Exact Measures of Income in Two Capital-Resource-Time Economies

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Running title. Measures of Income in Capital-Resource-Time Economies

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Abstract. Exact optimal paths are calculated for two closed economies, each with an accumulable capital, a non-renewable resource and exogenous technical progress. The first economy has hyperbolic discounting and (possibly) hyperbolic technical progress. On its optimal path, generally, welfare-equivalent income > wealth-equivalent income > Sefton-Weale income > NNP; and sustainable income = NNP only if consumption is constant. These results support the view that there can be no single definition of income. The Solow (1974) constant consumption solution is a special case; and for low enough discounting, growth is optimal even when technical progress is zero. The second economy has a non-linear frontier between consumption and investment goods. In it, Weitzman's (1997) technical progress premium formula works only if an upwards correction factor is applied to the rate of progress in production, to convert it to a rate of progress in NNP.

Key words. Income, NNP, sustainable, optimal growth, non-renewable resources, exogenous technical progress, hyperbolic discounting, non-linear production.

1. Introduction

This note derives exact formulae for optimal development paths which maximise the present value of utility in two economies with explicit functional forms. Both economies are closed, deterministic, have constant population, and a representative agent. In both, there are three inputs to production: the stock of human-made capital, the depletion of a finite stock of a non-renewable resource, and time in the form of exogenous technical progress. This explains the label of "capital-resource-time economy", though technical progress may be absent as a special case of the first economy. Special features of the first economy are that the utility discount factor and the technical progress factor are hyperbolic rather than exponential functions of time, so it will be called the Hyperbolic economy below. In the second, the division of output between consumption and investment goods is non-linear, so it will be called the NLO (Non-Linear Output) economy. These economies are thus in the tradition of Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974), but with some new twists.

Because of their explicit functional forms, these economies yield no new general theory. Their value is the way they illustrate yet reveal the often limited generality of existing theories, and suggest some new lines of enquiry. The Hyperbolic economy may prove a useful testbed for the recently renewed interest in hyperbolic and other non-constant discounting, especially for the far-distant future (see for example Henderson and Bateman 1995 and Laibson 1997), and it confirms that income is impossible to define uniquely. The NLO economy shows that Weitzman’s (1997) formula for a "technical progress premium" in calculating an economy’s income requires the rate of technical progress in NNP, not in production alone. Other more specific results are noted below. And the two economies may also prove useful in extending the range of algebraically exact, capital-resource economies which can be used to develop and check new theories − a range which otherwise appears to comprise only Solow's constant consumption solution, Stiglitz's asymptotic steady state, and Pezzey and Withagen's (1998) non-steady solution of a Dasgupta-Heal economy.

As a preliminary, Section 2 lists ten features of a capital-resource economy, some of which are always defined in a simplifying way in theoretical models, so that results almost never fully general. Section 3 defines the Hyperbolic economy and lists and interprets its results. Section 4 does likewise for the NLO economy. All calculations (as flagged by "it can be shown that...") are done using straightforward though tedious algebra starting from the necessary first order conditions of the optimal control problem; full details are available from the author. Section 5 concludes.

2. Ten sources of non-generality in theoretical results

Any new features in the Hyperbolic and NLO economies in Sections 3 and 4 spring from the inevitable lack of full generality found in theoretical models of capital-resource economies, even when these are confined to representative-agent models where population is constant, and consumption is the sole determinant of utility. For example, two of the best known results of the mid-1970s use significantly different assumptions, which conceals their interrelationship within a more general overarching theory. Weitzman's (1976) result, on the annuity-equivalent properties of net national product, assumes non-linear production, non-constant consumption, a linear utility function and a constant interest rate. Hartwick's (1977) rule, on constant consumption resulting from zero net investment, assumes linear production, constant consumption, and (implicitly) a non-linear utility function and a declining interest rate.

As a reminder of the simplifying assumptions that have to be made before most results can be found, Table 1 lists ten key features about production functions, utility functions, intertemporal objectives and trade, and typical simplifying assumptions which can be made about them. The notation used is fairly standard, but is fully defined in the next section. Our two exact economies in Sections 3 and 4 make quite different assumptions about features 1, 6, 7 and 8.

Table 1 Ten key features, some of which are simplified in almost all theoretical models of capital-resource economies

No.	General feature	Simplifying assumption
1	Non-linear consumption/ investment frontier (e.g. $F = (C^{\varepsilon}+\dot{K}^{\varepsilon})^{1/\varepsilon}$, $\varepsilon>1$	Linear consumption/ investment frontier (e.g. $F = C + \dot{K}$)
2	Resource extraction costs	No resource extraction costs
3	Capital depreciation	No capital depreciation
4	Unspecified returns to scale in production	Constant returns to scale in production
5	Exogenous technical progress	No exogenous technical progress
6	Non-linear utility function (e.g. $U = C^{1-\eta}/(1-\eta)$)	Linear utility function (e.g. $U = C$)
7	Non-constant utility discount rate, (e.g. discount factor $\phi(t) = (1+\theta t)^{-\rho}$)	Constant utility discount rate (i.e. discount factor $\phi(t) = e^{-\rho t}$, $\rho>0$ constant)
8	Non-constant interest rate $r(t)$	Constant interest rate r
9	No constant consumption goal	Constant consumption goal, $\dot{C} = 0$
10	Closed or large open economy (so prices are endogenous)	Small open economy (so prices are exogenous)

3. The Hyperbolic economy

3.1 Definition, and the optimal path

The economy is a special case of that described in the appendix of Asheim (1997). Population is constant; consumers are identical and have no age structure, with each generation represented as an instant in continuous time, which stretches from zero to infinity; and the economy is closed to trade. The variables below are non-negative quantities along any development path in the economy, using terminology similar to that in Asheim (2000). Less familiar terms, or ones which are often given different meanings in the literature, are highlighted in italics.

$K(t)$ is the non-depreciating, manmade capital stock, $K(0) = K_0 > 0$

$S(t)$ is the non-renewable, natural resource stock, $S(0) = S_0 > 0$

$C(t)$ is consumption of a single produced good

$R(t) = -\dot{S}(t)$ is the resource depletion flow, with zero extraction costs

$F(K,R,t)$ is output; $F = F(K,R)$ if technology is constant

$U(C)$ is instantaneous utility

$\phi(t)$ is the utility discount factor

$\Phi(t) := \phi(t)U_C(C)$ is the consumption discount factor

$W(t) := \int_t^\infty [\phi(s)/\phi(t)]U[C(s)]ds,\ t \geq 0$ is (current) *welfare*

$\Theta(t) := \int_t^\infty [\Phi(s)/\Phi(t)]C(s)ds,\ t \geq 0$ is (current) *wealth*

$\pi^K(t), \pi^S(t)$ are the co-state variables of $K(t)$ and $S(t)$

$\delta(t) := -\dot{\phi}(t)/\phi(t)$ is the *instant discount rate*

$\delta_\infty(t) := \phi(t) \,/ \int_t^\infty \phi(s)ds$ is the *infinite discount rate*

$r(t) := -\dot{\Phi}(t)/\Phi(t)$ is the *instant interest rate*

$r_\infty(t) := \Phi(t) \,/ \int_t^\infty \Phi(s)ds$ is the *infinite interest rate.*

Five definitions of income are then

$A(t)$:= $U^{-1}(\delta_\infty W)$ is *welfare-equivalent income* (Asheim 2000)

$Y_e(t)$:= $r_\infty(t)\Theta(t)$ is *wealth-equivalent income* (Asheim 2000)

$SW(t)$:= $[\int_t^\infty r(s)\Phi(s)C(s)ds]\ /\ \Phi(t)$ is *Sefton-Weale income* after Sefton and Weale (1996)

$Y(t)$:= $C(t) + [\pi^K(t)\dot{K}(t) + \pi^S(t)\dot{S}(t)]/U_C(t)$ is Net National Product

$M[K(t),S(t)]$:= max C s.t. $C(t') = C$ for all $t' \geq t$, i.e. *sustainable income* or the maximum sustainable consumption level. M is calculated only when there is no technical progress, as an analytic solution is not available when there is technical progress.

The representative agent acts to maximise welfare at all times, and the resulting path is called optimal. Existence and uniqueness are assumed.

The specific functional forms used in the Hyperbolic economy are:

Production:	$F = K^\alpha R^\beta(1+\theta t)^\nu = \dot{K}+C,\ \theta > 0,\ \nu \geq 0$)	
Instantaneous utility:	$U(C) = C^{1-\alpha}/(1-\alpha),\ \ 0 < \alpha < 1$)	[3.1]
Discount factor:	$\phi(t) = (1+\theta t)^{-\rho},\ \ \rho > 0$)	

Restrictions on parameters and algebraic abbreviations are:

$$\beta < \alpha < \alpha+\beta \leq 1 \qquad [3.2]$$

$$\rho > 1+\alpha-\beta+\nu \qquad [3.3]$$

$$\xi := (\rho-\alpha-\nu)/(1-\beta) \qquad [3.4]$$

$$\sigma := (\alpha+\nu-\beta\rho)/(1-\alpha)(1-\beta) \qquad [3.5]$$

$$\Rightarrow \xi+\sigma = \rho+\alpha\sigma = [\rho(1-\alpha-\beta)+\alpha(\alpha+\nu)]\ /\ (1-\alpha)(1-\beta)$$

$$\theta := [\alpha(\xi-1)^\beta S_0^{\ \beta}/(\xi+\sigma)K_0^{\ 1-\alpha}]^{1/(1-\beta)} \qquad [3.6]$$

Restriction [3.6] places the economy on a (hyperbolically) steady state path from time zero. Without it, only the asymptotically steady state can be computed analytically, much as in Stiglitz (1974).

It can be shown that the optimal paths are then as follows:

Consumption	$C(t)$	$= [(\rho-\alpha)\theta K_0/\alpha]\ (1+\theta t)^{\sigma}$	[3.7]
Capital	$K(t)$	$= K_0(1+\theta t)^{\sigma+1}$	[3.8]
Resource stock	$S(t)$	$= S_0(1+\theta t)^{-(\xi-1)}$	[3.9]
Resource flow	$R(t)$	$= (\xi-1)\theta S_0(1+\theta t)^{-\xi}$	
Output	$F(t)$	$= [(\xi+\sigma)/(\rho-\alpha)]C(t)$	
Instant interest rate	$r(t)$	$= (\xi+\sigma)\theta/(1+\theta t)$	[3.10]
Infinite interest rate	$r_\infty(t)$	$= (\xi+\sigma-1)\theta/(1+\theta t)$	

3.2 The five measures of income

From the above results, it can be shown that the five measures of income on the optimal path of the Hyperbolic economy are at any time:

For any rate of technical progress, $\nu \geq 0$*:*

Welfare-equivalent income	$A(t)$	$= [1+(1-\alpha)\sigma/(\xi-1)]^{1/(1-\alpha)}\ C(t)$	[3.11]
Wealth-equivalent income	$Y_e(t)$	$= [1+\sigma/(\xi-1)]\ C(t)$	[3.12]
Sefton-Weale income	$SW(t)$	$= (1+\sigma/\xi)\ C(t)$	[3.13]
Net national product	$Y(t)$	$= [1-\nu/(\rho-\alpha)](1+\sigma/\xi)\ C(t)$	[3.14]

For no technical progress, ν *= 0 only:*

Sustainable income	$M(t)$	$= [(\xi+\sigma)(\alpha-\beta)/(\xi-1)\alpha]^{\beta/(1-\beta)}(1+\sigma/\xi)C(t)$	[3.15]

Several features of these results are worth noting:

(a) Since all parameters are positive, as are $(1-\alpha)$, $(\xi-1)$ and $(\rho-\alpha)$ thanks to [3.2]-[3.4], the first four income measures are in the strict size order $A > Y_e > SW > Y$. This is of course consistent with the non-strict order given for a general economy in Asheim (2000).

(b) The presence of the $-\nu/(\rho-\alpha)$ term in net national product and its absence in wealth-equivalent income and Sefton-Weale income clearly suggests some kind of "technical progress premium", which is overlooked by the national accounting definition of income, but included in present-value-equivalent definitions. However, it remains to be seen if Weitzman’s (1997) formula for the technical progress premium, which holds for an economy with a constant interest rate, can be generalised to the non-constant interest rate here (see also Section 4.2).

(c) It can be shown that

$$\sigma \gtrless 0 \iff \text{economy optimally grows or declines} \iff M \gtrless Y \qquad [3.16]$$

confirming that in general, sustainable income M is only loosely related to net national product Y.

(d) From [3.5], if $\alpha+\nu-\beta\rho = 0$, then $\sigma = 0$ (and $\xi = \rho$), giving the Solow (1974) constant consumption path $C = (1-\beta)\{K^{\alpha-\beta}[(\alpha-\beta)S]^{\beta}\}^{1/(1-\beta)}$ $\forall t$ as a special case of the optimal path of the Hyperbolic economy. If also $\nu > 0$, then $A = Y_e = SW = C > Y$. The economy can perpetually consume (C) more than it "produces" (Y) because time is itself productive here, but the value of time (i.e. of exogenous technical progress) is omitted from Y. Only if $\sigma = 0$ and there is no technical progress, $\nu = 0$ (hence $\rho = \alpha/\beta$), are all five income measures defined, constant and identical to consumption: $A = Y_e = SW = Y = M = C$.

(e) An initial idea of how big the differences can be among the income definitions comes from a numerical example. If $\rho = 2$, $\alpha = 0.6$, $\beta = 0.05$, $\nu = 0.4$, $K_0 = 1000$ and $S_0 = 100$, then to 3 decimal places, $\xi = 1.053$, $\sigma = 2.368$ and $\theta = 0.010$. The various instantaneous exponential rates in the economy at time $t = 0$ are then

utility discount rate	$\rho\theta$	$= 0.019$
technical progress rate	$\nu\theta$	$= 0.004$

consumption growth rate	$\sigma\theta$	$= 0.023$
instant interest rate	$(\xi+\sigma)\theta$	$= 0.033$

These *initial* rates are the same order of magnitude as data for constant rates used by Weitzman (1997) and other authors, and so are not wildly implausible. Inserting the numbers into [3.11]-[3.14] then shows that the four income measures defined for $\nu > 0$ vary dramatically in this example, being (to one decimal place):

welfare-equivalent income	$A(t) =$	$1573.6\ C(t)$
wealth-equivalent income	$Y_e(t) =$	$46.0\ C(t)$
Sefton-Weale income	$SW(t) =$	$3.3\ C(t)$
net national product	$Y(t) =$	$2.3\ C(t)$

However, any empirical significance of these results is hard to judge, since all the rates here decline over time as $1/(1+\theta t)$, contrary to empirical experience in Western economies over the last two centuries or so. Perhaps more significant are the results from an exact solution of the Stiglitz (1974) economy, where it can be shown that for the parameter values $\rho = 0.025$, $\alpha = 0.6$, $\beta = 0.05$ and $\nu = 0.01$ (a fairly standard set of exponential rates, except for the role of α in $U(C)$), the income measures are $A = 3.2C$, $Y_e = SW = 2.5C$ and $Y = 1.5C$.

(f) A general view supported by the above results is that *there can be no single or "best" definition of income*. This case has been made by others, on the grounds that measuring income serves many different purposes, for example:

> "...charting business cycles, comparing prosperity among nations, observing industrial structure, measuring factor shares and so on. ...real income may be interpreted as a family of concepts, each member of which is best for some particular purpose." (Usher 1994, p124)

The results here remind us that even as a measure of prosperity, income is hard to define uniquely. Clearly, measuring current prosperity should take proper account of the future, and consumption alone is not a proper measure. But this leaves undefined what kind of future society may want. It can choose from an infinitude of intertemporal welfare objectives, and there is no shortage of unresolved arguments about which is the right one to maximise. Even when present value maximisation with a particular discount factor is chosen as the objective, there is still a difference, given diminishing marginal utility of consumption, between the welfare-equivalence and wealth-equivalence methods of accounting for the future. Hicks (1946, Ch 14) himself emphasised many different definitions of income. Moreover, he used a framework (an individual facing exogenous prices, rather than a closed economy facing endogenous prices, as above) in which some of the income definitions above are indistinguishable. So the continued use of the phrase "Hicksian income" (as for example in Nordhaus 2000) can easily be ambiguous, and has been deliberately avoided here.

3.3 Sustained growth

A result that could have been listed in the previous subsection, but seems worth giving special prominence to, is that optimal consumption in the Hyperbolic economy is steadily growing if the discount rate is low enough ($\rho < (\alpha+\nu)/\beta$ so that $\sigma = \dot{C}/C > 0$). Moreover, *sustained growth can be optimal even if there is no technical progress* ($\nu = 0$). This reflects the way that a hyperbolic utility discount rate declines over time, in a way that can match the declining returns to capital investment in a capital-resource economy. By contrast, in the main Dasgupta and Heal (1974) economy, the discount rate is constant, and ultimately becomes greater than the declining return to capital. Hence optimal consumption asymptotically falls toward zero there, no matter how small the discount rate.

4. The Non-Linear Output (NLO) economy

4.1 Definition, and the optimal path

The definition of the NLO economy is as for the Hyperbolic economy except for the specific functional forms, which are:

Production:	$F = K^\alpha R^\beta e^{\nu t} = (\dot{K}^2 + C^2)^{½}$; $\dot{K} > 0$	)
Instantaneous utility:	$U(C) = C$	) [4.1]
Discount factor:	$\phi(t) = e^{-\rho t}$	)

This could be viewed as a variant on the Stiglitz (1974) optimal economy, with non-linearity in the output function rather than in the utility function. All parameters are strictly positive, with other restrictions and algebraic abbreviations being:

$$\beta < \alpha < \alpha+\beta < 1 \qquad [4.2]$$

$$\beta\rho < \nu < (1-\alpha)\rho \qquad [4.3]$$

$$\psi := [(1-\alpha)\rho-\nu]/(1-\alpha-\beta) > 0 \qquad [4.4]$$

$$\gamma := (\nu-\beta\rho)/(1-\alpha-\beta) > 0 \quad \text{and } \psi + \gamma = \rho \qquad [4.5]$$

$$(\rho\gamma)^{½}K_0^{\,1-\alpha} = \alpha^{½}(\psi S_0)^\beta \qquad [4.6]$$

[4.2], [4.3] $\Rightarrow \alpha\nu < (1-\alpha)(1-\beta)\rho \Rightarrow \alpha(\nu-\beta\rho) < (1-\alpha-\beta)\rho \Rightarrow$

$$\rho > \alpha\gamma \qquad [4.7]$$

Restriction [4.6] is needed to put the NLO economy on a steady state, analytically soluble path from time zero. It can then be shown that optimal paths are as follows:

Consumption	$C(t)$	$= (\rho/\alpha-\gamma)^{½}\gamma^{½}K(t)$ where	[4.8]
Capital	$K(t)$	$= K_0e^{\gamma t}$	[4.9]
Resource stock	$S(t)$	$= S_0e^{-\psi t}$	[4.10]
Resource flow	$R(t)$	$= \psi S(t)$	
Output	$F(t)$	$= (\gamma\rho/\alpha)^{½}K(t)$	
Instant interest rate	$r(t)$	$= \rho$	

From [4.5], the condition for $\gamma \gtrless 0$, and hence for optimal consumption to be forever rising or falling, is $\nu/\beta \gtrless \rho$, which is identical to the asymptotic condition in Stiglitz (1974, p136).

4.2 The technical progress premium

Using the above results, it can be shown that the four measures of income based on the PV-optimal path of the NLO economy are

Welfare-equivalent income	$A(t)$	$= \rho C(t)/(\rho-\gamma)$	[4.11]
Wealth-equivalent income	$Y_e(t)$	$= \rho C(t)/(\rho-\gamma)$	[4.12]
Sefton-Weale income	$SW(t)$	$= \rho C(t)/(\rho-\gamma)$	[4.13]
Net national product	$Y(t)$	$= (1-\beta)\rho C(t)/(\rho-\alpha\gamma)$	[4.14]

There is no expression here for sustainable income $M(t)$ because [4.3] means that $\nu > 0$, and calculating M requires $\nu = 0$ (no technical progress). Moreover, $\gamma > 0$ in [4.5] means that the (fortuitous) case of constant consumption is not allowed, since $\gamma = 0$ would make $\dot{K} = 0$, and hence invalidate the proofs.

Thanks to the linear utility function and the constant interest rate, the first three income measures [4.11-13] are identical. The interest here lies in the "technical progress premium" (TPP) defined by Weitzman (1997) as:

$$\text{TPP} := (Y_e/Y)-1 \quad = \nu/(\rho-\gamma)(1-\beta) \text{ for the NLO economy.} \qquad [4.16]$$

Comparing this with Weitzman's formula that TPP = $\lambda/(r-g)$, where

r, the interest rate, = ρ here;

g, the growth rate of (inclusive or green) NNP, = $\dot{Y}/Y = \gamma$ here;

$\lambda = \int_t^\infty (\partial Y/\partial s) e^{-rs} ds \,/\, \int_t^\infty Y(s) e^{-rs} ds$

shows that

$$\lambda = \nu/(1-\beta) \qquad [4.17]$$

Weitzman described λ as the "average future growth rate of the...pure effect of time alone on enhancement of productive capacity not otherwise attributable to capital accumulation" (p7) or the "annual growth rate of total factor productivity" (p11). Given the production function $F = K^{\alpha}R^{\beta}e^{\nu t}$, one might then think that $\lambda = \nu$ in the NLO economy, rather than $\nu/(1-\beta)$ as in [4.17]. But since we have exponential growth here, the value of λ can be confirmed directly by calculating $(\partial Y/\partial t)/Y$, from:

$$Y = C + (\pi^K\dot{K} + \pi^S\dot{S})/U_C = C + \pi^K(F^2-C^2)^{\frac{1}{2}} - \pi^S R$$

$$\Rightarrow\ \partial Y/\partial t = \pi^K \tfrac{1}{2}(F^2-C^2)^{-\frac{1}{2}}2F\nu F = (\dot{K}/C)(\dot{K})^{-1}\nu F^2 = \nu F^2/C, \qquad \text{and also}$$

$$\Rightarrow\ Y = C + (\dot{K}/C)\dot{K} - (\beta F^2/CR)R = (1-\beta)F^2/C,$$

so $\lambda = (\partial Y/\partial t)/Y = \nu/(1-\beta)$,

where we have used the costate variables $\pi^K = \dot{K}/C$ and $\pi^S = \beta F^2/CR$. This emphasises that one must distinguish between technical progress (at rate ν) in *production F*, and technical progress (at rate λ) in *net national product Y*. Intuitively, in this economy the $e^{\nu t}$ term in $F(K,R,t)$ gives progress only in producing C and $\dot{K}$, but not in the resource rent $\pi^S R$, so the progress λ in making $C + \pi^K\dot{K} - \pi^S R$ is *higher* (by a factor $1/(1-\beta)$) than the progress ν in F alone. Exactly the same result (formula [4.17], and the underlying intuition) can be shown to apply to the Stiglitz (1974) asymptotic economy.

Note also that the upward adjustment factor $1/(1-\beta)$ needed to correct from technical progress in production to technical progress in NNP is greater, the greater is the power β of the resource in production. If this power β is less than 0.05, as is generally held to be the case in modern industrial economies, then the adjustment needed is probably smaller than the measurement error in all the other terms in NNP. But we have the mildly paradoxical result that if resources ever became more important in production, then it would become correspondingly more important not to forget the technical progress premium needed to convert NNP into a measure of wealth-equivalent income.

5. Conclusions

Exact solutions have been presented for the optimal paths of two economies with accumulable capital, a non-renewable resource, exogenous technical progress, and specific functional forms. They illustrate some significant points in recent literature on income and sustainability accounting, and should prove useful as testbeds for future theoretical enquiry. In the first, "Hyperbolic" economy, a combination of hyperbolic discounting and hyperbolic technical progress makes five measures of income − welfare-equivalent income, wealth-equivalent income, Sefton-Weale income, net national product and sustainable income − all quite different, and it is hard to view any one measure as "the" definition of income, Hicksian or otherwise. The first four are in descending size order, and a rough numerical example suggests that these differences may be substantial. The optimal (present-value-maximising) consumption path becomes the Solow (1974) constant consumption path for a specific discount rate. A low enough discount rate leads to the optimal consumption level growing forever, even if there is no technical progress.

The second, "Non-Linear Output (NLO)" economy combines a non-linear trade-off between consumption and investment outputs, with a linear utility function. This reveals a wrinkle in Weitzman’s (1997) result that a technical progress premium (TPP) must be added to net national product to give a true measure of an economy’s productive potential. The wrinkle is that the rate of technical progress in Weitzman’s formula for the TPP must be that for net national product. This is larger than the rate of progress in production alone, because of the deduction of resource rents in calculating

NNP; and the required adjustment gets larger as the power of the resource in production becomes larger. Further research on the empirical significance of the above results for both economies would appear worthwhile.

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