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Corruption and Auctions

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Abstract

We investigate the outcome of an auction where the auctioneer approaches one of the two existing bidders and offers an opportunity for him to match his opponent's bid in exchange for a bribe. In particular, we examine two types of corruption arrangements. In the first case, the auctioneer approaches the winner to offer the possibility of a reduction in his bid to match the loser's bid in exchange for a bribe. In the second arrangement, the auctioneer approaches the loser and offers him the possibility of matching the winner's bid in exchange for a bribe. While oral auctions are corruption free under the two arrangements, corruption affects both bidding behavior, efficiency and the seller's expected revenue in a first-price auction.

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1 Introduction

Standard auction theory does not distinguish between the seller of the item being auctioned off and the auctioneer. However, it is often the case that these two agents are indeed two distinct individuals. Government procurement is typically organized by a bureaucrat on behalf of the government. Private firms' procurement decisions are taken by firm officials belonging to the purchasing department and not by general managers. When selling a particular good, such as a house, car, painting, or bottle of wine, individual sellers usually do not conduct the auction themselves but instead turn to an auction house.

This distinction between the seller and the auctioneer creates the possibility of illicit behavior by the auctioneer as he might have incentives that differ from those of the seller. Whereas the seller wants the highest price for the object being sold, the auctioneer might want to illicitly solicit a bribe to somehow change the auction result.

Previous studies of illicit behavior in auctions have focused on collusion between bidders (bidding rings).[1] Laffont and Tirole (1991) examine the design of auctions to favor specific bidders, such as the case of government procurement favoring domestic suppliers. Graham and Marshall (1987) and Mailath and Zemsky(1991) show that in second-price sealed-bid and English oral auctions, when collusion among players is allowed, cooperative strategies are dominant. The optimal response of the auctioneer is to establish a reserve price that is a function of the coalition's size. Furthermore, they show that

[1] One exception is Beck and Maher (1986), who compare competitive bidding (in the form of a first price sealed bid auction) and a bribery competition, and find that the expected payoffs are the same under the two allocation mechanisms. In contrast, by allowing corruption between the auctioneer and one of the players, we show that seller's expected revenue and the expected bribe may depend on how the bribery market is organized.

the revenue equivalence between second-price and English auctions holds.

McAfee and McMillan (1992) examine collusion among players in first-price auctions where they show that the price paid is the minimum price. Again, sellers can react by increasing the reserve price. Notice that there is no clear cut way of ranking first- and second-priced auctions when collusion is possible, because one can construct examples where the revenue from oral auctions with collusion will be greater than the revenue from a first-priced auction, or vice-versa, depending on the underlying assumptions.

However, this is not the case when there is collusion involving the auctioneer (we refer to this as corruption), because the oral auction is immune to this type of corruption. We should point out that while McAfee and McMillan cite evidence for collusion between bidders for government contracts where "it has been often the case that all bids are identical to the last cent," this could also be evidence of corruption involving the auctioneer. We claim that any evidence based on little dispersion amongst observed bids can also be construed as evidence of corruption.

In this paper we investigate the outcome of an auction where the auctioneer approaches one of the existing bidders and offers an opportunity for him to change his bid in exchange for a bribe. In particular, we examine two arrangements. In the first case, the auctioneer approaches the winner of the auction to offer a reduction in his bid to match the loser's bid in exchange for a bribe. In the second case the auctioneer approaches the loser and offers him the possibility to match the winner's bid in exchange for a bribe. Therefore, if corruption does occur, the published bids will be identical. (In practice, the observed winner's bid might be a few cents above the observed loser's bid).

Oral and second-price auctions are corruption free in both cases. Corruption, however, affects both bidding behavior and the seller's expected revenue in a first-price auction. Under the first bribery arrangement, there is an increasing symmetric equilibrium bidding strategy and, therefore, the object always goes to the individual with the highest valuation. Thus, efficiency is not compromised. Under the second bribery arrangement, an increasing symmetric bidding strategy equilibrium does not exist in general and, thus, the object might not necessarily go to the individual with the highest valuation. We also provide examples where we explicitly compute equilibrium bidding strategies, the seller's expected revenue and the expected bribe.

2 An Example

In this section we provide a simple example to illustrate some of the implications of having a corrupt auctioneer. The next sections contain a more general and detailed analysis.

Consider for the moment two risk-neutral bidders, 1 and 2, whose valuations are independent draws from the uniform [0,1] distribution. Each individual's valuation is private information. For simplicity we focus on a particular type of corruption arrangement; the auctioneer approaches the individual with the highest bid and allows him to reduce his bid to match the second highest bid (i.e., the lowest of the two bids) in exchange for a bribe. In the next section we will analyze different types of bribe payments, but here we assume that the bribe is a proportion (ρ) of the difference between the highest and the lowest bid. We can write Bidder 1's expected profit as a function of his private value v_1, his choice of bid b_1, assuming that Bidder 2 follows some bidding strategy $b(y)$, as follows:

$$\pi(v_1, b_1, b(y)) = E\left(v_1 - b(y) - \rho(b_1 - b(y))\right)\chi_{b_1 > b(y)}. \tag{1}$$

Where $\chi_{b_1>b(y)}$ denotes an indicator function that has value one in the events where $b_1 > b(y)$, that is, when $y < b^{-1}(b_1)$.[2] Bidder 1 can win only if he is the highest bidder and if he does win he will pay Bidder 2's bid plus the bribe, as $b_1 + \rho(b_1 - b(y)) \leq b_1$. We can take the expected value of (1) by integrating over the appropriate interval, which yields:

$$\pi(v_1, b_1, b(y)) = \int_0^{b^{-1}(b_1)} (v_1 - b(y) - \rho(b_1 - b(y))\, dy.$$

Bidder 1 will then choose b_1 to maximize his expected profits. Differentiating the above expression with respect to b_1 and using $b_1 = b(\cdot)$, as we are looking for a symmetric equilibrium, yields the following differential equation:

$$\frac{v - b(v)}{b'(v)} - \rho v = 0$$

One can check directly that the solution to this differential equation – our symmetric equilibrium bidding strategy, which is increasing in v as needed, is given by

$$b(v) = \frac{v}{1+\rho}.$$

To help develop the intuition, consider some polar cases. For $\rho = 1$, the winner's total payment is equal to his bid as

$$b(y) + (b(v) - b(y)) = b(v)$$

where y denotes the loser's valuation. This is identical to the winner's total payment in a standard first-price sealed-bid auction. Accordingly, $b(v) = v/2$ in this case, where $v/2$ is the expected valuation of the other bidder conditional on v being the highest value. Note that the winner's total payment

[2] This last inequality follows only if $b(.)$ is an increasing function. We will check below that indeed the derived equilibrium bidding strategy is increasing and the analysis justified.

is split into a payment to the seller that is equal to the loser's bid and a payment to the auctioneer that is equal to the difference between the two bids.

For $\rho = 0$, the winner's total payment is equal to the second-highest bid, as in a standard second-price sealed-bid auction. Accordingly, $b(v) = v$ in this case, the winner pays the seller the lowest value and the auctioneer receives no bribe.

For $0 < \rho < 1$, the winner's total payment is equal to

$$b(y) + \rho(b(v) - b(y)) < b(v).$$

The intuition is that conditional on having the highest value, a bidder bids in such a way to outbid his opponent, just as he does in a standard first-price auction, but taking into account that a high bid might result in a higher payment to the auctioneer. Therefore, in equilibrium, a bidder bids less than what would in the absence of corruption. In the next section we will show how to use the revenue equivalence theorem in some instances to argue that the winner's expected payment is actually fixed across a variety of bribery arrangements, as long as there is an increasing equilibrium bidding strategy.

For this example, it is also possible to compute the seller's expected revenue and the expected bribe as a function of ρ The seller's expected revenue (ER) is simply the expected value of the second highest bid:

$$ER = \frac{1}{1+\rho} \cdot \frac{1}{3}$$

where $\frac{1}{3}$ is the expected value of the lowest valuation. The expected bribe (EB) is simply ρ times the difference between the highest and second-highest bids:

$$EB = \frac{\rho}{1+\rho} \cdot \frac{1}{3}$$

where $\frac{1}{3}$ is the difference between the highest and the second-highest expected values. Notice that ER and EB add up to 1/3, which is the expected total payment of the winner in any standard auction with two bidders with values drawn independently from the uniform [0,1] distribution.

In this environment, the first-price sealed-bid auction is still efficient as the object is allocated to the individual with the highest valuation. The winner's total payment, however, is now split between the seller and the auctioneer.

3 The Model

There are n risk-neutral bidders who are competing for an object to be auctioned off. According to the independent private values assumption, Bidder $i \in I$ knows his own value (v_i) for the object but only knows the distribution $F(v_j)$, $\forall j \neq i$, of the other bidder's value. It is assumed that values are independently drawn from the distribution F. We assume that F has a density $f(\cdot)$ strictly positive on its support $[0,1]$.

The auctioneer is corrupt. We consider two possible types of corrupt practices. In the first case, the auctioneer approaches the winner and offers him to reduce his bid to the second highest bid in exchange for a bribe. If this bidder agrees to pay the bribe, then he wins the object paying what is effectively the second highest bid (plus the bribe to the auctioneer). If this player does not agree to pay the bribe, then he still wins the object and pays accordingly to the auction rules — e.g., his bid when a first price auction is used. In the second case, the auctioneer approaches the loser and offers him to increase his bid to match the highest bid in exchange for a bribe. If this player agrees, he wins the auction and pays this highest bid. Otherwise, this player loses the auction. Note that the bribery arrangement is common knowledge. Moreover, both bribery arrangements are consistent with the evidence of little dispersion amongst observed bids as discussed in the previous section because under corruption the two observed bids will be identical.

Our formulation in the next sections considers different possible types of bribe payments, such as fixed bribe B or a function of the difference between the highest bid and the second highest bid. A fixed bribe B, for example, might have been determined by a social convention or norm. Similarly, it might be that the convention is to pay the auctioneer a percentage of the difference between the highest and second highest bids. Although a more complete model would have the size of the bribe as endogenous, the analysis presented already yields several interesting results.

We also ignore the principal-agent relationship between the seller and the auctioneer. In principle, the seller could design a contract that would reduce the likelihood of corruption but would involve some payment from the seller to the auctioneer. As noted earlier, our motivating example is that of a government official who has been delegated the authority to purchase a given good or service on behalf of the government or an auctioneer who has been given an object to sell on behalf of an individual seller.

4 Corruption in oral and second-price sealed-bid auctions

In the section we explain why oral and second-price auctions are corruption free. In the next section we investigate some consequences of corruption in a first price auction.

Consider a second-price auction and assume that the auctioneer approaches the winner of the auction. Let $\hat{b} = max\{b_2, ..., b_n\}$ where $b_2, ..., b_n$ are the bids made by bidders $2, ..., n$. We find Bidder 1's best reply. Suppose he bids $b_1 > v_1$. Bidder 1 does not gain anything by bidding more than his value (as he would win anyway by bidding somewhere in between b_1 and v_1) but may as well lose if $b_1 > \hat{b} > v_1$. Similarly, suppose Bidder 1 bids $b_1 < v_1$. Bidder 1 does not gain by bidding less than his value but may as well lose the auction (and consequently the object because the auctioneer approaches only the winner) when it should have won, that is, when $v_1 > \hat{b} > b_1$. Note that there is never corruption because the winner always pays the second highest bid (without the need to pay a bribe).

Now suppose that the auctioneer approaches the loser of the auction. Assume that Bidder 1 bids $b_1 > v_1$. Without loss of generality, suppose $\hat{b} = b_2$. As above, Bidder 1 does not gain anything by bidding more than his value but may as well lose if $b_1 > b_2 > v_1$ and Bidder 2 decides not to pay the bribe. Now suppose Bidder 1 bids $b_1 < v_1$. When $b_1 > \hat{b}$. Bidder 2 is offered the opportunity to match 1's bid and win the auction. In this case, by increasing his bid (up to v_1), Player 1 would increase his expected profits. If $v_1 > b_2 > b_1$, then Player 1 is offered to match 2's bid and pay the bribe. By increasing his bid, Player 1 would increase his expected profits by not having to pay the bribe. As in the previous case, there is never corruption because the winner always pays the second highest bid (without the need to pay a bribe).

A similar reasoning can be applied to show that oral auctions are also immune to the type of corruption we consider. This is summarized by the following proposition.

Proposition 1 *Under the two alternative bribery arrangements, oral and second-price auctions are corruption free.*

5 Corruption in First-Price Auctions

In this section we investigate the effects of corruption on bidding behavior in a first-price auction. We also provide examples where we explicitly compute equilibrium bidding strategies, the probability of corruption and the expected bribe, investigate the efficiency of first price auctions under corruption and compute the seller's expected revenue. We examine the two bribery arrangements separately.

5.1 The winner is approached by the auctioneer.

We consider the case when the auctioneer approaches the winner and offers him the possibility to match the second highest bid in exchange for a bribe. Denote the maximum of bidders $i = 2, \ldots, n$ values by $Y = \max_{j \geq 2} v_j$. This random variable has distribution $F_Y(x) = F^{n-1}(x)$ and density $f_Y(x) = (n-1) F^{n-2}(x) f(x)$. To find an equilibrium we could follow the usual approach used in Section 2 of finding the optimal bid $x^* \geq 0$ of Bidder 1, when his opponents are bidding according to a function $b(v_j)$, $j = 2, \ldots, n$, by means of the first-order condition of Bidder 1's expected utility maximization problem, and then using that in a symmetric equilibrium $x^* = b(v_1)$. Instead, we will follow a more general and neater approach[3]. First, note that if there is a symmetric equilibrium strictly increasing strategy $b(\cdot)$, the object will be allocated to the bidder with the highest value. As the bidder with the lowest possible value obtains a zero payoff, the winner's total payments is actually fixed across any efficient auction format. This is a consequence of the celebrated Revenue Equivalence Theorem.[4] The distinction here is that part of the bidders' payment now goes to the auctioneer as a

[3]We thank a referee for this suggestion.

[4]See for example a general argument in theorem 2 in Milgrom and Segal (2000).

bribe rather than to the seller. In the next proposition we characterize a symmetric equilibrium bidding strategy.

Proposition 2 *Suppose the auctioneer approaches the winner and for a bribe $B(\cdot)$ offers to change his bid to match the second highest bid. If the strictly increasing function $b : [0,1] \rightarrow \mathbb{R}_+$ is a symmetric equilibrium then for every $v \in [0,1]$,*

$$\int_0^v \min\{b(v), b(y) + B(\cdot)\} f_Y(y)\, dy = \int_0^v y f_Y(y)\, dy. \tag{2}$$

Proof: The winning bidder's payment is $b(v)$ if he does not accept the bribe and is $b(Y) + B(\cdot)$ if he accepts the bribe. Thus his expected payment $E[\min\{b(v), b(Y) + B(\cdot)\}|\, v > Y] = E[Y|\, v > Y]$. From the Revenue equivalence theorem, the winner's total payment in a first-price auction has to be identical to his expected payment in any efficient auction and, in particular, in a second-price auction. This is equivalent to (2). QED

Note that we have not specified the type of bribe payments. The idea is that for a given type of bribe payment – for example, a fixed bribe $B(\cdot) = B$ – one should try to find a strictly increasing strategy $b(\cdot)$ that satisfies (2). This yields a differential equation (with respect to v) that $b(\cdot)$ has to satisfy together with the boundary condition $b(0) = 0$.

Thus, the type of reasoning that leads to (2) does not ensure either uniqueness or existence. It is a necessary condition. However, the equation (2), as we will see from the reasoning below for various particular bribe payments, will imply a unique solution if there is one. Moreover, if such a strictly increasing equilibrium strategy exists, then the object will still be allocated to the individual with the highest valuation.

Corollary 1 *In the equilibrium characterized by (2) first-price auctions are still efficient.*

5.1.1 Fixed bribe

Consider the case where $B(\cdot) = B$. We will follow the procedure outlined above. That is, we will obtain $b(\cdot)$ so as to solve the differential equation that we obtain by differentiating (2) with respect to v. First assume that that $b(v) \leq B$. Then we have

$$\min \{b(v), b(y) + B\} = b(v)$$

and therefore that

$$b(v) F_Y(v) = \int_0^v y f_Y(y) \, dy.$$

Thus necessarily

$$b(v) = b_1(v) \text{ if } b(v) \leq B \text{ where } b_1(v) = \frac{\int_0^v y f_Y(y) \, dy}{F_Y(v)}. \tag{3}$$

If the bribe is sufficiently large, e.g. $B \geq E[Y] = b_1(1)$ then $b(v) = b_1(v), 0 \leq v \leq 1$. Suppose now that $0 < B < b_1(1) = E[Y]$. Then in general we have the following:

Theorem 1 *There is an increasing sequence $v_0 = 0 < v_1 < \ldots v_n < v_{n+1} < \ldots < v_m = 1$ and an associated sequence of functions $b_n(\cdot)$ defined in the interval $[v_{n-1}, v_n]$ that satisfy the differential equation*

$$b_n'(v) = \frac{(v - b_n(v)) f_Y(v)}{F_Y(v) - F_Y\left(b_{n-1}^{-1}(b(v) - B)\right)}, v_{n-1} \leq v \leq v_n \text{ and } b_n(v_{n-1}) = (n-1) B. \tag{4}$$

Moreover if $v_n < 1$ then $b_n(v_n) = nB$.

Since $v_n \geq b_n(v_n) = nB$ it is clear that $m \leq 1/B$. The proof is by induction and is in the appendix.

If the fixed bribe goes to zero the equilibrium bidding function must approach $b(v) = v$ since the highest bidder will pay the bribe and pay the second highest bid.

Example 1 *Consider the particular case of the uniform distribution with two bidders. We know already that if* $B \geq E[Y] = 1/2$ *the equilibrium is* $b_1(v) = \frac{\int_0^v ydy}{v} = \frac{v}{2}$. *Suppose* $\frac{1}{2} \geq B \geq \frac{1}{2+\sqrt{2}} = 0.29289$. *Then*

$$b(v) = \begin{cases} \frac{v}{2} & \text{if } 0 \leq v \leq 2B \\ \frac{v}{2} + B - \frac{\sqrt{4Bv - v^2}}{2} & \text{if } 2B \leq v \leq 1 \end{cases}$$

is an equilibrium when the auctioneer approaches the winner. The seller's expected revenue is $\frac{1}{3} - B \times (1 - 2B^2)$ *and the expected bribe* $B \times (1 - 2B^2)$.

That is, when the auctioneer approaches the winner, the seller's expected revenue can be quite dramatically reduced for sufficiently large values of B. This demonstrates the potential devastating effects on the government's revenue in very corrupt economies.

5.1.2 Proportional bribe

We now generalize the example of Section 2 where the auctioneer approaches the winner and ask for a bribe that is proportional to the difference between the highest and the second highest bids. Suppose $0 < \rho \leq 1$ so that if b_1 is the highest bid and b_2 is the second highest bid then the auctioneer asks for a bribe $B(\cdot) = \rho(b_1 - b_2)$. To find the equilibrium bidding strategy, we assume that symmetric equilibrium bidding strategy $b(\cdot)$ is a strictly increasing symmetric equilibrium strategy. Replacing this particular type of bribe payment into (2) yields:

$$\int_0^v [b(y) + \rho(b(v) - b(y))] f_Y(y)\, dy = \int_0^v y f_Y(y)\, dy.$$

Differentiating this equation with respect to v we obtain:

$$b(v) + \rho b'(v) \frac{F_Y(v)}{f_Y(v)} = v$$

The solution of this differential equation yields our equilibrium. For example, when there are n bidders with values distributed uniformly on [0,1], our strictly increasing symmetric equilibrium is $b(v) = \frac{n-1}{n-1+\rho}v$. One can check that this approach yields the same equilibrium as obtained in the example of Section 2 when $n = 2$.

5.1.3 General linear bribe

Suppose the bribe now takes the form $B + \rho\rho(b(v) - b(y))$ where $\rho \in (0,1)$. We now show that this case reduces to the fixed bribe case with a fixed bribe equal to$\frac{B}{1-\rho}$.To see this note first that equation (2) is now of the form

$$\int_0^v \min\{b(v), b(y) + B + \rho(b(v) - b(y))\} f_Y(y)\,dy = \int_0^v y f_Y(y)\,dy.$$

We have that $b(v) \leq b(y) + B + \rho(b(v) - b(y))$ if and only if $b(v) \leq \frac{B}{1-\rho} + b(y)$.Thus if $b(v) \leq \frac{B}{1-\rho}$ we obtain that $b(v) = b_1(v)$ where $b_1(v) = \frac{\int_0^v y f_Y(y)dy}{F_Y(v)}$.Define v_1 as the solution of $b_1(v_1) = \frac{B}{1-\rho}$ or as $v_1 = 1$ if $b_1(1) \leq \frac{B}{1-\rho}$. We now proceed analogously as in the appendix. Equation (6) changes to

$$b'(v) = \frac{(v - b(v)) f_Y(v)}{F_Y(v) - (1-\rho) F_Y(\alpha(v))} =$$

$$\frac{(v - b(v)) f_Y(v)}{F_Y(v) - (1-\rho) F_Y\left(b_1^{-1}\left(b(v) - \frac{B}{1-\rho}\right)\right)}$$

Comparing the expression above with equation (7) in the appendix allows us to conclude that the solution will be analogous to that of the fixed bribe case for $B(\cdot) = \frac{B}{1-\rho}$.

5.1.4 Comparing bribe payments

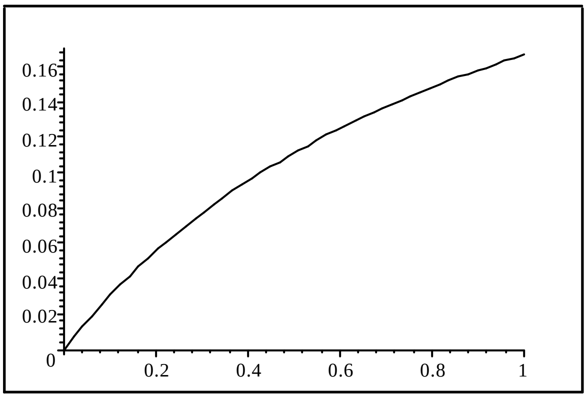
As different types of bribe payments yield different results for the corrupt auctioneer, one could ask the question of which type the auctioneer would choose.[5] We cannot provide a general result as to be able to make such comparison we have to obtain an explicit equilibrium bidding function for

[5] We thank a referee for raising this issue.

each type of bribe payments. However, we can compare the auctioneer's expected bribe under two of the bribe payments considered above. Consider the case of two players with values drawn from a uniform distribution on [0,1]. From the example in Section 2, the expected bribe under a proportional bribe is equal to:

$$EB^P = \frac{\rho}{1+\rho}\frac{1}{3}$$

The diagram below plots this function for $0 \leq \rho \leq 1$:

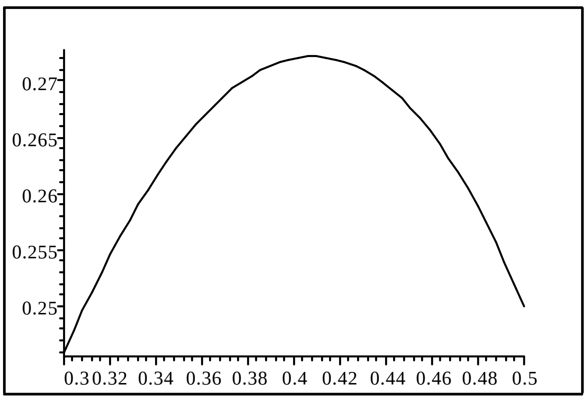


Expected bribe as a function of ρ

>From example 1 above, the expected bribe under a fixed bribe is given by

$$EB^F = B \times \left(1 - 2B^2\right).$$

This is plotted below:



Expected bribe as a function of B

It is clear from the above diagrams that the fixed bribe would always be preferred from the corrupt auctioneer's point of view. This conclusion can not be generalized as is dependent on the equilibrium bidding strategy that is in turn dependent on the distribution of values.

Finally, note that changing the way the auctioneer is compensated, say for example from a fixed salary to a fixed percentage of the sales revenue as commission, may not eliminate or even reduce corruption. Corruption can only be eliminated if the commission is sufficiently large so that the expected commission is larger than the expected bribery. This finding contrasts with that of the classical economic theory of corruption — e.g., Becker and Stigler (1974) – but it is consistent with recent findings (Mookherjee and Png, 1995) that to wipe out corruption a large discrete jump in the official's compensation is required.

5.2 The highest losing bidder is approached by the auctioneer

We now assume that the auctioneer approaches the bidder who lost the auction and offers him the possibility of revising his bid to match the highest bid and win the auction in exchange for a bribe. For simplicity consider a fixed bribe B. In the next theorem we show that there is no increasing symmetric bidding strategy equilibrium if the bribe is sufficiently small. If the bribe is large then the standard equilibrium (i.e., the equilibrium obtained in the absence of corruption) holds and the bribe is never accepted.

Theorem 2 *In general there is no increasing symmetric biding strategy equilibrium if the auctioneer approaches the highest losing bidder.*

The formal proof is in the appendix but the intuition is quite straightforward. Suppose bidders $2, ..., n$ follow a symmetric increasing bidding strategy. Consider Bidder 1's best response. If he has the highest value and chooses the same symmetric increasing bidding strategy he will win the auction but might lose the object as the auctioneer will approach the highest losing bidder. In such scenario, the highest valued bidder will actually wants to have the second highest bid and an increasing symmetric equilibrium bidding strategy will then not exist in general.

Of course, the inexistence of an increasing symmetric equilibrium bidding strategy does not mean that the object will never be allocated to the individual with the highest value. This is still possible, but not guaranteed, if this individual has the second highest bid.

6 Other Applications

Wall street lore has it that a target of a takeover will sometimes run a sealed-bid auction, release the results to a favored bidder and then permit rebidding.[6] Although there is no documented evidence of this phenomenon, target officer might do that in order to keep their jobs under new ownership or to receive favorable treatment if the takeover does occur.

Similarly, there are newspapers reports of a taped phone conversation between the government official in charge of selling the former state-owned Brazilian Telecomm company and one of the bidders. The government official, it is allegedly, was concerned that the highest standing bid was made by a consortium that did not have the appropriate technological expertise and urged this bidder to resubmit a higher bid.

[6] We thank the editor for suggesting this application.

These two examples can be analyzed in the context of Section 5, where the auctioneer approaches one of the losing bidders and offers him the opportunity to match the highest bid. Even if there is no bribe payment to the auctioneer, the fact that a losing bidder might be given a chance to match the highest bid and win the object creates the same incentives as outlined in the previous section, where bids may not be increasing in values. As a result, we cannot guarantee that the result will be efficient. This simple analysis suggests the very tentative policy implication that if such behavior from target officers or government officials is of concern, then an open auction might be a preferred option.

7 Conclusion

In this paper we analyzed the effects of corruption on auctions under the independent private values setting with risk neutral bidders. We examined two distinct arrangements where corruption might occur that are consistent with the evidence of little difference between observed bids. Under these two arrangements, oral and second-price auctions are shown to be corruption free as it is still a dominant strategy for a bidder to bid one's true valuation – thus rendering corruption ineffective.

However, bidding behavior in a first price auction is affected by corruption. When the auctioneer approaches the winner of the auction to offer him the possibility of reducing his bid to match his opponent's bid in exchange for a bribe. We show how to compute an increasing symmetric equilibrium bidding strategy for any type of bribe payment. If such equilibrium exists, then the auction is still efficient. For example, we show that in the case of a large fixed bribe, the probability of corruption is high and, consequently, the seller's expected revenue is low (as the winning bidder reduces his bid).

When the auctioneer approaches the highest losing bidder of the auction and offers him the possibility of matching the highest bid in exchange for a bribe, in general there is no increasing symmetric bidding strategy equilibrium. The reason is that a high valued bidder may want to lose the auction. As a result, the object may not go to the individual who values it the most.

These results are only suggestive of how the existence of corruption might affect both the seller's expected revenue and the efficiency of an auction. This seems to be a research line worth pursuing as developing countries, naturally more prone to corruption, are increasingly using auctions to allocate goods and services and to privatize government owned assets,

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A Appendix

Proof of Theorem 1: Define $v_1 \in (0,1)$ as the solution v of $b_1(v) = B$. We consider now (2) for $v_1 < v \leq 1$. Define $\alpha(v) \in [0,1]$ as the solution α of $b(v) = b(\alpha) + B$. It is clear that $\alpha(v)$ is increasing and $\alpha(v_1) = 0$. Define also $v_2 = 1$ if $b(1) \leq 2B$ and if $b(1) > 2B$ define $v_2 \leq 1$ as the solution v of $b(v) = 2B$. In the range $v_1 \leq v \leq v_2$ it is true that $b(v) = b_1(\alpha(v)) + B$. Thus

$$\int_0^v \min\{b(v), b(y) + B\} f_Y(y)\, dy = \tag{5}$$

$$\int_0^{\alpha(v)} (b(y) + B) f_Y(y)\, dy + \int_{\alpha(v)}^v b(v) f_Y(y)\, dy = \tag{6}$$

$$\int_0^{\alpha(v)} (b(y) + B) f_Y(y)\, dy + b(v)(F_Y(v) - F_Y\alpha(v)).$$

If we take the derivative in (2) and use (5) we obtain after simplification

$$b'(v)(F_Y(v) - F_Y(\alpha(v))) + b(v) f_Y(v) = v f_Y(v).$$

Or

$$b'(v) = \frac{(v - b(v)) f_Y(v)}{F_Y(v) - F_Y(\alpha(v))} = \frac{(v - b(v)) f_Y(v)}{F_Y(v) - F_Y\left(b_1^{-1}(b(v) - B)\right)}, v_1 \leq v \leq v_2. \tag{7}$$

Define $b_2(\cdot)$ the solution of (7) such that $b_2(v_1) = B$. Thus if $b_2(1) > 2B$, $b_2(v_2) = 2B$. Otherwise $v_2 = 1$. Proceeding in this way we obtain an increasing sequence $(v_n)_n$ which terminates if some $v_m = 1$. And we obtain as associated sequence of functions $b_n(\cdot)$ defined in the interval $[v_{n-1}, v_n]$ that satisfy the differential equation

$$b_n'(v) = \frac{(v - b_n(v)) f_Y(v)}{F_Y(v) - F_Y\left(b_{n-1}^{-1}(b(v) - B)\right)}, v_{n-1} \leq v \leq v_n \text{ and } b_n(v_{n-1}) = (n-1)B.$$

Moreover if $v_n < 1$ then $b_n(v_n) = nB$.

Proof of Theorem 2: Suppose $b(\cdot)$ is an increasing symmetric equilibrium strategy. Bidder 1 expected payment is

$$(v - b(v)) \Pr(b(v) > b(Y), b(v) + B > Y) + E[(v - b(Y) - B)^+ \chi_{Y>v}] = \int_0^v y f_Y(y)\, dy.$$

Since $b(\cdot)$ is increasing this is equivalent to

$$(v - b(v)) F_Y(\min\{v, b(v) + B\}) + \int_v^1 (v - b(Y) - B)^+ f_Y(y)\, dy = \int_0^v y f_Y(y)\, dy.$$

Efficiency requires that if $Y > v$ then $b(Y) + B > v$. Therefore

$$\int_v^1 (v - b(Y) - B)^+ f_Y(y)\, dy = 0$$

and

$$(v - b(v)) F_Y(\min\{v, b(v) + B\}) = \int_0^v y f_Y(y)\, dy.$$

Consider now $v = 1$.

$$(1 - b(1)) F_Y(b(1) + B) = \int_0^1 y f_Y(y)\, dy = E[Y].$$

If B is small enough this will not have a solution. For the uniform distribution and $n = 2$, there is no solution if $B < 1/4$:

$$\frac{1}{4} + (1 - b(1))\, B \geq (1 - b(1))\,(b(1) + B) = \frac{1}{2}$$

and therefore $B \geq (1 - b(1))\, B \geq 1/4$.