Assessing Health Inequalities Using a Dynamic Microsimulation Model

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Abstract

The paper describes stage 1 of the development of a Health Module to a dynamic microsimulation model which simulates individuals’ life cycles. The life cycle approach has been shown in the literature to be important when studying health inequalities. The enhanced model accounts for the links between Australians’ socio-economic status and their health. The full model, which is able to project 20 to 30 years into the future, is based on a 1 per cent unit record Census sample of the Australian population.

Health is measured using information on individuals’ disability status, which in turn is linked to their life expectancy. Both disability and life expectancy are estimated for each individual as a function of age, gender and socio-economic status. Socio-economic status is measured using the ‘Index of Relative Socio-economic Disadvantage’ (developed by the Australian Bureau of Statistics), and/or an index based on families’ incomes and assets (being especially developed for this project).

The paper also touches on proposed further developments of the Health Module and lists a range of studies that could be attempted once that Module is completed. Such studies could for example compare the demographic, labour force, financial and distributional impacts of proposed policies that aim to lower health inequalities – and do that more accurately and at a much greater level of detail than was possible previously.

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1 Introduction

In recent years health inequalities became a much-researched subject, with a number of developed countries having already announced policies that aim to reduce inequalities in health – eg the UK and Canada [1-3].

To date, much of the research in Australia and overseas concerned differences in mortality patterns across the socio-economic status (SES) spectrum at a particular point in time. The virtually universal finding was that people with low SES – usually the ‘poor’ – died younger than people with high SES – usually the ‘rich’ [4,5]. More recently the focus of research has shifted to the reasons for premature death by the socially disadvantaged, and to the study of individuals’ health and lifestyles as it affects mortality [6].

The aim of the full project is to make a unique contribution to this broadening of the way in which health inequalities are analysed. The full project involves the attaching of a health module, by SES groups, to an existing dynamic microsimulation model of the Australian population.

2 Scope of project and of this paper

Recently there has been increasing interest internationally in the study of health issues as they affect people throughout their life cycles – eg family, education, work, finances, independence in old age, etc [7].

This project aims to simulate the life paths of individuals over a 20 to 30 year period using a dynamic microsimulation model (Appendix, Section A1). In its first stage the project involves the modelling of health as a function of socio-economic status – the subject of this paper - with mortality and disability being the indicators of health.

In later stages we intend to add additional health indicators (such as the severity of disability and disease types), and to model health related decisions regarding issues such as remaining in (or exiting) the workforce, becoming a carer or, in old age, leaving home to enter an institution (eg hostel, nursing home).
3 Overview: model linkages between health and socio-economic status

In the Base population of the dynamic microsimulation model, DYNAMOD, disability status will be allocated to individuals, by socio-economic status, in line with historically observed patterns (section 6). For definition of disability see Appendix, section A3. Demographic, social and economic events occurring throughout the lives of people in this Base population are then simulated over a 20 to 30 year period (Appendix, section A1).

Figure 1 illustrates the way the link between health (ie mortality, disability) and SES will be handled in the model. Briefly, when a child is born, he/she will be allocated a date of death based on observed probabilities and on his/her parents’ socio-economic status. A disabled new born will be allocated an earlier date than an able-bodied new born. When considering probabilities of death, allowance will be made for increases in future life expectancies - based on predictions by the Australian Bureau of Statistics. The initially allocated date of death will remain unchanged throughout the life of that person, unless a change occurs in his/her disability or socio-economic status.

Figure 1: Model links between mortality and disability, by socio-economic status
Our modelling approach is similar to that in the original model [8], except that the already complex links between disability and mortality will now also account for socio-economic status. The complex set of equations that define the linkages between disability and mortality, when developing the model’s input data, are in the Appendix (sections A5 and A6).

4 Indicators of health status

4.1 Mortality

When studying health at the population level it is common to use various mortality rates and related life expectancies as proxies for health status – eg infant mortality, life expectancy at birth [4, 9].

However, health as indicated by mortality rates says nothing about how health problems developed over an individual’s life; how they impacted on a person’s family or employment prospects; what factors contributed to a particular mortality outcome; and how that outcome could have been altered through adoption of various interventions by individuals or government.

In effect, we know very little from standard statistical collections about the characteristics of people who just died. While such collections tell us when we are likely to die, they say nothing about the quality of our lives prior to death. In this project, by linking mortality to disability, we expect to fill some aspects of this knowledge gap.

We used mortality statistics that were especially extracted for this project by the Australian Institute of Health and Welfare (AIHW) from its 1995-97 Mortality Database. Apart from the variables often used in earlier mortality studies - age, gender and an index of socio-economic status based on area of residence – we also obtained data split into two groups based on cause of death. These were ‘External’ and ‘Non-external’ causes, which were separately identified so as to be able to distinguish between deaths due to accidents (eg drowning, car crashes) and deaths due to illnesses (eg heart attack). This distinction is important because people dying from external causes are generally much younger than people dying from diseases or old age – hence the former group, although much smaller, will be important when estimating the effects of policy or other changes in terms of ‘years of life lost’.
4.2 Disability

In search of information on the health related characteristics of Australians, we studied the health and disability statistics available from the Australian Bureau of Statistics (ABS). Although these are collected through large cross sectional surveys, it is possible to construct life cycle patterns by assuming – as is adopted as ‘default’ in the model - that health patterns for groups with a particular set of demographic and socio-economic characteristics will remain unchanged over time.

Our earlier research suggested that data from household surveys were unlikely to be satisfactory when studying health status throughout the life cycle. This was because such surveys excluded persons in institutions. Since it is older people who tend to reside in institutions – such as hospitals, hostels and nursing homes – using the household surveys would mean that we would need to limit the study’s coverage to the younger, healthier age groups.

In an earlier study of the 1995 National Health Survey we found that respondents aged 70 years or over appeared to have better health on average than did younger age groups [10]. This was thought to be because seriously ill or disabled Australians aged 70 or over who had moved into an institution were excluded from the survey. A similar pattern was found in ABS Household Expenditure Surveys (1993-94 and 1998-99) in that people aged 75 or over appeared to have spent less on prescribed pharmaceuticals than did younger age groups [11]. Thus, if in this project we relied on household surveys, we would have to exclude people over the age of 70 – the group that includes the majority of persons close to death[12].

While it could be argued that those who survived to age 70 may have had a stronger constitution than the ‘younger old’, this was not borne out by the statistics in the 1998 Disability Survey – a survey which included persons in institutions (Appendix, section A3). That survey showed the proportion of Australians with a disability and/or long term illness as increasing steadily well beyond age 70 [11].

In view of the above, use of the Disability survey in this project is clearly preferable to the household-based alternatives. The Disability survey also has linkages to illnesses, in the form of the ‘main disabling condition’ variable, as well as information on the severity of disability. Use of these variables is planned for the later stages of the project and, if necessary, additional disease-related information may be imputed from National Health Surveys (Appendix, section A2), but only to age 70.
5 Indicators of socio-economic status

Two common indicators of socio-economic status are used in Australian studies: the ABS’s socio-economic indexes for areas (SEIFA) and equivalent (or per person) family income. In the literature family income was found to be a good single ‘proxy’ for socio-economic disadvantage.

With mortality statistics only the SEIFA based indicator is available (Section 4.1). However, with the 1998 Disability survey we could choose either the SEIFA, or an income-based indicator of SES. The question is which of these two is preferable for our purposes.

Studying both mortality and disability as a function the SEIFA index has the advantage of consistency across different aspects of the project. This however needs to be weighed against the advantages of the income-based indicator, in that the latter can be applied to individuals while the SEIFA applies to all people residing in a particular geographical area. Thus households in a particular area – in our case a Collector District, which covers on average around 200 households – will all have the same SEIFA quintile attached to them, even though both rich and poor families may reside there. Since a ‘per person’ indicator cannot be obtained from the SEIFA, it is not possible to distinguish between large and small families with that index. For example, a two person couple family with annual income of $50,000 a year residing in a particular area will have the same SEIFA quintile than a five person family, such as a couple with three dependent children, living in the same area. By comparison, with the income index it is common practice to use the ‘equivalent family income’ measure - which can be thought of as being a ‘per person’ indicator of family income.

Thus, a priori one would expect equivalent income to be a more appropriate and precise indicator of SES than the geographically based SEIFA index. This is because family income reflects the purchasing power of the particular family being studied, and not the collective wealth of the geographic area in which that family happens to reside.

However, a key disadvantage of the income-based indicator is that it takes no account of assets. In a life cycle context many people start out by being ‘cash poor and asset poor’, then progress to a ‘cash rich - asset poor’ status, ending their lives as being ‘cash poor and asset rich’. This means that the income measure will tend to class older Australians into the lower SES groups which, for a significant proportion, may not be in line with what their life styles– in terms larger houses, expensive cars, overseas holidays, etc – would suggest. By comparison the SEIFA index has the advantage of taking account of - albeit indirectly - characteristics that tend to be stable over the life cycle (such as education and assets).
Using the 1998 Disability survey, we were able to compare how the allocation of the same population to SES quintiles differs across the two measures (SEIFA quintiles and income quintiles). For this exercise we used the ABS’s index of ‘Relative Socio-economic Disadvantage’ as the SEIFA Indicator. For income we computed ‘equivalent family income’ [16]. The dollar values related to each ‘equivalent family income quintile’ are shown in Table 1. For example, taking Quintile 3 families, their ‘per person’ gross family cash incomes ranged from $356.48 to $554.15 a week in 1998.

Table 1: Equivalent family incomes by income quintile, 1998 (gross weekly cash income, 1998 dollars)

<table>
<thead>
<tr>
<th>Quintile 1</th>
<th>Quintile 2</th>
<th>Quintile 3</th>
<th>Quintile 4</th>
<th>Quintile 5</th>
<th>All quintiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>$</td>
<td>$</td>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
<td>211.87</td>
<td>356.48</td>
<td>554.15</td>
<td>879.02</td>
</tr>
<tr>
<td>Maximum</td>
<td>211.87</td>
<td>356.48</td>
<td>554.15</td>
<td>879.02</td>
<td>1990.89</td>
</tr>
<tr>
<td>Mean</td>
<td>145.19</td>
<td>275.09</td>
<td>450.86</td>
<td>699.8</td>
<td>1095.53</td>
</tr>
</tbody>
</table>

Source: ABS, 1998 Disability Survey

Because the SEIFA index represents an average for a geographic area, one would expect its use to result in a ‘flattening’ of the inequality gradient compared with the individual family-based income indicator. This is supported by our findings when plotting the proportion of disabled by the two SES measures, with Figure 2 indicating a considerably greater difference in the proportion of disabled across income quintiles than across SEIFA quintiles.

With the SEIFA measure 26 per cent of persons in the lowest SES group (quintile 1) were disabled in 1998, compared with 14 per cent in the highest SES group (quintile 5). When using the ‘equivalent income’ measure, a much higher proportion of the lowest SES group was disabled (35 per cent), and a much lower proportion in the highest group (10 per cent).

This confirms our expectation that the geographically based SEIFA measure would flatten the inequality gradient compared with the income measure.

Further analyses showed the patterns in Figure 2 were linked to life cycle issues. One example was the distribution, by age, of the male population across income quintiles. In this respect we found that working age adults made up the bulk of each of
quintiles 3, 4 and 5. By comparison, amongst Australian men of retiring age, less than 2 per cent fell into these top three income categories. The patterns were found to be similar for females and with the SEIFA quintiles.

Figure 2: **Proportion disabled by equivalent income and SEIFA quintiles, 1998**

Although both measures (ie income and SEIFA) indicated higher proportions of older persons in the lower quintiles, with the SEIFA over 13 per cent of men aged 65 years or over fell into each of quintiles 3, 4 and 5, compared with less than 2 per cent with the income measure – reflecting the SEIFA’s ‘flattening’ effect noted earlier.

Overall, neither of the measures was found to be clearly superior. So for reasons of comparability, we chose the ABS’s index of ‘Relative Socio-economic Disadvantage’ (SEIFA) for both mortality and disability when preparing the input data to the dynamic microsimulation model.

As will be seen in Section 6.5, a more suitable income/asset-based socio-economic indicator can be constructed for use throughout the simulation years of the model.

**6 Modelling the health-SES link**

**6.1 Preparing the Base year data**

The input data needed for the model - by gender, SES and single years of age (0 to 104) – comprises:
• Probability of death – for the able-bodied, the disabled and the population in
general (ie able-bodied plus disabled). The probability of death is a function of
disability – ie the disabled have lower life expectancies than the able bodied;
• Mortality improvement rates covering three periods: 1987 to 1994, 1995 to 2004
and 2005 to 2050 – based on ABS predictions;
• Disability prevalence (computed by as the number disabled divided by the total
number of persons in that class);
• Disablement rate (function of the number of people becoming disabled, the
number of recoveries, the able-bodied population, and the number of deaths in
the able bodied population – see equation A.7 in section A5 of the Appendix);
• Recovery rate (with values chosen so that the number of persons becoming
disabled remains positive for all ages within the SES quintile being studied).

The data sources used were an extract from the AIHW’s Mortality database (1995-
97), the ABS’s SEIFA estimates from the 1996 Census, and the ABS’s Disability
survey (1998, as the income data in the 1993 survey could not be used due the non-
response rate being exceptionally high). For much of the survey based analyses we
used the SAS programming language.

Because the mortality and disability statistics were in 5-year age groups (to age 74,
with a final 75+ grouping), the data initially derived for the model needed to be
converted into single years of age. To do this we used the GAM spline program
available in SAS. The ‘fit’ in all cases was statistically significant (with Pr > F less
than 0.0001).

6.2 Mortality rates

To compute separate mortality rates for the able-bodied and disabled populations by
SEIFA quintiles, we used the equations detailed in the Appendix (section A5). The
mortality data needed for the equations was obtained from the AIHW extract, and
the disability prevalence data from the 1998 Disability Survey – as extracted by the
author for each SEIFA quintile. Note that it was only possible to obtain these
separate mortality rates by using both data sources. This is because – as the
equations indicate - the disability prevalence rate mathematically links the able-
bodied population to the disabled population.

The mortality rates for the able-bodied, disabled and total populations were initially
computed by gender, SEIFA quintile and 5-year age groups. These data were then
converted into single years of age (section 6.1). However, in Figures 3, 4 and 5 the
results are presented by 5-year age groups because with that grouping the charts are easier to read when covering each of the five SEIFA quintiles.

Figure 3: **Mortality rates, external causes, males by SEIFA quintiles, 1995-7**

![Mortality rates, external causes, males by SEIFA quintiles, 1995-7](image)

**Sources:** computations for this projects using AIHW mortality data extract

Figure 4: **Mortality rates, all causes, males by SEIFA quintiles, 1995-7**

![Mortality rates, all causes, males by SEIFA quintiles, 1995-7](image)

**Sources:** computations for this projects using AIHW mortality data extract
The greatest differences across quintiles were shown for males dying from external causes (such as car accidents and suicides) – Figure 3. As expected, that Figure shows that males aged 15-39 years had exceptionally high mortality rates due to ‘external causes.

Figure 5: Mortality rates, all causes, females by SEIFA quintiles, 1995-7

Sources: computations for this projects using AIHW mortality data extract

Across all three Figures, quintile 1 people (Q1, the most disadvantaged) consistently had the highest mortality rates, and Q5 the lowest - with Q2, 3 and 4 falling in between. Although not always shown in the paper, this ranking remained remarkably steady not only with all mortality graphs, but also across all the other aspects of the input data required by the model. This is remarkable, given that the various rates were computed using complex equations drawing variables from several distinct datasets.

Figures 4 and 5 - which chart all cause mortality - are very similar to the charts for non-external causes (not reproduced here). This is because mortality rates from non-external causes – ie generally due to diseases – reach very much higher levels and affect many more older people than the rates from external causes.

These three charts illustrate that the key age groups affected by mortality from external causes – young males - and non-external causes - persons aged 40 years - differ considerably. Hence our efforts to model mortality from external and non-external causes separately.
Distinction between the genders was found to be important, since all cause mortality rates were considerably lower for women (Figure 5), than for men (Figure 4). Differences across quintiles for women were also found to be significantly smaller than for men.

### 6.3 Disability prevalence

As noted above, the indicator of socio-economic status for disability prevalence was chosen to be the same as for the mortality analyses (ie SEIFA quintiles). Figure 6 plots the proportion of the disabled by single years of age. For sake of readability, this graph does not present results by SEIFA quintile.

**Figure 6:** Proportion of population disabled by gender and single-years of age, 1998

![Image of disability prevalence graph](image)

*Sources:* Computations for this project using the 1998 Disability survey (ABS)

Figure 6 shows that disability prevalence increases rapidly after age 40, and that the prevalence rate for men tends to be somewhat higher for men than for women.

### 6.4 Disability decrement rates

In the model, disability decrement rates are derived using standard multiple decrement techniques. Briefly, the able bodied population is subject to exits due to death and disability and entrants due to recovery, while the disabled population is subject to exits due to death and recovery and entrants due to onset of disability [8].
We assumed that, for all SEIFA quintiles, the recovery patterns specified in the original DYNAMOD input dataset remained unchanged. For the new input data we then only needed to compute, by SEIFA quintile, the number - and rate - of people becoming disabled. We did this using equations A.5, A.6 and A.7 in section A6 of Appendix A. The computed disablement rates for all quintiles were similar to the rates initially specified for the model.

6.5 Proposed income-wealth measure as indicator of SES in projection years

Although the projection years of the model are outside the scope of this paper, the proposed SES measure for that part of the project is briefly mentioned in this section. As seen from the research presented above, an index which can be attached to individuals, and which accounts for both cash income and assets, seems preferable to the alternatives considered in this paper (ie those that could be constructed from published statistics).

Because dynamic microsimulation is ideally suited to take account of wealth accumulation, and because assets are already covered in the model, we are able to propose the use of a combined income-wealth indicator during the model’s projections years. It is currently envisaged that such an index would be the sum of equivalent (or per person) family cash income – from sources such as earnings and social security payments – and an annualised value of ‘equivalent’ family assets.

7 Concluding remarks

Once the embedding of the input statistics in the simulation part of the model is complete, a limited number of applications will become possible. For example:

- what would be the impact on Australians’ health (indicated initially by mortality/disability) of policies through which the health of the most disadvantaged was lifted to the level of, say, the national average, in terms of lower mortality rates. (In later stages of the project the effects on a range of other factors, such as welfare payments, workforce exits due to health - based on family level decisions – and nursing home entries/costs could also be analysed).

This initial illustrative simulation could be thought of as being similar to, but considerably more complex than earlier analyses [20, 21]. Examples of relevant findings from [20] are:
• 7597 lives could be saved (7% of all deaths under age 65) if wealth redistribution patterns in the UK were reduced to those of the early 1980s;

• 92% of avoidable child deaths could be prevented in areas where the death rates were higher than the national average if child poverty were eradicated;

• 2504 lives could be saved through achievement of full employment.

Another example, from [21] is that:

“If it were possible to reduce death rates .. to a level equivalent to that of the least disadvantaged area, premature all-cause mortality for males in each age group would be lower by 22%, 28% and 26% respectively, and for females, 35%, 70% and 56%.”

An example of a possible application once the full project is completed would be a study of the impact of future health improvements on people remaining in the workforce after age 65. The impact could be measured, for example, in terms of changes in government expenditures on aged care and on the age pension.

Overall, the ability to model the SES-health link in a life cycle context - as is attempted in this project - would improve the tools currently available to support complex life cycle related policy decisions.
APPENDIX

A1 DYNAMOD

The dynamic micro-simulation model to which the health module will be added - DYNAMOD - was developed at the National Centre for Social and Economic Modelling, University of Canberra. DYNAMOD is a full population model able to project the entire Australian population forward. The model’s Base year dataset is based on a 1 per cent representative sample of the Australian population (150,000 persons), extracted by the Australian Bureau of Statistics from its 1986 Census.

The model – written in C++ - then simulates future events occurring in these persons’ lives: forming couples, birth of a child, education, leaving home, migration, divorcing, being employed, income from work and government, wealth accumulation, becoming disabled, recovering from disability, and death [8].

A2 National Health Surveys (ABS)

The National Health Surveys are large surveys covering a very wide range of demographic, socioeconomic and health variables. The 1995 survey is the latest available, although the 2001 survey is expected in 2002. The 1995 survey involved Australia-wide interviews of some 23,800 non-institutionalised households, and obtained detailed information on 57,633 people [17].

A3 Disability surveys (ABS)

In this project we mainly used the 1998 survey. The sample for this survey included 15,316 private dwellings, 399 non-private dwellings and 626 cared accommodation establishments. Overall, 37,580 persons were interviewed from the households and 5,761 from cared accommodations [18].

Disability is defined by the ABS as a limitation, restriction or impairment, which has lasted, or is likely to last, for at least six months and restricts every day activities [19].

The survey contains a wide range of questions on the disabled and their carers. Of particular interest to this project are questions on the degree of disability (profound, severe, moderate and mild), and on the main disabling condition.
### A4 Socio-economic indexes for areas (SEIFA)

The geographically based index of Relative Socio-economic Disadvantage (SEIFA) used in this study was derived by the ABS from the 1996 Census. This ABS index reflects attributes such as low income, low educational attainment, high unemployment and jobs in relatively unskilled occupations.

Geographical areas with a high proportion of people with such attributes will fall into the lower SEIFA quintiles and, conversely those with a low proportion of such persons into the higher SEIFA quintiles. In deriving these indices, the ABS used the technique of Principal Component Analysis.

### A5 Equations for computing mortality rates: able bodied and disabled populations

Mortality rates, at age \( x \) and with SES \( y \), for the disabled may be approximated by:

\[
q_{x,y}^d = \frac{\text{no. of deaths of those aged } x \text{ in quintile } y \text{ due to non-external causes}}{\text{disabled population aged } x \text{ in quintile } y}
\]

As above, the mortality rates covering deaths form all causes, \( q_{x,y} \), were calculated as:

\[
q_{x,y} = \frac{\text{total no. of deaths of those aged } x \text{ in quintile } y}{\text{total population aged } x \text{ in quintile } y}
\]

Thus,

\[
q_{x,y}^d = q_{x,y} \times \frac{\text{no. of deaths (} x, y \text{) due to non-external causes}}{\text{total no. of deaths (} x, y \text{)} \times \frac{\text{total population (} x, y \text{)}}{\text{disabled population aged (} x, y \text{)}}}
\]

If \( p_{x,y} \) is the prevalence of disability at age \( x \) then

\[
\frac{\text{total population aged (} x, y \text{)}}{\text{disabled population aged (} x, y \text{)}} = \frac{1}{p_{x,y}}
\]

So, since the prevalence \( p_{x,y} \) is known, we only need to compute the proportion of deaths due to non-external causes to obtain an estimate for \( q_{x,y}^d \).
The equation for the able bodied population is similar, except that the mortality statistics are for external causes and the prevalence is for the able bodied is:

\[
\frac{\text{total population aged (x, y)}}{\text{able bodied population aged (x, y)}} = \frac{1}{1 - p_{x,y}}
\]

A6 Equations linking disability and mortality

Disability decrements rates are derived using standard multiple decrement techniques [8]. Briefly, the able bodied population is subject to exits due to death and entrants due to recovery, while the disabled population is subject to exits due to death and recovery and entrants due to disability onsets.

Let:

- \( \ell^a_{x,y} \) be the able-bodied population aged \( x \), with family SES quintile \( y \)
- \( \ell^d_{x,y} \) be the disabled population aged \( x \), with family SES quintile \( y \)
- \( R_{x,y} \) be the number of recoveries aged \( x \), with family SES quintile \( y \)
- \( D_{x,y} \) be the number of people becoming disabled aged \( x \), with family SES quintile \( y \)

Then, in calculating independent death rates for the able-bodied population, those initially exposed to risk are given by:

\[
E^a_{x,y} = I^a_{x,y} + \frac{1}{2} R_{x,y} - \frac{1}{2} D_{x,y}
\]

and similarly for the disabled population

\[
E^d_{x,y} = I^d_{x,y} - \frac{1}{2} R_{x,y} + \frac{1}{2} D_{x,y}
\]

Then if \( q^a_{x,y} \) is the mortality rate for the able-bodied population and \( q^d_{x,y} \) the mortality rate for the disabled population, we have

\[
E^a_{x,y} q^a_{x,y} + E^d_{x,y} q^d_{x,y} = E_{x,y} q_{x,y}
\]

where \( E_{x,y} \) is the initial exposed risk for all persons within that particular SES quintile, \( y \).

Thus
\[
q_{x,y}^d = \frac{(l_{x,y}^a + l_{x,y}^d)q_{x,y} - (l_{x,y}^a + \frac{1}{2} R_{x,y} - \frac{1}{2} D_{x,y})q_{x,y}^a}{l_{x,y}^d - \frac{1}{2} R_{x,y} + \frac{1}{2} D_{x,y}}
\]  
(A.1)

Furthermore
\[
l^a_{(x+1),y} = l_{x,y}^a - D_{x,y} + R_{x,y} - \left(l_{x,y}^a + \frac{1}{2} R_{x,y} - \frac{1}{2} D_{x,y}\right)q_{x,y}^a
\]  
(A.2)

and
\[
l^d_{(x+1),y} = l_{x,y}^d - R_{x,y} + D_{x,y} - \left(l_{x,y}^d - \frac{1}{2} R_{x,y} + \frac{1}{2} D_{x,y}\right)q_{x,y}^d
\]  
(A.3)

The able bodied and disabled populations can be related at any point in time using the prevalence rate for the appropriate age within the SES quintile being studied, so that for example
\[
l^d_{(x+1),y} = l_{(x+1),y}^d \frac{p_{(x+1),y}}{1 - p_{(x+1),y}}
\]  
(A.4)

which, upon substituting from expressions (A.1), (A.2), (A.3), gives for each SES quintile, \( y \)
\[
l_{x,y}^d - R_{x,y} + D_{x,y} - \left(l_{x,y}^d - \frac{1}{2} R_{x,y} + \frac{1}{2} D_{x,y}\right)q_{x,y}^d \frac{p_{x,y} - p_{x+1} - \left(l_{x,y}^d - \frac{1}{2} R_{x,y} + \frac{1}{2} D_{x,y}\right)q_{x,y}^d}{1 - p_{x+1} - \left(l_{x,y}^d - \frac{1}{2} R_{x,y} + \frac{1}{2} D_{x,y}\right)q_{x,y}^d}
\]

This equation is linear in \( D_{x,y} \) and therefore can be solved as follows
\[
D_{x,y} = p_{(x+1),y} l_{x,y}^a + R_{x,y} - q_{x,y}^a \left(l_{x,y}^a + \frac{1}{2} R_{x,y}\right) - \left(1 - p_{(x+1),y}\right) q_{x,y}^d \left(l_{x,y}^a + q_{x,y}^a - l_{x,y}^a q_{x,y}^a\right)
\]  
(A.5)

For any given values of \( R_{x,y} \) this formula can be used to build up a double decrement table for death and disability. Values for \( R_{x,y} \) have been chosen to ensure that \( D_{x,y} \) remains positive for all ages within the SES quintile being studied and start a 4% of the disabled population at age 0, increase to 15% of the disabled population over the age range 11 to 40 and then gradually decline to zero by age 94.

Independent rates of increments due to disability and recovery, \( r_{x,y} \) and \( d_{x,y} \) can be calculated from \( R_{x,y} \) and \( D_{x,y} \) using the relationship
\[
r_{x,y} = \frac{R_{x,y}}{l_{x,y}^d + \frac{1}{2} D_{x,y} - \frac{1}{2} \theta_{x,y}^d}
\]  
(A.6)

and
\[ d_{x,y} = \frac{D_{x,y}}{l_{x,y} + \frac{1}{2} R_{x,y} - \frac{1}{2} \theta_{x,y}} \] (A.7)

References


16 The methodology is described in Walker, A. and Abello, 2000, A., ibid, Section 3.4.


