed of $K_i$ and $N_i$ |
| $TB$ | = | trade balance |
| $TX$ | = | lump sum tax on household |
| $U$ | = | instantaneous utility function |
| $W$ | = | real wage rate |
| $X_i$ | = | intermediate input in sector *i*, composed of goods *1* and *2* |

| | | |
|---|---|---|
| $X_{ji}$ | = | the amount of good *j* used as intermediate goods in sector *i* |
| $Y_i$ | = | output of sector *i* |
| $\delta_H$ | = | deprecation rate of commercial building |
| $\delta_{Hi}$ | = | deprecation rate of *i*-type of residential housing; $i = G, V$ |
| $\delta_K$ | = | deprecation rate of physical capital |
| $\lambda_j$ | = | shadow price of variable *j* |
| $\Pi_i$ | = | profit function of sector *i* |
| $(1-\theta_1)$ | = | subsidy rate of public residential housing investment goods |
| $\phi_H$ | = | adjustment cost parameter of commercial building |
| $\phi_{Hi}$ | = | adjustment cost parameter of *i*-type of residential housing; $i = G, V$ |
| $\phi_K$ | = | adjustment cost parameter of physical capital |
| $\eta$ | = | tie down parameter of risk premium |
| $\mu$ | = | domestic subjective discount rate |
| $\xi$ | = | risk premium |

## Table A2: The National Account

| | Value | % of sub-total | % of GDP |
|---|---|---|---|
| Consumption of manufacturing goods | 90.44 | 48.54 | 17.90 |
| Consumption of services | 95.87 | 51.46 | 18.97 |
| **Sub-total of consumption of non-durables** | **186.31** | **100** | **36.87** |
| Investment on residential housing | 53.23 | 26.03 | 10.33 |
| Investment on physical capital | 122.36 | 59.85 | 24.21 |
| Investment on commercial building | 28.87 | 14.12 | 5.71 |
| **Sub-total of private investment** | **204.46** | **100** | **40.46** |
| Government spending on public housing | 13.42 | | 2.66 |
| Net exports | 0 | | 0 |
| Intermediate inputs | 101.15 | | 20.02 |
| **GDP** | **505.23** | | **100** |

## Table A3: Relative Sizes of Industries

| | Gross output | % of sector output | % of GDP |
|---|---|---|---|
| Manufacturing export | 122.36 | 46.53 | 24.22 |
| Manufacturing intermediate | 50.15 | 19.07 | 9.93 |
| Manufacturing consumption | 90.44 | 34.4 | 17.90 |
| **Output of manufacturing industry** | **262.95** | **100** | **52.05** |
| Service intermediate | 50.89 | 34.66 | 10.07 |
| Service consumption | 95.87 | 65.34 | 18.98 |
| **Output of service industry** | **146.76** | **100** | **29.05** |
| Output of construction industry | 95.52 | | 18.91 |
| **GDP** | **505.23** | | **100** |

## Table A4: Relative Sizes of Property Markets

| | Gross value | % of sub-total | % of total |
|---|---|---|---|
| Private housing | 219.18 | 32.89 | 22.95 |
| Public housing | 447.30 | 67.11 | 46.83 |
| **Sub-total of residential housing** | **666.48** | **100** | **69.77** |
| Manufacturing | 150.26 | 52.05 | 15.73 |
| Services | 83.86 | 29.05 | 8.78 |
| Construction | 54.58 | 18.91 | 5.71 |
| **Sub-total of commercial building** | **288.70** | **100** | **30.22** |
| **Total** | **955.19** | | |

## Table A5: Household's Balance Sheet

| Item | Value | % of pre-tax income |
|---|---|---|
| Interest payment | 164.11 | 64.84 |
| Lump sum tax (negative = subsidy) | -37.10 | 14.66 |
| Wage payment | 89.00 | 35.16 |
| Pre-tax income | 253.11 | 100 |
| After-tax income | 290.21 | 114.66 |

## Table A6: Relative Prices

| | Relative Price |
|---|---|
| Capital investment goods | 1 (numeraire) |
| **Output Prices** | |
| Manufacturing goods | 0.33 (exogenous) |
| Services | 0.31 |
| Construction (aggregate housing investment goods) | 0.30 |
| **Property Prices (including adjustment cost)** | |
| Private housing | 0.48 |
| Public housing (subsidized rate) | 0.48 |
| Commercial building | 0.60 |
| **Others** | |
| Consumer price index | 0.39 |
| Wage | 11.73 |
| Interest rate | 0.05 |

## Table A7: Values of the Parameters and Exogenous Variables of the Model

| | | | | |
|---|---|---|---|---|
| $r^* = 0.05$ | $\theta_1 = 0.7$ | $\beta_1 = 1.0$ | $\alpha_1 = 0.7$ | $\alpha_9 = 0.5$ |
| $P_K^* = 1.0$ | $\eta = 0.05$ | $\beta_2 = 0.33$ | $\alpha_2 = 1.0$ | $\alpha_{10} = 2.0$ |
| $P_1^* = 0.33$ | $\delta_K = 0.1$ | $\beta_3 = 1.0$ | $\alpha_3 = 0.5$ | $\alpha_{11} = 0.5$ |
| $L_i = 20.0;$ | $\delta_H = 0.1$ | $\beta_4 = 2.0$ | $\alpha_4 = 0.5$ | $\alpha_{12} = 0.2$ |
| $(i = 1–3)$ | $\phi_K = 20.0$ | $\beta_{31} = 12.5$ | $\alpha_5 = 0.5$ | $\alpha_{13} = 0.25$ |
| | $\phi_H = 20.0$ | $\beta_{32} = 0.6$ | $\alpha_6 = 0.2$ | $\alpha_{14} = 0.25$ |
| | | $\beta_{33} = 0.2$ | $\alpha_7 = 0.7$ | $\alpha_{15} = 0.50$ |
| | | $\beta_{34} = 0.1$ | $\alpha_8 = 2.0$ | |
| | | $\beta_{35} = 0.1$ | | |