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WORKING PAPERS IN ECONOMICS AND ECONOMETRICS

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Working Paper No. 406

December, 2001

ISBN: 086831 406 4

Production Externalities and Multi-task Principal-Agent Problems

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December 12, 2001

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In relationships between principals and agents, such as those between employers and employees, shareholders and directors, insurers and insured businesses, regulators and the regulated, and so on, agents frequently undertake multiple tasks, some of which produce observable outputs, while others do not. Modelling such multi-task agency problems in the presence of principal-agent problems has, in general, proved difficult. One approach, developed by Holmström and Milgrom (1987, 1994), is to assume that the agent's space of actions is so rich that the principal is confined to linear payment structures.

Recently, Quiggin and Chambers (1998) have proposed an alternative representation of the principal-agent problem based on the assumption of an underlying state-contingent technology, which allows for a closed-form representation of the agency cost function, and a simple solution to the principal's problem. Their analysis, however, confines attention to problems involving scalar outputs. The objective of this paper is to extend their analysis by considering the more difficult class of problems where some outputs are observable and others are not.

Following Chambers and Quiggin (1996), we consider a concrete example drawn from the generic class of problems where producers produce both socially desirable and socially undesirable outputs. In particular, we consider the production choices facing a farmer producing two outputs— an agricultural output (corn) and chemical runoff. Because chemical runoff is a by-product of the output and farmers receive direct benefits from producing corn, chemical runoff can have a positive shadow value to farmers, even though they may not value it directly. Runoff also has socially undesirable effects because it pollutes water and other ecosystems. Therefore, although the farmer has private incentives to emit runoff, the rest of society wants to control runoff. In this paper the interests of society are represented by a planner, who develops a payment rule for the farmer, conditional on observable information on the output, runoff, the inputs committed, and the state of nature.

In the informational structure considered by Chambers and Quiggin (1996), farmers' outputs of corn and runoff were unobservable, so that the only policy instrument available to the planner was a state-contingent payment. In the present paper, we consider a range of alternative informational assumptions on the two kinds of output and the state of nature.

Our main focus is the multi-task case where output of corn is observable to the planner, but runoff pollution and the state of nature are not. Relative to the first-best, a social planner then faces two problems – an externality problem associated with the fact that runoff pollution is unobservable and an agency problem associated with the fact that the state of nature is unobservable. We are interested in the interaction between these problems. In general, as Chambers and Quiggin (1996,1999) observe, the nature of the interaction will depend on whether runoff is 'risk-increasing' or, more precisely, on whether runoff is a complement for increasing riskiness in the state-contingent output plan for corn. To simplify the analysis of this issue, we focus on a special case of the production technology, in which runoff is associated solely with output in the more favourable state of nature, and is therefore strongly risk-complementary in the sense of Chambers and Quiggin (1999). The implication is that, whereas the externality encourages producers in a private optimum to produce too much output in the more favourable state, the absence of full insurance encourages them to produce too little output in that state. It is the planner's job to balance these tensions in an incentive-compatible manner.

We are particularly interested in the question of whether a fixed standard level of output will ever be superior to an incentive-based output scheme. There exists a long-standing debate in the regulatory literature on the relative efficacy of taxes versus standards in achieving environmental policy goals. When production is stochastic, the conventional wisdom, which does not consider agency effects, is that a standard can dominate a tax as a policy device, and that the relative desirability of taxes versus standards hinges upon the relative impact of uncertainty on environmental damage and productivity (Weitzman, 1974). We demonstrate that under the assumption of risk-complementary pollution, standards can only be optimal when they are associated with a complete moratorium on production of the crop output. That is, it is better to shut down the entire production operation rather than to expose Society to the damage associated with the chemical runoff.

The paper is organised as follows. First, we describe the state-contingent production technology and derive the characteristics of the farmer's private cost function. Next we derive the optimal solutions under a range of informational assumptions. Finally, we undertake comparative static analysis and compare the characteristics of the alternative solutions.

1 The Model

There are two types of individuals: the planner and a farmer who produces two outputs, runoff from land and an agricultural crop, under conditions of production uncertainty. Only the farmer engages in productive activity, and uncertainty is modelled by 'Nature' making a choice from a set of two alternatives, $\Omega = \{1, 2\}$. Production relations are governed by a state-contingent input correspondence (Chambers and Quiggin, 1999), $X : \Re_+^2 \times \Re_+ \to \Re_+^n$, defined by

$$X(\mathbf{z},p) = \{\mathbf{x} \in \Re_+^{\mathbf{n}} : \mathbf{x} \text{ can produce } (\mathbf{z},p)\ \}, \quad \mathbf{z} \in \Re_+^2, p \in \Re_+.$$

Here $\mathbf{x}$ represents an input vector that is committed prior to the resolution of uncertainty, i.e., before Nature makes its choice from Ω, and $\mathbf{z}$ is a vector of state-contingent agricultural output also chosen before Nature makes its choice. If Nature picks state j then the *ex post* or realized value of the agricultural output is z_j. p is a non-stochastic level of runoff associated with the production of the state-contingent output vector and the input committal.

The planner is risk-neutral. The farmer's preferences only depend directly upon his remuneration from farming, which we denote y, and the vector of inputs that he commits to farming. The farmer is assumed to have no direct preferences over either the level of runoff that he emits or the level of output that he produces[1]. His *ex post* utility function is given by

$$w(y, \mathbf{x}) = u(y) - g(\mathbf{x})$$

where $u : \Re \to \Re$ is a strictly increasing, strictly concave, and twice differentiable function, and $g : \Re_+^n \to \Re$ is a continuous, strictly increasing, and strictly convex effort-evaluation function. Both u and g are taken to be satisfy the von Neumann-Morgenstern postulates. The planner and

[1] In other words, the farmer does not value runoff pollution for pathological reasons. He only values it because its committal brings with it the possibility of lowering the cost of producing the stochastic output.

the farmer share the same probabilities of a state j occurring and that probability is denoted by $\pi_j \in \Re_{++}$ $(j = 1, 2)$ with $\pi_1 + \pi_2 = 1$.

Two indirect representations of the state-contingent input correspondence prove useful. The first is the farmer's *cost function* for a given vector of state-contingent output and a pollution level. It is defined by

$$c(\mathbf{z}, p) = \min \{g(\mathbf{x}) : \mathbf{x} \in X(\mathbf{z}, p)\}$$

if there exists an $\mathbf{x} \in X(\mathbf{z}, p)$ and ∞ otherwise. To focus our analysis, we assume that $c(\mathbf{z}, p)$ assumes the following particular form[2]:

$$c(\mathbf{z}, p) = c_1 z_1 + c^2(z_2, p),$$

where $c_1 > 0$ and $c^2(z_2, p)$ is positively linearly homogeneous in its arguments and strictly increasing in z_2. Hence, the overall cost function is also positively linearly homogeneous. This particular technology is a special case of what Chambers and Quiggin (1999) refer to as a state-allocable technology. Specifically, it implies that the input vector $\mathbf{x}$ can be targetted at two production activities: preparing the first state-contingent output and preparing the second state-contingent output and the runoff. (More formally, this separability implies that $X(\mathbf{z}, p) = X_1(z_1) + X_2(z_2, p)$.) Runoff, therefore, is most naturally associated with the production of the second state-contingent output.

The second cost function that we consider is the one that is relevant when the farmer privately chooses the level of runoff associated with a given vector of state-contingent outputs. Denote the farmer's *private cost function* by

$$C(\mathbf{z}) = Min_p \{c(\mathbf{z}, p)\}.$$

Let

$$\rho = \arg\min_{\frac{p}{z_2}} \left\{ c^2\left(1, \frac{p}{z_2}\right) \right\}.$$

[2]As later discussion will reveal, this form is chosen to emphasize runoff's role as a risk-complement in the farmer's private-cost minimization problem.

It follows immediately that

$$
\begin{aligned}
C(\mathbf{z}) &= c_1 z_1 + Min_p \left\{ c^2 (z_2, p) \right\} \\
&= c_1 z_1 + z_2 Min_{\frac{p}{z_2}} \left\{ c^2 \left(1, \frac{p}{z_2} \right) \right\} \\
&= c_1 z_1 + z_2 c^2 (1, \rho) \\
&\equiv c_1 z_1 + c_2 z_2.
\end{aligned}
$$

2 The Planner's Problem

Given the occurrence of state s, the farmer's information set consists of four items; the state s, the output z_s and pollution p, and the farmer's committal of effort $\mathbf{x} \in \Re^n_+$. Government intervention is feasible only if the planner can observe and act upon some of this information.

In what follows, we shall typically assume that the government cannot observe the farmer's commitment of inputs. Chambers and Quiggin (1996) have already treated the case where only the state s is observable. The case when z_s and p are both observable is equivalent to the point-source, principal-agent problem considered by Quiggin and Chambers (1998), with pollution being subject to a Pigovian tax and therefore regarded as a standard input to production. The case when only p is observable is a standard externality problem. When s and p are observable the planner can infer $\mathbf{z}$ from knowledge of the farmer's technology and preferences. This case is, therefore, equivalent to full information.

In this paper, we consider the remaining four informational assumptions. In the zero-information or *private-maximization* case, the planner has no information and is, therefore, irrelevant to the problem. The farmer simply maximises private benefits. In the other polar case, referred to as the *full-information* case, the planner observes the entire information set. Two intermediate cases are considered. First, there is the case when the planner can observe s and z, but not p. This is referred to as the *externality with insurance problem.* Second, there is the case when the planner can observe only z. This is referred to as the *limited information* case.

The possible cases are depicted in Table 1 documenting what is and what is not observable to the government:

Table 1: Problem Taxonomy

Observables			Class of Problem
Corn	Runoff	State of Nature	
yes	yes	yes	full information
yes	yes	no	moral hazard
yes	no	yes	externality with insurance
yes	no	no	limited information
no	yes	yes	equivalent to full information
no	yes	no	externality
no	no	yes	Chambers and Quiggin (1996)
no	no	no	private optimum

We are mainly interested in the limited information social optimum. In this multitask problem, the planner must use one piece of information, the observed output, to pursue two social goals, the achievement of an optimal level of pollution mitigation and the provision of appropriate insurance to the farmer.

3 Farmer's Private Maximization Problem

In the absence of government intervention, the farmer simply sells his product on the market, the price of which is normalized to one. He chooses his state-contingent output vector according to:

$$Max_{\mathbf{z}} \left\{ \pi_1 u\left(z_1\right) + \pi_2 u\left(z_2\right) - c_1 z_1 - c_2 z_2 \right\}.$$

It will be convenient to make a simple change of variables, $u_i = u(z_i)$ whence $z_i = u^{-1}(u_i) = h(u_i)$. Because u has been assumed to be strictly increasing and strictly concave, h will be strictly increasing and strictly convex. After making this change of variables, the corresponding first-order conditions can be written

$$\pi_1 - c_1 h'\left(u_1^p\right) = 0, \quad (1)$$

$$\pi_2 - c_2 h'\left(u_2^p\right) = 0, \quad (2)$$

where superscript p denotes the solution value for the farmer's private optimization problem. These conditions simply require the farmer to equate his private marginal benefit from a unit of state-contingent production, $\frac{\pi_i}{h'(u_i)}$, to his private marginal cost. We assume that

$$\frac{c_2}{\pi_2} < \frac{c_1}{\pi_1}. \tag{3}$$

Expression (3), or its converse, must hold and can be viewed as a normalization which, by the strict convexity of h, orders states of nature so that state 2 is the 'good' state of nature in the sense that in the absence of government intervention its state-contingent output (and accordingly the farmer's return) would always be higher than the state-contingent output in state 1, i.e.,

$$z_1^p = h\left(u_1^p\right) < h\left(u_2^p\right) = z_2^p.$$

Notice, in particular, that the farmer doesn't act to stabilize production across states of nature. The reason, of course, is that any income insurance generated by the farmer must be in the form of potentially costly self insuring actions taken by the farmer in arranging his production plans. Thus, one naturally expects that the farmer's preferences towards risk play an important role in determining the farmer's optimal choice of z_2, and, thus of the level of runoff pollution that he or she emits. For example, one intuitively expects a decrease in the cost of producing the second state-contingent output to lead to an increase in the opportunity cost of self insurance because a fall in that marginal cost offers a marginal inducement for the farmer to expand z_2, thus increasing the dispersion of the state-contingent output vector. As this dispersion increases, the level of runoff also increases. More formally, notice that upon differentiating (2) logarithmically, we have

$$\hat{z}_2 = -\frac{1}{r\left(z_2\right)}\hat{c}_2,$$

where circumflexes over variable denote percentage change and $r\left(z_2\right)$ is the Arrow-Pratt relative risk aversion coefficient. The more risk averse is the farmer in this sense, the less one expects him to expand his output as a result of a fall in c_2.

Condition (3), by ordering the states of Nature, also determines the role that runoff plays in enhancing or mitigating the riskiness of the production process. Consider how the farmer adjusts his runoff in response to an increase in the riskiness of $\mathbf{z}^p$ arrived at by taking a multiplicative spread of the state-contingent output from (z_1^p, z_2^p) to $\left(z_1^p - \frac{\pi_2}{\pi_1}\delta, z_2^p + \delta\right)$ for suitably small $\delta > 0$. Because runoff varies directly with state-2 output, this increase in the riskiness of the production bundle leads the farmer to increase his runoff. In the terminology of Chambers and Quiggin (1996, 1999), runoff is a *risk complement* to production of the agricultural product in the neighborhood of $\mathbf{z}^p$. If the reversal of (3) held, then state-2 output would always be lower than state-1 output in the farmer's private maximization problem, and runoff would then be viewed as a *risk substitute* because lower usage levels would be associated with more risky production patterns in the neighborhood of $\mathbf{z}^p$.[3] Although we do not directly consider the case of risk substitutes in what follows, it is a straightforward extension of our results to consider such a relationship between runoff and the state-contingent outputs.

The assumption that runoff is most naturally associated with technologically favorable states of nature appears particularly plausible in the case of chemical fertilizers, because application of chemical fertilizers primarily acts to increase yields when production conditions, for example rainfall, are favorable. Intuitively, therefore, we would expect such inputs to be risk complements. By contrast, some forms of runoff, for example, pesticide contamination, may be more closely associated with attempts to control damage to output in, otherwise, unfavorable states of nature, and they would seem more reasonably deemed as risk substitutes.

4 Full-information Social Optimum

We now discuss the planner's problem when he or she has complete and symmetric information with the farmer, and seeks to maximise social welfare. Then the planner's problem is to:

$$Max_{\mathbf{x},\mathbf{z},\mathbf{y},p} \left\{ \begin{array}{c} \pi_1 u(y_1) + \pi_2 u(y_2) + \pi_1 (z_1 - y_1) + \pi_2 (z_2 - y_2) - m(p) \\ -g(\mathbf{x}) : \mathbf{x} \in X(\mathbf{z}, p) \end{array} \right\}.$$

where $m(p) > 0$ is a strictly increasing and strictly convex damage function associated with runoff.

[3] In Chambers and Quiggin (1996), the terminology risk increasing and risk reducing was used in place of risk complementary and risk substituting, respectively.

Because the farmer is the true residual claimant for his or her effort $\mathbf{x}$, this problem can be rewritten

$$Max_{\mathbf{z},\mathbf{y},p}\left\{\begin{array}{c}\pi_1 u(y_1)+\pi_2 u(y_2)+\pi_1(z_1-y_1)+\pi_2(z_2-y_2)-m(p)\\ -c(\mathbf{z},p)\end{array}\right\}. \tag{4}$$

This version of the the planner's problem (4) establishes the well-known fact that when the state of Nature and both outputs are fully observable, observability of the inputs does not enhance the planner's ability to achieve the full-information optimum (Harris and Raviv, 1979).

First-order conditions for this problem require that:

$$\begin{aligned} u'(y_1) &\leq 1, \quad y_1 \geq 0 \\ u'(y_2) &\leq 1, \quad y_2 \geq 0 \\ \pi_1 &\leq c_1, \quad z_1 \geq 0 \\ \pi_2 &\leq c_z^2(z_2,p), \quad z_2 \geq 0 \\ -m'(p) &\leq c_p^2(z_2,p), \quad p \geq 0 \end{aligned} \tag{5}$$

in the notation of complementary slackness. Denote the solution to these first-order conditions by a superscript F for full information. To simplify our analysis, we assume that $\pi_1 < c_1$, so that in the full-information case, the planner would arrange production so that no output occurs in state 1, hence $z_1^F = 0$.[4] We also assume that $u'(0) > 1$ so that with full information, an interior solution is required for payments in both states of nature. Hence by the strict concavity of u, we conclude immediately that in the full information case $y_1^F = y_2^F = y^F$ where y^F is defined by the implicit solution to

$$u'\left(y^F\right) = 1.$$

From these first-order conditions, not surprisingly, we can also ascertain that in the full-information case, the farmer emits less runoff per unit of output than he does privately. By the homogeneity properties of the cost function, the first-order condition for the optimal level of runoff

[4] If this condition were not satisfied, output of the first state-contingent output would not be well defined in the full-information case.

in the full-information case can be rewritten as

$$-m'\left(p^F\right) \leq c_p^2\left(1, \frac{p^F}{z_2^F}\right), \quad p \geq 0,$$

while the first-order condition for the private-cost minimization problem can be written as

$$0 \leq c_p^2\left(1, \rho\right), \quad \rho \geq 0.$$

By the convexity of the cost function in runoff, it follows immediately, therefore, that:

Result 1

$$\frac{p^F}{z_2^F} \leq \rho.$$

Moreover, we observe that in the full-information case the optimal pattern of state-contingent production and the level of runoff emitted are all independent of the farmer's attitude towards risk.

5 Externality with Insurance problem

We now turn to the case where the state of nature is observable, but runoff pollution is not. Denote the (constrained) optimal payment vector as $\mathbf{y}^E$, the optimal output vector as $\mathbf{z}^E$ and the optimal pollution as p^E. Because the state of nature and output produced are both observable, the planner can specify contracts in terms of state-contingent output levels and state-contingent payments. It is easy to see that the state-contingent payments satisfy:

$$y_1^E = y_2^E = y^F.$$

However, because the planner cannot directly observe pollution, the producer will choose

$$p^E = \rho z_2^E.$$

Hence the planner's problem reduces to choose (z_1^E, z_2^E) to

$$Max_{\mathbf{z}}\left\{\pi_1 z_1 + \pi_2 z_2 - \pi_2 m(\rho_2 z_2) - c_1 z_1 - c_2 z_2\right\}.$$

The optimal solution must satisfy:

$$z_1^E = 0 \tag{6}$$

$$\pi_2 - \rho m'(\rho z_2^E) \leq c_2 \quad z_2^E \geq 0. \tag{7}$$

State-1 output is still set to zero because its marginal cost of production exceeds its marginal benefit. The second state contingent output is chosen so that its marginal cost of production equals its marginal social benefit. Its marginal social benefit is its direct societal benefit less the pollution damage it causes. Not surprisingly, therefore, the optimal level of the second state-contingent output varies inversely with ρ. However, just as in the full-information problem, the optimal levels of pollution and both state-contingent outputs are independent of the producer's attitude toward income risk. This happens, of course, because the planner's optimal response is to stabilize income across states, thus effectively providing the producer with perfect income insurance.

6 Limited information Social optimum

We now presume that the planner cannot observe either the state of nature or the amount of pollution emitted. Hence, the informational asymmetry between the planner and the farmer, as in traditional moral-hazard analyses, encompasses both hidden action (input committal, pollution) and hidden knowledge (the state of nature that occurs) on the part of the farmer. What is observable, and by assumption contractible to both parties, is the level of *ex post* output.

The planner's task is to design a contract structure that awards the farmer on the basis of this realized output while coming as close as possible to maximizing social surplus. Let S be the class of all functions $s : \Re_+ \to \Re$ that the planner can choose from in designing a farmer reward scheme. The reward scheme works as follows: If the farmer realizes an output of z then his income is set at $s(z)$ by the planner. In picking such a reward scheme, the planner must realize that if she wants to implement a particular state-contingent production structure (z_1, z_2), that state-contingent production structure must be both technically feasible and consistent with the agent's private optimization in the sense that

$$\begin{aligned}(z_1, z_2) &\in \arg\max_{p,\mathbf{z}} \{\pi_1 u(s(z_1)) + \pi_2 u(s(z_2)) - Min_p\{c(\mathbf{z}, p)\}\} \\ &= \arg\max_{\mathbf{z}} \{\pi_1 u(s(z_1)) + \pi_2 u(s(z_2)) - C(\mathbf{z})\}.\end{aligned}$$

Notice, in particular, that here we use the farmer's private-cost function because pollution is not observable or contractible.

It is analytically convenient to solve this problem in stages. To that end, notice that the maximization problem can be rewritten as:

$$\max_{s\in S,\overline{u}} \left\{ \begin{array}{c} \overline{u} + \pi_1 (z_1 - s(z_1)) + \pi_2 (z_2 - s(z_2)) - m(\rho z_2) : \\ (z_1, z_2) \in \arg\max \{\pi_1 u(s(z_1)) + \pi_2 u(s(z_2)) - C(z_1, z_2)\}, \\ \pi_1 u(s(z_1)) + \pi_2 u(s(z_2)) - C(z_1, z_2) \geq \overline{u} \end{array} \right\}.$$

Under relatively weak conditions (Quiggin and Chambers, 1998), this version of the planner's is equivalent to designing a state-contingent payment structure subject to a set of constraints which make it privately rational for the farmer to pick the desired state-contingent production structure. As discussed by Quiggin and Chambers (1998), the payment scheme operates in the following way: When the farmer realizes an *ex post* output of, say, z she receives an *ex post* payment of y_1 if $z = z_1$, a payment of $z = z_2$, and an arbitrarily large negative payment otherwise. Formally, the planner's problem becomes:

$$\max_{\overline{u},y,z} \left\{ \begin{array}{c} \overline{u} + \pi_1 (z_1 - y_1) + \pi_2 (z_2 - y_2) - m(p(z_1, z_2)) : \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq \overline{u}, \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq u(y_1) - C(z_1, z_1), \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq u(y_2) - C(z_2, z_2) \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq \pi_1 u(y_2) + \pi_2 u(y_1) - C(z_2, z_1) \end{array} \right\}.$$

The last three constraints in this problem are the ones that ensure that the farmer finds it privately rational to pick the state-contingent production structure in return for the state-contingent reward structure offered by the planner.

We make several observations at this point. First, manipulating the second and third constraints demonstrates that contracts must be monotonic in the sense that whichever state has the highest output associated with it must also have the highest payment. Otherwise, the farmer will always find it optimal to misrepresent which state of nature actually occurs. When combined with the fact that our earlier assumptions on the technology (3) ensure that in the full information case output should be highest in state 2, it then follows immediately that the payment also should be the highest in state 2 (Quiggin and Chambers, 1998). We summarize by:

Lemma 1 An optimal contract must have:

$$\begin{aligned}(y_1, z_1) &< (y_2, z_2), \text{ or} \\ (y_1, z_1) &= (y_2, z_2).\end{aligned}$$

Following Quiggin and Chambers (1998), one can show that the planner's problem can now be rewritten as:

$$\max_{\overline{u},\mathbf{z}} \left\{ \overline{u} + \pi_1 z_1 + \pi_2 z_2 - m(\rho z_2) - Y(z_1, z_2, \overline{u}) : z_2 \geq z_1 \right\}, \tag{8}$$

where $Y(z_1, z_2, \overline{u})$ represents what Quiggin and Chambers (1998) term the *agency-cost function* and is defined as the least costly way in an expected value sense for the planner to get the farmer to adopt (z_1, z_2). Mathematically,

$$Y(z_1, z_2, \overline{u}) = \min_{\mathbf{y}} \left\{ \begin{array}{c} \pi_1 y_1 + \pi_2 y_2 : \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq \overline{u}, \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq u(y_1) - C(z_1, z_1) \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq u(y_2) - C(z_2, z_2) \\ \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq \pi_1 u(y_2) + \pi_2 u(y_1) - C(z_2, z_1) \end{array} \right\}.$$

By making the change of variables, $u_i = u(y_i)$ whence $y_i = u^{-1}(u_i) = h(u_i)$, this agency-cost problem can be translated into a simple convex programming problem subject to four linear constraints (Grossman and Hart, 1983; Quiggin and Chambers, 1998). Using Result 6 in Quiggin and Chambers (1998) while substituting in the exact form for the farmer's private cost function yields the following explicit solution for the agency-cost function:

$$Y(z_1, z_2, \overline{u}) = \pi_1 h(\overline{u} + (c_1 + c_2) z_1) + \pi_2 h\left(\overline{u} + \frac{c_1 z_1 + c_2 z_2 - \pi_1 (c_1 + c_2) z_1}{\pi_2}\right). \tag{9}$$

Thus, $u_1 = \overline{u} + (c_1 + c_2) z_1$ represents the farmer's optimal *ex post* utility in state 1 and $u_2 = \overline{u} + \frac{c_1 z_1 + c_2 z_2 - \pi_1 (c_1 + c_2) z_1}{\pi_2}$ the same in state 2.

6.1 Characterizing the Limited-Information Solution

Using (9) in (8) and introducing the auxiliary variable $\gamma \geq 0$

$$z_2 = z_1 + \gamma$$

gives the following first-order conditions for the planner's problem:

$$\begin{aligned} 1 - \pi_1 h'(u_1) - \pi_2 h'(u_2) &= 0, && && (10) \\ 1 - \rho m'(\rho z_2) - (c_1 + c_2)\left(\pi_1 h'(u_1) + \pi_2 h'(u_2)\right) &\leq 0 && z_1 \geq 0 && (11) \\ \pi_2 - \rho m'(\rho z_2) - h'(u_2) c_2 &\leq 0 && \gamma \geq 0, && (12) \end{aligned}$$

in the usual complementary slackness notation.

The first first-order condition (10) requires that the expected marginal cost of the farmer's welfare level be set to its marginal social welfare weight which is one. Substituting (10) into (11) gives

$$\begin{aligned} 1 - \rho m'(\rho z_2) - (c_1 + c_2) &\leq 0 && z_1 \geq 0 && (13) \\ \pi_2 - \rho m'(\rho z_2) - h'(u_2) c_2 &\leq 0 && \gamma \geq 0. && (14) \end{aligned}$$

Condition (13) requires that the marginal social cost of producing an extra unit of z_1 be at least as large as its marginal social benefit. Both the benefit and the cost have two components, which we might think of as *direct* and *agency-induced* benefits and cost. The direct benefit of increasing z_1 by one unit is the probability of state one occuring while the direct cost is c_1. Hence, the direct benefit-cost ratio is less than one, and in the absence of agency effects output in the first state would equal zero as in the full-information case.

However, because the principal cannot directly monitor pollution and thus control it, he must rely on manipulation of the farmer's payment structure as a next-best method of controlling pollution. Therefore, changes in z_1 bring with them cost and benefits induced by the principal's need to control pollution through the output-reward scheme. In particular, the reward scheme must preserve the monotonicity conditions outlined in Lemma 1. If it is to be optimal to produce anything in the first state, the agency-induced net benefit must be strictly positive and equal to the direct net loss associated with producing a marginal unit, i.e.,

$$\pi_2 - \rho m'(\rho z_2) - c_2 = c_1 - \pi_1 > 0. \tag{15}$$

Necessarily, therefore, there is an interior solution for z_1 only if the direct net benefit associated with state-2 output is positive, i.e., $\pi_2 - c_2 > 0$. Assuming that marginal pollution damage at the origin is negligible then (15) reduces to

$$1 > c_1 + c_2.$$

This condition, which can now be recognized as necessary for the existence of an interior solution for z_1, has the straightforward interpretation of requiring that increasing both state-contingent outputs at the origin by the same small amount brings with it a net social benefit. By Lemma 1, z_2 must be at least as large as z_1. Hence, producing a single unit of z_1 can only be optimal if it is also optimal to produce a single unit of z_2, thus the requirement that the cost producing a single unit of both state-contingent outputs at the origin be less than the expected return from producing a single unit of both state-contingent outputs.

Result 2 The optimal incentive scheme has $z_1 > 0$ only if $\pi_2 - c_2 > 0$. If $\lim_{p \to 0} m'(p) = 0$ and $1 > c_1 + c_2$, the optimal incentive scheme requires $z_1 > 0$.

Condition (14) essentially repeats (7) for the current problem. At the margin, the optimal incentive scheme is structured so that the farmer internalizes the marginal cost of runoff caused by the marginal unit that he produces. Effectively, the incentive scheme is structured so that the farmer is the residual claimant for social surplus associated with crop production in the good state of nature. Notice, however, that in (7) the farmer's payment has been set so that $h'\left(u^E\right) = 1$ whereas in (14) $h'(u_2) > 1$ by (10) reflecting the fact that in the limited-information optimum the planner must balance her risk sharing with the farmer against the potential pollution damage that it entails.

For a strictly interior solution:

$$1 - \rho m'(\rho z_2) - (c_1 + c_2) = 0 \tag{16}$$

$$\pi_2 - \rho m'(\rho z_2) - h'(u_2)\, c_2 = 0. \tag{17}$$

An immediate consequence of (16) is that, just as in the full information case, but unlike the private-information case, the optimal level of z_2, and hence of pollution, is independent of the producer's risk preferences. Instead both depend only upon the parameters of his private-cost structure and the effect that runoff has on social welfare. The optimal level of production is obtained by setting marginal pollution damage equal to the net benefit from increasing both state-contingent outputs by the same amount. Combining these observations with some simple calculus leads us to conclude:

Result 3 For a strictly interior solution, the optimal level of pollution and state-2 production is independent of the farmer's risk preferences and varies inversely with c_1, c_2, and ρ.

It turns out, however, that while the level of state-2 output can be determined parametrically in the terms established in Result 3, the dispersion of the state-contingent outputs and hence the degree of risk to which the farmer is optimally exposed in the limited information case does depend upon the producer's attitudes toward risk. This is a necessary consequence of the incentive problems caused by the presence of hidden action and hidden knowledge on the part of the farmer. If the planner fully insured the farmer against his production risk, she would give him adverse incentives for runoff control. Therefore, she balances the socially desirable goal of insuring the farmer against that of preventing runoff. Consequently, the optimal dispersion of state-contingent outputs will depend upon the producer's attitudes towards risk. In particular, we can solve for $h'(u_1)$ and $h'(u_2)$ parametrically as

$$h'(u_1) = 1 - \frac{\pi_2 (c_1 - \pi_1)}{c_2 \pi_1} \tag{18}$$

$$h'(u_2) = 1 + \frac{c_1 - \pi_1}{c_2} \tag{19}$$

7 Standards versus incentives

The central contribution of Quiggin and Chambers (1998) is to recognize that the hidden-action, principal-agent problem is isomorphic to a hidden-knowledge agency problem. Their solution approach, therefore, closely parallels methods developed in the hidden-knowledge, nonlinear taxation literature (Weymark, 1986). The deployment of standards in the limited-information case corresponds to the principal demanding a fixed level of production from the agent while providing him with a fixed payment across all states of nature. In the terminology of adverse selection, there would be a pooling rather than a separating equilibrium. The difference from the adverse selection literature, of course, is that the pooling and separating equilibria are across states of nature as opposed to across individuals. Thus for a production standard to be optimal, the contract structure cannot be strictly monotonic, and we must have $z_1 = z_2$. Mathematically, this reduces to requiring a corner solution for the auxiliary variable γ in (14).

We first examine the case where the standard is imposed at a positive level of production $z_2 = z_1 > 0$. In that case, conditions (13) and (14) require:

$$\begin{aligned} 1 - \rho m'(\rho z_1) - (c_1 + c_2) &= 0 \qquad (20) \\ \pi_2 - \rho m'(\rho z_1) - c_2 &< 0. \end{aligned}$$

Solving (20) for $\rho m'(\rho z_1)$ and rearranging shows that a standard with positive production is, therefore, optimal only if

$$\pi_1 > c_1.$$

which contradicts our assumption that $z_1^F = 0$. Hence, it is not possible to have a standard with positive production. This observation and (10) imply that:

Result 4 A production standard is socially optimal only if it involves a complete moratorium on production and making a fixed payment to the farmer equalling y^F.

Production standards, if they are to be optimally designed, therefore must be relatively blunt instruments that should only be applied in the most dire circumstances, where it is better to avoid any production of the run-off generating activity rather than suffer the consequences of runoff. Social optimality also then requires paying the farmer what he would receive in the full-information case while not requiring him to undertake any productive activity. Effectively, therefore, the production standard must be coupled with a welfare scheme for farmers. Hence, one concludes that when runoff pollution is a risk complement in production, standards generally will not be a useful instrument in achieving the constrained optimal outcome except in the most hazardous conditions. More typically, social optimality will require some type of incentive-based scheme.

Result 4 depends critically upon the assumption that runoff pollution is a risk complement in production. If the model were to be restructured to permit pollution to be a risk substitute in production, then one can envision cases in which production standards could be optimal at low but positive levels of productive activity on the part of the farmer.

8 Comparison of solutions

We have derived solutions for four different informational assumptions. An exhaustive comparison of the solutions would therefore require twelve separate pairwise comparisons, and would be tedious in the extreme. We can simplify this task by observing that, only limited comparisons are possible between the private optimization solution and the solutions involving government intervention. Government intervention typically involves both the provision of insurance and measures to reduce pollution. Generally such measures work in opposite directions.[5] It is possible, however to obtain some results on the pattern of payments and production.

Combining (10) and Lemma 1 establishes:

$$h'(u_2) > 1 > h'(u_1).$$

The strict convexity of h, therefore, establishes $u_2 > u_1$ from which we conclude:

$$y_2 > y^F = y^E > y_1. \tag{21}$$

[5]A set of counterexamples, showing that no general ranking of state-contingent outputs between the private-maximization case and other solutions, is available on request.

Expression (21) reflects the incentive problems that provision of insurance causes in the limited-information case. Because fully insuring the farmer in the absence of the ability to monitor his level of effort provides him with adverse production incentives, the optimal limited-information contract, which links payments to output levels, must induce the farmer to take some self-insuring actions. This is accomplished by exposing him to some income risk.

We now compare the farmer's return in the private and limited-information cases. Solving (17) while using (17) and (2) gives:

$$h'\left(u_2^p\right) - \frac{\rho m'\left(\rho z_2\right)}{c_2} = h'\left(u_2\right).$$

Accordingly, the difference between the farmer's private state-contingent return in state 2 and the one he receives from the limited-information incentive scheme reflects the marginal pollution damage associated with z_2 in the limited-information case. Assuming an interior solution in (11), we have as a consequence (2), Result 2 and (5) that:

$$h'\left(u^F\right) = 1 < \frac{\pi_2}{c_2} = h'\left(u_2^p\right).$$

We, therefore, conclude for a strictly interior optima that:

$$z_2^p > y_2 > y^F = y^E.$$

From (1),

$$h'\left(u_1^p\right) = \frac{\pi_1}{c_1} < 1 = h'\left(u^F\right),$$

where the inequality follows from the assumption that $c_1 - \pi_1 > 0$. Using this fact and (18), one can write:

$$\begin{aligned} h'\left(u_1^p\right) - h'\left(u_1\right) &= \frac{\pi_1}{c_1} - 1 + \frac{\pi_2\left(c_1 - \pi_1\right)}{c_2\pi_1} \\ &= \left(\pi_1 - c_1\right)\left(\frac{1}{c_1} - \frac{\pi_2}{c_2\pi_1}\right) > 0, \end{aligned}$$

where the inequality follows from (3) and the assumption that $c_1 - \pi_1 > 0$. We, therefore, immediately conclude:

$$y^F = y^E > z_1^p > y_1.$$

Summarizing, we have:

Result 5 For strictly interior solutions:

$$z_2^p > y_2 > y^F = y^E > z_1^p > y_1.$$

We now turn our attention to ranking the levels of runoff and output in the externality and limited-information cases. Presuming an interior production equilibrium in the limited information case, it follows from (15) that in the optimum:

$$\pi_2 - c_2 - (c_1 - \pi_1) = \rho m' (\rho z_2),$$

while optimality for the externality case requires:

$$\pi_2 - c_2 = \rho m' \left(\rho z_2^E\right).$$

Using the fact that $c_1 - \pi_1 > 0$ by assumption, we conclude that $\rho m' \left(\rho z_2^E\right) > \rho m' (\rho z_2)$. Hence,

Result 6 For strictly interior solutions, the level of runoff and z_2 is larger in the externality with insurance case than in the limited-information case.

Result 6 reflects the fact that in the externality with insurance case, the planner can provide the farmer with full production insurance without worrying about providing him adverse incentives for the committal of effort. This happens because in that case the planner can base his contract upon both the observed state of nature and the observed output. Thus, the planner has a finer ability to detect the farmer shirking on the application of inputs devoted primarily to crop growth than in the limited-information case. Consequently, the planner will be less willing to forego crop output in the good state of nature than he is in the limited-information case. The result is a higher level of runoff than in the limited-information case.

Result 6 may also be derived from (19), which reflects the fact, previously observed by Quiggin and Chambers (1998) that, for the limited-information solution, the marginal cost of output in the good state of nature is set equal to the marginal utility of additional income for the farmer, just as in the externality with insurance case. However, because the limited information solution does not involve full insurance, the marginal utility of income in the good state is lower in the limited information case. Consequently, the optimal level of output is also lower.

We now turn our attention to ranking the level of outputs produced in the full-information case and in the externality with insurance cases. By (5) and (6)

$$z_1^E = z_1^F = 0.$$

Now notice by the definition of the full-information optimum that z_2^F solves

$$Max_{z_2} \left\{ \pi_2 z_2 - z_2 c^2 (1, \theta) - m (z_2 \theta) \right\},$$

at $\theta = \frac{p^F}{z_2^F}$ while z_2^E the same problem at $\theta = \rho$. The first-order condition for an interior solution to this problem is

$$\pi_2 - c^2 (1, \theta) - \theta m' (z_2 \theta) = 0.$$

Totally differentiating this first-order condition establishes that the optimal solution must satisfy:

$$\frac{dz_2}{d\theta} = \frac{-c_p^2 (1, \theta) - m' (z_2 \theta) - \theta z_2 m'' (z_2 \theta)}{\theta^2 m'' (z_2 \theta)}.$$

Evaluating this expression at $\theta = \rho$ yields

$$\frac{dz_2}{d\theta} = \frac{-m' (z_2 \rho) - \rho z_2 m'' (z_2 \rho)}{\rho^2 m'' (z_2 \rho)} < 0.$$

Now applying Result 1 to this last expression establishes that $z_2^F > z_2^E$.

Summarizing, in conjunction with Lemma 1 and Result 6 we have established:

Result 7 For strictly interior solutions:

$$z_2^F > z_2^E > z_2 > z_1 \geq z_1^E = z_1^F = 0.$$

It trivially follows from Result 7 that

Corollary Expected output is higher in the full-information case than in the externality with insurance case.

The intuition here is fairly clear cut. In the full-information case, the ability of the planner to observe all aspects of the productive operation allows her to fully insure the farmer against his production risk while still requiring that he maximize the expected profit from farming. It also allows the planner to insist upon a lower level of runoff per unit of output than in either the private-information or externality with insurance problems. Hence, the planner can legitimately insist upon a higher output in state-2 in the full-information case than in the externality with insurance case without creating adverse incentives for the creation of runoff. The result is a greater output dispersion in the full-information case than in the externality with insurance case.

An alternative way of viewing the externality with insurance case is that the farmer bears the full cost of pollution through an effective Pigovian tax, but cannot commit to the socially (and therefore, in the presence of the tax, privately) optimal ratio of pollution to output. Hence, the farmer is forced to adopt an inefficient production technology, using too much of the pollution 'input'. Not surprisingly, the result is a lower level of output.

The output dispersion is the least in the limited-information case. Because of the farmer's ability in that case to shirk on the application of crop-enhancing inputs, it is generally not optimal for the planner to insist upon a zero output in state-1. To do so would give the farmer adverse incentives to shirk totally in the application of such inputs. On the other hand, the planner also realizes that expanding the output dispersion by increasing the desired output in state-2 gives the farmer an adverse incentive to create chemical runoff. These tendencies towards increased runoff and shirking in the provision of crop-enhancing inputs must always be balanced against the legitimate social goal of having the risk-neutral planner bear all of the farmer's production risk. The end result is a less dispersed output pattern than in either the full-information or externality with insurance cases.

9 Concluding comments

The state-contingent representation of agency problems has largely been neglected since the observation by Mirrlees (1974), that, if the technology available to the agent could be represented by a stochastic production function, it was always possible to achieve the first-best outcome in such problems. Quiggin and Chambers (1998) showed that, with a general representation of the state-contingent production technology, the solution to the problem was both tractable and non-trivial. In this paper, the analysis has been extended to cover a class of multitask agency problems where some outputs are observable and others are not. Particular emphasis is placed on the production choices facing a producer producing two outputs—one that is socially desirable and observable and one that is socially desirable but unobservable.

We characterize the equilibrium outcomes under a series of different informational settings that we refer to as the private-maximization case, the full-information case, the externality with insurance case, and the limited-information case. For the limited-information case, which corresponds to an agency problem where only the production of the desirable output is observable, we show if the unobservable and undesirable output is risk complementary in production, standards are preferable to incentive-based schemes be optimal when they are associated with a complete moratorium on production. We also show that the full-information case has the most disperse output distribution and the limited-information case the least with the externality with insurance case having an output distribution that is less disperse than the full-information case and more disperse than the limited-information case.