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Special Majorities Rationalized*

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Abstract: Complaints are common about the arbitrary and conservative bias of special-majority rules. Such complaints, however, apply to asymmetrical versions of those rules alone. Symmetrical special-majority rules remedy that defect, albeit at the cost of often rendering no determinate verdict. Here we explore what is formally at stake, both procedurally and epistemically, in the choice between those two forms of special-majority rule and simple majority rule; and we suggest practical ways of resolving matters left open by symmetrical special-majority rules, such as judicial extrapolation or 'subsidiarity' in a federal system.

'Our Constitution ... is called a democracy because power is in the hands ... of the greater number', the draft Constitution for Europe begins, invoking Pericles' Funeral Oration. But instead of rule purely by 'the greater number' (simple majority rule) draft Article 24 actually prescribes 'qualified majority' rule. Depending on circumstances, decisions of the European Council or Council of Ministers would require the consent of a majority or two-thirds of the Member States, 'representing at least three fifths of the population of the Union' (Giscard d'Estaing 2003, pp. 5, 21).

Such 'special' (or 'qualified' or 'super') majority requirements are not uncommon. It may take only a majority vote of both houses of the US Congress to declare war; but it takes a two-thirds majority to override a President's veto or three-fifths to close Senate debate (Mayhew 2002). Increasing taxes requires a three-fifths, two-thirds or even three-quarters majority of the legislature in nearly a third of American states (Rafool 1998). Criminal verdicts must be unanimous or nearly so (ten-to-two, in England and some American states) (Abrahamson 1994, p. 180).

Super-majorities are sometimes seen as second-best compromises, employed where decisions ought ideally to be unanimous but where the costs of securing unanimity would be too high. Libertarians, for example, hold that people should not have authority exercised over them, or money taken from them, except with their consent: pragmatically, they concede, securing the consent of absolutely everyone to absolutely every enactment and absolutely every tax would preclude the efficient operation of government; still, they say, we should at least require substantial majorities for those purposes (Buchanan and Tullock 1962; Mueller 2003, pp. 137-44). Likewise with the criminal jury: ideally, we should convict someone only by a unanimous verdict; but where that cannot be obtained, at least we should demand a large (ten-to-two) majority. And likewise in amending constitutions: every state's consent was required in joining the union, so ideally every state's consent should be required in changing the terms of association; it may be impractical to demand unanimous consent to every amendment, but at least a large majority (three-quarter, in the US) ought to be required.

The plain political fact is that the larger the majority required, the less likely it can be secured. Special-majority rules ordinarily leave existing arrangements in place unless there is some positive decision to change them. Hence special-majority rules of the sort with which we are most familiar have a powerful conservative bias.

That is precisely their attraction, for those attracted to them. Most of us think it right that there be a presumption of innocence in criminal trials, and that it should be hard to overcome that presumption. Libertarians think there should be a presumption in favour of letting people do as they please, particularly with their own money, and that it should be hard for government to interfere with that. Economists think that there should be a presumption in favour of sticking to our long-term economic objectives, rather than succumbing to short-term temptations, and they recommend central bankers operate according to super-majority rules to ensure that (Dal Bó 2002).

Justifying special-majority rules of the familiar sort is thus largely a matter of justifying the conservative bias built into them. Sometimes, as with the presumption of innocence in criminal cases, the bias seems justified; other times it is controversial, justified in the view of some but not others. Sometimes there is not even a single status quo. (Think of heterogeneous federations like the European Union: the status quo varies from one member state to the next; any EU-wide policy picking out some one of them would be arbitrary.) Still other times, there seems no more reason for decision procedures to be biased in one direction rather than the other. Civil trials, for example, are decided on the 'balance of probabilities'. The same standard of proof falls on both parties, rather than (as in criminal cases) one side having to establish its case 'beyond a reasonable doubt' and the other side winning by default if not.

The reluctance to let social decisions be determined by some arbitrary bias built into super-majority decision rules can be captured in the complaint, 'It would be impossible to get the requisite majority for the status quo, either!' (Barry 1965, pp 244, 312-16). Or as Rae (1975, p. 1279) puts it, 'If *all* outcomes were subject to unanimity, then we would risk [being in] the position in which we both refused to change policy and refused to keep it the same'.

What is ordinarily taken to follow from that line of thought is that, where we have no grounds for 'a presumption one way or the other', then we must 'surely abide by a simple majority' (Barry 1965, p. 312). To implement a super-majority rule in such circumstances would be to privilege, arbitrarily, whichever outcome is identified as the default option.

That problem of arbitrariness derives from the *asymmetrical* nature of the familiar sorts of special-majority rules. One option is identified as the 'default option'; and it prevails if no other option secures the requisite 'special majority'.[1] The default option can thus prevail without itself commanding the support of anything approaching the requisite 'special majority', whereas any other option can prevail only if it does have the support of such a 'special majority'. That is the sense in which ordinary special-majority rules are actually *Asymmetrical Special-Majority* rules, although they are not usually so labelled. And that asymmetry is the source of the complaint made above. Unless the choice of the default option can itself be justified, privileging that option is purely arbitrary.

There is, however, an alternative way of specifying a special-majority rule without privileging any option. Just remove the asymmetry. Under a *Symmetrical Special-Majority* rule, the same special majority would be required to install *any* option as the social decision. If no option can secure such a majority then *no* option is chosen. This rule is symmetrical in holding each option to the same standard: each option has to get exactly the same 'special majority' of votes in order to be installed as the outcome; no option is installed by default,

[1] The *status quo* is typically the default option, but logically any option could be so designated. Specifying one option as the 'default' which wins (even if no one votes for it!), so long as no other option gets the requisite 'special majority', is just one case within the larger class of generalized majority rules which hold different options to different (‘super’, ‘simple’, or ‘sub’) ‘majoritarian’ standards. See Appendix 1.

merely because no option (including the default one) got the requisite number of votes. It is this version of special-majority voting that this article is concerned to develop.

Whereas the great bugbear of Asymmetrical Special-Majority rules is arbitrariness, the great bugbear of Symmetrical Special-Majority rules is that they may leave too much open. For those matters that cannot be left open, some supplementary mechanism is required for settling things that Symmetrical Special-Majority voting cannot. We sketch proposals along those lines in Section V below.

We precede those more practical remarks with some more formal ones identifying the distinctive characteristics of those two alternative forms of special-majority voting. Democratic decision procedures can, in general, be defended either on grounds of procedural fairness or on grounds of their epistemic truth-tracking capacities or both (List and Goodin 2001). We analyze the formal characteristics of the two forms of special-majority voting, first from a procedural point of view (Section I) and then from an epistemic one (Section II). The epistemic standards are of a Bayesian sort, growing out of related work on the Condorcet Jury Theorem (Goodin 2002; List 2003); the procedural standards are variations on those which May (1952) famously showed to characterize Simple Majority Voting itself.

We identify a 'trilemma', in both the procedural and epistemic domains. In each, there are three properties we might like a decision rule to display, but any given rule can display at most two of them at once. Our choice among decision rules – Simple Majority rule, Asymmetrical Special-Majority rule or Symmetrical Special-Majority rule – depends on which of the three desiderata we are prepared to sacrifice. What is at stake in this choice is summarized in Section III.

I. Procedural Properties of Special-Majority Rules

Within the social choice literature, May's Theorem 'is deservedly considered a minor classic' (Barry and Hardin 1982, p. 298; see similarly Luce and Raiffa 1957, pp. 357-8; Sen 1970, pp. 68. 70-1; Mueller 2003, pp. 133-6). In a literature replete with negative (impossibility) results, May's Theorem tells us what *positively* can be said on behalf of Simple Majority rule.

It shows that Simple Majority rule – and it alone among all decision procedures – simultaneously satisfies four conditions, each of which seems independently desirable on democratic grounds.

Here we assess both forms of special-majority rule against analogous criteria. To foreshadow our conclusions: Both forms of special-majority rule require a relaxation of one of those criteria, but different ones. The Symmetrical Special-Majority rule relaxes the responsiveness criterion (permitting more ties), whereas the more familiar Asymmetrical Special-Majority rule relaxes the symmetry criterion. Which, if either, of the special-majority rules is attractive in a given context depends on whether good grounds can be adduced there for relaxing the relevant criterion.

A. An Informal Statement

The conditions which May (1952) shows to be uniquely satisfied by Simple Majority rule are stated formally in Section II.B below. We here describe them informally and suggest some grounds for their appeal as features we would like a democratic decision procedure to display.

The first condition is what May calls 'decisiveness' and what others (including us below) call 'universal domain'. The condition stipulates that the social decision rule renders a decision (where a tie is a decision, too) for every logically possible combination – or 'profile′ – of votes. A democratic decision procedure should surely aspire to being open in this way to all the possible combinations of votes that might be entered into it. Insofar as the decision procedure is ‘indecisive’ in that way, yielding no decision when confronted with certain constellations of votes, those preference profiles are effectively disenfranchised.

The second condition May calls 'equality', and others (including us below) call 'anonymity'. The condition says that it should not matter *who* votes for what; all that should matter is *how many* of us vote for each option.

All majoritarian voting rules considered here – simple and special ones alike – satisfy those first two conditions. May's third condition he calls 'neutrality', and others (including us below) call 'symmetry'. Just as the last condition requires that all voters be treated equally, this condition requires that all alternatives be treated equally. A given number of votes for one alternative should yield the same social decision that it would yield if it were for any other alternative.

The conditions of ‘anonymity’ and ‘symmetry’, 'taken together... embody an interpretation of the basic idea of popular will theories of political fairness – any fair method for aggregating individual preferences should treat each person's preference equally' (Beitz 1989, p. 59). They embody the principle that each citizen's 'opinion is at least as good as any other's' (Ackerman 1980, p. 279, see also pp. 11-12, 44-4, 277-85).[2]

Asymmetrical Special-Majority rule violates May’s third condition, thus leading to the complaints about arbitrariness mentioned above. Symmetrical Special-Majority rules, by contrast, are designed to satisfy the condition.

May calls his fourth condition 'positive responsiveness'. This condition subsumes 'monotonicity', which we split out as a separate condition. The ‘monotonicity’ part says that, if some votes change in a certain direction (say, from 'against' to 'for' a proposition) while all other votes remain fixed, then the social decision should not change in the opposite direction. The 'positive responsiveness' part says that, starting in a situation in which the decision is one of social indifference, if one person who initially opposes a proposition changes to vote in favour of it, then the social decision should also change to favour the proposition.

May’s fourth condition does a lot of work. The ‘monotonicity’ part is what makes social decisions a positive (or, anyway, non-negative) function of how people vote, which is of course the essence of democracy. The ‘positive responsiveness’ part – which stipulates that the social decision always changes with the change of a single vote, given antecedent social indifference – is what makes Simple Majority Voting the decision rule that uniquely satisfies May’s four conditions simultaneously.

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[2] Reimer (1951, p. 17) similarly complains that (implicitly, Asymmetrical) special-majority rules violate the 'egalitarian premise' central to democratic rule 'that each citizen ... has the right to have his vote for elected officials counted equally with others'. Under an (Asymmetrical) special-majority rule, 'the views of the individual members of the minority would be more heavily weighted than those of the individuals composing the majority'.

While all special-majority rules meet the ‘monotonicity’ part of May’s condition, only Asymmetrical Special-Majority rule meets the part about ‘positive responsiveness to the change of a single vote’ (albeit at the expense of violating ‘symmetry’). A Symmetrical Special-Majority rule, in contrast, is responsive only to the change of enough votes to constitute a 'special majority'.

Of course, whatever reasons we have for requiring a 'special majority' to make a decision, those might also constitute reasons for reconstruing the responsiveness requirement accordingly. In our formal exposition below, we introduce a generalized responsiveness condition, called ‘positive k-responsiveness’, where k is the number of votes sufficient to break a tie. May’s condition corresponds to the special case of k=1.

Symmetrical Special-Majority rule satisfies all of May’s conditions, so long as ‘positive responsiveness’ is replaced with the less demanding condition of ‘positive k-responsiveness’ for a suitable k. This modification comes at the price of a proliferation of what we call 'non-trivial ties'. Where no option obtains the requisite 'special majority', the Symmetrical Special-Majority rule deems the decision to be a 'tie'; such ties may occur even if there are more votes for one option rather than any other (just insufficiently many more).[3]

In the case of Asymmetrical Special-Majority rules, the more demanding version of 'positive responsiveness' can be satisfied, but the 'symmetry' condition has to be sacrificed. To justify an Asymmetrical Special-Majority rule, therefore, we need some justification for the asymmetry (for the 'bias' in favour of the default option, in the earlier examples) — and also for the strength of that bias (as reflected in the size of the special majority required for any other option to prevail).

The formal analytics in Section II.B below point to a 'trilemma'. If we want to give minorities a 'veto power' (which is what special-majority rules in effect do), then we must sacrifice either the condition of 'symmetry' or the condition of 'no non-trivial ties'. We prove that a decision rule can satisfy any two of those conditions – veto powers; symmetry; no non-

[3] Colloquially, this might be deemed a failure of 'decisiveness', but technically (in May's sense) it is not so: a 'tie' is still a decision; no options are left unranked, even if many options stand in a relation of indifference to one another.

trivial ties – but no decision rule can satisfy all three of them simultaneously. Simple Majority rule satisfies the last two but forsakes the first (it allows no vetoes). The Asymmetrical Special-Majority rule satisfies the first and last but forsakes the middle (it lacks symmetry). The Symmetrical Special-Majority rule satisfies the first and second but forsakes the last (it allows non-trivial ties).

B. Formal Analytics[4]

1. The Framework

Let there be n individuals, labelled 1, 2, …, n. Let there be two alternatives, labelled 1 and -1. The two alternatives might be interpreted as the acceptance or rejection of a proposal, or as two alternatives or candidates in an election.

The vote of individual i is represented by v_i, where v_i takes the value 1 or -1; $v_i = 1$ means that individual i votes for alternative 1, and $v_i = -1$ means that individual i votes for alternative -1. For simplicity, we assume that no individual is indifferent between the two alternatives. A *profile* is a vector $v = \langle v_1, v_2, \ldots, v_n \rangle$ of votes across the n individuals.

Given a profile v, we write Σv_i as an abbreviation for $v_1+v_2+\ldots+v_n$. Then Σv_i is the absolute margin between the number of votes for 1 and the number of votes for -1, i.e. [number of 1s in v] - [number of -1s in v].

An *aggregation function* is a function f that maps each profile v in a given domain to an outcome $f(v)$, where $f(v)$ takes the value -1, 0 or 1:

$f(v) = 1$ means that 1 is collectively chosen (a *positive decision*);

$f(v) = -1$ means that -1 is collectively chosen (a *negative decision*);

$f(v) = 0$ means that 1 and -1 are tied (a *tie*).

This allows the group to be indifferent between the two alternatives.

Examples of aggregation functions are:

[4] Proofs of the results in this section are given in Appendix 2.

Simple Majority Voting. For any v,

$$f(v) = \begin{cases} 1 & \text{if } \Sigma v_i > 0 \\ 0 & \text{if } \Sigma v_i = 0 \\ -1 & \text{if } \Sigma v_i < 0 \end{cases}$$

Dictatorship. For any v, $f(v) := v_i$, where i is some antecedently fixed individual.

Imposed Acceptance. For any v, $f(v) := 1$.

Imposed Rejection. For any v, $f(v) := -1$.

Imposed Indifference. For any v, $f(v) := 0$.

A *procedural argument* for a particular aggregation function is an argument that this rule has certain desirable procedural properties.

2. May's Theorem

The conditions of May's theorem are as follows.

Universal domain (U). The domain of f is the set of all logically possible profiles.

Anonymity (A). For any two profiles v and w, if v and w are permutations of each other, then $f(v) = f(w)$.

Symmetry (S). For any profile v, $f(-v) = -f(v)$.

We write $v \geq w$ if, for every i, $v_i \geq w_i$. We write $v > w$ if $v \geq w$ and not $v = w$.

Monotonicity (M). For any two profiles v and w, $v \geq w$ implies $f(v) \geq f(w)$.

Positive responsiveness (PR). For any two profiles v and w, if $f(w) = 0$ and $v > w$, then $f(v) = 1$.

Now May's theorem states that Simple Majority Voting is the unique aggregation function satisfying these five conditions.

Proposition 1 (May's theorem). Let f be any aggregation function. Then f satisfies (U), (A), (S), (M) and (PR) <u>if and only if</u> f is Simple Majority Voting.

To the extent that the conditions of May's theorem are desirable procedural properties, May's theorem provides a procedural argument for Simple Majority Voting. Let us briefly consider the properties of Simple Majority Voting.

3. Symmetry

We have already highlighted the fact that Simple Majority Voting satisfies condition (S). Swapping all votes for 1 and -1 will imply that the outcome is swapped correspondingly.

4. Responsiveness, Ties and Tie-breaking

Simple Majority Voting is highly responsive to individual votes in the following sense. In the event that there is a tie between the two alternatives, i.e. $f(v) = 0$, Simple Majority Voting has the property that the change of even a single vote will break the tie in the direction of that change. This property is captured by condition (PR).

Condition (PR) is a special case of a more general responsiveness condition:

Positive *k*-responsiveness (PR_k). For any two profiles v and w, if $f(w) = 0$, $v \geq w$, and there are at least k individuals i such that $v_i > w_i$, then $f(v) = 1$.

An aggregation function f satisfies positive k-responsiveness if, in the case of a tie, the change of k votes (all in the same direction, specifically from -1 to 1) will break the tie in the direction of that change (also from -1 to 1). Condition (PR), as used in May's theorem, is the special case of (PR_k) where k=1.

By being highly responsive to individual votes, Simple Majority Voting disallows most ties. A profile v leads to a tie under the aggregation function f if $f(v) = 0$. If $f(v) = 0$ and $\Sigma v_i = 0$, we say that the tie is *trivial*. Such ties occur where the number of individuals voting for 1 equals the number of individuals voting for -1. On the other hand, if $f(v) = 0$ and $\Sigma v_i \neq 0$, we say that the tie is *non-trivial*. In that case, there is a tie although one of the two alternatives receives more votes than the other. Simple Majority Voting does not allow any non-trivial ties. It satisfies:

No non-trivial ties (NT). For any profile v, $f(v) = 0$ implies $\Sigma v_i = 0$.

We can get a full characterization of Simple Majority Voting by replacing May's condition (PR) – what we call (PR_1) – with condition (NT). Then we have:

Proposition 2. Let f be any aggregation function. Then f satisfies (U), (A), (S), (M) and (NT) if and only if f is Simple Majority Voting.

In other words, under conditions (U), (A), (S) and (M), Simple Majority Voting is the unique aggregation function satisfying (NT).

5. Veto powers

Simple Majority Voting has the property that a group of $n/2$ or more of the individuals can veto a positive decision; and also that a group of $n/2$ or more of the individuals can veto a negative decision. Formally, consider the following two conditions:

Veto over positive decisions for a group of size *k* (PV_k). For any profile v, if there are at least k individuals i such that $v_i = -1$, then $f(v) \neq 1$.

Veto over negative decisions for a group of size *k* (NV_k). For any profile v, if there are at least k individuals i such that $v_i = 1$, then $f(v) \neq -1$.

Simple Majority Voting satisfies both (PV_k) and (NV_k) with $k = n/2$. It does not, however, satisfy either condition for any $k < n/2$. So no minorities – groups of size less than $n/2$ – have any veto powers under Simple Majority Voting.

6. The Trilemma between Symmetry, No Non-trivial Ties and Minority Veto Powers

We have seen that Simple Majority Voting satisfies symmetry and no non-trivial ties, but it does not give any veto powers to minorities. This raises the question of whether there are any *other* aggregation functions satisfying all of

(i) symmetry,

(ii) no non-trivial ties,

(iii) giving certain veto powers to minorities.

The following result gives a negative answer to this question.

Proposition 3. Let f be any aggregation function satisfying (U). Suppose that f satisfies (NT), (S), (PV_{k_1}) and (NV_{k_2}). Then $k_1 \geq n/2$ and $k_2 \geq n/2$.

We are faced with a trilemma. No aggregation function can satisfy all three of (i), (ii) and (iii), but any two of (i), (ii) and (iii) can be simultaneously satisfied. Simple Majority

Voting satisfies (i) and (ii) while violating (iii). In fact, Simple Majority Voting is the unique aggregation function satisfying (U), (NT), (S), ($PV_{1/2}$) and ($NV_{1/2}$).

Proposition 4. Let *f* be any aggregation function. Then *f* satisfies (U), (NT), (S) and ($PV_{1/2}$) (and ($NV_{1/2}$)) if and only if *f* is Simple Majority Voting.

If we want to ensure certain minority veto powers, we need to relax either (i) or (ii). Asymmetrical Special-Majority Voting – the classical solution – satisfies (ii) and (iii) while violating (i). Symmetrical Special-Majority Voting – the solution explored here – satisfies (i) and (iii) while violating (ii).

7. Asymmetrical Special-Majority Rules

If we relax symmetry but do not permit non-trivial ties, not only is one alternative always privileged over the other; the minority veto powers the special-majority rule grants are then themselves also asymmetrical. There is always, in that case, a trade-off between minority veto powers over negative decisions and minority veto powers over positive decisions.

Proposition 5. Suppose *f* satisfies (U), (PV_{k_1}), (NV_{k_2}) and (NT). Then $k_1+k_2 \geq n$.

If we give a minority of size $k_1 < n/2$ veto power over positive decisions, then only a supermajority of size greater than $n-k_1 > n/2$ has veto power over negative decisions. Likewise, if we give a minority of size $k_2 < n/2$ veto power over negative decisions, then only a supermajority of size greater than $n-k_2 > n/2$ has veto power over positive decisions.

An Asymmetrical Special-Majority rule can be defined as follows:

Asymmetrical Special-Majority Voting with parameter *m*. For any v,

$$f(v) = \begin{cases} 1 & \text{if } \Sigma v_i \geq m \\ -1 & \text{if } \Sigma v_i < m \qquad (m > n \text{ or } m < -n \text{ is admissible}). \end{cases}$$

If $m > 0$ (if n is even) or $m > 1$ (if n is odd), the Asymmetrical Special-Majority rule is biased in favour of -1. In that case, a minority of size greater than $(n-m)/2$ can veto a positive decision; but only a supermajority of size at least $(n+m)/2$ can veto a negative decision. If $m \leq 0$ (if n is even) or $m \leq -1$ (if n is odd), the rule is biased in favour of 1. In that case, any minority of size greater than $(n+m-1)/2$ can veto a negative decision; but only a supermajority of size at least $(n-m+1)/2$ can veto a positive decision.[5]

Proposition 6. Suppose f is Asymmetrical Special-Majority Voting with parameter m. Then f satisfies (PV_k) <u>if and only</u> if $k > (n-m)/2$. And f satisfies (NV_k) <u>if and only if</u> $k > (n+m-1)/2$.

8. Symmetrical Special-Majority Rules

If we keep symmetry, but permit non-trivial ties, then it is the case not only that no alternative is privileged over the other, but also that the minority veto powers that the special majority rule grants are always symmetrical.

Proposition 7. Suppose f satisfies (U) and (S). For any k, f satisfies (PV_k) <u>if and only if</u> f satisfies (NV_k).

A Symmetrical Special-Majority rule can be defined as follows:

Symmetrical Special-Majority Voting with parameter *m* (*m* > 0). For any v,

$$f(v) = \begin{cases} 1 & \text{if } \Sigma v_i \geq m \\ 0 & \text{if } m > \Sigma v_i > -m \\ -1 & \text{if } \Sigma v_i \leq -m \end{cases} \qquad (m > n \text{ is admissible}).$$

The limiting case $m=1$ corresponds to Simple Majority Voting. The condition $\Sigma v_i \geq m$ means that there is a special majority for 1 with a margin of at least m between the majority and the minority. The condition $\Sigma v_i \leq -m$ means that there is a special majority for -1 with a margin of at least m between the majority and the minority. The condition $m > \Sigma v_i > -m$ means that there is no sufficient special majority for either 1 or -1.

It turns out that the class of symmetrical special majority rules can be fully characterized by May's conditions (U), (A), (S), (M), where condition (PR) is relaxed.

Proposition 8. Let f be any aggregation function. Then f satisfies (U), (A), (S), (M) <u>if and only if</u> f is Symmetrical Special-Majority Voting for some value of m ($m>0$).

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[5] To make the special majority more demanding than a simple majority, [$m > 1$ or $m \leq -1$] if n is odd, and [$m > 2$ or $m \leq -2$] if n is even.

Note that proposition 8 characterizes a whole class of aggregation functions. This class includes, for example, Simple Majority Voting ($m = 1$), the Unanimity Rule ($m = n$), the Imposed Indifference rule ($m > n$). For a suitable choice of $m > 1$, minorities have veto powers over *both* positive *and* negative decisions (recall proposition 7 above).

Is it possible to specify conditions that characterize not just the class of all Symmetrical Special-Majority rules, but a specific such rule, for a specific parameter *m*? We provide such a characterization in two steps. In a first step, we use a minority veto condition to impose a lower bound on the parameter *m*. In a second step, we use a responsiveness condition to impose an upper bound on *m*. May's conditions (U), (A), (S), (M) together with these veto and responsiveness conditions will then characterize a Symmetrical Special-Majority rule for a specific parameter *m*.

Proposition 9. Suppose *f* is Symmetrical Special-Majority Voting with parameter *m*. Then *f* satisfies (PV_k) (and, by symmetry, (NV_k)) <u>if and only if</u> $m \geq n\text{-}2k\text{+}1$.

By proposition 9, condition (PV_k) (equivalently, (NV_k)) imposes a lower bound on the parameter *m*. An aggregation function satisfies (U), (A), (S), (M) and (PV_k) if it is Symmetrical Special-Majority Voting with parameter $m \geq n\text{-}2k\text{+}1$.

Proposition 10. Suppose *f* is Symmetrical Special-Majority Voting with parameter *m*. Then *f* satisfies (PR_k) <u>if and only if</u> $m^* \leq k\text{+}1$, where

$$m^* \text{ is } \begin{cases} \text{the smallest even number greater than or equal to } m \text{ if } n \text{ is even} \\ \text{the smallest odd number greater than or equal to } m \text{ if } n \text{ is odd.} \end{cases}$$

By proposition 10, condition (PR_k) imposes an upper bound on the parameter *m*. An aggregation function satisfies (U), (A), (S), (M) and (PR_k) if it is Symmetrical Special-Majority Voting with parameter $m^* \leq k\text{+}1$, where m^* is as defined in proposition 10.

Propositions 9 and 10 together allow us to describe the trade-off between minority veto powers and responsiveness under Symmetrical Special-Majority Voting.

Proposition 11. Suppose *f* is a Symmetrical Special-Majority rule satisfying both (PV_{k_1}) and (PR_{k_2}). Then $k_2 > n\text{-}2k_1$.

This means, the smaller the value of k_2 in 'positive k_2-responsiveness' (i.e. the more responsive the aggregation function), the larger the group size k_1 that is required for vetoing a (positive or negative) decision. Condition (PR_1) (where $k_2 = 1$), as satisfied by Simple Majority Voting, implies n-2$k_1 < 1$, i.e. $k_1 > n/2-1/2$, ruling out minority veto powers.

Finally, we are in a position to characterize a Symmetrical Special-Majority rule, for a specific parameter *m*:

Proposition 12. Let *f* be any aggregation function. Let *m* be any integer greater than 0 (where *m* is even if and only if *n* is even). Then *f* satisfies (U), (A), (S), (M), ($PV_{(n-m)/2+1}$) and (PR_{m-1}) if and only if *f* is Symmetrical Special-Majority Voting with parameter *m*.

By proposition 12, a Symmetrical Special-Majority rule with parameter *m* is the unique aggregation function satisfying (U), (A), (S) and (M), giving veto powers to minorities of size at least (n-m)/2+1, and satisfying positive (m-1)-responsiveness.

II. Epistemic Properties of Special-Majority Rules

Democratic procedures commend themselves not only on the grounds of procedural fairness, such as those formalized in May's Theorem. They also commend themselves on epistemic grounds, in terms of their truth-tracking power.

Aristotle's loose talk of the 'wisdom of the multitude' was formalized in the Condorcet Jury Theorem in the eighteenth century and has been intensively explored in recent years (Grofman, Owen and Feld 1983; Mueller 2003, pp. 128-33). That theorem shows that, if individuals cast their votes independently of one another and each voter is more than 0.5 likely to be correct in a two-option choice, the probability that the majority vote is correct is an increasing function of the size of the electorate, approaching one as the number of individuals tends to infinity. Majority voting is, in that sense, a good truth-tracker.

A. An Informal Statement

Here we explore a Bayesian version of the familiar Condorcet Jury model, to reveal an epistemic trilemma analogous to the procedural one revealed above.[6] The role of a 'minority veto' in the procedural case is, in the epistemic case, taken by a 'no reasonable doubt' criterion. Such an issue arises in various circumstances, legal (Kaplan 1968), medical (Scheff 1964) and administrative (Goodin 1985).

Sometimes we want to make very certain we are right before acting. The criminal jury is charged with convicting only if convinced 'beyond a reasonable doubt' of the defendant's guilt: something like a 95 percent probability that the defendant is guilty. In civil trials, in contrast, the standard of proof is merely the 'more likely than not': just over 50 percent, either way, is decisive.

Sometimes we think that the evidentiary burden ought to weigh disproportionately in one direction. In the criminal jury case, the prosecution has to prove its case beyond a reasonable doubt; the defense does not. Other times, we think that the evidentiary burden ought to be symmetrical, as in civil cases.

Sometimes, yet again, we think that the standard of proof ought to be 'no reasonable doubt', but that that standard ought to apply symmetrically to both sides of the proposition. Suppose, for example, we are dealing with a drug that would, at worst, have only mildly unpleasant side-effects and, at best, that drug would alleviate a condition which is only mildly unpleasant. There we might suppose: (1) the state should allow the sale of the drug under the imprimatur of a 'licenced and approved therapeutic agent' only upon production of evidence that it is 90 percent certain that the drug alleviates the condition; (2) the state should prohibit the sale of the drug only if it is 90 percent certain that it does more harm than good; and (3) the state should allow the drug to be sold over the counter as a 'folk remedy', but without any official imprimatur, if neither of those conditions is met.

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[6] Analogous results are restated in Appendix 4 using statistical models of hypothesis testing of a non-Bayesian sort, so the argument can be developed without referring to Bayesian prior probabilities at all.

The form that the trilemma takes in the epistemic case is this. There are three properties we might like to see in our epistemic decision procedure. One is 'symmetry' in the epistemic sense: positive decisions are held to the same standard of proof as negative ones. A second is an epistemic equivalent of 'no non-trivial ties' (ties occur only where the probability of the truth of a proposition equals that of its negation). The third is a 'no reasonable doubt' standard, requiring more than a mere 'more-likely-than-not' threshold to be crossed before we decide for or against some proposition.

The trilemma, epistemically, is that any two of those criteria can be met – but not all three at once. Assuming independent voters each of whom is more likely to be right than wrong, Simple Majority Voting meets the first and second criteria but not the third. Asymmetrical Special-Majority Voting meets the second and third but not the first. Symmetrical Special-Majority Voting meets the first and third but not the second.

Here again, we sometimes have grounds for sacrificing one of those desiderata. Which decision rule we want to adopt, on epistemic grounds, follows from those independent reasons we have for thinking one or another desideratum is more important, in any given situation.[7]

<u>B. Formal Analytics[8]</u>

<u>1. The Framework</u>

We begin by stating Condorcet's classical model of jury decisions. We assume that there are two possible states of the world, represented by the variable X, which takes the value 1 or -1. The two possible states of the world might be, respectively, the guilt or innocence of a

[7] As we have pointed out, the drawback of Symmetrical Special-Majority Voting, epistemically as well as procedurally, is that it allows non-trivial ties. But in the epistemic case, non-trivial ties turn out to be less of a problem. In a sufficiently large electorate, the probability of non-trivial ties under Symmetrical Special-Majority Voting can be proven to be vanishingly small. This follows from the fact that Symmetrical Special-Majority Voting, as defined here (in terms of a required absolute margin of votes between the majority and the minority) satisfies the condition of *truth-tracking in the limit*: the probability of obtaining a special majority for 1 if $X = 1$ converges to 1 as n increases; likewise, the probability of obtaining a special majority for -1 if X=-1 converges to 1 as n increases (List 2003).

[8] Proofs of the results in this section are given in Appendix 3.

defendant, or the truth or falsity of some factual proposition. Again, we assume that there are n individuals, labelled 1, 2, …, n. The individuals are treated as diagnostic devices whose votes are signals about the state of the world. The stochastic process by which individual i generates his or her vote is represented by the random variable V_i, where V_i takes the value 1 or -1. Let V denote the vector $<V_1, V_2, \ldots, V_n>$ of such random variables across the n individuals. For each individual i, a specific value of V_i – i.e. a specific vote of that individual – is represented by v_i. As before, a *profile* is a vector $v = <v_1, v_2, \ldots, v_n>$ of specific such votes. Condorcet’s model makes two assumptions, which we will tentatively retain throughout the following discussion and results.[9]

First, if the state of the world is 1, the individuals each have a greater than random chance of voting for 1; and if it is -1, they each have a greater than random chance of voting for -1.

Competence. For each individual i, $Pr(V_i = 1 \mid X = 1) = Pr(V_i = -1 \mid X = -1) = p > 1/2$, where the value of p is the same for all individuals.

The probability $Pr(V_i = 1 \mid X = 1)$ (respectively $Pr(V_i = -1 \mid X = -1)$) is the conditional probability that individual i votes for 1 (respectively -1), given that the state of the world is 1 (respectively -1). The parameter p is called the *individual competence level*.

Secondly, once the state of the world is given, the votes of different individuals are independent from each other.

Independence. The votes of different individuals $V_1, V_2, \ldots, V_n$ are independent, given the state of the world X.

In short, the votes of different individuals are independent identically distributed signals about the state of the world, where each signal is noisy but biased towards the truth.

[9] Many important modifications of Condorcet’s model have been discussed in the literature. Cases where different jurors have different competence levels – i.e. where the present homogeneous competence assumption does not hold – are discussed in Grofman, Owen and Feld (1983), Borland (1989), Kanazawa (1998). Cases where there are certain dependencies between different jurors’ votes – i.e. where the present independence assumption does not hold – are discussed in Ladha (1992) and Estlund (1994). Cases where jurors vote strategically rather than sincerely – i.e. where jurors may not vote their private signals about the state of the world – are

The key idea of an epistemic account of voting is that a particular voting pattern provides evidence about the state of the world, and that a good evaluation of that evidence – using a suitable aggregation function – allows a group to make decisions that track the state of the world reliably. An *epistemic argument* for a particular aggregation function is an argument that a group using this function will be good at making decisions that track the state of the world reliably. Let us first address the properties of Simple Majority Voting from an epistemic perspective.

2. No Reasonable Doubt

Suppose we assign an equal prior probability of 1/2 to each of the two states of the world, 1 and -1. This need not be an objective probability; in the absence of more precise information, we might justify this equiprobability assumption by using some normative principle ('no bias') or some methodological principle (Laplace's 'principle of insufficient reason'). Condorcet own presentation implicitly relied on this assumption (List 2003). While the present exposition uses Bayesian notions and therefore requires a prior probability assignment over the different states of the world, we present a classical (non-Bayesian) statistical variant of the present results in Appendix 4, which requires no assumption about prior probabilities at all.

The first thing to note is that, other things being equal, observing an individual vote for 1 (respectively -1) should increase our degree of belief in the hypothesis that the state of the world is 1 (respectively -1). Observing more such votes should increase our degree of belief in that hypothesis further. Whenever we observe a majority of votes for 1, this should lead us to believe that $X = 1$ is more likely to be true than $X = -1$. Likewise, whenever we observe a majority for -1, this should lead us to believe that $X = -1$ is more likely to be true than $X = 1$. In short, under Simple Majority Voting, a positive decision is made <u>if and only if</u> $X = 1$ is more likely to be true than $X = -1$; a negative decision is made <u>if and only if</u> $X = -1$ is more likely to be true than $X = 1$.

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discussed in Austen-Smith and Banks (1996). But, for the purposes of this article, However, in many situations, we require that a positive decision be made, not as soon as $X = 1$ is more likely to be true than $X = -1$, but only if we believe, *beyond any reasonable doubt*, that $X = 1$ is true. Consider the following two conditions:

A standard of proof of *c* for positive decisions (PP_c). For any profile v, $f(v) = 1$ if and only if $Pr(X = 1 \mid V = v) > c$.

A standard of proof of *c* for negative decisions (NP_c). For any profile v, $f(v) = -1$ if and only if $Pr(X = -1 \mid V = v) > c$.

The probability $Pr(X = 1 \mid V = v)$ (respectively $Pr(X = -1 \mid V = v)$) is the conditional probability that the state of the world is 1 (respectively -1), given that the pattern of votes across the n individuals is precisely the profile v. The parameter c captures the requisite standard of proof. The conditions require that a positive (respectively negative) decision be made if and only if the conditional probability that $X = 1$ (respectively $X = -1$), given the voting pattern, exceeds the threshold c.

As we have noted, Simple Majority Voting satisfies (PP_c) and (NP_c) for $c = 1/2$. But Simple Majority Voting does not satisfy either (PP_c) and (NP_c) for any value of c sufficiently greater than 1/2. We will say, in a technical sense, that c is *sufficiently greater* than 1/2 if

$$c \geq \begin{cases} p & \text{if } n \text{ is odd} \\ p^2/(p^2+(1-p)^2)) & \text{if } n \text{ is even.} \end{cases}$$

Intuitively, only a value of c close enough to 1 and thus typically sufficiently greater than 1/2 – say $c = 0.95$ – will capture the requirement of “no reasonable doubt”. So Simple Majority Voting is an unsuitable aggregation function if we demand a threshold of “no reasonable doubt” that is sufficiently greater than 1/2.

3. Ties

We have seen that Simple Majority Voting permits no non-trivial ties in a *procedural* sense: it allows ties only when the number of individuals voting for 1 equals the number of individuals voting for -1. There is also an *epistemic* sense in which Simple Majority Voting permits no non-trivial ties. If $f(v) = 0$ and $Pr(X = 1 \mid V = v) = 1/2$, we say that the tie is *trivial*.

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we tentatively use Condorcet’s model in its simplest, classical form.

In that case, the tie occurs in a situation where we consider the two possible states of the world equally probable. If $f(v) = 0$ and $Pr(X = 1 \mid V = v) \neq 1/2$, on the other hand, we say that the tie is *non-trivial*. In that case, there is a tie although we consider one of the two possible states of the world more probable than the other. Simple Majority Voting does not allow any non-trivial ties in this epistemic sense. It satisfies:[10]

No non-trivial ties (NT*). For any profile v, $f(v) = 0$ implies $Pr(X = 1 \mid V = v) = Pr(X = -1 \mid V = v) = 1/2$.

4. Symmetry

In the procedural case, we defined symmetry as the requirement that swapping all votes for 1 and -1 will imply that the outcome of the aggregation is swapped correspondingly. But symmetry can also be defined in epistemic terms, namely as the requirement that the standard of proof for positive decisions should be exactly the same as that for negative decisions; in other words, that an aggregation function should satisfy (PP_c) for some value of c <u>if and only if</u> it satisfies (NP_c) for the same value of c. Under Condorcet's assumptions – including, crucially, the assignment of an equal prior probability to the two states of the world[11] – Simple Majority Voting satisfies symmetry in this sense.

5. The Trilemma between Symmetry, No Non-trivial Ties and No Reasonable Doubt

We have seen that Simple Majority Voting satisfies both symmetry and no non-trivial ties in the epistemic sense, but it cannot implement a threshold of "no reasonable doubt" sufficiently greater than 1/2 for either positive or negative decisions. In analogy with the procedural case, this raises the question of whether there are any *other* aggregation functions satisfying all of

(i) symmetry in the epistemic sense,

(ii) no non-trivial ties in the epistemic sense,

[10] Assuming an equal prior probability of the two states of the world – this assumption is not needed in the classical (non-Bayesian) version of the argument in Appendix 4.

(iii) no reasonable doubt.

The following result gives a negative answer to this question.

Proposition 13. Let *f* be any aggregation function satisfying (U). Suppose that *f* satisfies (U), (NT*), (PP_c) and (NP_c). Then c is not 'sufficiently greater' than 1/2.

Again, we are faced with a trilemma. No aggregation function can satisfy all three of (i), (ii) and (iii), but any two of (i), (ii) and (iii) are satisfiable. Simple Majority Voting satisfies (i) and (ii) while violating (iii). In fact, Simple Majority Voting is the unique aggregation function satisfying ($PP_{1/2}$) and ($NP_{1/2}$) together with universal domain:

Proposition 14. Let *f* be any aggregation function. In Condorcet's model, *f* satisfies (U), ($PP_{1/2}$) and ($NP_{1/2}$) <u>if and only if</u> *f* is Simple Majority Voting.

If we want to ensure a standard of proof sufficiently greater than 1/2 – i.e. a threshold of "no reasonable doubt" – we need to relax either (i) or (ii). Asymmetrical Special-Majority Voting – the familiar solution – satisfies (ii) and (iii) while violating (i). Symmetrical Special-Majority Voting satisfies (i) and (iii) while violating (ii).

<u>6. Asymmetrical Special-Majority Voting</u>

If we relax symmetry but do not permit non-trivial ties, we are faced with a trade-off between standards of proof for positive and negative decisions.

Proposition 15. Suppose *f* satisfies (U) and (NT*), and suppose c_1 is sufficiently greater than 1/2. If *f* satisfies (PP_{c_1}), then *f* does not satisfy (NP_{c_2}) for any $c_2 \geq 1/2$; and if *f* satisfies (NP_{c_1}), then *f* does not satisfy (PP_{c_2}) for any $c_2 \geq 1/2$.

If we demand a standard of proof for positive decisions that is sufficiently greater than 1/2, then we cannot also demand a standard of proof for negative decisions that is greater than or equal to 1/2. Likewise, if we demand a standard of proof for negative decisions that is sufficiently greater than 1/2, then we cannot also demand a standard of proof for negative decisions that is greater than or equal to 1/2.

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[11] Again, the assumption is not needed in the classical (non-Bayesian) version of the argument in Appendix 4.

In jury decisions this seems acceptable, as the standard of proof for conviction should be higher than that for acquittal. But in other decision problems, where there is no antecedently privileged alternative, we may require a symmetrical standard of proof. And if we require not only a standard of proof that is symmetrical, but also one that is sufficiently greater than 1/2, then we are led to Symmetrical Special-Majority Voting.[12]

7. Symmetrical Special-Majority Voting

We can now provide a characterization result on Symmetrical Special-Majority Voting. If we permit non-trivial ties in the epistemic sense, then Symmetrical Special-Majority Voting is the unique aggregation function satisfying universal domain and a symmetrical standard of proof. When the required standard of proof c and the individual competence parameter of p are given, the parameter m of the corresponding symmetrical special majority rule can be determined by the expression $log(^1/_c\text{-}1)/log(^1/_p\text{-}1)$.

Proposition 16. Let $c \geq 1/2$. The aggregation function f satisfies (U), (PP_c) and (NP_c) if and only if f is Symmetrical Special-Majority Voting, where m is the smallest integer strictly greater than $log(^1/_c\text{-}1)/log(^1/_p\text{-}1)$.

The case $c = 1/2$ corresponds to Simple Majority rule. The case c sufficiently greater than 1/2 but less than $p^n/(p^n+(1\text{-}p)^n))$ corresponds to a Special-Majority (up to Unanimity) rule. The case c greater than or equal to $p^n/(p^n+(1\text{-}p)^n))$ corresponds to Imposed Indifference.

III. Choosing Among Decision Rules

The parallel trilemmas identified in Sections I-II help us see what is at stake in the choice among alternative decision rules. That choice is summarized in the decision tree in Figure 1.

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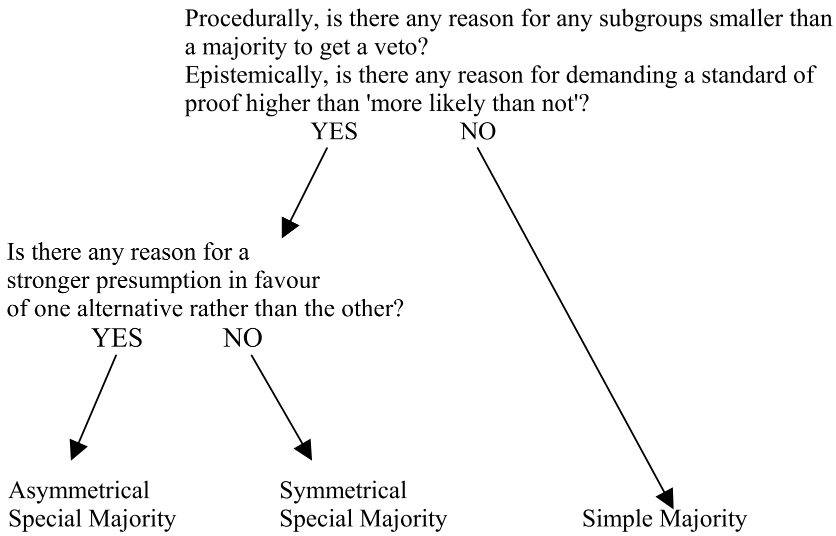
[12] Once again, it is important to note the assumption of an equal prior probability of both states of the world. If that assumption is given up, then, in a Bayesian framework, a *symmetrical* standard of proof may lead to (and therefore justify) an *asymmetrical* Special-Majority rule. In the classical framework in Appendix 4, by contrast, no assumption about prior probabilities is needed to justify a Symmetrical Special-Majority rule on the basis of a symmetrical standard of proof requirement.

Figure 1: Grounds for choosing among decision rules



Procedurally, the great disadvantage of Simple Majority rule is that it runs the risk of majority tyranny. If there is some 'discrete and insular' subset of the population, with distinctive interests but a minority of the votes, the majority might ride roughshod over its interests under Simple Majority Voting. In such circumstances, we may wish to protect those minorities by requiring that decisions affecting them be taken by 'special majorities' sufficiently large to, in effect, give the minorities in question a veto.

Often, of course, there is no such problem. There may be no group that is so discrete and insular to be at any real risk of being tyrannized by majorities. And in the limiting case in which that is true (the case of 'no factions': every pair of voters is as likely to vote with one another as against one another), Simple Majority Voting is then the decision rule that uniquely maximizes each voter's probability of being on the winning side of an election (Rae 1969; Taylor 1969; Mueller 2003, pp. 136-137).

Thus, Simple Majority Voting works fine where there are no factions or any other special reason to think that any submajority group deserves veto power over the social outcome. But where there are sufficiently cohesive submajority groups with sufficiently strong and distinctive interests at stake, we may well want to give them veto power. To do that, we need to abandon Simple Majority Voting in favour of some form of special-majority voting.

Epistemically, similarly, Simple Majority Voting is ideal so long as we merely want to identify propositions that are 'more likely than not' to be true. If we require more confidence than that, we need some form of special-majority voting.

The great disadvantage of the ordinary Asymmetrical Special-Majority rule is precisely its asymmetry. It privileges one option over others, by requiring a lower 'special majority' to install it as the social decision than the others. Standardly, it identifies one option as the 'default' which will prevail in the absence of the requisite 'special majority' for any other option.

Again, sometimes that is not a problem. Sometimes there are good grounds for thinking that one option *should* be privileged in that way. There are good grounds for a presumption of innocence in criminal trials, and for adopting an asymmetrical decision rule that makes it harder there to convict than to acquit. There are good grounds for supposing that, if a mixed constitution is to provide genuine checks-and-balances, a larger legislative majority must be required to overturn a president's veto than was required to pass the bill in the first instance.

Thus, there clearly are cases in which the asymmetries built into Asymmetrical Special-Majority rules are not arbitrary. But the burden must be on advocates of the differential treatment of the various options to provide a justification for the asymmetry. Without some convincing justification, the asymmetry would be arbitrary.

Symmetrical Special-Majority rules solve that problem by treating all options symmetrically, requiring the same 'special majority' for each option in order for it to be chosen. The great disadvantage of the Symmetrical Special-Majority rule is that it may generate many 'non-trivial ties'. It chooses no option if none achieves the requisite majority, even if some options get more votes than others.

Sometimes this disadvantage of the Symmetrical Special-Majority rule may not be much of a problem. Sometimes it does no harm to leave the matter unsettled. But in general, we put something to a vote only when we genuinely need to have the issue resolved; and hence a decision procedure that leaves too many things unsettled seems problematic. It is to that problem that we now turn.

<u>IV. The Problem of Ties</u>

The problem with leaving things formally unsettled is that, as we have long been aware, 'non-decisions' are decisions too (Bachrach and Baratz 1963). Leaving matters unsettled is itself a decision, and one with potentially serious consequences.

Douglas Rae's reflections on 'The Limits of Consensual Decision' merit quoting at length, in this connection:

> It is shallow at best to imagine that ... legal choices can be put off: to 'not decide' must imply one of two outcomes: (a) tacit permission ...
> or (b) continuance of an explicit *status quo* mandate, prohibition or permission.
> - To see the point's obviousness try to devise a way of effectively postponing a decision on the legality of abortion which can be explained to a pregnant woman.
> - In the choice of candidates for office, either someone is elected or nobody is (yet) elected at any given moment: a final choice can be put off only by accepting an interval of indeterminacy or 'provisional' control which is a very real outcome (consider the Greek colonels or the Portuguese Communists).
> - International agreements can be put off, but a very real and often painful status quo remains to be experienced.
>
> These cases suggest a ubiquitous forcing of choice: even when explicit outcomes can be postponed, tacit ones are *ipso facto* chosen (Rae 1975, p. 1279; formatting altered).

For one particularly dramatic practical example, reflect upon this tale from the Ukraine:

> Parliamentary elections were ... held in 1994, contested by a large number of parties, many of which were concentrated in particular regions. The elections were conducted under the requirement that, to be elected, a candidate had to secure the support of over 50 percent of *registered* voters, either in the first round or in a runoff between the top two candidates in a constituency. The results were hardly conclusive. [Low turnout meant that m]any constituencies could not elect a representative under this system... (Dryzek and Holmes 2002, p. 117, emphasis added).

Such non-election inevitably favours one side or another. In the Ukraine, it in effect served the purposes of the Communist Party and the old *nomenklatura*. They won, not exactly by default, but nonetheless precisely because other seats went unfilled.

Leaving things undecided always has practical consequences, of some sort or another.[13] Something has to be done, or not done, in consequence of things being left undecided; and some interests will be well-served, and others ill-served, by what is done or not done in consequence. There can be no pretence that leaving things undecided is without distributional consequences.

While ties are more frequent under Symmetrical Special-Majority voting, however, they are nowise unique to that decision rule. Most decision rules – including Simple Majority Voting with an even number of voters – have to face the problem of what to do in the case of tied votes.

Under Simple Majority Voting, ties will presumably be rare and 'trivial', in both the technical sense introduced above and the more colloquial sense as well. But even though it may make no *democratic* difference which way the issue is decided, where there are equal numbers on each side of the issue, the divisions may nonetheless be deep and it may matter enormously to each side which way the tied vote is resolved. In that sense, how we resolve tied votes might be anything but trivial, even under Simple Majority rule.

Generically, there are three ways of doing so. Either: (1) we can privilege one of the options; or (2) we can privilege one of the voters; or (3) we can settle issues on which there are ties by some wholly separate procedure.

Cursory inspection of actual decision procedures reveals many examples of (1). The most familiar is the rule that 'the *status quo* remains in force unless some alternative to it is enacted'.

There are not many cases of (2). One example rather like that might be the practice of the Speaker of the US House of Representatives casting the deciding vote in cases of a tie. But even that is not a completely clean case of (2). It is not as if the Speaker has a 'golden

[13] Sunstein (1996) famously uses the same phrase, note, to refer to something slightly different: leaving the 'grounds' for a judicial decision open, as judges content themselves with simply rendering a decision in the case at hand (with each judge deciding the case on different grounds, perhaps).

vote'. Rather (by convention) the Speaker only votes when his vote is 'golden', i.e., in cases of a tie when his vote will indeed decide the matter.[14]

More interesting from our point of view are examples of (3). Note the procedure set down in the original text of Article II, Section 1 of the US Constitution for choosing the President:

> The person having the greatest number of votes [in the Electoral College] shall be the President, if such a number be a majority of the whole number of Electors appointed; and if there be more than one who have such majority, and have an equal number of votes, then the House of Representatives shall immediately chuse by ballot one of them for President...[15]

The details of those arcane arrangements matter less than their structure, for present purposes. The structure of the decision rule is this: in case of a tie, decide the issue through some other procedure altogether.

V. Tie-Breakers under Symmetrical Special-Majority Rules

Here we explore two possible variations on that 'other procedure' strategy for breaking the ties generated by Symmetrical Special-Majority voting. The first, which can be used only in very special circumstances, would be for authorities (perhaps most naturally 'judicial authorities') to extrapolate further decisions from ones that have already been made under a suitably-specified Symmetrical Special-Majority rule. The second mechanism, which can be used in any federal system, is to let lower levels of government settle issues that are left open by Symmetrical Special-Majority voting at the higher level.

A. Judicial Extrapolation

Under certain special circumstances, it might be permissible for some authoritative agent – perhaps most naturally judicial authorities – to extrapolate from what 'special majorities' have

[14] The Speaker's case is interestingly different from that of the President of the Senate (the Vice-President of the US), whom the US Constitution (Article I, Section 3) stipulates 'shall have no vote, unless they be equally divided'.

[15] Before the Twelfth Amendment, Electors cast ballots in this way for two persons, and the person with most votes became President and the person with the next-most votes Vice-President. Thus it was logically possible for two candidates both to have more than a majority of the number of Electors.

agreed to other propositions that are implied by what has been agreed, and to treat those 'implied' propositions as having the same status as those that had actually been agreed to.

One common principle of extrapolation along these lines, for example, is that 'those who intend an end intend the means strictly necessary to attain that end'. So a legislature that has enacted a statute that assigns powers and responsibilities to the 'Bureau of the Census' will be construed as having endorsed the existence of such a Bureau, even if the legislature neglected to stipulate that in the statute itself.

One of the worries surrounding such 'judicial legislation' in general is that the judges might well make laws that are contrary to ones the legislature itself would have enacted. In ordinary circumstances, that is a genuine worry. It is particularly worrisome wherever decisions involving multiple connected propositions are made by Simple Majority rule (List and Pettit 2002).

To illustrate, imagine three decision-makers who each have a certain set of views over three propositions: *P*; '*P* implies *Q*'; and *Q*. The first believes that *P* is true, that *P* implies *Q*, and therefore that *Q* is also true. The second believes that *P* is true, but does not believe that *P* implies *Q*, so is consistent in also believing *Q* is false. The third accepts that *P* implies *Q* but believes that *P* and *Q* are both false. Suppose these individuals vote on each of these propositions. A majority (two out of three) hold that *P* is true; a majority (two out of three) hold that *P* implies *Q*; and yet a majority (two out of three) hold that *Q* is false. Were the judiciary to extrapolate from the majority acceptance of *P* and '*P* implies *Q*' that *Q* should also be law, this would be opposite to what a majority vote on *Q* would have yielded.

Symmetrical Special-Majority Voting can assuage those worries. For that to be the case, the 'special majority' supporting each of the component propositions needs to be high: it must be more than $(s-1)/s$ of the individuals, where s is the number of distinct propositions on which decisions are to be made (including 'atomic' propositions like *P* and *Q* and 'connection rules' like '*P* implies *Q*'). It can be proven that, if propositions (and connection rules) are accepted only if they command a special majority of that size, then there will not be a similarly large special majority against any inferences drawn from the accepted propositions through the accepted connection rules.[16]

Thus, judges can draw, and impose as law, the logical implications of the laws that the legislature has passed by 'special majorities' of the requisite size. (Note well, those implications must be derived by inference rules that the legislators themselves do – or presumably would – agree to, by a similar-sized 'special majority'.) Judges can do that, confident that their verdicts will not be contrary to the will of any similarly large special majority among the legislature.

(Remember the context, here. Judicial extrapolation is being offered as a way of 'breaking ties', filling in matters that are left open by Symmetrical Special-Majority Voting in the legislative forum. The scenario in view is one in which the legislature is operating under the rule that 'a special majority of members is required to decide for or against any proposition'. Anything less, and the outcome would be considered a 'tie' and left open, legislatively. Judicial extrapolation in those circumstances would therefore not fall afoul of the will of the legislature, operating under the rules here in view.[17])

Of course, the requisite 'special majority' – $(s\text{-}1)/s$ – increases with the number of propositions (s) involved in the inference. If there are 3 propositions involved in the inference (including connection rules), those propositions and connection rules would each have to be endorsed by more than 2/3 of the legislature (or judges have to think they would be so endorsed); if there are 4, each by more than 3/4; if 8, each by more than 7/8. Hence this is a mechanism that can plausibly be employed only where the inferences are fairly simple and

[16] The theorem can be stated as follows. Suppose there are s propositions to be voted on (counting each proposition-negation pair as one proposition), and each individual has a consistent set of views over these propositions. Then proposition-wise supermajority voting with a supermajority size of more than $(s\text{-}1)/s$ of the individuals generates a consistent (but not necessarily complete and deductively closed) collective set of views over the propositions. For a more technical exposition and a proof, see List (2002, theorem 7).

[17] The parameter m of the legislature's Symmetrical Special-Majority rule (where m is the required margin between the majority and the minority) must satisfy the inequality $m > n(2(^{s-1}/_{s})- 1)$. To illustrate, if the number of propositions is $s = 3$, then $(s\text{-}1)/s = 2/3$, and so in a 100-member legislature m must be at least 34, corresponding to a majority of at least 67 out of 100.

straightforward, involving few steps. Otherwise the special majority requirement approaches unanimity.

Still, there are such cases. Consider the principle, mentioned above, that 'endorsing the ends implies endorsement of the necessary means'. In terms of our example of the three propositions above: *P* is the end; '*P* implies *Q*' is the proposition that 'if *P* is the end then *Q* is a necessary means to that end'; and so *Q* is the means in question. Since we have three propositions, any special majority size greater than 2/3 would be sufficient.

B. Federal Subsidiarity

A second mechanism for resolving ties generated by Symmetrical Special-Majority Voting can be found in the division of decisional responsibilities within a federal system.

To help motivate our proposal here, consider a familiar aspect of appellate court practice. In courts of appeal, it is standard for the judgment of the lower court to be allowed to stand, if the higher court splits evenly on the appeal.

Barry and Hardin (1982, p. 297) describe that as a violation of May's (1952) 'neutrality' condition. Certainly that rule privileges one outcome over the other: whichever outcome was the one chosen by the lower court. But the 'whichever' in that last sentence is crucial. It is not any particular outcome that is privileged by this rule, as the 'default option' is in the Asymmetrical Special-Majority rule. Instead, what is privileged is a particular decision-maker – the lower court. Furthermore, the decision-maker thus privileged is not one of the participants in the appeals court (not the senior judge of the panel, for example), which would violate the 'anonymity' condition. What is actually privileged by this rule is a wholly separate decision process, to be used for making the decision when this process yields no determinate choice: when the higher court is tied, the decision of the lower court stands.

Of course, that particular rule actually operates only for the very special case of a literally 'tied' verdict in an appeals court that operates according to Simple Majority rule. But we propose that model, suitably generalized, as a good strategy for solving the much more frequent 'ties' that can be expected under Symmetrical Special-Majority Voting.

The generalized version of this rule, as applied to a federal system (like the European Union, for example), might be this. The higher-level authority can be empowered to act only on the basis of a 'special majority' of requisite size. There is no 'default' option that wins, unless some other commands that number of votes; if no option commands the requisite 'special majority', the higher-level authority has simply not decided in favour of any option. From the point of view of the higher-level authority, it remains a completely open question. But in the context of a federal system operating under a rule of 'subsidiarity' such irresolution at the higher level of authority can simply leave the matter open to determination by authorities at the lower levels.

Determinations of higher-level authorities can still preempt determinations of lower-level ones. But where the higher-level authority makes no determinations – here, where there is no 'special majority' of the requisite size in favour of any option, at the higher level – the higher-level authority has not preempted any decisions by lower-level authorities. Under the doctrine of 'subsidiarity', the issue then falls to lower-level authorities (member states, local communities, private individuals) to resolve.

Such strategies resonate particularly powerfully in the EU context, because of its strong traditions of subsidiarity (Delors 1991; Bermann 1994). But they are common enough even where the central authorities are stronger. The US federal government often intentionally leaves matters that are formally within its jurisdiction open to determination on a state-by-state basis, hoping that the 'laboratory of federalism' might throw up better solutions than any of the federal legislators or bureaucrats can think of for now. The US welfare reforms of 1996 explicitly invited different states to come up with different rules for governing Temporary Assistance to Needy Families, for example. And in its refusal either to endorse a right to physician-assisted suicide or to prohibit it, the US Supreme Court left the matter open for resolution on a state-by-state basis.[18]

[18] The Court's rationale follows Sunstein (1997). The US Supreme Court's actions later that year seem to be acting on that principle: it ruled in 1997 that there is no constitutionally guaranteed right to die, which states are required to respect (in *Washington v. Glucksberg* and *Vacco v. Quill*); later that same year, however, the

Of course, something gets decided, somewhere. Some people win, others lose, in consequence of those decisions. Leaving things undecided at one level just shifts the effective decision point elsewhere.

The point remains, however, that the decision is at least a decision, rather than just the automatic consequence of some arbitrary 'default'. There are reasons for the decision, which people can discuss and debate in those other fora. The Asymmetrical Special-Majority rule, in contrast, would simply impose some outcome merely by default.

In the course of breaking the 'ties' left by the Symmetrical Special-Majority rule at the higher level, lower-level jurisdictions will often choose different options one from another. Indeed, often they already have.

Such situations pose a compelling case for employing the Symmetrical Special-Majority rule. If different lower-level jurisdictions have already chosen different options, then to let any one of those options serve as the 'default option' under an Asymmetrical Special-Majority rule would arbitrarily favour some jurisdictions and their choices over others. There may well be good grounds for thinking that the higher-level jurisdiction needs to have a 'special majority' of some sort or another to overrule the determinations of the lower-level jurisdictions, in a regime like the EU. But if so, it surely ought to be a Symmetrical Special-Majority that treats all jurisdictions and all options identically. And if the requisite 'special majority' cannot be attained by any of the options at the higher level, then there is a good case to be made in terms of 'subsidiarity' to leave each of the lower-level jurisdictions to determine its own choices for itself.[19]

VI. Conclusion

Majoritarian democrats are ordinarily wary of super-majority requirements. Where we have good grounds for setting the presumption one way rather than another, as we sometimes do, it may be perfectly legitimate to let the outcome often be determined by the defaults built into

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Court also refused to hear a challenge to an Oregon law granting people a right to die (*Lee v. Harcleroad*, #96-1824, certeriori denied).

[19] We are grateful to Dennis Mueller for this observation.

Asymmetrical Special-Majority rules. But more often there seems to be no good reason for favouring one outcome over any other in that way, in advance.

The Asymmetrical version of the Special-Majority rule is the most familiar form. Were it the only form, unease at the arbitrariness of setting a presumption one way rather than some other (absent any grounds for setting a presumption either way) would incline us toward Simple Majority rule instead. That is certainly the standard way of avoiding any arbitrary bias in favour of one 'default' option.[20]

The asymmetrical form is not the only possible form of special-majority voting, however. Special-majority requirements can be specified in such a way as to apply to all options the same. Under the Symmetrical Special-Majority rule, no option is privileged as the default option. Arbitrariness is avoided, just as it is under Simple Majority Voting. And something else is achieved as well: procedurally, the special protection that comes with the 'special majority' requirement is afforded to minority interests; epistemically, a higher standard of proof for making decisions is implemented. Such special minority protection or such a special standard of proof may not always be warranted, and where they are not Simple Majority rule works well. But where we have good reasons for thinking that larger-than-bare majorities ought to be required for decisions in any direction, we ought to use a Symmetrical Special-Majority rule instead. And with the symmetrical version of that rule, we can do so in full confidence that no option will be arbitrarily privileged by being the default option.

That decision rule has not heretofore received the attention it deserves, owing perhaps to doubts as to its practical feasibility. Symmetrical Special-Majority rules run the risk of leaving too many things open, which might be undesirable. That practical concern can be assuaged, however, if we supplement the Symmetrical Special-Majority rule with other decision procedures, such as venue-shifting options available in the judicial or federal realms.

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[20] Except to break truly 'trivial' ties.

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APPENDICES: 'SPECIAL MAJORITIES RATIONALIZED'

Appendix 1: Generalized Majority Voting and Its Properties

Generalized Majority Voting with parameters k_- and k_+ ($k_+ > k_-$). For any v,

$$f(v) = \begin{cases} 1 & \text{if } \Sigma v_i \geq k_+ \\ 0 & \text{if } k_+ > \Sigma v_i > k_- \\ -1 & \text{if } \Sigma v_i \leq k_- \end{cases} \qquad (k_+ > n \text{ or } k_- < -n \text{ are admissible}).$$

Proposition A1. Let f be any aggregation function. Then f satisfies (U), (A), (M) if and only if f is Generalized Majority Voting for some values of k_+ and k_- ($k_+ > k_-$).

Proof. Suppose f satisfies (U), (M) and (A).

Step 1. I show that, for any v, w, $\Sigma v_i \geq \Sigma w_i$ implies $f(v) \geq f(w)$. For each v, let $\#v$ denote the number of 1s in the profile v. Suppose $\Sigma v_i \geq \Sigma w_i$. Then $\#v \geq \#w$ (since v_i and w_i only take the values -1 or 1).

Let $v' := \langle \underbrace{1, \ldots, 1}_{\#v}, \underbrace{-1, \ldots, -1}_{n-\#v} \rangle$ and $w' := \langle \underbrace{1, \ldots, 1}_{\#w}, \underbrace{-1, \ldots, -1}_{n-\#w} \rangle$.

Since v' and w' are, respectively, permutations of v and w, by (A), we have $f(v)=f(v')$ and $f(w)=f(w')$. Since $\#v \geq \#w$, $v' \geq w'$. By (M), $f(v') \geq f(w')$. But since $f(v)=f(v')$ and $f(w)=f(w')$, we have $f(v) \geq f(w)$, as required.

Step 2. Step 1 implies that there exists an increasing function $g : \{-n, -n+1, \ldots, n-1, n\} \to \{-1, 0, 1\}$ such that, for any v, $f(v) = g(\Sigma v_i)$. This implies that there exist $k_-, k_+ \in \{-n, -n+1, \ldots, n-1, n\}$, where $k_- < k_+$, such that

$$f(v) = \begin{cases} 1 & \text{if } \Sigma v_i \geq k_+ \\ 0 & \text{if } k_+ > \Sigma v_i > k_- \\ -1 & \text{if } \Sigma v_i \leq k_- \end{cases} \qquad (k_+ > n \text{ or } k_- < -n \text{ are admissible}). \blacksquare$$

The following three propositions state some of the properties of a generalized majority aggregation function.

Proposition A2. A Generalized Majority aggregation function f never takes the value -1 if $k_- < -n$; it never takes the value 1 if $k_+ > n$; it never takes the value 0 if $k_+ - k_- = 1$ or [$k_+ - k_- = 2$ and k_+ and n are both odd].

Proposition A3. A Generalized Majority aggregation function f satisfies (NT) <u>if and only if</u> either (i) [$k_+ - k_- = 1$ or [$k_+ - k_- = 2$ and $k_+ (\neq 1)$ and n are both odd]] or (ii) [$k_+ = 1$ and $k_- = -1$].

Case (ii) is the special case of Simple Majority Voting. Case (i) is the case of Asymmetrical Special-Majority Voting.

Proposition A4. Suppose f is Generalized Majority Voting. Then f satisfies (PV_k) <u>if and only</u> if $k_+ > n-2k$. And f satisfies (NV_k) <u>if and only if</u> $k_- < -(n-2k)$.

<u>Appendix 2: Proofs of the Results in Section I</u>

Proposition 8 (stated above). Let f be any aggregation function. Then f satisfies (U), (A), (S), (M) <u>if and only if</u> f is Symmetrical Special-Majority Voting for some value of m ($m > 0$).

Proof. It is easy to see that, if f is Symmetrical Special-Majority Voting for any value of m (m>0), then f satisfies (U), (A), (S), (M). Suppose, conversely, that f satisfies (U), (A), (S), (M). By proposition A1, there exist k_+ and k_- (where $k_+ > k_-$) such that, for any v,

$$f(v) = \begin{cases} 1 & \text{if } \Sigma v_i \geq k_+ \\ 0 & \text{if } k_+ > \Sigma v_i > k_- \\ -1 & \text{if } \Sigma v_i \leq k_- \end{cases} \qquad (k_+ > n \text{ or } k_- < -n \text{ are admissible}).$$

Now it is easy to see that, since f satisfies (S), $k_+ = -k_-$. Since $k_+ > k_-$, we must therefore have $k_+ > 0 > k_- = -k_+$. Put $m := k_+$. Then the desired result follows. ■

Proposition 1 (May's theorem) and proposition 2 can now be deduced straightforwardly from proposition 8, by observing that conditions (PR) and NT) are each satisfied by a Symmetrical Special-Majority Voting rule if and only if $m = 1$.

Proposition 5 (stated above). Suppose f satisfies (U), (PV_{k_1}), (NV_{k_2}) and (NT). Then $k_1+k_2 \geq n$.

Proof. Assume, for a contradiction, that f satisfies the conditions of the proposition with $k_1+k_2 < n$. So we cannot have both $k_1 = n/2$ and $k_2 = n/2$. Without loss of generality, assume that $k_1 \neq n/2$. Consider a profile v where $v_1, \ldots, v_{k_1} = -1$, and $v_{k_1+1}, \ldots, v_n = 1$. Then $|\{i \in N : v_i = -1\}| = k_1$, and so, by ($PV_{k_1}$), $f(v) \neq 1$. Also, $|\{i \in N : v_i = 1\}| = n-k_1 > k_2$ (since

$k_1+k_2 < n$) and so, by (NV_{k_2}), $f(v) \neq -1$. Hence $f(v) = 0$. By (NT), this implies $\Sigma v_i = 0$. But $\Sigma v_i = n-2k_1 \neq 0$, a contradiction. Hence $k_1+k_2 \geq n$. ■

Proposition 7 (stated above). Suppose f satisfies (U) and (S). For any k, f satisfies (PV_k) if and only if f satisfies (NV_k).

Proof. This proposition follows immediately from condition (S). ■

The conjunction of propositions 5 and 7 immediately entails proposition 3.

Proposition 4 (stated above). Let f be any aggregation function. Then f satisfies (U), (NT), (S) and ($PV_{1/2}$) (and ($NV_{1/2}$)) <u>if and only if</u> f is Simple Majority Voting.

Proof. If f is Simple Majority Voting, f clearly satisfies the relevant conditions. I prove the reverse implication. Suppose f satisfies (U), (NT), (S) and ($PV_{1/2}$). Suppose $\Sigma v_i < 0$. This implies $|\{i \in N : v_i = -1\}| \geq n/2$, and hence, by ($PV_{1/2}$), $f(v) \neq 1$. But, since $\Sigma v_i < 0$, (NT) implies $f(v) \neq 0$, and therefore $f(v) = -1$. By (S), if $\Sigma v_i > 0$, this implies that $f(v) = 1$. If $\Sigma v_i = 0$, we must have $|\{i \in N : v_i = -1\}| = n/2$. By ($PV_{1/2}$), $\Sigma v_i \neq 1$. But by (S), we must also have $\Sigma v_i \neq -1$, i.e. $\Sigma v_i = 0$. Hence f is Simple Majority Voting. ■

Propositions 6 and 9 follows immediately from proposition A4.

Proposition 10 follows from the definition of a symmetrical special majority aggregation function.

Proposition 11 (stated above). Suppose f is a Symmetrical Special-Majority rule satisfying both (PV_{k_1}) and (PR_{k_2}). Then $k_2 > n-2k_1$.

Proof. Suppose f is a symmetrical special majority aggregation function with parameter m satisfying both (PV_{k_1}) and (PR_{k_2}). By proposition 9, $m \geq n-2k_1+1$. By proposition 10, $m^* \leq k_2+1$, where

$$m^* \text{ is } \begin{cases} \text{the smallest even number greater than or equal to } m \text{ if } n \text{ is even} \\ \text{the smallest odd number greater than or equal to } m \text{ if } n \text{ is odd.} \end{cases}$$

Note that $n-2k_1+1$ is even if and only if n is odd. So, when n is even, the smallest even number greater than or equal to $n-2k_1+1$ is $n-2k_1+2$. Likewise, when n is odd, the smallest odd number greater than or equal to $n-2k_1+1$ is $n-2k_1+2$. So, when $m \geq n-2k_1+1$, we must have $m^* \geq n\text{-}2k_1\text{+}2$. But, since $m^* \leq k_2\text{+}1$, we must have $k_2\text{+}1 \geq$

$n\text{-}2k_1\text{+}2$, i.e. $k_2 > n\text{-}2k_1$. ■

Proposition 12 (stated above). Let *f* be any aggregation function. Let *m* be any integer greater than 0 (where *m* is even <u>if and only if</u> *n* is even). Then *f* satisfies (U), (A), (S), (M), ($PV_{(n-m)/2+1}$) and (PR_{m-1}) <u>if and only if</u> *f* is Symmetrical Special-Majority Voting with parameter *m*.

Proof. By propositions 8, 9 and 10, if *f* is Symmetrical Special-Majority Voting with parameter *m*, then it satisfies (U), (A), (S), (M), ($PV_{(n-m)/2+1}$) and (PR_{m-1}). Conversely, suppose *f* satisfies (U), (A), (S), (M), ($PV_{(n-m)/2+1}$) and (PR_{m-1}). By proposition 8, *f* is Symmetrical Special-Majority Voting for some value of $m' > 0$. By proposition 9, since *f* satisfies ($PV_{(n-m)/2+1}$), $m' \geq n\text{-}2((n\text{-}m)/2\text{+}1)\text{+}1$, i.e. $m' \geq m\text{-}1$. By proposition 10, since *f* satisfies (PR_{m-1}), $m'^* \leq (m\text{-}1)\text{+}1 = m$, where

$$m'^* \text{ is } \begin{cases} \text{the smallest even number greater than or equal to } m' \text{ if } n \text{ is even} \\ \text{the smallest odd number greater than or equal to } m' \text{ if } n \text{ is odd.} \end{cases}$$

So we have $m\text{-}1 \leq m' \leq m'^* \leq m$, and hence $m' = m\text{-}1$ or $m' = m$. (If $m = 1$, then $m' = 1$, since $m' > 0$.) But, since *m* is even <u>if and only if</u> *n* is even, Symmetrical Special-Majority Voting with parameter $m' = m\text{-}1$ or $m' = m$ is equivalent. ■

<u>Appendix 3: Proofs of the Results in Section II</u>

Proposition 13 (stated above). Let *f* be any aggregation function satisfying (U). Suppose that *f* satisfies (U), (NT*), (PP_c) and (NP_c). Then *c* is not sufficiently greater than 1/2.

Proof.

Let $v := \langle \underbrace{1, \ldots, 1}_{\#v}, \underbrace{-1, \ldots, -1}_{n-\#v} \rangle$, where

$$\#v := \begin{cases} n/2\text{+}1 & \text{if } n \text{ is even} \\ (n\text{+}1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

Then $\Sigma v_i = 2$ if n is even, and $\Sigma v_i = 1$ if n is odd. By a formula by Condorcet (List 2003), $Pr(X = 1 \mid V = v) = p^{\Sigma v_i}/ (p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i})$. So $Pr(X = 1 \mid V = v) = p^2/(p^2+(1\text{-}p)^2))$ if n is even, and $Pr(X = 1 \mid V = v) = p$ if n is odd. Note that $Pr(X = 1 \mid V = v) \neq 1/2$. Since f satisfies (NT*), we must have $f(v) \neq 0$. So $f(v) = 1$ or $f(v) = -1$. Since f satisfies (PP_c), if $f(v) = 1$, then $Pr(X = 1 \mid V = v) > c$; so $c < p^2/(p^2+(1\text{-}p)^2))$ if n is even, and $c < p$ if n is odd. Since f satisfies (NP_c), if $f(v) = -1$, then $Pr(X = -1 \mid V = v) > c$, i.e. $1\text{-}Pr(X = 1 \mid V = v) > c$; so
$c < 1\text{-} p^2/(p^2+(1\text{-}p)^2)) < p^2/(p^2+(1\text{-}p)^2))$ if n is even, and $c < 1\text{-}p < p$ if n is odd. Therefore c is not sufficiently greater than 1/2, in the technical sense defined above. ■

Proposition 14 (stated above). Let f be any aggregation function. In Condorcet's model, f satisfies (U), ($PP_{1/2}$) and ($NP_{1/2}$) <u>if and only if</u> f is Simple Majority Voting.

Proof. It is easy to check that Simple Majority Voting satisfies (U), ($PP_{1/2}$) and ($NP_{1/2}$) (under Condorcet's assumptions). Suppose f satisfies (U), ($PP_{1/2}$) and ($NP_{1/2}$). Let v be any profile. By ($PP_{1/2}$) and ($NP_{1/2}$),

$f(v) = 1$ <u>if and only if</u> $Pr(X = 1 \mid V = v) > 1/2$,

$f(v) = -1$ <u>if and only if</u> $Pr(X = -1 \mid V = v) > 1/2$.

By (U), we must therefore have $f(v) = 0$ <u>if and only if</u> $Pr(X = 1 \mid V = v) = 1/2$.

By a formula by Condorcet (List 2003), $Pr(X = 1 \mid V = v) = p^{\Sigma v_i}/ (p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i})$ and $Pr(X = -1 \mid V = v) = 1 - p^{\Sigma v_i}/(p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i})$. So

$f(v) = 1$ <u>if and only if</u> $p^{\Sigma v_i}/ (p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i}) > 1/2$, and

$f(v) = -1$ <u>if and only if</u> $1 - p^{\Sigma v_i}/(p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i}) > 1/2$,

i.e. $p^{\Sigma v_i}/(p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i}) < 1/2$.

But

$p^{\Sigma v_i}/ (p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i}) > 1/2$ <u>if and only if</u> $p^{\Sigma v_i} > (1\text{-}p)^{\Sigma v_i}$,

i.e. $\Sigma v_i > 0$ (since $p > 1/2$)

$p^{\Sigma v_i}/(p^{\Sigma v_i} + (1\text{-}p)^{\Sigma v_i}) < 1/2$ <u>if and only if</u> $p^{\Sigma v_i} < (1\text{-}p)^{\Sigma v_i}$,

i.e. $\Sigma v_i < 0$ (since $p > 1/2$).

Finally, $p^{\Sigma v_i} / (p^{\Sigma v_i} + (1-p)^{\Sigma v_i}) = 1/2$ if and only if $p^{\Sigma v_i} = (1-p)^{\Sigma v_i}$, i.e. $\Sigma v_i = 0$ (since $p \neq 1/2$). We conclude that f is Simple Majority Voting. ■

Proposition 15. Suppose f satisfies (U) and (NT*), and suppose c_1 is sufficiently greater than 1/2. If f satisfies (PP_{c_1}), then f does not satisfy (NP_{c_2}) for any $c_2 \geq 1/2$; and if f satisfies (NP_{c_1}), then f does not satisfy (PP_{c_2}) for any $c_2 \geq 1/2$.

Proof. Suppose f satisfies (U), (NT*) and (PP_{c_1}), where c_1 is sufficiently greater than 1/2. Suppose further that f satisfies (NP_{c_2}) for some $c_2 \geq 1/2$. Define v as in the proof of proposition 13. Then $c_1 \geq Pr(X = 1 \mid V = v) > 1/2$. By ($PP_{c_1}$), $f(v) \neq 1$. By (NT*), $f(v) \neq 0$. But $Pr(X = -1 \mid V = v) = 1-Pr(X = 1 \mid V = v) < 1/2 \leq c_2$, so by ($NP_{c_2}$), $f(v) \neq -1$. This contradicts (U). The proof that if f satisfies (NP_{c_1}) then f does not satisfy (PP_{c_2}) for any $c_2 \geq 1/2$ is analogous. ■

Proposition 16 (stated above). Let $c \geq 1/2$. The aggregation function f satisfies (U), (PP_c) and (NP_c) if and only if f is Symmetrical Special-Majority Voting, where m is the smallest integer strictly greater than $log(^1/_c\text{-}1)/log(^1/_p\text{-}1)$.

Proof. Suppose f satisfies (U), (PP_c) and (NP_c), where m is the smallest integer above $log(^1/_c\text{-}1)/log(^1/_p\text{-}1))$. Then

$f(v) = 1$ if and only if $Pr(X = 1 \mid V = v) > c$,

$f(v) = -1$ if and only if $Pr(X = -1 \mid V = v) > c$.

By (U), we must therefore have $f(v)=0$ if and only if $c \geq Pr(X = 1|V = v) \geq 1-c$.

By a theorem in List (2003),

$Pr(X = 1|V = v) > c$ if and only if $\Sigma v_i > log(^1/_c\text{-}1)/log(^1/_p\text{-}1))$,

i.e. $\Sigma v_i \geq m$, with m as defined above.

Similarly, $Pr(X = -1|V = v) > c$ if and only if $-\Sigma v_i > log(^1/_c\text{-}1)/log(^1/_p\text{-}1))$,

i.e. $\Sigma v_i \leq -m$, with m as before.

Finally, $c \geq Pr(X = 1 \mid V = v) \geq 1-c$ if and only if $m > \Sigma v_i > -m$. We conclude that f is Symmetrical Special-Majority Voting with parameter m. It is easy to check that, if f is

Symmetrical Special-Majority Voting with parameter *m*, then *f* satisfies (U), (PP_c) and (NP_c) with *m* as defined above. ■

Appendix 4: A Version of the Epistemic Case in Terms of the Degree of Support

Again we make Condorcet's assumptions, except that no assumption about prior probabilities is required here. The degree of support some evidence *E* gives to some hypothesis *H* is defined to be $l(H, E) := log(Pr(E|H)/Pr(E|\neg H)$ (Fitelson 2001). The degree of support the voting pattern $V = v$ gives to the hypothesis $X = 1$ is $l(X = 1, V = v) = \Sigma v_i\ log(p\ /\ (1\text{-}p))$. The degree of support the voting pattern $V = v$ gives to the hypothesis $X = \text{-}1$ is $l(X = \text{-}1, V = v) = \Sigma v_i\ log((1\text{-}p)/p) = \text{-}\Sigma v_i\ log(p/(1\text{-}p))$.

Now a classical (non-Bayesian) way of stating the standard of proof conditions is as follows.

A standard of proof of *c* for positive decisions (PP^*_c). For any profile v, $f(v) = 1$ if and only if $l(X = 1, V = v) > c$.

A standard of proof of *c* for negative decisions (NP^*_c). For any profile v, $f(v) = \text{-}1$ if and only if $l(X = \text{-}1, V = v) > c$.

The standard of proof corresponding to Simple Majority Voting is $c = 0$. We can now state corollaries of propositions 13 to 16.

Proposition 14*. Let *f* be any aggregation function. In Condorcet's model, *f* satisfies (U), (PP^*_0) and (NP^*_0) if and only if *f* is Simple Majority Voting.

The classical (non-Bayesian) statement of no non-trivial ties, then, is

No non-trivial ties (NT).** For any profile v, $f(v) = 0$ implies $l(X = 1, V = v) = l(X = \text{-}1, V = v) = 0$.

Clearly, Simple Majority Voting satisfies (NT**).

Let us say that the threshold *c* is *sufficiently greater* than 0 if

$$c \geq \begin{cases} log(p\ /\ (1\text{-}p)) & \text{if } n \text{ is odd} \\ 2log(p\ /\ (1\text{-}p)) & \text{if } n \text{ is even.} \end{cases}$$

Proposition 13*. Let f be any aggregation function satisfying (U). Suppose that f satisfies (U), (NT**), (PP*$_c$) and (NP*$_c$). Then c is not sufficiently greater than 0.

Proposition 15*. Suppose f satisfies (U) and (NT**), and suppose c_1 is sufficiently greater than 0. If f satisfies (PP*$_{c_1}$), then f does not satisfy (NP*$_{c_2}$) for any $c_2 \geq 0$; and if f satisfies (NP*$_{c_1}$), then f does not satisfy (PP*$_{c_2}$) for any $c_2 \geq 0$.

Proposition 16*. Let $c \geq 0$. The aggregation function f satisfies (U), (PP*$_c$) and (NP*$_c$) if and only if f is Symmetrical Special-Majority Voting, where m is the smallest integer strictly greater than $c/log(p/(1\text{-}p))$.

The case $c = 0$ corresponds to Simple Majority rule. The case c sufficiently greater than 0 but less than $n\ log(p/(1\text{-}p))$ corresponds to a Special-Majority rule. The case c greater than or equal to $n\ log(p/(1\text{-}p))$ corresponds to Imposed Indifference.