THE AUSTRALIAN NATIONAL UNIVERSITY

Working Paper Series in Finance 00-02

THE ADJUSTMENT OF THE YULE-WALKER RELATIONS IN VAR MODELING: THE IMPACT OF THE EURO ON THE HONG KONG STOCK MARKET

Jack H.W. Penm, T.J. Brailsford and R.D. Terrell

Faculty of Economics and Commerce
Australian National University

JEL Classification	G10, G13
Keywords	VAR models; Yule-Walker relations; Foreign exchange market; Financial time series
Last revision date:	May 2000

Contact author: Tim Brailsford
Faculty of Economics and Commerce
Australian National University
Canberra 0200 AUSTRALIA
Telephone: 02 6249 3596 (International +61 2 6249 3596)
Facsimile: 02 6249 0414 (International +61 2 6249 0414)
Email: Dean.Ecocomm@anu.edu.au

THE ADJUSTMENT OF THE YULE-WALKER RELATIONS IN VAR MODELING: THE IMPACT OF THE EURO ON THE HONG KONG STOCK MARKET

Jack H.W. Penm, T.J. Brailsford and R.D. Terrell

The Australian National University, Canberra, 0200, Australia

Abstract

VAR models are increasingly being used in the analysis of relationships between financial markets. In such models, there are circumstances that require zero entries in the coefficient matrices. Such circumstances can be particularly relevant in the context of emerging markets given their characteristics. We show that a direct extension of the use of the Yule-Walker relations for fitting VAR models with zero-non-zero patterned coefficient matrices is inconsistent with statistical procedures as the resultant estimated variance-covariance matrix of the white noise process becomes non-symmetric. This inconsistency has biased consequences for financial theory. The paper provides a theoretically consistent adjustment which fits with theory. The paper applies the procedure to a vector system comprising variables from the Hong Kong stock market and foreign exchange markets. The results indicate that the euro exchange rate contains leading information for the other components in the system.

1. Introduction

Vector autoregressive (VAR) models provides a device which has proved to be efficient, and therefore a less costly tool than conventional financial and econometric time-series models. In recent years the use of VAR models as a means of modeling financial time series, signal processing and producing *ex-ante* forecasts has become widespread.

Specifically, VAR modeling has increasingly been employed to examine relationships in stock markets. For instance, Eun and Shim (1989) estimate a VAR using index returns on nine stock markets to examine interactions among the markets. In the context of emerging stock markets, Bekaert et al (1999) estimate a VAR using capital flows, equity returns, dividend yields and interest rates to examine the degree to which lower interest rates contribute to increased capital flows. In a similar study, Froot et al (1998) also employ a VAR estimation to examine the relationship between capital flows and equity returns in emerging markets.

However, VAR models have a wider application. Hsiao (1981) has suggested a stepwise VAR modeling method of testing the supply of money and aggregate nominal income for Granger-causality. Hsiao estimates the variance-covariance matrices of various restricted and unrestricted VAR models to calculate the log-likelihood values. Likelihood ratio tests are then carried out to select the optimal VAR model which is used as a basis for detecting Granger-causality. Bar-Yosef *et al* (1987) employ Hsiao's method to examine the linkage between corporate earnings and corporate investment.

Geweke (1982) has derived a means of measuring the linear dependence and feedback present in multiple time series. The variance-covariance matrices of various VAR models with both exogenous variables and restricted VAR models are estimated for measuring two-way linear causality and instantaneous linear causality. Geweke also shows how the notions of causality relate to exogeneity in the context of a complete dynamic simultaneous equation model.

Further, financial, monetary and macroeconomic variables have been tested for the existence and direction of Granger causal relations. Bhattacharya *et al* (2000) utilise a

sequential hypothesis testing procedure using standard likelihood ratio tests to analyse the causal direction of volatility transmission between two different classes of shares in the framework of a VAR. Lee (1992) uses a VAR approach to investigate causal relations and dynamic interactions among asset returns, real activity, and inflation in postwar United States. Bekaert and Hodrick (1992) examine the predictability of excess returns on equity and foreign exchange markets using a six-variable VAR model. Whitelaw (1994) investigates a VAR for assessing the time series properties of the expectation and volatility of stock market returns. Thorbecke (1997) uses impulse-response functions from a VAR to analyse the relationship between monetary policy and stock returns. Bernanke and Blinder (1992) employ a VAR in which the federal funds rate is used to measure monetary policy. Also, Malliaris and Urrutia (1992), and Hassapis *et al* (1999) examine related issues.

It is appropriate to note that recent cointegration work suggests that, if cointegrating relations exist between the variables, then the use of the vector error-correction model, which is associated with the VAR model with unit roots, may be more effective for testing Granger-causality.

Generally, econometric research into VAR models has in part been driven by the desire to provide users with a relatively simple forecasting procedure accessible to non-specialists. However early researchers realised that heavy parameterisation of their VAR models resulted in poor *ex-ante* forecasting performance. Their proposed procedures rested on the assumption that the coefficient matrices of the VAR model had all non-zero entries. In practice the assumed specification for this full order VAR model could be quite different from the actual specification, as there could in fact be some zero entries in the coefficient matrices of the model. Therefore, overcoming the problem of over-parameterisation needs to be carefully considered when assessing the value of the proposed VAR modeling system. Avoiding over-parameterisation has the benefits of improved efficiency with a reduced computational burden and greater numerical reliability.

Penm and Terrell (1984) propose a search algorithm, using the Yule-Walker relations for fitting VAR models in conjunction with model selection criteria, to select the optimal VAR models with zero-non-zero patterned coefficient matrices. These zero-

non-zero patterned VAR models allow for possible zero entries in the coefficient matrices. The optimal AR models have been utilised to construct multi-layered neural networks for conducting *ex-ante* forecasting (see Penm *et al* 1999), and improved forecasting performance has been reported.

The zero-non-zero patterned optimal VAR models can also used as a basis for detecting Granger-causality and instantaneous causality among time series variables (Penm and Terrell 1984). Granger-causality and instantaneous causality have been defined by Granger (1969), and are based entirely on the predictability of the objective variables such that they make no explicit use of economic and financial laws to provide *a priori* restrictions on the structure.[1]

The use of the Yule-Walker relations for fitting of vector autoregressive (VAR) models in the modelling of stationary vector time series is well known. Much of the background of the Yule-Walker relations for the scalar case is detailed in Box and Jenkins (1976). Whittle (1963) has extended these relations to fit VAR models. DeJong (1976) uses Whittle's results for the recursive fitting of VAR models of successively higher order. To date, computing programs using the Yule-Walker relations for fitting of VAR models have been incorporated into a variety of powerful and well used software packages, including GAUSS®, S-Plus® and MATLAB®.

Attempts to make a direct extension of the Yule-Walker relations to fit the VAR model with zero-non-zero patterned coefficient matrices have been unsuccessful. Specifically, after all (i,j)-th zero entries (for specified i and j, in the coefficient matrices in a VAR model) are imposed as the result of prior knowledge, they are then applied to the Yule-Walker relations for fitting purposes. However, the resulting estimated variance-covariance matrix of the white noise process, as explained later in Section 3, becomes non-symmetric. This result is inconsistent with theory which states that the estimated variance-covariance matrix of the white noise process is certainly symmetric. This problem creates motivation for developing an adjustment to

[1] For example, Caines *et al* (1981) propose procedures to test sales and advertising for Granger-causality in a class of VAR models. In the application to supermarket sales analysis, a model is constructed to aid managers in their decision-making. Caines *et al* use likelihood ratio tests to discriminate between various pairs of restricted and unrestricted VAR models held as the null and alternative hypotheses respectively. In the course of tests, the variance-covariance matrices of restricted

the Yule-Walker relations for fitting of VAR models with zero-non-zero patterned coefficient matrices.

The importance of this adjustment is clear if the time series under examination behaves in such a way where zero-non-zero patterns in the coefficient matrices may be expected to occur. This may be the case across many financial time series, such as those in emerging markets.

Research into emerging stock markets has shown that they exhibit patterns different to those of developed stock markets and potentially relationships between time series in these markets may not be expected to produce coefficient matrices of full order. Sewell et al (1993) provide evidence of non-linear dependencies in five emerging markets. Bekaert and Harvey (1997) study volatility in emerging stock markets and report it to be substantially higher than that in developed markets and show that it is influenced by particular emerging market characteristics. Returns and risk in emerging stock markets have been found to be higher relative to developed markets[2] [Wilcox (1992); Claessens, Dagupta and Glen (1993); and Harvey (1995)]. Moreover, the returns in emerging stock markets exhibit stronger mean reversion properties [Harvey (1993,1995); Bekaert (1995); and Bekaert and Harvey (1995)], with a higher degree of autocorrelation.

The remainder of this paper is organised as follows. Section 2 revisits Whittle's (1963) method of fitting VAR models using the Yule-Walker relations. Section 3 shows the theoretical inconsistency by using a two-variable VAR example. Section 4 outlines the necessary adjustment. In Section 5, we provide an application concerning the Hong Kong stock market and the foreign exchange market. Some concluding remarks are provided in Section 6. A technical appendix is included that shows that the generalised least squares (GLS) coefficient estimator for the zero-non-zero patterned VAR is an approximation of the maximum likelihood estimator.

---

and unrestricted VAR models are estimated to compute the log-likelihood values.

[2] However it is questionable as to whether these high and somewhat predictable returns are transitory in nature as evidenced by the substantial declines in market capitalisation in emerging markets that occurred during 1997 and 1998.

2. Whittle's Method Revisited

Let $\mathbf{u}(t) = \{u_1(t),\ u_2(t),\ \ldots,\ u_m(t)\}'$ be a zero mean, wide-sense stationary time series of dimension m. We consider the vector AR (p) model of the form

$$\sum_{k=0}^{p} A_k \mathbf{u}(t-k) = \varepsilon(t) \tag{2.1}$$

where A_0=I, A_k, k=1,...,p, are the mxm parameter matrices and $\varepsilon(t)$ is an mx1 stationary vector process with E{ $\varepsilon(t)$ }=0.

$$E\{\varepsilon(t)\varepsilon'(t-k)\} \begin{matrix} = V & \text{as} & k=0 \\ = 0 & \text{as} & k>0 \end{matrix} \tag{2.2}$$

and uncorrelated with $u(t-p-r),\ \ r>0.$ The sample lag covariance matrices

$$\Gamma_k = \frac{1}{N}\sum_{t=1}^{N-k} \mathbf{u}_{t+k}\mathbf{u}_t' \tag{2.3}$$

obey the following Yule-Walker relations proposed by Whittle (1963)

$$\Gamma_j + \sum_{k=1}^{p} \hat{A}_k \Gamma_{j-k} = 0 \quad (j=1,\ldots,p) \tag{2.4}$$

$$\Gamma_0 + \sum_{k=1}^{p} \hat{A}_k \Gamma_{-k} = \hat{V} \tag{2.5}$$

where N is the sample size. $\hat{A}_k$ and $\hat{V}$ are the estimates of A_k and V respectively.

Following Penm and Terrell (1984), Equation (2.4) can be expressed as

$$\Lambda_p R_p = -\Pi_p \tag{2.6}$$

where $\Lambda_p = \{\hat{A}_1 \quad \hat{A}_2 \quad \ldots \quad \hat{A}_p\}$, $\Pi_p = \{\Gamma_1 \quad \Gamma_2 \quad \ldots \quad \Gamma_p\}$ and

$$R_p = \begin{bmatrix} \Gamma_0 & \Gamma_1 & \cdots & \Gamma_{p-1} \\ \Gamma_{-1} & \Gamma_0 & \cdots & \Gamma_{p-2} \\ \vdots & \vdots & & \vdots \\ \Gamma_{1-p} & \Gamma_{2-p} & \cdots & \Gamma_0 \end{bmatrix}.$$

Applying the vectorisation rule $\mathrm{vec}(ABC) = [C' \otimes A]\mathrm{vec}(B)$, we then transpose and vectorise Equation (2.6) to be

$$\{I_m \otimes R_p\}\mathrm{vec}\{\Lambda'_p\} = -\mathrm{vec}\{\Pi'_p\} \tag{2.7}$$

where the vec operation is obtained by stacking the columns of a matrix, and $\otimes$ denotes the Kronecker product.

3. The Inconsistency

In this section we show the theoretical inconsistency of the use of the Yule-Walker relations for fitting of VAR models with zero-non-zero patterned coefficient matrices using a two-asset example.

In considering a VAR model with zero-non-zero patterned coefficient matrices, as defined in Penm and Terrell (1984), we allow for zero entries in the parameter matrices A_k of (2.1). Let returns of the assets, $\Delta y_{1,t}$, $\Delta y_{2,t}$, be jointly determined by the following two equations

$$\Delta y_{1,t} + a_{12}\Delta y_{2,t-1} = \varepsilon_{1,t} \tag{3.1}$$

$$\Delta y_{2,t} + a_{21}\Delta y_{1,t-1} + a_{22}\Delta y_{2,t-1} = \varepsilon_{2,t} \tag{3.2}$$

where $y_{1,t}$ and $y_{2,t}$ are the log prices of the assets. In this two-equation system the first equation shows that $\Delta y_{1,t}$ is caused by $\Delta y_{2,t-1}$, while the second equation demonstrates that $\Delta y_{2,t}$ is caused by both $\Delta y_{1,t-1}$ and $\Delta y_{2,t-1}$.

The equivalent VAR model of this system can then be expressed as

$$\begin{bmatrix} \Delta y_{1,t} \\ \Delta y_{2,t} \end{bmatrix} + \begin{bmatrix} 0 & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta y_{1,t-1} \\ \Delta y_{2,t-1} \end{bmatrix} = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \tag{3.3}$$

where the white noise process comprises two components $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, with

$E\{\varepsilon_{1,t}\} = E\{\varepsilon_{2,t}\} = 0$, and

$$E\left\{ \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-k} & \varepsilon_{2,t-k} \end{bmatrix} \right\} \begin{matrix} = V & \text{as} & k = 0 \\ = 0 & \text{as} & k > 0. \end{matrix}$$

It is noteworthy that V is symmetric according to the above definition and that the measurements of Granger-causality and the instantaneous causality are crucially dependent on the estimate of V (see Geweke 1982). We now utilise Equation (2.7) to estimate the non-zero coefficients in this VAR model. Since $a_{11} = 0$ in Equation (3.3) as prior knowledge, we place 0 in the first row elements of Λ'_p and Π'_p, place 1 in the first diagonal element of $\{I_2 \otimes R_1\}$, and 0 everywhere else in the first row and column of $\{I_2 \otimes R_1\}$. Thus the following relation is established

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \tau_{22}(0) & 0 & 0 \\ 0 & 0 & \tau_{11}(0) & \tau_{21}(0) \\ 0 & 0 & \tau_{12}(0) & \tau_{22}(0) \end{bmatrix} \begin{bmatrix} 0 \\ \hat{a}_{12} \\ \hat{a}_{21} \\ \hat{a}_{22} \end{bmatrix} = -\begin{bmatrix} 0 \\ \tau_{12}(1) \\ \tau_{21}(1) \\ \tau_{22}(1) \end{bmatrix} \tag{3.4}$$

where $\tau_{ij}(k) = \frac{1}{N}\sum_{t=1}^{N-k} \Delta y_i(t+k)\Delta y_j(t) = \tau_{ji}(-k)$, and $\hat{a}_{ij}$ are the estimates of a_{ij}.

By multiplying Equation (3.4) with the inverse of the 4x4 coefficient matrix in Equation (3.4), we have

$$\hat{a}_{12} = -\frac{\tau_{12}(1)}{\tau_{22}(0)},\ \hat{a}_{21} = \frac{\tau_{21}(0)\tau_{22}(1) - \tau_{22}(0)\tau_{21}(1)}{\tau_{11}(0)\tau_{22}(0) - \tau_{21}(0)\tau_{12}(0)} \text{ and } \hat{a}_{22} = \frac{\tau_{12}(0)\tau_{21}(1) - \tau_{11}(0)\tau_{22}(1)}{\tau_{11}(0)\tau_{22}(0) - \tau_{21}(0)\tau_{12}(0)}.$$

As a result the estimate $\hat{V}$ in Equation (2.5) becomes

$$\begin{bmatrix} \tau_{11}(0) & \tau_{12}(0) \\ \tau_{12}(0) & \tau_{22}(0) \end{bmatrix} + \begin{bmatrix} \hat{a}_{12}\tau_{12}(1) & \hat{a}_{12}\tau_{22}(1) \\ \hat{a}_{21}\tau_{11}(1) + \hat{a}_{22}\tau_{12}(1) & \hat{a}_{21}\tau_{21}(1) + \hat{a}_{22}\tau_{22}(1) \end{bmatrix}, \tag{3.5}$$

which is non-symmetric. However V is symmetric in the true model of (3.3) and there is a need for the estimate $\hat{V}$ to conform to the behaviour of V, therefore the estimate $\hat{V}$ must be a symmetric matrix. That means the result in Equation (3.5) does not fit with statistical assumptions of importance in financial theory. An adjustment to the Yule-Walker relations is therefore required which is presented in the next section.

4. The Adjustment

The necessary adjustment to the Yule-Walker relations for fitting of VAR models with zero-non-zero patterned coefficient matrices follows directly from the inconsistency demonstrated above.

With the definition of the variance-covariance matrix of the vector white noise, we have

$$V = E\left\{\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}\begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} \end{bmatrix}\right\}$$

$$= E\left\{\left\{\begin{bmatrix} \Delta y_{1,t} \\ \Delta y_{1,t} \end{bmatrix} + \begin{bmatrix} 0 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} \Delta y_{1,t-1} \\ \Delta y_{2,t-1} \end{bmatrix}\right\}\left\{\begin{bmatrix} \Delta y_{1,t} & \Delta y_{1,t} \end{bmatrix} + \begin{bmatrix} \Delta y_{1,t-1} & \Delta y_{2,t-1} \end{bmatrix}\begin{bmatrix} a_{22} & a_{21} \\ a_{12} & 0 \end{bmatrix}\right\}\right\}.$$

Then the estimate $\hat{V} = \begin{bmatrix} \tau_{11}(0) & \tau_{12}(0) \\ \tau_{12}(0) & \tau_{22}(0) \end{bmatrix} +$

$$\begin{bmatrix} \hat{a}_{12}\tau_{12}(1) & \hat{a}_{12}\tau_{22}(1) \\ \hat{a}_{21}\tau_{11}(1) + \hat{a}_{22}\tau_{12}(1) & \hat{a}_{21}\tau_{21}(1) + \hat{a}_{22}\tau_{22}(1) \end{bmatrix} + \begin{bmatrix} \hat{a}_{12}\tau_{12}(1) & \hat{a}_{21}\tau_{11}(1) + \hat{a}_{22}\tau_{12}(1) \\ \hat{a}_{12}\tau_{22}(1) & \hat{a}_{21}\tau_{21}(1) + \hat{a}_{22}\tau_{22}(1) \end{bmatrix} +$$

$$\begin{bmatrix} 0 & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{bmatrix}\begin{bmatrix} \tau_{11}(0) & \tau_{12}(0) \\ \tau_{12}(0) & \tau_{22}(0) \end{bmatrix}\begin{bmatrix} \hat{a}_{22} & \hat{a}_{21} \\ \hat{a}_{12} & 0 \end{bmatrix} \tag{4.1}$$

Since the first matrix of Equation (4.1) is symmetric, the second matrix is the transpose of the third matrix, and the remaining product matrix is also symmetric, therefore the matrix $\hat{V}$ is symmetric.

Noting that the matrix $\hat{V}$ of Equation (3.5), obtained from the Yule-Walker relations, is the result of cancelling the third matrix from the fourth product matrix of Equation (4.1). Although this cancellation is acceptable for fitting of the full order VAR (if $a_{11} \neq 0$), it is unsuitable for fitting VAR with zero-non-zero patterned coefficient matrices.

In considering the use of the Yule-Walker relations for fitting of vector autoregressive models of Equation (2.1) with zero-non-zero patterned coefficient matrices, the coefficient estimates obey the following relationship

$$Z(C_r)\alpha(C_r) = \gamma(C_r) \tag{4.2}$$

where $Z = \{I_m \otimes R_p\}$. $\alpha = \mathrm{vec}\{\Lambda_p'\}$, $\gamma = \mathrm{vec}\{\Pi_p'\}$, C_r is an integer set which contains $c_1, c_2, \ldots, c_r$, and the $(c_1, c_2, \ldots, c_r)$ th entries of α are constrained to zero. Then $\alpha(C_r)$ and $\gamma(C_r)$ are formed by placing 0 in the $(c_1, c_2, \ldots, c_r)$ th row entries of α and γ, and $Z(C_r)$ is formed by placing 1 in the $\{(c_1, c_1), (c_2, c_2), \ldots, (c_r, c_r)\}$ diagonal entries of Z and 0 everywhere else in the $(c_1, c_2, \ldots, c_r)$ rows and columns of Z.

However, should anyone attempt to use these relations for fitting purposes, the resulting estimated $\hat{V}$ in Equation (2.5) becomes non-symmetric analogously to the example in Section 3. The necessary adjustment to Equation (2.5) of the Yule-Walker relations is then conducted to ascertain that the resulting estimated variance-covariance matrix of the white noise process is a symmetric one.

An analogous approach of Equation (4.1) is feasible. From Equation (2.2), we have

$$V = E\left\{\left[\sum_{k=0}^{p} A_k \mathbf{u}(t-k)\right]\left[\sum_{j=0}^{p} \mathbf{u}'(t-j)A_j'\right]\right\}$$

Then the estimate $\hat{V} = \Gamma_0 + \sum_{k=1}^{p} \hat{A}_k \Gamma_{-k} + \sum_{j=1}^{p} \Gamma_j \hat{A}'_j + \sum_{j=1}^{p}\sum_{k=1}^{p} \hat{A}_k \Gamma_{j-k} \hat{A}'_j$. (4.3)

The proof is straightforward that the matrix $\hat{V}$ of Equation (4.3) is symmetric. Since $\Gamma'_{-j} = \Gamma_j$, Γ_0 is symmetric. If we redefine k as j, the second matrix, $\sum_{k=1}^{p} \hat{A}_k \Gamma_{-k}$, becomes $\sum_{j=1}^{p} \hat{A}_j \Gamma_{-j}$, which is the transpose of the third matrix, $\sum_{j=1}^{p} \Gamma_j \hat{A}'_j$. If we redefine j as k and k as j, the fourth matrix, $\sum_{j=1}^{p}\sum_{k=1}^{p} \hat{A}_k \Gamma_{j-k} \hat{A}'_j$ becomes $\sum_{k=1}^{p}\sum_{j=1}^{p} \hat{A}_j \Gamma_{k-j} \hat{A}'_k$, which is the transpose of the fourth matrix itself. Thus we have established that the matrix $\hat{V}$ is symmetric.

In considering a subset VAR, Penm and Terrell (1982) use well-established criteria to choose the best subset of lags between 0 and p for the full order VAR of Equation (2.1). It is also straightforward to observe that Equation (4.3) will reduce to Equation (2.5) for fitting of a subset VAR or of a full order VAR by cancelling $\sum_{j=1}^{p} \Gamma_j \hat{A}'_j$ from $\sum_{j=1}^{p}\left(\sum_{k=1}^{p} \hat{A}_k \Gamma_{j-k}\right)\hat{A}'_j$, using Equation (2.4). However this cancellation is unsuitable for fitting VAR with zero-non-zero patterned coefficient matrices. Thus Equation (2.5) requires adjustment to transform into Equation (4.3).

In addition, comparing Equation (4.3) with the direct definition of V in Equation (2.2), the structure of Equation (4.3) is computationally efficient in terms of execution time and storage requirements, and provides the obvious relations to link the covariance matrices with different lags.

It is noteworthy that consideration of the contemporaneous correlation in $\varepsilon(t)$ cannot be ignored. A zero-non-zero patterned VAR model can be viewed as a system of 'seemingly unrelated regressions' as originally proposed by Zeller (1962). As the regressors in each equation of the VAR model are no longer necessarily the same, the

GLS coefficient estimates are more efficient than the least squares coefficient estimates. Since $\hat{V}$ is positive definite, there exists an mxm non-singular matrix $\hat{K}$, such that $\hat{V}^{-1} = \hat{K}\hat{K}'$. As described in Penm and Terrell (1984), we premultiply **u**(t) by $\hat{K}^{-1}$, and then follow the proposed method of using the Yule-Walker relations for fitting of VAR models, and so obtain the GLS coefficient estimates of the zero-non-zero patterned VAR model. The appendix shows that this GLS estimator is an approximation of the maximum likelihood coefficient estimator.

5. An Application to Hong Kong

In this section we present an application to illustrate the practical use of the algorithm and comment on the results. First, consider a potential causality relationship between stock and exchange rate markets. Flows of capital are related to exchange rate movements and such flows have been shown to be related to equity returns[3] (eg. Froot et al 1998). Of course, the relationship between exchange rates and stock prices is more complex than implied here and involves consideration of parity conditions and inflationary expectations. Nevertheless, while exchange rate risk should not be separately priced if purchasing power parity holds, in the short-to-medium term, deviations from PPP have been reported [Adler and Lehman (1983); Frenkel (1981)]. Under these conditions, deviations from purchasing power parity will be priced to the extent that they represent exchange rate risk that must be borne by investors [Jorion (1991); Dumas and Solnik (1995)]. In any event, our purpose here is mainly empirical and illustrates how this analysis can provide insights into Granger-causal relationships among financial variables.

In the case Hong Kong, it is an important Asian banking and financial centre. For instance, the Hong Kong stock market is the second largest in Asia after Tokyo. The monetary authority of Hong Kong still maintains a currency pegged to the US dollar to encourage stability and investor confidence during and after the unification of Hong Kong with China. Nevertheless, Hong Kong generally allows unrestricted flows of international funds and it has been suggested that international agencies often park their funds in Hong Kong stocks while waiting for other investment opportunities.

[3] Under perfect purchasing power parity conditions, exchange rates should adjust to reflect relative inflation levels, and the law of one price will be upheld. Hence, exchange rate risk will not be

On January 1, 1999 the introduction of Europe's single currency, the euro, was a significant event in the globalisation of financial markets. The euro is expected to play a significant role in Asian financial development, challenging the existing role of the US dollar. Hence, movements in both the US dollar and the euro have potential implications for the Hong Kong stock market.

Within this context, the following three variables are studied contemporaneously in a stochastic vector system using the ZNZ patterned vector AR modelling proposed above:

(i) Euro dollar to US dollar - exchange rate (EUFX)
(ii) Hong Kong's Hang Seng - stock price index (HSI)
(iii) Hong Kong dollar to US dollar - exchange rate (HKFX)

The Hang Seng Index is the main stock market indicator in Hong Kong. This index comprises 33 constituent stocks which are representative of the market. The aggregate market capitalisation of these stocks accounts for about 70% of the total market capitalisation on Hong Kong's stock exchange. At the beginning of 1999 the HSI was 9,000. However it climbed to 17,000 by the end of 1999, closing with a 90% gain over the year. This abnormal growth is characteristic of emerging markets (Price 1994) although we note that making such a classification involves many considerations (see Goetzmann and Jorion 1999).

While our interest is in detecting Granger-causality in the proposed three variable EUFX-HSI-HKFX system using the ZNZ patterned VAR modeling, we also seek to answer other questions such as whether the relationship between the Hong Kong-US dollar and Hong Kong's stock market is important in the presence of the euro? Further, can variations in the euro provide leading information on variations in the Hong Kong stock market?

All data are sampled daily between 1 January and 31 December 1999. Graphs of EUFX, HSI and HKFX are shown in Figures 1 to 3 respectively. The variables are

separately priced.

log transformed such that $u_1(t)=\log(EUFX)$, $u_2(t)=\log(HSI)$ and $u_3(t)=\log(HKFX)$. Following Penm and Terrell (1984) we initially use Forsythe's (1957) method for generating orthogonal polynomials to assess the data for suitable detrending to produce stationarity. The results show that detrending using a first-order polynomial is required before fitting the VAR models. The standard errors of estimates of coefficients are reported in Table 1.

After detrending, we assign a maximum order of 36 and undertake the search procedures proposed in Penm and Terrell (1984) to obtain the optimal ZNZ patterned VAR model. Each of three order selection criteria - Akaike, Schwarz and Hannan - is used to determine the best specification. The ability of these three order selection criteria to determine the true specification of a stationary VAR has been examined using a simulation approach by Penm and Terrell (1984). Their results suggest that the Schwarz criterion (SC) is superior in order-identification to the other two alternatives in ZNZ patterned VAR modelling for causality studies. Therefore we emphasise only the specification determined by SC and use it as the benchmark model for analysing lead-lag relations.

The OLS coefficient estimates of the specification using the adjusted Yule-Walker relations are presented in Table 2. The procedures outlined in Section 4 to obtain the GLS coefficient estimates are then carried out. These results are also reported in Table 2. Table 3 then presents the associated causal pattern and causal relationships.

The relationships identified by the three selection criteria are markedly similar. All the determined specifications consistently indicate that euro exchange rate is the major variable which provides leading information for other components of the system. The lagged euro exchange rate enters not only its own equation but also those for HSI and HKFX. In all the determined specifications, the lagged level of HSI does not enter any of the exchange rate equations, indicating that variations in the Hong Kong stock market index provide little leading information for the exchange rate markets, as expected. Also no lagged HKFX components enter the equation of the HSI and EUFX, indicating that this variable contains little leading information for either the stock market or euro exchange rate movements. Given that the HKFX is essentially

pegged, this result is not surprising. Hence, the identified casual relationships are consistent with economic intuition.

A more complete analysis would include other economic and financial variables such as net capital flows, interest rates and money supply, which could all play a significant role in these markets. Indeed, our model could be extended to incorporate the recent work of Bekaert et al (1999) who propose a larger system. The importance of this application is that it shows the procedures which can be applied to any set of variables. In the context of emerging markets where traditional models and theories have met with little success, such exercises are likely to provide valuable insights into the relationships and causality between financial variables.

6. Conclusion

The use of VAR modeling in financial economics has become common. However, the models are typically constrained through problems of over-parameterisation. In this paper, we have presented an adjustment to the Yule-Walker relations for fitting of VAR models with zero-non-zero patterned coefficient matrices. The adjustment is consistent with statistical procedures in theory and has the advantages of computational efficiency and reliability.

The procedure has been applied to the Hong Kong stock market, focussing on its relationship with international foreign exchange markets. The results of this exercise are helpful in understanding linkages between various markets and/or financial variables. As indicated above, in the area of emerging markets where there is often no clear consensus concerning relationships among financial variables and each market appears to exhibit almost unique characteristics, this procedure can potentially yield important insights.

Figure 1: Euro dollar to US dollar - exchange rate (EUFX), daily: 1 January 1999 to 31 December 1999

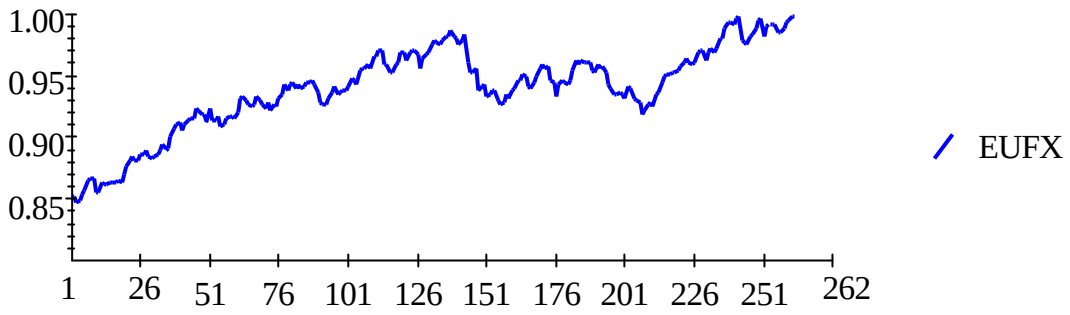


Figure 2: Hang Seng stock price index (HSI), daily: 1 January 1999 to 31 December 1999

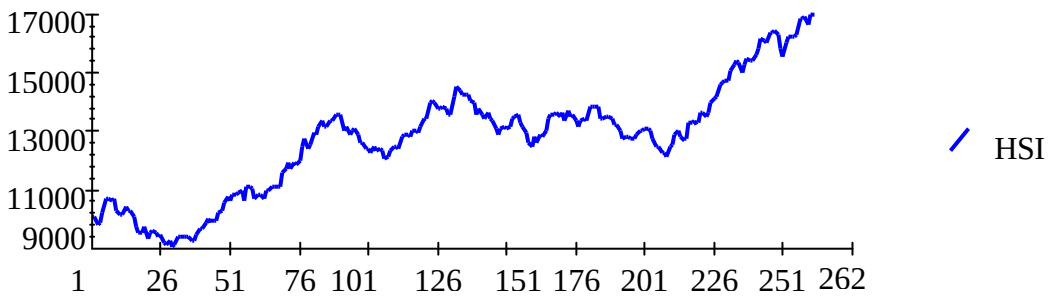


Figure 3: Hong Kong dollar to US dollar - exchange rate (HKFX), daily: 1 January 1999 to 31 December 1999

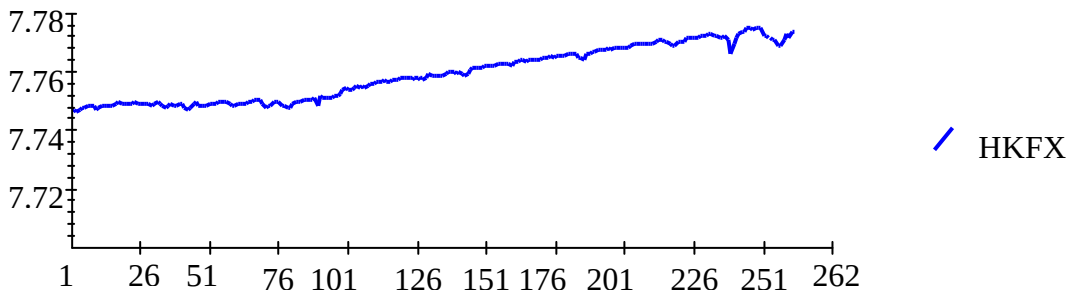


Table 1

Orthogonal polynomial regression on testing data for examining stationarity[a]

	Intercept	Orthogonal polynomial P_1
log(EUFX)	-0.11	3.98E-04
	(2.83E-03)	(1.87E-05)
log(HSI)	9.20	1.75E-03
	(9.32E-03)	(6.14E-05)
log(HSFX)	2.04	1.50E-05
	(3.17E-05)	(2.09E-07)

[a]The values in parentheses are standard errors of the coefficient estimates

Table 2

The optimal ZNZ patterned VAR selected by SC[a,b]

$$\mathbf{u}(t) = \{\log \text{EUFX}, \quad \log \text{HSI}, \quad \log \text{HKFX}\}'$$

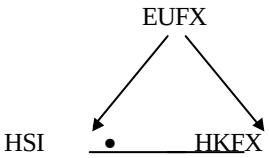
The maximum order assigned for search	36	
The optimal VAR selected	1	
Coefficient Estimator	LS	GLS
The type of coefficient matrices selected	$\hat{A}_1 \begin{bmatrix} -0.963 & 0 & 0 \\ (0.017) & & \\ -0.168 & -0.942 & 0 \\ (0.059) & (0.018) & \\ 0.001 & 0 & -0.870 \\ (0.0004) & & (0.031) \end{bmatrix}$	$\begin{bmatrix} -0.963 & 0 & 0 \\ (0.017) & & \\ -0.006 & -0.941 & 0 \\ (0.002) & (0.017) & \\ 0.005 & 0 & -0.870 \\ (0.002) & & (0.030) \end{bmatrix}$
The estimate of residual variance-covariance matrix by LS times 10^{-4}	$\begin{bmatrix} 0.3783 & 0.1239 & -0.0007 \\ 0.1239 & 2.732 & 0.0006 \\ -0.0007 & 0.0006 & 0.00009 \end{bmatrix}$	

[a]SC is also applied to the residual vector. The results support that the residual vector is a white noise process.
[b]The values in parentheses are standard errors of the non-zero coefficient estimates.

Table 3

Lead-lag relations in the three variable system selected by SC

Panel A Causal pattern:



[a] x → y denotes that x Granger-causes y only and not instantaneously ;
x -•- y denotes that no causal relation between x and y.

Panel B Causal relations:

Caused by the following variables	EUFX	HSI	HKFX
EUFX	-	one-day causal effect	one-day causal effect
HSI	nil	-	nil
HKFX	nil	nil	-

APPENDIX: Approximation of the Maximum Likelihood Estimator

In scalar AR modelling Box and Jenkins (1972) showed that all currently used autoregressive coefficient estimators, including the Yule-Walker coefficient estimator and the least squares (LS) estimator, are approximations of the maximum likelihood estimator. In VAR modelling the Yule-Walker coefficient estimation can be achieved by using the LS estimation.

To a set of vector data,{**u**(t),..., **u**(N)}, suppose we assume that **u**(t) = 0 for t < 1 and t > N. We can write Equation (2.1) in the following form of a linear regression model by extending the index range for $\varepsilon(t)$ from 1 to s, s=N+p:

$$\overbrace{\begin{bmatrix} \varepsilon(1) \\ \vdots \\ \varepsilon(p) \\ \vdots \\ \varepsilon(N) \\ \vdots \\ \varepsilon(N+p) \end{bmatrix}}^{\mathbf{E}} = \overbrace{\begin{bmatrix} u(1) & & & & & & \\ \vdots & \ddots & & & & 0 & \\ u(p) & \cdots & \cdots & \cdots & \cdots & \cdots & u(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u(N) & \cdots & \cdots & \cdots & \cdots & \cdots & u(N-p) \\ & & & & & & \vdots \\ & 0 & & & & \ddots & u(N) \end{bmatrix}}^{\mathbf{U}} \overbrace{\begin{bmatrix} I \\ A_1 \\ A_2 \\ \vdots \\ \vdots \\ \vdots \\ A_p \end{bmatrix}}^{\mathbf{A}} \qquad (A.1)$$

or $\mathbf{E} = \mathbf{UA}$.

It is noteworthy that the full-windowed design matrix **U** in (A.1) is constructed by extending the sample in both directions by adding p zero vector data. As a result the normal equations matrix, $\frac{\mathbf{U'U}}{N}$, becomes block Toplitz. Therefore this system can be solved in a very efficient manner using Whittle's recursive method.

In considering the zero-non-zero patterned VAR modelling, a VAR can be treated as a system of "seemingly unrelated regressions (SUR)" as originally proposed by Zeller (1962).

Let β_1 denote a (k_1x1) vector containing the non-zero coefficients that appear in the first equation of the VAR:

$$\bar{u}_1 = X_1\beta_1 + \varepsilon_1,$$

where the vectors u_1 and ε_1 are (sx1) vectors and the full-windowed design matrix X_1 is of order (s)x k_1. Hence the VAR has a set of m equations

$$\begin{aligned} \bar{u}_1 &= X_1\beta_1 + \varepsilon_1 \\ \bar{u}_2 &= X_2\beta_2 + \varepsilon_2 \\ &\cdots\cdots\cdots\cdots \\ \bar{u}_m &= X_m\beta_m + \varepsilon_m \end{aligned} \tag{A.2}$$

, where $\bar{u}_j, X_j, \beta_j$ and ε_j are associated with the j-th equation of the VAR.

Then the system of equations in (A.2) can be written in 'stack form' as

$$\overbrace{\begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \vdots \\ \bar{u}_m \end{bmatrix}}^{\bar{U}} = \overbrace{\begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & X_m \end{bmatrix}}^{X} \overbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}}^{ß} + \overbrace{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix}}^{?} \tag{A.3}$$

or $\quad \bar{\mathbf{U}} = \mathbf{?}\,\mathbf{ß} + \mathbf{?}$. Then $E(\mathbf{??'}) = \Omega = V \otimes I_s$.

The objective is to choose **ß** and V so as to maximise the log likelihood function

$$L = \text{const} + \frac{1}{2}\log\left|\Omega^{-1}\right| - \frac{1}{2}\mathbf{?}'\Omega^{-1}\mathbf{?}$$

This is equivalent to choosing B so as to minimise $\frac{1}{2}\mathbf{?}'\Omega^{-1}\mathbf{?}$ (see Magnus 1978).

The resulting estimator of B is Zeller's GLS estimator. However Zeller's GLS method requires knowledge of V, but V is not known and needs to be estimated. Thus the GLS coefficient estimator is an approximation of the maximum likelihood coefficient estimator. For brevity Zeller's procedure is not discussed, though interested readers are referred to Zeller (1962) for details. This GLS coefficient estimator is also identical to the GLS coefficient estimator obtained by using the Yule-Walker relations for fitting the zero-non-zero VAR as described in Section 4, since the design matrices, $X_1, X_2, \ldots$ and X_m in (A.2) are selected as the full-windowed case and the zero-non-zero VAR is treated as a system of SUR.

References

Adler, M. and Lehmann, B. (1983), Deviations from purchasing power parity in the long run, *Journal of Finance* 38, 1471-1487.

Assoe, K.G. (1998), Regime-switching in emerging stock market returns, *Multinational Finance Journal* 2, 101-132.

Bar-Yosef, S., Callen, J.L. and Livnat, J. (1987), Autoregressive modeling of earnings-investment causality, *Journal of Finance* 42, 11-28.

Bekaert, G. (1995), Market integration and investment barriers in emerging equity markets, *World Bank Economic Review* 9, 75-107.

Bekaert, G. and Harvey, C. (1995), Time-varying world market integration, *Journal of Finance* 50, 403-44.

Bekaert, G. and Harvey, C.R. (1997), Emerging equity market volatility, *Journal of Financial Economics* 43, 27-77.

Bekaert, G., Harvey, C.R. and Lumsdaine, R.L. (1999), The dynamics of emerging market equity flows, Working paper, National Bureau of Economic Research.

Bekaert, G. and Hodrick, R.J. (1992), Characterizing predictable components in excess returns on equity and foreign exchange markets, *Journal of Finance* 47, 467-509.

Bernanke, B. and Blinder, A. (1992), The federal funds rate and the channels of monetary transmission, *American Economic Review* 82, 901-921.

Bhattacharya, U., Daouk, H., Jorgenson, B. and Kehr, C. (2000), When an event is not an event: The curious case of an emerging market, *Journal of Financial Economics* 55, 69-101.

Box, G.E.P. and Jenkins, G.M. (1976), *Time Series Analysis: Forecasting and Control*, 2nd edition, Holden-Day, San Francisco.

Caines, P.E., Keng, C.W. and Sethi, S.P. (1981), Causality analysis and multivariate autoregressive modeling with an application to supermarket sales analysis, *Journal of Economic Dynamics and Control* 3, 267-298.

Claessens, S., Dasgupta, S. and Glen, J. (1995), Return behaviour in emerging markets, *World Bank Economic Review* 9, 131-151.

DeJong, P. (1976), The recursive fitting of autoregressions, *Biometrika* 63, 525-530.

Dumas, B. and Solnik, B. (1995), The world price of foreign exchange risk, *Journal of Finance* 50, 445-479.

Eun, C. and Shim, S. (1989), International transmission of stock market movements, *Journal of Financial and Quantitative Analysis* 24, 241-256.

Enders, W. (1996), *RATS Handbook for Econometric Time Series*. Wiley, New York.

Frenkel, J.A. (1981), The collapse of purchasing power parities during the 1970s, *European Economic Review* 16, 145-166.

Froot, K.A., O'Connell, P. and Seasholes, M. (1998), The portfolio flows of international investors, Working paper, Harvard Business School.

Geweke, J. (1982), Measurement of linear dependence and feedback between multiple time series, *Journal of the American Statistical Association* 77, 304-313.

Goetzmann, W.N. and Jorion, P. (1999), Re-emerging markets, *Journal of Financial and Quantitative Analysis* 34, 1-32.

Granger, C.W.J. (1969), Investigating causal relations by econometric models and cross-spectral methods, *Econometrica* 37, 424-438.

Harvey, C.R. (1995b), Predictable risk and return in emerging markets, *Review of Financial Studies* 8, 773-816.

Hassapis, C., Pittis, N. and Prodromidis, K. (1999), Unit roots and Granger causality in the EMS interest rates: the German Dominance Hypothesis revisited, *Journal of International Money and Finance* 18, 47-73.

Hsiao, C. (1981), Autoregressive modeling and money-income causality detection, *Journal of Monetary Economics* 7, 85-106.

Jorion, P. (1991), The pricing of exchange rate risk in the stock market, *Journal of Financial and Quantitative Analysis* 26, 363-376.

Lee, B.S. (1992), Causal relations among stock returns, interest rates, real activity, and inflation, *Journal of Finance* 47, 1591-1603.

Magnus, J.R. (1998), Maximum likelihood estimation of the GLS model with unknown parameters in the disturbance covariance matrix, *Journal of Econometrics* 7, 281-312.

Malliaris, A.G. and Urrutia, J.L. (1992), The international crash of October 1987: Causality Tests, *Journal of Financial and Quantitative Analysis* 27, 353-364.

Penm, J.H.W., Brailsford, T.J. and Terrell, R.D. (1999), A robust algorithm in sequentially selecting subset time series systems using neural networks, *Journal of Time Series Analysis*, forthcoming.

Penm, J.H.W. and Terrell, R.D. (1982), On the recursive fitting of subset autoregressions, *Journal of Time Series Analysis* 3, 43-59.

Penm, J.H.W. and Terrell, R.D. (1984), Multivariate subset autoregressive modeling with zero constraints for detecting causality, *Journal of Econometrics* 24, 311-330.

Price, M. (1994), *Emerging Stock Markets: A complete investment guide to new markets around the world*, New York: McGraw-Hill.

Sewell, S., Stansell, S., Lee, I. and Pan, M. (1993), Non-linearities in emerging foreign capital markets, *Journal of Business Finance and Accounting* 20, 237-248.

Thorbecke, W. (1997), On stock market returns and monetary policy, *Journal of Finance* 52, 635-654.

Whittle, P. (1963), On the fitting of multivariate autoregressions and the approximate factorization of the spectral density matrix, *Biometrika* 50, 129-134.

Whitelaw, R.F. (1994), Time variations and covariations in the expectation and volatility of stock market returns, *Journal of Finance* 49, 515-541.

Wilcox, J.W. (1992), Global investing in emerging markets, *Financial Analysts Journal*, 15-19.

Zeller, A. (1962), An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias, *Journal of the American Statistical Association* 57, 348-368.