A ONE-SIDED SUSTAINABILITY TEST WITH MULTIPLE CONSUMPTION GOODS

by

John C. V. Pezzey

Centre for Resource and Environmental Studies, Australian National University
Canberra, ACT 0200, Australia
http://cres.anu.edu.au/~pezzey

and

Visiting Fellow, Department of Economics, University of Bath, U.K.

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Abstract: In an economy with multiple consumption goods (including environmental amenities) that uniquely maximises the present value of utility with constant discounting, constant or falling augmented green net national product, or zero or negative augmented net investment, at any time implies that the economy is unsustainable then. "Augmented" means that time is treated as a productive stock, so augmented net investment includes the value of time. This allows future exogenous technical progress and changes in world prices to be included in a unified accounting framework, along with features such as resource depletion, pollution and foreign investment. The practical and philosophical rationale for testing sustainability in a present-value maximising, and therefore fully prescribed, development path are discussed.

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1. INTRODUCTION

In his comprehensive survey of the relationships among several concepts of income and sustainability, Asheim (2000, p39) concluded that despite a number of contrary early remarks in the literature on national income accounting, there is

> "no general result...available on the relation between [green net national product] and [sustainable income] when neither consumption nor the interest rate is constant".

This lack of a theoretical relationship is troubling, given the substantial interest during the last decade in measuring the sustainability of real economies by estimating green net national product (hereafter GNNP) from available national accounting data (see for example Pearce and Atkinson 1993, Pearce et al 1993, Atkinson et al 1997). Is all this empirical work without theoretical foundation?

Some relationship does exist, however, and we give here a one-sided test of sustainability in a perfectly competitive, present-value-maximising economy with a constant utility discount rate. We show that falling or constant augmented GNNP at some time, or a negative or zero level of augmented net investment (including changes in non-marketed environmental resource stocks), implies that the level of instantaneous utility (wellbeing) is unsustainable at that time. That the test is one-sided − for it shows if an economy is *un*sustainable, but not if it is sustainable − can be guessed from remarks that:

> "...it cannot be shown that [instantaneous utility at some instant in time] is sustainable if [net investment] is zero at [that] time." (Asheim 1994, p262)

> "...even when GNNP exceeds consumption (implying that the value of net investments is positive), consumption may be at an unsustainable level." (Asheim 2000, p40)

Similar observations were made by Vellinga and Withagen (1996, p511) and

Aronsson and Lofgren (1998, p213). However, our sustainability test has not been formally stated before.

The test also contributes two extensions, an observation, and a correction to existing theory. One extension is implied by the word "augmented". Augmentation adds an extra term to GNNP to account for any exogenous changes in the economy's set of production possibilities, whether by (exogenous) technical progress, or by changing world interest rates and terms of trade. It thereby includes more specific results by Weitzman (1997) on technical progress, and Sefton and Weale (1996) and Vincent et al (1997) on resource trade. A second extension is to include multiple 'consumption' goods, which includes anything such as amenity that directly affects utility. With multiple consumption goods, sustainable income is hard to define without restrictive assumptions, but using results in Asheim and Weitzman (2001), hereafter AW, no direct reference to sustainable income is necessary.

The observation is that the well-known (if augmented) result built into the sustainability test − that the change in augmented GNNP equals the interest on augmented net investment − can in principle be tested empirically, and allows a practical choice between measures with different data requirements. The correction is that "defensive" costs to be deducted from gross domestic product (GDP) to arrive at GNNP include not just pollution abatement costs, but also the costs of discovering and extracting natural resources. What the test does not contribute is any solution to the enormous practical difficulties of finding dollar values for all environmental services. Indeed, it adds to the difficulties by requiring that the course of technical progress and terms of trade be predicted for far enough into the future to no longer matter, thanks to discounting.

The structure of the paper is as follows. Section 2 describes the economy, and establishes the one-sided sustainability test and related results. Section 3 illustrates the test for a specific economy with several realistic features: resource discovery and renewability, accumulation and abatement of pollution, exogenous technical progress and changing terms of trade. Section 4 illustrates both the sustainability tests, and some of the less familiar concepts (a price of consumption different from 1, an interest rate not equal to the marginal product of capital) using an exact algebraic example. Section 5 considers how accurate the theory's assumptions are likely to be in practice, and why sustainability measurement should be of interest anyway in an economy where present-value-maximisation already prescribes a unique development path, and so apparently leaves no role for sustainability concerns. Section 6 concludes.

2. TESTING FOR SUSTAINABILITY

2.1 Nature of the economy considered, and the time derivative of augmented GNNP

We consider an economy as in AW, but with the important additions of treating time t, the cause of exogenous shifts in production possibilities, as a productive stock,[1] and of defining and investigating its sustainability properties. A vector $\mathbf{C}(t)$ represents a consumption bundle of everything, including environmental amenities, that influences current well-being, denoted by instantaneous utility $U(\mathbf{C}(t))$.[2] The economy's main stocks are

1. This device is mentioned by Aronsson, Johansson and Lofgren (1997, p54) and developed by Pemberton and Ulph (2001).

2. s is used throughout as a variable time, e.g. within integrals.

denoted by a vector $\mathbf{K}(t)$ (of maybe different dimension to $\mathbf{C}(t)$), representing stocks of built capital, natural resources, environmental assets, human capital from education, and knowledge produced by R&D. At any time, the combination of consumption $\mathbf{C}(t)$ and all *net* investments $\dot{\mathbf{K}}(t)$ is constrained by the convex production possibilities set $\Pi\{.\}$, which depends on both endogenous stocks $\mathbf{K}$ and the exogenous stock of time t:

$$[\mathbf{C}(t),\dot{\mathbf{K}}^{\dagger}(t)] \in \Pi\{\mathbf{K}^{\dagger}(t)\} \quad \text{where} \quad \mathbf{K}^{\dagger} := (\mathbf{K},t) \qquad [1]$$

Any variable with the superscript † will henceforth have the label *augmented* and will contain time as a stock, or some variable corresponding to time as a stock.

We assume, crucially but controversially in the context of sustainability, as discussed below in Section 5, that the economy at all times solves the problem of maximising the remaining present value (PV) of utility using a constant discount rate $\rho > 0$. That is, it solves

$$\underset{\mathbf{C},\dot{\mathbf{K}}}{Max} \quad W(t) := \int_t^{\infty} U[\mathbf{C}(s)]e^{-\rho(s-t)}ds \quad \text{s.t. } [\mathbf{C}(s),\dot{\mathbf{K}}^{\dagger}(s)] \in \Pi\{\mathbf{K}^{\dagger}(s)\}, \qquad [2]$$

The current-value Hamiltonian of problem [2] is

$$H(\mathbf{C},\dot{\mathbf{K}}^{\dagger};\Psi^{\dagger}) := U(\mathbf{C}) + \Psi^{\dagger}.\dot{\mathbf{K}}^{\dagger}, \qquad [3]$$

where the co-state vector $\Psi^{\dagger} = (\Psi,\Psi^{t})$ includes Ψ^{t} as the co-state variable of the stock of time. It is then trivial to show (so no proof is given) that the (time-)augmented version of AW's equation (5) is as follows. At all times on the maximised development path, the Hamiltonian satisfies

$$\dot{H}(t) = \rho\Psi^{\dagger}(t).\dot{\mathbf{K}}^{\dagger}(t) = \rho[H(t)-U(t)] \quad \text{by [3].} \qquad [4]$$

We next define several measures of prices and production. We would like to use the shadow utility prices of consumption goods, $(\partial U/\partial \mathbf{C})(t)$, and of net investments, $\Psi^\dagger(t)$, that come from [2], but these are unobservable. What can actually be observed are nominal prices $\mathbf{p}(t)$ and $\mathbf{q}^\dagger(t) = (\mathbf{q}(t),q^t(t))$ denominated in money like dollars. They are proportional to utility prices, formally $\mathbf{p}(t) = [(\partial U/\partial \mathbf{C})(t)]/\lambda(t)$ and $\mathbf{q}^\dagger(t) = \Psi^\dagger(t)/\lambda(t)$, where $\lambda(t) > 0$ is the marginal utility of a dollar at time t. If we further deflate the nominal prices by some (yet to be calculated) *price index* $\pi(t) > 0$, we get *real dollar* prices $\mathbf{P}(t) := \mathbf{p}(t)/\pi(t)$ and $\mathbf{Q}^\dagger(t) := \mathbf{q}^\dagger(t)/\pi(t) =: (\mathbf{Q}(t),Q^t(t))$; hence

$$\mathbf{P}(t) = [(\partial U/\partial \mathbf{C})(t)]/\lambda(t)\pi(t) \quad \text{and} \quad \mathbf{Q}^\dagger(t) = \Psi^\dagger(t)/\lambda(t)\pi(t). \qquad [5]$$

$Q^t(t) := \Psi^t(t)/\lambda(t)\pi(t)$ will be called the *value of time*. Intuitively, it measures the real value flow to the economy at t of time passing, just as each element of $\mathbf{Q}(t)$ measures the value at t of another economic stock (one which can be affected by human choices, which time cannot) growing. We derive further expressions for Q^t later, to give it a clearer meaning. To use AW's results, we choose the index $\pi(t)$ to satisfy the Divisia property, that defines the sense in which the overall real price level is constant:

$$\dot{\mathbf{P}}(t).\mathbf{C}(t) = 0 \text{ at all times on the optimal path.} \qquad [6]$$

The *real consumption discount factor* $\Phi(t)$ is defined as the utility discount factor, times the marginal utility of money, times the price index:

$$\Phi(t) = e^{-\rho t}\lambda(t)\pi(t) \qquad [7]$$

The (real) *consumption discount rate*, which in a perfectly informed and maximising economy is the (real) *interest rate* $r(t)$, is then defined as

$$r(t) = -\dot{\Phi}(t)/\Phi(t) = \rho - \dot{\lambda}(t)/\lambda(t) - \dot{\pi}(t)/\pi(t), \qquad [8]$$

and is assumed always to be strictly positive, $r(t) > 0$ for all t.

Green Net National Product (GNNP) $Y(t)$ is defined as the real value of consumption plus investment expenditures:

$$Y(t) := \mathbf{P}(t).\mathbf{C}(t) + \mathbf{Q}(t).\dot{\mathbf{K}}(t). \qquad [9]$$

Augmented GNNP is GNNP plus $Q^t(t)$ (recall that $\mathbf{K}^\dagger := (\mathbf{K},t)$, so that $\dot{\mathbf{K}}^\dagger = (\dot{\mathbf{K}},1)$):

$$Y^\dagger(t) := Y + Q^t = \mathbf{P}.\mathbf{C} + \mathbf{Q}^\dagger.\dot{\mathbf{K}}^\dagger. \qquad [10]$$

Using the above assumptions and definitions, we can conclude the foundations of the paper by stating (without proof, which is trivial) our first result, which is the augmented version of AW's Proposition 3:

Proposition 1: The time derivative of augmented GNNP (after AW)

The time derivative of augmented GNNP $Y^\dagger(t)$ is always the interest rate $r(t)$ times augmented net investment $\mathbf{Q}^\dagger(t).\dot{\mathbf{K}}^\dagger(t)$:

$$\text{For all } t,\ \dot{Y}^\dagger(t) = r(t)\mathbf{Q}^\dagger(t).\dot{\mathbf{K}}^\dagger(t) = r(t)[Y^\dagger(t) - \mathbf{P}(t).\mathbf{C}(t)] \text{ by [10].} \qquad [11]$$

2.2 The one-sided sustainability test

To find a sustainability test, sustainability must be defined. We define the (maximum) *sustainable utility* $U_m(t)$ at any time, which depends on the economy's stocks at t, as

$$U_m(t) := \max U \text{ s.t. } U(\mathbf{C}(s)) \geq U \text{ for all } s \geq t, \qquad [12]$$

and then use as our sustainability definition:

$$\text{an economy is } \textit{sustainable} \text{ at time } t \iff U(t) \leq U_m(t). \qquad [13]$$

Going straight to this definition ducks all debate about the notoriously prolific and often vague meanings of sustainability (see Pezzey 1989 for a historical collection); but contributing to that (often semantic) debate is not our aim here. We will merely say that definition [13] seems a fairly natural, exact translation of the word "sustainable" into a mathematical formula. If the current level of wellbeing (utility) $U(t) > U_m(t)$, the economy is unable to sustain this level from t forever after, since this would contradict the definition of $U_m(t)$, and wellbeing will decline at some finite time in the future. Given this definition, we can now state our main result. Here and in Section 2.3, "Extra assumptions" are what is required in addition to those in Section 2.1.

Proposition 2: A one-sided sustainability test

Extra assumptions: The optimal utility path is unique and non-constant.

Result: At t, a non-rising augmented GNNP or non-positive augmented net investment means that the economy is *un*sustainable at t. That is:

$$\{\dot{Y}^{\dagger}(t) \leq 0 \text{ or } \mathbf{Q}^{\dagger}(t)\mathbf{.}\dot{\mathbf{K}}^{\dagger}(t) \leq 0\} \Rightarrow \{U(t) > U_m(t)\} \qquad [14]$$

or equivalently (from [10])

$$\{\dot{Y}(t) + \dot{Q}^{t}(t) \leq 0 \text{ or } \mathbf{Q}(t)\mathbf{.}\dot{\mathbf{K}}(t) + Q^{t}(t) \leq 0\} \Rightarrow \{U(t) > U_m(t)\} \qquad [15]$$

Proof: See Appendix.

Comments:

(a) Positive augmented net investment, $\mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger} > 0$, does not necessarily mean the economy is sustainable. No general test *for* sustainability is known. For example, there is no known generalisation of the $\beta < \alpha$ condition that Solow (1974) discovered to be necessary for sustainable utility U_m to exist in an economy with production function $F = K^{\alpha}R^{\beta}$.

(b) In any economy where the welfare-maximising path is *not* unique, the first two inequalities in [14] and [15] must be *strict* ($\mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger} < 0$, etc) to be able to conclude that the economy is unsustainable.

(c) Closely related to, but different from, Proposition 2, is an augmented form of Hartwick's rule: augmented net investment $\mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger} = 0$ forever means that utility $U(t)$ is constant forever. For by (5) and (4), $\rho\lambda\pi\mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger} = \rho\Psi^{\dagger}.\dot{\mathbf{K}}^{\dagger} = \dot{H} = \rho(H-U)$; hence $\{ \mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger} = 0 \text{ forever} \} \Rightarrow \{ H = U \text{ forever} \} \Rightarrow \{ \dot{U} = \dot{H} = 0 \text{ forever} \}$.

To round out our main result, we develop the meaning of Q^t, the "value of time", by showing it is the generalised present value of the partial time derivative of GNNP:

Proposition 3: The value of time

$$Q^t(t) = \int_t^{\infty}[\partial Y(s)/\partial s]\exp[-\int_t^s r(z)dz]ds$$

$$\left(= \int_t^{\infty}[\partial Y(s)/\partial s]e^{-r(s-t)}ds \quad \text{if } r \text{ constant} \right) \qquad [16]$$

Proof: See Appendix.

Q^t is thus forward looking, and likely to be much harder to calculate than other elements of $\mathbf{Q}^{\dagger}$, which are based on current values only. Formula [16] takes on a more digestible form in Proposition 7 in Section 3 below, which shows how Q^t adds to GNNP because of future exogenous technical progress and future changes in world resource prices (and hence terms of trade).

2.3 Further results

The first result here slightly extends the generality of the classic Weitzman (1970, 1976) result on the present-value equivalence of GNNP, which itself has nothing to do with sustainability as defined here. However, in the case of a constant interest rate, it does have implications for the relationship between wealth and sustainability. The second and third results both assume a constant interest rate. The second restates the Weitzman (1997) result on technical progress and income in terms of our value of time Q^t. The third uses this result to give a simpler sustainability test in the case when the rates of overall GNNP growth and the rate of GNNP growth due to time alone are both constant.

For the first result, we need two further definitions.

Real wealth $\Theta(t)$ is defined as the present value of consumption expenditures **P**.**C** on the optimal path, using the real consumption discount factor:

$$\Theta(t) := \int_t^{\infty} \mathbf{P}(s).\mathbf{C}(s)[\Phi(s)/\Phi(t)]ds. \qquad [18]$$

Wealth-equivalent income, $Y_e(t)$, is defined as the consumption level which, if held constant from t to ∞, gives the same present value as wealth on the optimal path:

$$Y_e(t) := \int_t^{\infty} \mathbf{P}(s).\mathbf{C}(s)\Phi(s)ds \,/\, \int_t^{\infty}\Phi(s)ds$$

$$\Rightarrow \int_t^{\infty} Y_e(t)[\Phi(s)/\Phi(t)]ds = \Theta(t). \qquad [19]$$

A further assumption, which does not apply to previous results, is needed for wealth-equivalent income Y_e to be well defined: the utility function $U(\mathbf{C})$ must be homothetic. Otherwise, the Divisia index $\pi(t)$ is path-dependent, and cannot be used to compare expenditure **P**.**C** on different development paths. This is not a problem when we assume a constant interest rate, since this effectively assumes a constant utility discount rate ρ, and a linear homogeneous utility function $U(.)$ which is a stronger restriction than homotheticity.[3]

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3. I thank Geir Asheim for these insights. Note that Asheim (1997) developed PV-equivalence results for the cases of varying interest *and* utility discount rates.

Proposition 4(a) below uses Proposition 1 to find an expression for augmented GNNP $Y^\dagger$ which is closely related to wealth. It allows the terms of trade and the interest rate to vary (as did Sefton and Weale in their equation (8)), and also for technical progress (which they excluded). Proposition 4(b) shows that in the special case of a constant interest rate, augmented GNNP $Y^\dagger$ is the same as wealth-equivalent income Y_e, and both can be seen as a return (at rate r) on wealth Θ.

Proposition 4: The present-value equivalence of GNNP (after Sefton and Weale 1996)

Extra assumption: The utility function $U(.)$ is homothetic.

Result (a): $Y^\dagger(t) = \int_t^\infty r(s)\mathbf{P}(s).\mathbf{C}(s)\exp[-\int_t^s r(z)dz]ds.$ [20]

Proof: [20] follows directly by integrating $\dot{Y}^\dagger(s) = r(s)[Y^\dagger(s) - \mathbf{P}(s).\mathbf{C}(s)]$ from [11] from time t to ∞, and assuming that the integral converges.

Extra assumptions: The utility function $U(.)$ is homothetic, and the interest rate r is constant. Using [19] then gives:

Result (b): $Y^\dagger(t) = r\int_t^\infty \mathbf{P}(s).\mathbf{C}(s)e^{-r(s-t)}ds = r\Theta(t) = Y_e(t).$ [21]

Comments:

(a) In a *small*, *open economy*, where all prices are exogenous world prices, and with just one consumption good, wealth-equivalent income Y_e equals (maximum) sustainable consumption C_m (Asheim 2000, p38), which is defined in an analogous way to (maximum) sustainable utility in (12). The sustainability test is then *two-sided*, i.e. $\mathbf{Q}^\dagger.\dot{\mathbf{K}}^\dagger \gtrless 0 \Leftrightarrow C \lessgtr C_m$, etc, and comment (a) to Proposition 2, which underlay the concern expressed in Asheim (1994) and Pezzey (1994) that there was no direct connection between GNNP and sustainability, no longer applies. However, with multiple consumption goods, this deduction cannot be made, because the Divisia price index defining prices **Q** is generally different on the PV-optimal and maximum sustainable paths. Also, the result obviously cannot work for the sum of all open economies (i.e. the world economy), for prices are then no longer exogenous.

(b) There is a common view in the policy literature that non-declining wealth or aggregate capital will ensure sustainability, dating perhaps to well-known but rather informally worded claims to this effect (Solow 1986, 1993; Pearce, Markandya and Barbier 1989, Pearce et al 1993). However, there are models where consuming interest on wealth (and thus "keeping wealth constant" will not lead to a sustainable path. For example, on the maximin (constant consumption) path in Solow (1974), consumption is smaller than the interest on wealth, and wealth is increasing. Here we have a different example of the same problem. If the interest rate is constant, then $Y^{\dagger} = Y_e = r\Theta$ from Proposition 4(b); and from [14], $\dot{\Theta}(t) = 0$, i.e. *constant* wealth at t, then implies *un*sustainability at t. At an instant of constant wealth on an optimal path with varying utility, consumption expenditures are the same as wealth-equivalent income, but this exceeds sustainable consumption because optimal consumption expenditures are not constant.

Proposition 5, proved by Weitzman (1997, p9), is stated next to show how it fits in, and for use in Proposition 6. It actually applies to an open as well as closed economy, because the time dependence of the production possibilities set in [1] allows for the effect of exogenous changes in the terms of trade.

Proposition 5: The time premium (Weitzman 1997)

Extra assumption: The interest rate r is constant.

Result: The augmented GNNP (or the wealth-equivalent income) equals GNNP Y increased by a "time premium" Q^t/Y:

$$Y^{\dagger}(t) = Y(t)[1 + Q^t(t)/Y(t)] = Y\,\{1 + \chi(t) \,/\, [r-\Gamma(t)]\} \qquad [22]$$

where $\Gamma(t) := \int_t^{\infty}\dot{Y}(s)e^{-rs}ds \,/\, \int_t^{\infty}Y(s)e^{-rs}ds$ [23]
is the time-averaged overall growth rate of GNNP,

and $\chi(t) := \int_t^{\infty}[\partial Y(s)/\partial s]e^{-rs}ds \,/\, \int_t^{\infty}Y(s)e^{-rs}ds$ [24]
is the time-averaged growth in GNNP due to time alone.

The growth rate $\chi(t)$ in [24] is the result of any exogenous change in production possibilities caused by time alone, so we will call $Q^t/Y = (Y^{\dagger}/Y)-1$ the "time premium", rather than Weitzman's more specific "technological progress premium". A trivial corollary of Propositions 2 and 5 is then the following.

Proposition 6: The one-sided sustainability test with constant rates of GNNP growth and exogenous technical progress.

Extra assumptions: The optimal utility path is unique and non-constant. The interest rate r, instantaneous rate of overall GNNP growth, and instantaneous rate of GNNP growth due to time alone are all constant (hence the last two are Γ and χ respectively).

Result: The assumptions and [22] $\Rightarrow$ $Q^t(t) = \chi Y(t)/(r-\Gamma)$ and $\dot{Y}^{\dagger}/Y^{\dagger} = \dot{Y}/Y = \Gamma$. Proposition 2 then simplifies to

$$\{\dot{Y}(t)/Y(t) = \Gamma \le 0 \quad \text{or} \quad \mathbf{Q}(t).\dot{\mathbf{K}}(t)/Y(t) \le -\chi/(r-\Gamma)\} \;\Rightarrow\; \{U(t) > U_m(t)\} \qquad [25]$$

Comments:

(a) $\mathbf{Q}.\dot{\mathbf{K}}/Y \leq 0$, i.e. net investment (as a proportion of GNNP) without the time premium Q^t/Y, is the test used for most empirical measurements of sustainability. Weitzman (1997) and Vincent, Panayotou and Hartwick (1997) are the only papers we know that include part of the time premium $\chi/(r-\Gamma)$ in their calculations. [25] shows that, if $\chi > 0$, sustainability is more likely than indicated by the $\mathbf{Q}.\dot{\mathbf{K}}/Y \leq 0$ test that applies when $\chi = 0$. By contrast, the $\dot{Y}(t)/Y(t) \leq 0$ test is formally unaffected, and still means that any non-positive growth rate of GNNP implies unsustainability. Weitzman estimated $\chi/(r-\Gamma)$ to be about 0.4 for the USA, while the natural resource components of $\mathbf{Q}.\dot{\mathbf{K}}$ make up only about 0.03 of Y. However, Hamilton, Atkinson and Pearce (1998) suggested that some of $\chi/(r-\Gamma)$ could be endogenous technical progress; and some of it could also be changes in the terms of trade.

(b) Of the two conditions for unsustainability in each of Propositions 2 and 6, the second (net investment) conditions may be easier to compute than the first (GNNP change) conditions in the time-autonomous ($Q^t = 0$) case, because net investment ($\mathbf{Q}.\dot{\mathbf{K}}$) does not require estimation of the comprehensive consumption vector **C** and its associated price vector **P**. If $Q^t \neq 0$, the comparison is not so simple. Hanley et al (1999) is the only empirical work we know of, done for Scotland in their case, which has tried to compare the GNNP growth ($\dot{Y}$) and net investment ($\mathbf{Q}.\dot{\mathbf{K}}$) measures. However, several trade terms that could be significant for a small, open economy like Scotland, as well as the more difficult value of time Q^t, were omitted from their analysis.

3. A MORE SPECIFIC EXAMPLE

How can augmented GNNP $Y^{\dagger}$ be calculated in a real economy, in order to make practical use of the one-sided sustainability test in Proposition 2? We need to know $\dot{\mathbf{K}}^{\dagger}$, the changes in the economy's stocks. This is straightforward in theory, if often very difficult in practice. But even in theory, how do we find $\mathbf{Q}$, the optimal prices of stocks, and Q^t, the value of time, in terms of variables familiar to policy analysts, such as the cost of resource extraction, and or of pollution abatement? We answer these questions here by using a more detailed (and hence less general) economy, which is an amalgam and extension of the examples in Vellinga and Withagen (1996).[4]

Importantly, the economy will have two "consumption" goods: material consumption C and environmental quality B, which determine utility $U(C,B)$. This creates real prices of both consumption, P^C, and environmental quality, P^B, in terms of a weighted average of consumption and environmental quality. At the end of this section we give a formula for the growth of P^C; it uses measurements of environmental quality and its marginal value, though these are already needed for other parts of the GNNP formula. The model also has investment and depreciation in both productive and abatement capital; instantaneous and cumulative forms of environmental pollution; discovery, extraction, and renewal of n different natural resource stocks; and foreign trade in resources and consumption, resulting in a stock of foreign capital. Such features can occur in endless different ways (flow versus cumulative pollutants, current versus capital spending on abatement, to take

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4. Seminal models here are Hartwick (1990) and Maler (1991), with a notable contribution also by Hamilton (1994, 1996).

just one example), and our results show that some methods of accounting for pollution or resource depletion in GNNP measurement are far from general.

We designate the example economy's stocks of the n renewable and non-renewable resource stocks by the n-vector $\boldsymbol{S}(t)$. They are discovered at rate $\boldsymbol{D}(t)$, grow naturally at a stock-dependent rate $\boldsymbol{G}(\boldsymbol{S})$, and are depleted at rate $\boldsymbol{R}(t)$, all n-vectors, so:

$$\dot{\boldsymbol{S}} = \boldsymbol{D} + \boldsymbol{G}(\boldsymbol{S}) - \boldsymbol{R}; \quad \boldsymbol{S}(0) = \boldsymbol{S}_0, \text{ given} \qquad [26]$$

Two capital stocks in the domestic economy are productive capital $K(t)$ and abatement capital $K_a(t)$. They each increase at the rate of investment (respectively $I(t)$ and $I_a(t)$) minus depreciation ($\delta K(t)$ and $\delta K_a(t)$):

$$\dot{K} = I - \delta K; \qquad K(0) = K_0, \text{ given} \qquad [27]$$

$$\dot{K}_a = I_a - \delta K_a; \qquad K_a(0) = K_{a0}, \text{ given} \qquad [28]$$

Production $F(.)$ depends positively on inputs of capital $K(t)$, domestic resource use (extraction $\boldsymbol{R}(t)$ minus net exports $\boldsymbol{R}_x(t)$) and time t (exogenous technical progress). The sum of production, plus net imports $M(t)$ of the consumption/investment good, is distributed among consumption $C(t)$, productive and abatement investments $I(t)$ and $I_a(t)$, abatement current expenditure $a(t)$, discovery costs $V(\boldsymbol{D},\boldsymbol{S},t)$ with $V_{\boldsymbol{D}} > \boldsymbol{0}$, $V_{\boldsymbol{S}} < \boldsymbol{0}$, $V_t < 0$, and extraction costs $X(\boldsymbol{R},\boldsymbol{S},t)$ with $X_{\boldsymbol{D}} > \boldsymbol{0}$, $X_{\boldsymbol{S}} < \boldsymbol{0}$, $X_t < 0$ (note the flow and stock dependences in the last two costs, and the allowance for exogenous technical progress in both of them):

$$F(K,\boldsymbol{R}-\boldsymbol{R}_x,t) + M = C + I + I_a + a + V(\boldsymbol{D},\boldsymbol{S},t) + X(\boldsymbol{R},\boldsymbol{S},t) \qquad [29]$$

To calculate net national rather than net domestic product, we must include terms for the economy's trade. The economy owns a stock $K_f(t)$ of foreign capital which earns a return at the exogenous world interest rate $r(t)$; and its net resource exports $\boldsymbol{R}_x(t)$ are sold at exogenous world prices $\boldsymbol{Q}^x(t)$ (x signifies exports, while X is the resource extraction cost). The foreign capital stock then changes as follows (where only exogenous dependences on time t are shown):[5]

$$\dot{K}_f = r(t)K_f + \boldsymbol{Q}^x(t).\boldsymbol{R}_x - M; \quad K_f(0) = K_{f0}, \text{ given} \qquad [30]$$

Instantaneous utility U depends on consumption $C(t)$ and environmental quality or amenity $B(t)$ (so the "consumption vector" is $\mathbf{C} = (C,B)$):

$$U(t) = U[C(t),B(t)], \quad U_C, U_B > 0 \qquad [31]$$

Environmental quality $B(t)$ is, in this particular model, assumed to be some pristine level of quality B_0, minus ε^B times the quality lost from flow pollution $E^B(t)$ (which depends on output, abatement expenditure, abatement capital and time), and minus ε^Ω times the quality lost from stock pollution, as measured by the drop in some environmental absorption capacity $\Omega(t)$ below its pristine, pre-industrial level Ω_1:

$$B = B_0 - \varepsilon^B E^B[F(K,\boldsymbol{R}-\boldsymbol{R}_x,t),a,K_a,t] - \varepsilon^\Omega(\Omega_1-\Omega); \quad \varepsilon^B, \varepsilon^\Omega > 0 \qquad [32]$$

To aid comparison with other authors it will be convenient to denote:

$b(t) := 1 / (\partial B/\partial a)$ as the marginal cost of improving environmental quality by abating emissions. [33]

Finally, absorption capacity $\Omega(t)$ − say the gap between the current average environmental concentration in the economy of a long-lived pollutant and a maximum acceptable concentration − rises at assimilation rate $\gamma(\Omega)$, $\gamma' < 0$,

5. There is no distinction here between private or government ownership of foreign capital (or debt). For a large open economy, the interest rate r would depend also on the level of capital K_f, and the resource price $\boldsymbol{Q}^x$ would depend also on net exports $\boldsymbol{R}_x$.

and falls with emissions E^{Ω} which depend positively on domestic resource use $\boldsymbol{R}$-$\boldsymbol{R}_x$. Emissions can be abated only by reducing $\boldsymbol{R}$-$\boldsymbol{R}_x$:[6]

$$\dot{\Omega} = \gamma(\Omega) - E^{\Omega}(\boldsymbol{R}\text{-}\boldsymbol{R}_x); \quad \Omega(0) = \Omega_0\text{, given; } \Omega(-\infty) = \Omega_1 > \Omega_0 \qquad [34]$$

All functional forms are assumed to be as smooth and convex as is needed for generalised present value $W(t)$ in [1] to converge, and for partial derivatives below (denoted by subscripts) to exist. As before, we make the heroic assumption that society chooses its control variables, which here are $C(t)$, $I_a(t)$, $a(t)$, $\boldsymbol{D}(t)$, $\boldsymbol{R}(t)$, $\boldsymbol{R}_x(t)$ and $M(t)$, with $I(t)$ being given by [29], to maximise $W(t)$, which means that optimal environmental policies to internalise all externalities are in place. We then have the following.

Proposition 7: A detailed formula for augmented GNNP in a specific case

Result: $$Y^{\dagger} = P^C \{ C + bB + \dot{K} + \dot{K}_a + \dot{K}_f + (bB_{\boldsymbol{R}}+F_{\boldsymbol{R}}-\boldsymbol{Q}^x)(\gamma-E^{\Omega})/E^{\Omega}_{\boldsymbol{R}} + (\boldsymbol{Q}^x-X_{\boldsymbol{R}}).[\boldsymbol{D}+\boldsymbol{G}(\boldsymbol{S})-\boldsymbol{R}] \} + Q^t, \quad \text{where} \qquad [35]$$

$$Q^t(t) := \int_t^{\infty} P^C(s) \{bB_s+F_s-V_s-X_s + \dot{r}K_f + \dot{\boldsymbol{Q}}^x.\boldsymbol{R}_x\}(s) \exp[-\int_t^s r(z)dz]\, ds, \qquad [36]$$

and vector "divisions" like $B_{\mathbf{R}}/E^{\Omega}_{\mathbf{R}}$ are defined element by element, i.e. as

$$\{B_{\mathbf{R}}/E^{\Omega}_{\mathbf{R}}\}_i := \{B_{\mathbf{R}}\}_i/\{E^{\Omega}_{\mathbf{R}}\}_i. \qquad [37]$$

Proof: See Appendix.

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6. We model absorptive capacity rather than cumulative pollution so all stocks in the model are goods, and so can satisfy the "free disposal" assumption of Asheim (1997). We ignore the complication that total emissions of the most important pollutants, greenhouse gases in the global atmosphere, can in reality be controlled only by the global economy, not by our example, open economy.

Comments:

(a) GNNP $Y^{\dagger}$ is thus the value sum of the "consumption" vector, $C + bB$; the changes in the three capital stocks, $\dot{K} + \dot{K}_a + \dot{K}_f$, where the price is the same as for consumption; the change in the absorption capacity, $\dot{\Omega} = \gamma - E^{\Omega}$, valued at a price $(bB_{\boldsymbol{R}} + F_{\boldsymbol{R}} - \boldsymbol{Q}^x)/E^{\Omega}{}_{\boldsymbol{R}}$ which reflects the various roles in the economy played by the resource flow $\boldsymbol{R}$; and the change in the resource stocks, $\dot{\boldsymbol{S}} = \boldsymbol{D} + \boldsymbol{G}(\boldsymbol{S}) - \boldsymbol{R}$, valued at the marginal cost of discovery, $V_{\boldsymbol{D}}$; *plus* the value of time Q^t. Q^t is in turn the discounted present value of the various sources of exogenous technical progress, as represented by the pure time derivatives $bB_t + F_t - V_t - X_t$ plus the "capital gains" from exogenously changing world prices. These gains, a specific example of the more general analysis in Asheim (1996), are here the change in interest rate $\dot{r}$ times the economy's foreign capital K_f, and the changes in world resources prices $\dot{\boldsymbol{Q}}^x$ times the economy's resource exports $\boldsymbol{R}_x$.

(b) Environmental valuation, in the form of the marginal cost of abatement b, occurs three times in [35]. If there is no stock pollution, and no technical progress in abatement ($B_t = 0$), then only the first occurrence remains, which is not part of the net investment terms. There is then a practical difference between the two conditions in the one-sided sustainability tests: the $\dot{Y}^{\dagger}/Y$ test does require environmental evaluation, but the $\mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger}$ test does not. (See also comment (b) to Proposition 6 above.)

(c) The Proposition shows which "defensive" expenditures to abate pollution should be explict parts of augmented GNNP. Abatement net investment $\dot{K}_a$ should be included, but abatement current spending a should not; and nor should extraction costs X or discovery costs V.

Note also that only the capital gains in resource exports $\boldsymbol{R}_x$, not in resource extractions $\boldsymbol{R}$, are counted.

(d) With no technical progress, $B_t = F_t = V_t = X_t = 0$, and Q^t becomes

$$Q^t = \int_t^\infty P^C(s)[\dot{r}(s)K_f(s)+\dot{\boldsymbol{Q}}^x(s).\boldsymbol{R}_x(s)] \exp[-\int_t^s r(z)dz]\ ds, \qquad [38]$$

which makes [35] similar to the open economy results of Sefton and Weale (1996, Section 4). If we also exclude domestic capital K, environmental quality B (hence $P^C = 1$), abatement capital K_a, domestic production F, domestic resource use $\boldsymbol{R}-\boldsymbol{R}_x$ (so $\boldsymbol{R}_x = \boldsymbol{R}$), discoveries $\boldsymbol{D}$ and resource growth $\boldsymbol{G}$, and assume a constant interest rate r, [35] then becomes

$$Y^\dagger = C + \dot{K}_f - (\boldsymbol{Q}^x-X_{\boldsymbol{R}}).\boldsymbol{R} + \int_t^\infty \dot{\boldsymbol{Q}}^x(s).\boldsymbol{R}(s)e^{-r(s-t)}ds, \qquad [39]$$

which with [11] rapidly gives the main result (8) of Vincent, Panayotou and Hartwick (1997).

(e) The problem of translating results from utility units to consumption units, bypassed in Hartwick (1990) with an approximate "linearisation of the Hamiltonian", is here transformed to the problem of finding how $P^C(t)$, the real price of consumption, changes over time, as mentioned at the start of the section. This must be inferred using $P^B = bP^C$ from the Appendix, and the Divisia property [6] which defines the price level, $\dot{\mathbf{P}}.\mathbf{C} = \dot{P}^C C + \dot{P}^B B = \dot{P}^C C + (b\dot{P}^C+\dot{b}P^C)B = 0$

$$\Rightarrow \ \dot{P}^C/P^C = -\dot{b}B/(C+bB). \qquad [40]$$

The problem is transformed rather than solved, because of the difficulties of calculating both marginal abatement cost $b(t)$ and environmental quality $B(t)$. However, we now have a precise formula [40] to aim at, rather than an unknown linearisation error. And a constant marginal abatement cost, $\dot{b} = 0$, means that P^C is constant and so can be ignored in the expressions [35] and [36] for augmented

GNNP and the value of time. In this case, we can further simplify [35] by noting from [27]-[30] that

$$C + \dot{K} + \dot{K}_a + \dot{K}_f = C + I + I_a - \delta(K+K_a) + rK_f + \boldsymbol{Q}^x.\boldsymbol{R}_x - M$$
$$= F - a - V - X - \delta(K+K_a) + rK_f + \boldsymbol{Q}^x.\boldsymbol{R}_x$$

hence

$$Y^\dagger = F - \delta(K+K_a) + rK_f + \boldsymbol{Q}^x.\boldsymbol{R}_x + bB + (bB_{\boldsymbol{R}}+F_{\boldsymbol{R}}-\boldsymbol{Q}^x)(\gamma-E^\Omega)/E^\Omega{}_{\boldsymbol{R}}$$
$$+ (\boldsymbol{Q}^x-X_{\boldsymbol{R}}).[\boldsymbol{D}+\boldsymbol{G}(\boldsymbol{S})-\boldsymbol{R}] + Q^t - a - V - X \qquad [40a]$$

showing the adjustments that must be made to F (i.e. gross domestic product, GDP) in order to arrive at augmented GNNP. These include *deducting abatement, discovery and extraction costs (a, V and X) from GDP*. This was overlooked by, among others, Hartwick (1990, p294) and Hamilton (1994, p159), who incorrectly identified consumption plus investment ($C + \dot{K}$) with GNP or NNP (which were equivalent, since their models had no depreciation, and no abatement or foreign capital stocks either).

4. AN EXACT ALGEBRAIC EXAMPLE

Consider a case of the Section 3 model where the economy is closed, and with a non-renewable resource stock $S(t)$, $S(0) = S_0 > 0$, and depletion $R = -\dot{S}$. There is no abatement capital, just abatement current spending a, so the choice variables are consumption C, abatement a and depletion R. Production is Cobb-Douglas in capital $K(t)$, $K(0) = K_0 > 0$ and resource flow R, with exogenous technical progress at rate ν:

$$F(K,R,t) = K^\alpha R^\beta e^{\nu t} = \dot{K} + C + a, \quad 0 < \alpha,\ \beta < \alpha+\beta \le 1; \quad \nu > 0 \qquad [41]$$

Utility U and environmental quality B are given by

$$U = \alpha \ln(C) + \varepsilon \ln(B), \quad \text{where } \varepsilon > 0 \text{ and } B = a/R. \qquad [42]$$

The functional form of environmental quality in [42] is rather contrived, since a balanced growth path with abatement a growing at the same rate as C, and with resource flow R inevitably declining towards zero, will have environmental quality growing faster than consumption, and hence without bound. This means that consumption becomes more scarce relative to consumption-environment index of utility, and so its price P^C rises, while environmental quality becomes relatively less scarce, and its price P^B falls. With a utility discount factor of $\phi(t) = e^{-\rho t}$ as before, if the parameters happen to obey the relationship

$$(\rho+\omega)K_0^{1-\alpha} = (\rho S_0)^{\beta} \quad \text{where} \quad \omega := (\nu-\beta\rho)/(1-\alpha), \qquad [43]$$

then the following results can be shown to hold (proofs of any unexplained results are available from the author).

The economy's optimal path is given at all times (not just asymptotically) by the balanced growth forms

$$K(t) = K_0 e^{\omega t}, \quad C(t) = \alpha a(t)/\varepsilon = \{[\rho+(1-\alpha)\omega]/(\alpha+\varepsilon)\}K(t), \qquad [44]$$

$$R(t) = \rho S(t) = \rho S_0 e^{-\rho t}, \qquad [45]$$

and $$B(t) = a(t)/R(t) = \{[\rho+(1-\alpha)\omega]/(\alpha+\varepsilon)\}\ (\varepsilon/\alpha\rho)\ (K_0/S_0)e^{(\rho+\omega)t}. \qquad [46]$$

The marginal abatement cost is $b = 1/(\partial B/\partial a) = R$, hence

$$bB = a = (\varepsilon/\alpha)C \quad \text{and} \quad C+bB = (1+\varepsilon/\alpha)C. \qquad [47]$$

Combining [45], [47] and [40] (which comes from the Divisia relationship $\dot{P}^C C + \dot{P}^B B = 0$), the consumption price P^C rises at rate

$$\eta := \dot{P}^C/P^C = -(\dot{b}/b)\ /\ (1+C/bB) = \rho\ /\ (1+\alpha/\varepsilon) > 0, \qquad [48]$$

while the price of environmental quality falls:

$$\dot{P}^B/P^B = \dot{b}/b + \dot{P}^C/P^C = -\rho/(1+\varepsilon/\alpha) < 0. \qquad [49]$$

The rate of interest is the marginal product of capital ($F_K = \rho + \omega$) *plus* the rate of growth η of the consumption price;

$$r = \rho + \omega + \eta; \qquad [50]$$

the value of time [16] is

$$Q^t = \{\nu(\alpha+\varepsilon)(\rho+\omega) \;/\; \rho\alpha[\rho+(1-\alpha)\omega]\}\; P^C C; \qquad [51]$$

and wealth-equivalent income [19] is

$$Y_e = \{\varepsilon/\alpha + (\alpha+\varepsilon)(\rho+\omega)/\alpha\rho\}\; P^C C. \qquad [52]$$

Since the interest rate is constant, from [10], [21] and [52] we can then calculate GNNP:

$$Y = Y_e - Q_t = \{\varepsilon/\alpha + (1-\beta)(\alpha+\varepsilon)(\rho+\omega)/\alpha[\rho+(1-\alpha)\omega]\}\; P^C C. \qquad [53]$$

GNNP can also be calculated from the appropriate "consumption plus net investment" definition like [35], namely $Y = P^C(C+bB) + Q^K\dot{K} - Q^S R$. Here, the shadow price Q_S/Q_K for the resource flow relative to the consumption/investment good must be not its marginal product F_R, but

$$Q^S/Q^K = F_R + U_R/U_C = (\beta F - \varepsilon C/\alpha)/R. \qquad [54]$$

The intuition in [54] is that the unit value of resource flow is its marginal effect on production, plus its marginal effect on utility (here a negative externality) converted to consumption units.

In a balanced growth economy, sustainability means that the growth rate of utility (not consumption), which is

$$\dot{U} = \alpha\dot{C}/C + \varepsilon\dot{B}/B = (\alpha+\varepsilon)\rho\; [\omega/\rho + \varepsilon/(\alpha+\varepsilon)], \qquad [55]$$

must be positive. The augmented net investment terms here are

$$\mathbf{Q}^{\dagger}.\dot{K}^{\dagger} = Q^K\dot{K} - Q^S R + Q^t = (1+\varepsilon/\alpha)\; [\omega/\rho + \varepsilon/(\alpha+\varepsilon)]P^C C; \qquad [56]$$

while from using [43] and [48] with either [53], or [52] and [21], the growth rate of GNNP and augmented GNNP is

$$\dot{Y}/Y = \dot{Y}^{\dagger}/Y^{\dagger} = \dot{P}^{C}/P^{C} + \dot{F}/F = \eta + \omega = \rho\ [\omega/\rho + \varepsilon/(\alpha+\varepsilon)]. \qquad [57]$$

The sign of [56] or [57] is the same as the sign of $\dot{U}$ in [55], which confirms the accuracy of the one-sided sustainability tests in Proposition 2.

5. THE PRACTICAL AND PHILOSOPHICAL BASIS OF TESTING FOR SUSTAINABILITY WHILE ASSUMING PV-OPTIMALITY

Despite its mathematical fineries set out above, the neoclassical approach to sustainability contains some key approximations and apparent paradoxes, which may limit its use as a practical tool for policy makers. The approximations stem from two unavoidable facts about prices and quantities. Firstly, because of significant *externalities*, *current* prices and quantities observed in the market, including those estimated with non-market valuation techniques for the externalities, are significantly different from *optimal* prices that would be observed after policy intervention shifted development to the (PV-)optimal path. Secondly, if the sustainability goal [13] imposes a significantly binding constraint on optimality, then the *sustainability* prices and quantities, that would apply after further intervention achieves the constraint (presumably with minimum loss of PV), are different again.[7]

7. Even if the "right" (i.e. sustainability) prices could be calculated, combining them with current or optimal quantities would not say anything exact about sustainability, despite suggestions in Solow (1993) and Hartwick (1997) that this is so.

As set out above,[8] the neoclassical approach to sustainability thus:

(i) *Assumes* that current prices and quantities adequately approximate optimal prices;

(ii) *Proves* a theoretical inequality relationship (Proposition 2) between measures with optimal prices and quantities, and sustainability;

(iii) *Assumes* that the goal of policy intervention is PV-maximisation, subject to a sustainability constraint or modified by a public sustainability concern, even though individuals are not assumed to seek sustainability, only PV-optimality, in their private actions. Indeed, individuals must in fact believe there will be no policy intervention in favour of sustainability, or else they would modify their plans for the future, causing prices today not to be PV-optimal.[9]

Assumption (i) will not be explored further here. What we will comment on is the apparent paradox at the heart of assumption (iii) (although it is usually hidden and hence not discussed, since most literature focuses on measuring rather than implementing sustainability). Why should the government be interested in sustainability if private agents maximise PV? Maximising PV, at least for consumers rather than firms, in general has nothing to do with sustainability, and it gives a complete and unique prescription for the time paths of every decision that ever has to be made in the economy. So there is no apparent motive for using result (ii) to measure sustainability on a PV-optimal path! Likewise, if unsustainability is thus found, there is no apparent justification for the policy intervention which will

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8. This is an important qualification, which applies throughout this disccusion. Obviously other writers may choose a different but still neoclassical approach. For example, Asheim (1997, 2000) includes non-constant utility discounting.

9. I thank Geir Asheim for this insight.

in general be needed to make the economy depart from its optimal path to achieve sustainability.[10]

The resolution of this paradox has to lie in some kind of split between private and public concerns about the far future. We must assume that the individual chooses his or her own actions to maximise some form of present value, but votes for a government which applies a sustainability concern, both by measuring sustainability, and taking action to achieve it if necessary. People are thus in some sense schizophrenic, treating private economic decisions as the domain of Economic Man, and governmental decisions as the domain of the Citizen (Marglin 1963, p98).[11] One good reason for this is that individuals cannot provide personally for their distant descendants, because of the mixing of bequests that occurs over several generations (Daly and Cobb 1989, p39; Howarth and Norgaard 1993, p351).

This paper's philosophical basis of neoclassical sustainability economics thus rejects classical utilitarianism, which prohibits any discounting; it rejects neoclassical utilitarianism, which sees maximising PV (with a constant ρ) as

10. The apparent paradox is not as direct in an overlapping generations context. Society may have a view on distributing resources across generations to achieve an intergenerational sustainability objective, but this need not imply a constraint on any generation's maximisation of PV over its own, finite lifetime.

11. We would not go as far as Marglin by saying that "The Economic Man and the Citizen are for all intents and purposes two different individuals," since Economic Man can still maximise self-interest (seek optimality) within the bounds (sustainability) that the Citizen lays down. However, recent protests over fossil fuel prices in many Western countries, despite those countries having recently signed the Kyoto Protocol which aims to limit greenhouse gas emissions, may perhaps be seen as a sign that the schizophrenia is real and can lead to quite disconnected behaviour.

a complete prescription for intertemporal equity; and it rejects purely rights-based view that it is future generations' resource opportunities, not utility outcomes that matter (see Pezzey and Toman 2002 for further discussion).

We see roots of this somewhat schizophrenic approach to sustainability in the long discussion in mainstream economics about the purpose and methods of calculating *income*. The most general economic concept of income is some measure of what the economy is now doing or enjoying that includes an adequate provision for the future, just as individuals make provision for the future by investing some of their current money income rather than consuming it all. Asking what an "adequate" provision for the future should be, immediately raises questions about intergenerational equity. And, to risk controversy with scholars of the work, one can see a certain degree of schizophrenia in the classic discussion of income in Hicks (1946, Chapter 14). He sees income there as a guide to "prudent" behaviour that will avoid "impoverishment", but also as something that, held constant, is equivalent to the present value of future receipts. This allows some writers to interpret "Hicksian income" as some measure of sustainable income, and others to interpret it as wealth-equivalent income. As a further testimony to the schizophrenia, one can add GNNP and two further measures: welfare-equivalent income, as defined by Asheim (2000 p30), and Sefton-Weale income. All five income measures have been shown to be strictly different in a specific example with a non-constant utility discount rate (Pezzey 2002).

6. CONCLUSIONS

By applying recent developments in the theory of national income and welfare measurement when there are multiple consumption goods to a fairly general, representative agent, present-value-maximising economy, we have derived a one-sided test for the sustainability of an economy. If augmented green net national product (GNNP) is constant or falling at a moment in time, then the economy is enjoying a level of utility which cannot be sustained forever. An alternative condition for this result is that the value of augmented net investments is zero or negative. In both cases the "green" in GNNP means that all environmental stocks and flows are fully included in the measure. The "augmented" means that the value of time in allowing exogenous shifts of production possibilities, whether through technical progress or shifts in the terms of trade, is also fully included. An illustration in a more specific economy showed how the test fully incorporates many issues in national income accounting which have previously been tackled separately, such as the value of environmental quality, resource renewal and discovery, investment and current spending on pollution abatement, trade in goods and resources, changes in the terms of trade, and exogenous technical progress. It also shows that resource exploration and extraction costs must be deducted from GDP as part of the adjustments to reach GNNP, an adjustment which has often been overlooked.

The practical difficulties in using the test are obvious. It offers no escape from the need to put dollar values on small changes in all environmental resources, no matter how disconnected these resources are from current markets. Indeed, the price of material consumption can in general no longer be constant (and therefore set at unity), but will change over time in terms of a weighted average of material consumption goods and environmental goods. It also reminds us of the need to account for future prospects for technical progress and changing terms of trade alongside the environmental resource questions usually addressed by "green" accounting. However, the test should help to avoid some of the more obvious theoretical shortcomings of national income accounting. The philosophical limitations of the test are that it remains unexplained why sustainability should be of interest in a present-value-maximising economy. However, this is a limitation shared by most previous literature, and treating sustainability formally as a public good is suggested, but not analysed, as a solution to this conundrum.

APPENDIX

Proof of Proposition 2

The proof varies slightly for the two conditions. Starting with the condition on the sign of net investment,

$$\mathbf{Q}^{\dagger}(t).\dot{\mathbf{K}}^{\dagger}(t) \leq 0 \;\Rightarrow\; \lambda(t)\pi(t)\mathbf{Q}^{\dagger}(t).\dot{\mathbf{K}}^{\dagger}(t) \leq 0$$

$$\Rightarrow\; \Psi^{\dagger}(t).\dot{\mathbf{K}}^{\dagger}(t) \leq 0 \;\text{ by [5] }\; \Rightarrow\; H(t) \leq U(t) \;\text{ by [4].}$$

Now, integrating $\dot{H}(s) = \rho[H(s)-U(s)]$ in [4] from time t to ∞ gives

$$H(t) \;=\; \rho\int_t^{\infty} U(\mathbf{C}(s))e^{-\rho(s-t)}ds \;=\; \int_t^{\infty} U(\mathbf{C}(s))e^{-\rho(s-t)}ds \,/\, \int_t^{\infty} e^{-\rho(s-t)}ds.$$[12]

$$\Rightarrow \int_t^{\infty} H(t)e^{-\rho(s-t)}ds = \int_t^{\infty} U(\mathbf{C}(s))e^{-\rho(s-t)}ds \qquad \text{[A1]}$$

The non-constancy and uniqueness of the optimal path then means that

$$H(t) > U_m(t).$$

Otherwise, following the feasible constant utility path $U(s) = U_m(t)$ for all $s \geq t$ would, using the PV-equivalence result [A1], give at least the same PV as the optimal utility path, a contradiction of a unique optimum. Combining $H \leq U$ and $H > U_m$ gives the result that current $U(t)$ is unsustainable:

12. This step depends on the constancy of the utility discount rate, ρ. When the utility discount rate is a function of time, $\rho(s)$, it cannot appear be taken outside the first integral sign as occurs here. There is then no present value interpretation.

$U(t) \geq H(t) > U_m(t)$ as required in [14].

Starting from the sign of GNNP change requires in addition Proposition 1 (after AW), and an assumption that the real interest rate $r(t) > 0$, to be able to deduce the same sign for net investment:

$$\dot{Y}^\dagger(t) \leq 0 \;\Rightarrow\; r(t)\mathbf{Q}^\dagger(t).\dot{\mathbf{K}}^\dagger(t) \leq 0 \text{ by [11]} \;\Rightarrow\; \mathbf{Q}^\dagger(t).\dot{\mathbf{K}}^\dagger(t) \leq 0 \text{ by } r > 0.$$

The rest of the proof follows as above.

Proof of Proposition 3

$Y^\dagger(\mathbf{K}^\dagger,\mathbf{Q}^\dagger)$ is defined as max $\mathbf{P}.\mathbf{C} + \mathbf{Q}^\dagger.\dot{\mathbf{K}}^\dagger$ for $(\mathbf{C}.\dot{\mathbf{K}}^\dagger) \in \Pi(\mathbf{K}^\dagger)$

Hence from Proposition 1,

$$\begin{aligned} r\mathbf{Q}^\dagger.\dot{\mathbf{K}}^\dagger &= (d/dt)[Y^\dagger(\mathbf{K}^\dagger,\mathbf{Q}^\dagger)] \\ &= (\partial Y^\dagger/\partial \mathbf{K}^\dagger).\dot{\mathbf{K}}^\dagger + (\partial Y^\dagger/\partial \mathbf{Q}^\dagger).\dot{\mathbf{Q}}^\dagger \\ &= (\partial Y^\dagger/\partial \mathbf{K}^\dagger).\dot{\mathbf{K}}^\dagger + \dot{\mathbf{Q}}^\dagger.\dot{\mathbf{K}}^\dagger, \end{aligned}$$

and since this is true for any general $\dot{\mathbf{K}}^\dagger$, for all components with $\dot{K}_i^\dagger \neq 0$,

$$rQ_i^\dagger - \dot{Q}_i^\dagger = \partial Y^\dagger/\partial K_i^\dagger. \quad \text{[A2]}$$

Since $\dot{t} = 1$, $\dot{K}_i^\dagger \neq 0$ for the time component of [A2], which is

$$rQ^t - \dot{Q}^t = \partial Y^\dagger/\partial t = \partial Y/\partial t = Y_t \quad \text{[A3]}$$

because $Q^t = Y^\dagger - Y$ has no exogenous time dependence. [A3] can be integrated from time t to ∞ to give

$$Q^t(t) = \int_t^\infty [\partial Y(s)/\partial s]\exp[-\int_t^s r(z)dz]ds$$ which is [16].

Proof of Proposition 7

The current value Hamiltonian of the dynamic optimisation problem of maximising wealth[13] is

$$Y^{\dagger}(t) := Y(t) + Q^{t} = P^{C}C + P^{B}B + \mathbf{Q}^{\dagger}.\dot{\mathbf{K}}^{\dagger} \qquad \text{[A4]}$$

where

$\mathbf{K}^{\dagger} := (K, K_a, K_f, \boldsymbol{\Omega}, \boldsymbol{S}, t)$ is the vector of all state variables; [A5]

$\mathbf{Q}^{\dagger} := (Q^K, Q^a, Q^f, Q^{\Omega}, Q^{\boldsymbol{S}}, Q^t)$ is the vector of corresponding co-state variables (shadow consumption prices of stocks). [A6]

The prices and investment flows defined by [26]-[34] then make

$$Y^{\dagger}(t) = P^{C}C + P^{B}B + Q^{K}\dot{K} + Q^{a}\dot{K}_a + Q^{f}\dot{K}_f + Q^{\Omega}\dot{\Omega} + Q^{\boldsymbol{S}}.\dot{\boldsymbol{S}} + Q^{t} \qquad \text{[A7]}$$

$$\begin{aligned} &= P^{C}C + P^{B}\{B_0 - \varepsilon^{B}E^{B}[F(K,\boldsymbol{R}\text{-}\boldsymbol{R}_x,t),a,K_a,t] - \varepsilon^{\Omega}(\Omega_1 - \Omega)\} \\ &\quad + Q^{K}[F(K,\boldsymbol{R}\text{-}\boldsymbol{R}_x,t) + M - C - \delta K - a - I_a - V(\boldsymbol{D},\boldsymbol{S},t) - X(\boldsymbol{R},\boldsymbol{S},t)] \\ &\quad + Q^{a}[I_a - \delta K_a] + Q^{f}[r(t)K_f + \boldsymbol{Q}^{x}(t).\boldsymbol{R}_x - M] \\ &\quad + Q^{\Omega}[\gamma(\Omega) - E^{\Omega}(\boldsymbol{R}\text{-}\boldsymbol{R}_x)] + Q^{\boldsymbol{S}}.[\boldsymbol{D} + \boldsymbol{G}(\boldsymbol{S}) - \boldsymbol{R}] + Q^{t} \end{aligned} \qquad \text{[A8]}$$

so the first order conditions with respect to the control variables are

$\partial Y^{\dagger}/\partial C = P^{C} - Q^{K} = 0 \quad \Rightarrow \quad Q^{K} = P^{C}$ [A9]

$\partial Y^{\dagger}/\partial a = P^{B}B_a - Q^{K} = 0 \quad \Rightarrow \quad P^{B}/P^{C} = 1/B_a = b$ [A10]

$\partial Y^{\dagger}/\partial I_a = -Q^{K} + Q^{a} = 0 \quad \Rightarrow \quad Q^{a} = Q^{K} = P^{C}$ [A11]

$\partial Y^{\dagger}/\partial \boldsymbol{D} = -Q^{K}V_{\boldsymbol{D}} + Q^{\boldsymbol{S}} = \mathbf{0} \quad \Rightarrow \quad Q^{\boldsymbol{S}}/Q^{K} = V_{\boldsymbol{D}}$ [A12]

$\partial Y^{\dagger}/\partial \boldsymbol{R} = P^{B}B_{\boldsymbol{R}} + Q^{K}(F_{\boldsymbol{R}} - X_{\boldsymbol{R}}) - Q^{\Omega}E^{\Omega}_{\boldsymbol{R}} - Q^{\boldsymbol{S}} = \mathbf{0}$

$\Rightarrow \; Q^{\Omega}E^{\Omega}_{\boldsymbol{R}}/Q^{K} = (P^{B}/P^{C})B_{\boldsymbol{R}} + F_{\boldsymbol{R}} - X_{\boldsymbol{R}} - Q^{\boldsymbol{S}}/Q^{K}$

$\Rightarrow \; Q^{\Omega}/Q^{K} = (bB_{\boldsymbol{R}} + F_{\boldsymbol{R}} - X_{\boldsymbol{R}} - V_{\boldsymbol{D}}) \,/\, E^{\Omega}_{\boldsymbol{R}}$ [A13]

$\partial Y^{\dagger}/\partial M = Q^{K} - Q^{f} = 0 \quad \Rightarrow \quad Q^{f} = Q^{K} = P^{C}$ [A14]

$\partial Y^{\dagger}/\partial \boldsymbol{R}_x = -P^{B}B_{\boldsymbol{R}} - Q^{K}F_{\boldsymbol{R}} + Q^{f}\boldsymbol{Q}^{x} + Q^{\Omega}E^{\Omega}_{\boldsymbol{R}} = 0$; then use [A10], [A14], [A12]:

13. We are assuming that the optimal (welfare-maximising) path is regular, in the sense of Asheim (1997, p368), so that it maximises wealth as well as welfare.

$$\Rightarrow\ P^B B_{\boldsymbol{R}} + Q^K F_{\boldsymbol{R}} - Q^{\Omega} E^{\Omega}{}_{\boldsymbol{R}} = P^C \boldsymbol{Q}^x = Q^K X_{\boldsymbol{R}} + Q^{\boldsymbol{S}} \qquad [A15]$$

$$\Rightarrow\ bB_{\boldsymbol{R}} + F_{\boldsymbol{R}} - (Q^{\Omega}/Q^K) E^{\Omega}{}_{\boldsymbol{R}} = \boldsymbol{Q}^x = X_{\boldsymbol{R}} + V_{\boldsymbol{D}} \qquad [A16]$$

For Q^t, first use [A4] and [A8] to get

$$\partial Y/\partial t = P^B B_t + Q^K(F_t - V_t - X_t) + Q^f(\dot{r}K_f + \dot{\boldsymbol{Q}}^x . \boldsymbol{R}_x)$$

which, after using [A9], [A10] and [A14] becomes

$$\partial Y/\partial t = P^C(bB_t + F_t - V_t - X_t + \dot{r}K_f + \dot{\boldsymbol{Q}}^x . \boldsymbol{R}_x)$$

hence from [16],

$$Q^t(t) := \int_t^{\infty} P^C(s)\ \{bB_t + F_t - V_t - X_t + \dot{r}K_f + \dot{\boldsymbol{Q}}^x . \boldsymbol{R}_x\}(s)\ \exp[-\int_t^s r(z)dz]\ ds$$

which is [36].

Inserting [A9]-[A16] into a cross between [A7] and [A8] then gives

$$Y^{\dagger} = P^C C + P^C bB + Q_K\{\dot{K} + \dot{K}_a + \dot{K}_f\} + Q^{\Omega}[\gamma(\Omega) - E^{\Omega}(\boldsymbol{R}\text{-}\boldsymbol{R}_x)]$$
$$+ Q^{\boldsymbol{S}} . [\boldsymbol{D} + \boldsymbol{G}(\boldsymbol{S}) - \boldsymbol{R}] + Q^t$$

which using [16], [A12] and [A16] gives

$$= P^C\ \{C + bB + \dot{K} + \dot{K}_a + \dot{K}_f + (bB_R + F_R - \boldsymbol{Q}^x)(\gamma - E^{\Omega})/E^{\Omega}{}_{\boldsymbol{R}}$$
$$+ V_{\boldsymbol{D}} . [\boldsymbol{D} + \boldsymbol{G}(\boldsymbol{S}) - \boldsymbol{R}]\ \} + Q^t$$

which is [35].

If the problem is autonomous, time is "unproductive", so its value Q^t, the last term of [35], disappears.

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