Miller’s Equilibrium and Uncertainty

Chris Jones
Department of Economics
The Faculties
The Australian National University

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Abstract

This paper highlights the arbitrage activity by firms in Miller’s (1977) equilibrium when consumers face (short) selling constraints to restrict tax arbitrage. In this competitive equilibrium firms create risky tax-preferred securities that divide investors into strict tax clienteles; any changes in debt-equity ratios by individual firms have no real effects on consumers because other firms undo them. While DeAngelo and Masulis obtain this equilibrium with a full set of primitive bonds and a full set of primitive shares, the formalisation here relies only on a full set of conventional securities for firms to buy and sell. Once firms are constrained (for example, when the capital market is incomplete), Kim et al. (1979), Taggart (1980), Kim (1982) and Auerbach and King (1983) identify investor leverage clienteles. This paper demonstrates the arbitrage activity by firms that is implicit in Sarig and Scott (1982) who argue these clienteles are eliminated by standard portfolio theory.

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*Key Words: capital structure choice; investor leverage clienteles; arbitrage.*

Chris Jones
Department of Economics
The Faculties
The Australian National University
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1. Introduction

Modigliani and Miller (1963) showed how firms facing a classical corporate tax can raise their market values with higher leverage, where this can lead ultimately to an all debt equilibrium. There are a number of explanations for the interior debt-equity ratios that we observe. For example, expected bankruptcy costs and lost corporate tax shields increase with leverage, but they do not appear to be large enough in isolation to offset the tax advantage of debt. Miller (1977) identifies personal taxes for high tax investors that can favour equity by an amount sufficient to offset the corporate tax bias against it. This occurs when capital gains on shares are taxed less than cash distributions as dividends and interest. By itself, the corporate tax drives firms into debt, but progressive personal taxes can leave high tax investors with a preference for equity and low tax investors with a preference for debt. This leads to an interior aggregate debt-equity ratio for the corporate sector as a whole with debt specialists, equity specialists and marginal investors. Miller preserves these tax clienteles in equilibrium by imposing selling constraints on investors to restrict tax arbitrage, and then demonstrates that leverage is irrelevant to the market value of individual firms.

A general equilibrium model is used in this paper to provide a simple geometric analysis of Miller’s equilibrium in an uncertainty setting. This isolates the important arbitrage activity by corporations when investors face selling constraints. In their original derivations of the irrelevance propositions without taxes, Modigliani and Miller (1958) focus on the arbitrage activity by investors who re-bundle their portfolios to undo any changes in the financial policies of firms. In Miller’s equilibrium, however, ‘homemade leverage’ is restricted, so firms must perform this activity in a competitive capital market. When reflecting on the 30th anniversary of the Modigliani-Miller (MM) propositions, Miller (1988) acknowledges how important this is by noting that “much of what passes these days for corporate-raiding-cum-restructuring is just MM leverage arbitrage, but channeled through the raider’s corporate, 

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1 Warner (1977) presents estimates of these costs for US railway firms. DeAngelo and Masulis (1980a) provide a nice formal presentation of the optimal capital structures which arise from bankruptcy costs and lost corporate tax shields in a state preference model. Haugen and Senbet (1978) argue that bankruptcy costs can be avoided when financial intermediaries repackage the capital structures of firms.

2 Miller’s analysis is undertaken in a classical finance model with common information between agents in a competitive capital market. Common information rules out the agency costs that arise when investors and firm managers have different information, and competition invokes price-taking behaviour on all agents. No dividends are paid in Miller’s equilibrium.

3 Unless tax arbitrage is constrained, investors have infinite demands for their tax preferred securities when tax rates are independent of income, or their tax preferences are eliminated when marginal tax rates are continuous increasing functions of income.

4 Note also, that with marginal investors, the aggregate debt-equity ratio is indeterminate within upper and lower bounds determined by the demands of debt and equity specialists, where the range of indeterminacy depends on the amount of wealth marginal investors allocate to corporate securities.

5 Individual firms cannot change the risk spreading opportunities available to investors in a competitive capital market because there are perfect substitutes for every security they supply. To maximize profit, firms issue securities with the lowest cost, and this drives them to provide the securities that investors prefer.
Conventional securities have payouts in more than one state. A full set of conventional securities requires as many linearly independent securities as the states of nature. Senbet and Taggart (1984) extend this result in the presence of taxation to other forms of capital market imperfections and incompleteness. They argue that when firms have a comparative advantage in dealing with these imperfections they have an incentive to act as financial intermediaries and to complete the market for consumers in a competitive equilibrium.

Rather than personal, investment account.” In fact, this arbitrage occurs in a less obvious way when firms (or, as Ross (1988) observes, financial intermediaries who write derivatives on corporate assets) buy and sell each others securities.

The simplest place to begin a formalisation of Miller’s equilibrium is with certainty where investors choose between debt and equity solely on the basis of their tax preferences. In this setting, changes in firm debt-equity ratios do not impact on real household consumption when they can be absorbed by marginal investors. This, however, conceals the arbitrage that occurs in a more general setting. To see why, consider what happens without marginal investors; when one firm increases its debt-equity ratio there is an excess supply of debt and an excess demand for equity that puts downward pressure on bond prices and upward pressure on share prices. Other firms respond to these price changes by shifting from debt into equity, and this preserves the initial aggregate debt-equity ratio that supplies consumers with their tax-preferred securities.

Uncertainty makes the analysis more complicated because investors have tax and risk preferences for securities. DeAngelo and Masulis (1980b) obtain Miller’s equilibrium with a complete set of primitive, or Arrow, securities for both debt and equity (i.e., a full set of primitive bonds and a full set of primitive shares.) This allows investors to satisfy their risk preferences by holding just tax-preferred securities, but it raises the question as to whether or not we can replicate Miller’s equilibrium with conventional securities. When investors face (short) selling constraints they cannot create their tax-preferred risky securities, even with a complete set of conventional securities. Instead, they may hold debt and equity to spread risk, where this increases the tax paid from corporate income.

Kim et al. (1979), Taggart (1980), Kim (1982) and Auerbach and King (1983) identify investor leverage clienteles when this happens; firms use leverage decisions to change the riskiness of their securities, and investors gravitate toward firms that supply the securities they most prefer. Harris, Roenfeldt and Cooley (1983) and Kim (1982) find empirical evidence to support the existence of these leverage clienteles, but Sarig and Scott (1985) use standard portfolio theory to argue investor leverage clienteles do not arise in a competitive equilibrium similar to the one described by Miller. In this paper we formalise their argument by making explicit the arbitrage activity by firms who face incentives in a competitive capital market to provide investors with securities that fully satisfy their risk and tax preferences. Once again, this gives rise to strict tax clienteles, but without the capital market being “double complete”.

In a frictionless market with common information, which is the setting for Miller’s equilibrium, firms know what types of securities investors desire, and it is costless for them to create these securities. If, for whatever reason, the capital market is incomplete (which is assumed

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6 Conventional securities have payouts in more than one state. A full set of conventional securities requires as many linearly independent securities as the states of nature.

7 Senbet and Taggart (1984) extend this result in the presence of taxation to other forms of capital market imperfections and incompleteness. They argue that when firms have a comparative advantage in dealing with these imperfections they have an incentive to act as financial intermediaries and to complete the market for consumers in a competitive equilibrium.
explicitly in Kim, Kim et al., and Taggart, and implicitly in Auerbach and King), strict tax clienteles are replaced by financial leverage clienteles where firms offer a restricted set of securities to attract investors with similar risk and tax preferences. Clearly, this possibility arises with costly information, but not in the confines of the classical model adopted by Miller.

Aivazian and Callen (1987) use Edgeworth box diagrams to illustrate the equilibrium obtained by DeAngelo and Masulis. They focus on the demand side of the model and illustrate the equilibrium relationships between security returns when tax rates are continuous functions of income and compare this to the returns when tax rates are endowed on investors facing selling constraints. The geometric analysis in this paper differs by examining the security trades by firms and consumers to highlight the arbitrage activity that lies behind Miller’s equilibrium.

We begin the analysis in section 2 by formalising a two-period state-preference model where a competitive equilibrium is characterised in the absence of taxes. This equilibrium is presented using diagrams in section 3 of the paper. Taxes, both corporate and personal, are included in section 4 to obtain Miller’s equilibrium. The paper concludes in section 5 with a brief summary of the main findings.

2. Equilibrium in a Competitive Corporate Capital Market

All activity takes place at two points in time, t=0 and t=1, and there is a single non-storable good, x, which consumers are endowed with in the first period; they consume some of it then, and use the rest to buy financial securities from corporate firms who invest it to produce output of good x in the second period. The proceeds from this output are used to buy back financial securities from consumers. In other words, consumers transfer some of their current endowment of good x into the future period with corporate securities.

There is technological uncertainty in the model where firm output is uncertain and depends on the state of nature, s, which is outside the control of all agents. We assume a finite possible number of states of nature s = 1,…….S which are mutually exclusive. All agents agree on the state space and assign subjective probabilities to each state; they have common information.

Consumers and firms are small in the capital market, which is assumed to be competitive, and there are no leverage related costs. The only securities that trade are corporate securities, and there are K of them. The demand, supply and equilibrium conditions will now be formalised, initially in the absence of taxes.

**Demand:** There are a large number of consumers, i=1,…….I, who maximise utility by choosing consumption of good x in both periods. For a representative individual i we have:
Maximize \( U^i \{ x_0^i, x_1^i, x_2^i, \ldots, x_s^i \} \)

\( s.t. \) (1a) \( W^i = p_0 \bar{x}_0^i = \sum_k p_k a_k^i + p_0 x_0^i, \)

(1b) \( p_s x_s^i = \sum_k a_k^i R_{ks} \quad \forall s = 1, \ldots, S; \)

(1c) \( a_k^i \geq \bar{g}_k^i \quad \forall k = 1, \ldots, K. \)

Notation:
\( \bar{x}_0^i \) - \( i \)'s endowment of the consumption good at \( t=0; \)
\( x_0^i \) - \( i \)'s consumption at \( t=0; \)
\( p_0 \) - market price of good x at \( t=0; \)
\( a_k^i \) - units of security k purchased by \( i \) at \( t=0; \)
\( p_k \) - market price of security k at \( t=0 \) in state s;
\( x_s^i \) - \( i \)'s consumption at \( t=1 \) in state s;
\( p_s \) - market price of good x at \( t=1 \) in state s;
\( R_{ks} \) - payouts to security k in state s; and,
\( \bar{g}_k^i \) - is the lower bound on the units of security k sold by individual \( i \), with \( \bar{g}_k^i \leq 0. \)

The amount of good x each individual consumes and the amount they spend on corporate securities in the first period, is constrained by their initial endowment in (1a). Their consumption of good x in each state \( s \) is constrained by the payouts to securities in (1b), while the selling constraints in (1c) will apply later when taxes are introduced.

The first order condition for current consumption is:

\[
\frac{\partial U^i}{\partial x_0^i} - \lambda^i p_o = 0,
\]

while the \( K \) first order conditions for the securities, are:\textsuperscript{10}

\textsuperscript{10} The first order conditions are obtained by substituting the constraints in (1b) directly into the utility function where the consumer problem in (1) becomes:

(1') Maximize \( U^i \{ x_0^i, \sum_k \frac{a_k^i R_{ks}}{p_k}, \ldots, \sum_k \frac{a_k^i R_{ks}}{p_s} \} \)

\( s.t. \) (1'a) \( W^i = p_0 \bar{x}_0^i = \sum_k p_k a_k^i + p_0 x_0^i, \)

(1'c) \( a_k^i \geq \bar{g}_k^i \quad \forall k = 1, \ldots, K. \)
where \( q_s^i \) is \( i \)'s personalised discount factor for state \( s \) dollars. The inequalities will apply in (3) when selling by investors is constrained. \( \lambda_s^i \) is the lagrange multiplier for the budget constraint in (1a); it is \( i \)'s marginal utility for a dollar of income at \( t=0 \). Thus, \( q_s^i \) is the rate \( i \) is willing to substitute between dollars at \( t=0 \) and dollars in state \( s \) at \( t=1 \).

To simplify the analysis, define \( R \) as the \((S \times K)\) matrix of state-contingent payouts on the \( K \) traded securities; \( \tilde{q}^i \) as the \((1 \times S)\) row vector of personalised discount factors; and \( \tilde{p}_k \) as the \((1 \times K)\) column vector of security prices. Using this notation the first order conditions in (3) are written more compactly as:

\[
(3') \quad \tilde{q}^i R \leq \tilde{p}_k.
\]

When \( R \) is full rank (with \( \text{rank}(R) = K = S \)) households can trade across all the states by bundling securities of different risk together. This requires:
(i) As many linearly independent securities as states of nature; and,
(ii) Investors can buy or sell securities without constraint.

These conditions make the capital market complete for consumers, where the personalised discount factors in (3') are equated across all \( i \), with:

\[
\tilde{q} = \tilde{q}^i = \tilde{p}_k R^{-1} \quad \forall i.
\]

**Supply:** There are a large number of price-taking firms, \( j=1, \ldots, J \), who sell securities at \( t=0 \) to buy inputs of the current consumption good from consumers. These inputs are used at \( t=0 \) to produce state-contingent outputs at \( t=1 \) which are paid to consumers when firms redeem their securities. The representative firm will therefore:

\[
\sum_s q_s^i R_{ks} - p_k \leq 0 \quad \forall k=1, \ldots, K,
\]

This formulation reflects the fact that security demands are derived from underlying consumer demands for state-contingent consumption.

---

\( ^{11} \) Formally, \( q_s^i = \frac{\partial U^i}{\partial x_s^i} \frac{1}{\lambda p_s} \).

\( ^{12} \) In expanded form (3') is:

\[
\begin{pmatrix}
q_1^i & \cdots & q_S^i
\end{pmatrix}
\begin{pmatrix}
R_{11} & \cdots & R_{K1} \\
\vdots & \ddots & \vdots \\
R_{1S} & \cdots & R_{KS}
\end{pmatrix}
\leq
\begin{pmatrix}
p_1 & \cdots & p_K
\end{pmatrix}
\]
Maximize $V^j = \sum_k p_k a_k^j - p_0 x_0^j$

s.t.  

(4a) $p_s^j x_s^j (x_0^j) = \sum_k a_k^j R_{ks} \quad \forall s = 1, \ldots, S$

(4b) $a_k^j \geq \tilde{g}_k^j \quad \forall k = 1, \ldots, K$

Additional notation:

- $x_0^j$ - firm $j$'s investment at $t=0$;
- $x_s^j$ - firm $j$'s output at $t=1$ in state $s$;
- $a_k^j$ - units of security $k$ supplied by firm $j$ at $t=0$; and,
- $\tilde{g}_k^j$ - is the lower bound on the units of security $k$ purchased by firm $j$, with $\tilde{g}_k^j \leq 0$.

State contingent payouts to securities are bounded by firm $j$’s net outputs in (4a), while (4b) captures any buying constraints on firm $j$ when $\tilde{g}_k^j \leq 0$. The optimal investment decision satisfies:

\[ \sum_s \lambda_s^j p_s \frac{\partial x_s^j}{\partial x_0^j} - p_0 = 0 \]

while the efficient supply of each security $k$ satisfies:

\[ \sum_s \lambda_s^j R_{ks} - p_k \leq 0 \quad \forall k = 1, \ldots, K, \]

where $\lambda_s^j = q_s^j$ is firm $j$’s *personalised discount factor* for dollars in state $s$; it is the state-contingent Lagrange multiplier for the problem in (4). This provides the familiar asset valuation equation where, in the absence of constraints, the price of each security $k$ is equal to the discounted present value of its state-contingent payouts. Using vector notation for the first order conditions in (6) we obtain:

(6') $\tilde{q}^j R \leq \tilde{p}_k$.

The securities market is **complete for firms** when $R$ has full rank and there are no constraints on their security trades. Accordingly, they can arbitrage away any difference in the returns to debt and equity in each state, where this equates the discount factors on future dollars across firms, with:

\[ \tilde{q} = \tilde{q}^j = \tilde{p}_k R^{-1} \quad \forall j. \]

When the capital market is complete for both consumers and firms, we have, in the absence of taxes:

\[ \tilde{q} = \tilde{q}^i = \tilde{q}^j = \tilde{p}_k R \quad \forall i, j. \]

This means all agents in the economy discount future dollar payouts to assets identically at the margin.
Equilibrium: A competitive equilibrium is characterised by a vector of security prices $\tilde{p}_k^*$, state-contingent prices $\tilde{p}_s^*$, and current price $p_0^*$, such that:¹³

(i) $\tilde{\alpha}^i*$ - solves the consumer problem for each $i = 1, \ldots, I$;

(ii) $\tilde{\alpha}^j*$ - solves the firm problem for each $j = 1, \ldots, J$;

(iii) $\sum_i \tilde{\alpha}^i* = \sum_j \tilde{\alpha}^j*$; and,

(iv) $\sum_i \tilde{x}_0^{i*} = \sum_i x_0^{i*} + \sum_j x_0^{j*}$.

The security trades in (i) and (ii) solve the respective optimisation problems for all consumers and firms, while conditions (iii) and (iv) require market clearing in the respective financial and commodity markets.

Before proceeding to the geometric analysis it is worth noting that in a competitive capital market no individual firm or consumer can affect the payouts to securities i.e., they cannot supply new securities because perfect substitutes exist or can be created by bundling together existing securities. Thus, all agents effectively have access to a full set of primitive securities, with:

$$R_s = \sum_k \alpha_k \tilde{R}_k \quad \forall s \in S,$$

where $R_s$ is the primitive security that pays only in state $s$, and $\tilde{R}_k$ the vector of payouts across all the states for each conventional security $k$. By arbitrage, the price of each primitive security, $p_{k,s}$, is the linear weighted sum of the prices of its derivative conventional securities with weights equal to the amount of each security held in the derivative portfolio.

Thus, each firm implicitly supplies a bundle of primitive securities that are written over given net cash flows, where the debt-equity choice simply relabels these primitive securities without having any real effects on consumers. In a (frictionless) competitive capital market, financial assets are a veil over the real economy.

3. M-M Leverage Irrelevance in the Absence of Taxes

To facilitate the diagrammatic analysis that follows we group corporate securities together as debt (b) and equity (e) instruments i.e., debt is a bundle of risky bonds, and equity is a bundle of risky shares. The following table summarises the first order conditions (FOC’s) for the optimal supply of debt and equity by a representative firm $j$ using (6).

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¹³ There are also questions of uniqueness and stability of equilibrium. The equilibrium is unique when preference sets are strictly convex and firm production technologies are strictly concave.
It is important to note that when firms change their debt-equity ratios they are repackaging a given set of payouts across states (with investment held constant). Thus, debt and equity are equally risky at the margin. Using the FOC’s in the table above we obtain the familiar condition for an optimum, with:

\[ MRT_{je}^j = \frac{\sum_s q^j_s R_{bs}}{\sum_s q^j_s R_{es}} = \frac{p_b}{p_e}. \]

The possible equilibrium outcomes are illustrated in Figure 1. The amount of debt and equity firm \( j \) supplies is constrained by its net outputs and the payouts it makes on these securities. The line \( r_jr_j \) summarises this constraint, and its slope \( (MRT_{be}^j) \) is equal to the relative payouts on debt and equity; this is the marginal cost of leverage to firm value. The iso-profit lines \( \pi_j \) have slopes equal to the relative price of debt and equity; they measure the marginal benefit of leverage to firm value. At point A the firm sells only equity when \( \pi_j \) is flatter than the asset production frontier \( r_jr_j \). At point B the firm sells only debt when \( \pi_j \) is steeper than \( r_jr_j \). In a complete capital market arbitrage by firms will equate the slope of the iso-profit line to the slope of the asset production frontier, with:

\[ MRT_{be}^j = \frac{\sum_s q^j_s R_{bs}}{\sum_s q^j_s R_{es}} = \frac{p_b}{p_e}. \]

The following table summarises the FOC’s for the optimal amount of debt and equity held by the representative consumer \( i \) using (3).
When firms change their debt-equity ratio they are reshuffling the same set of state-contingent payouts between the two types of securities. From the FOC’s above, consumer i’s valuation for this debt-equity change is:

\[
MRS_{be}^i = \frac{\sum_s q_s^i R_{bs}}{\sum_s q_s^i R_{es}} > \frac{p_b}{p_e}
\]

This is illustrated in Figure 2, where \( r_j \) is the budget constraint that summarises the possible combinations of debt and equity consumer i can hold with given initial net wealth \( W^i = p_0(\bar{x}_0^i - x_0^i) \). The indifference schedules are linear because debt and equity are equally risky at the margin. When the MRS between debt and equity along indifference schedule \( U^i \) is less than the slope of the budget constraint the consumer prefers only equity, while for the MRS along indifference schedule \( U'' \) the consumer prefers only debt.

In a complete capital market arbitrage by consumers equates their marginal rates of substitution to the relative price of debt and equity, where in Figure 1 the indifference schedule for consumer i lies along the budget constraint, with:

\[
MRS_{be}^i = \frac{\sum_s q_s^i R_{bs}}{\sum_s q_s^i R_{es}} = \frac{p_b}{p_e}
\]

---

14 We analyse consumer preferences for changes in firm debt-equity ratios which shuffle the same payouts between debt and equity at the margin.
Taking all consumers and firms together in equilibrium we therefore have:

\[
MRS_{be}^i = \frac{\sum_s q_s^i R_{bs}}{\sum_s q_s^i R_{es}} = 1 = \frac{\sum_s q_s^j R_{bs}}{\sum_s q_s^j R_{es}} = MRT_{be}^j \quad \forall i,j.
\]

This is illustrated in figure 3 where the budget constraints and indifference schedules have been aggregated across all firms and consumers who face the same security payouts and security prices. The Asset Production Frontier (RR) measures the aggregate payouts to debt and equity by all firms, while UU is the aggregate indifference schedule for all consumers. The linearity of these schedules follows from the fact that the underlying net outputs are fixed by given investment.

It is clear from Figure 3 that leverage irrelevance holds at both the aggregate level and for individual firms in the absence of taxation. Any debt-equity choices along the asset production frontier RR are a matter of indifference for all consumers because they are willing to hold either type of security in equilibrium.

4. Miller’s Equilibrium

Miller examines the way personal taxes offset the classical corporate tax bias against equity. The corporate tax exempts interest on corporate bonds and is assumed to be a constant rate \( t_c \) for all firms. In contrast, personal taxes apply to all income received by investors, but at different rates to capture the progressive rate scales that typically apply to personal income with taxes on cash distributions (as interest and dividends), \( t_b^i \), being greater than the taxes on capital gains, \( t_g^i \), for all consumers.\(^{15}\)

4.1 Equilibrium with Just a Classical Corporate Tax

As a first step toward the Miller equilibrium we consider the effects of the corporate tax on its

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\(^{15}\) In many countries the statutory tax rates on capital gains are lower than the tax rates on cash distributions. In Australia, for example, only real capital gains are taxed.
own. It is assumed throughout the analysis that taxes apply to economic income i.e., there is no tax on capital, but since capital is returned to consumers with income in the second period, the two components need to be separated by writing market payouts as \( R_{ks} = p_k(1 + r_{ks}) \). These are payouts after corporate tax has been paid by firms, and are what consumers receive. The only significant change in notation occurs in (4a), where the constraint for the representative firm becomes:

\[
p_s x^j_s(x^j_0)(1-t_c) = \alpha^j_b p_b (1 + r_{bs})(1-t_c) + \alpha^j_e p_e [(1-t_c) + r_{es}] \quad \forall s \in S.
\]

Notice how interest income and the initial capital invested in debt and equity are tax deductible expenses under the classical corporate tax base; only income paid on equity is subject to tax. This is more easily recognised by rewriting the constraint as:

\[
r_{es}\alpha^j_e p_e = (1-t_c) \left[ p_s x^j_s - (1+r_{bs}) \alpha^j_b p_b - \alpha^j_e p_e \right] \quad \forall s \in S.
\]

The first order conditions for the representative consumer and firm in the presence of the corporate tax, are:

<table>
<thead>
<tr>
<th>Security</th>
<th>Consumer i</th>
<th>Firm j</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_b )</td>
<td>( \sum_s q_s^j (1+r_{bs}) \leq 1 )</td>
<td>( \sum_s q_s^j (1+r_{bs})(1-t_c) \leq 1 )</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>( \sum_s q_s^j (1+r_{es}) \leq 1 )</td>
<td>( \sum_s q_s^j [(1-t_c) + r_{es}] \leq 1 )</td>
</tr>
</tbody>
</table>

This leads to:

\[
MRS_{be}^i = \frac{\sum_s q_s^j (1+r_{bs})}{\sum_s q_s^j (1+r_{es})} \geq 1 \quad \text{for consumers; and,}
\]

\[
MRT_{be}^j = \frac{\sum_s q_s^j (1+r_{bs})(1-t_c)}{\sum_s q_s^j [(1-t_c) + r_{es}]} \leq 1 \quad \text{for firms.}
\]

16. We follow the familiar practice of assuming tax revenue is not returned to the economy. While this matters for the equilibrium prices and consumption it does not change the way the taxes affect the slopes of consumer indifference schedules and the asset production frontier in the debt-equity space. Our interest here is not in the welfare effects of the taxes, but the way they distort relative prices.

17. In practice the taxes are levied on measured (or accounting) income which differs from economic income. A major reason for the difference lies in the way depreciation is measured.

18. The net output of each firm \( j \) measures its residual value at \( t=1 \). In a multi-period model there would also be the value of the firm at \( t=1 \). This would allow us to measure depreciation in the conventional way by calculating the change in the value of the firm over each period of time.
Firms issue both debt and equity when $r_{bs} (1-t_c) = r_{es}$ in each state $s$ (with $MRT^j_{be} = 1$), while consumers hold both securities when $r_{bs} = r_{es}$ in each state $s$ (with $MRS^i_{be} = 1$). Clearly, these conditions cannot hold simultaneously because $MRS^i_{be} > MRT^j_{be}$. Instead, consumers prefer debt over equity because it exempts corporate income from tax.

This is illustrated in Figure 4 where the slope of the asset production frontier RR is less (in absolute value terms) than the slope of the aggregate indifference curve UU. The tax reduces corporate income payable to consumers when firms sell equity; where this shifts the intercept of RR from A to A'. Since no tax is collected on debt, the intercept at B is unchanged. No equilibrium exists in this setting unless tax arbitrage is restricted, and this can be achieved by restricting security sales by consumers, or restricting security purchases by firms. If either of these constraints is prohibitive, an all debt equilibrium exists at point B in the diagram, where no equity is issued by firms or consumers, and no corporate tax revenue is collected.\(^{19}\)

At this point it is worth noting that the choice between the constraints to restrict tax arbitrage can cause subtle differences in the analysis. If firms face buying constraints, consumers can spread risk with a full set of conventional bonds; in essence, they can create a full set of primitive securities. Alternatively, if consumers face selling constraints, they cannot create a full set of primitive securities with a full set of conventional bonds.\(^{20}\) Instead, they rely on firms to supply the risky bonds that satisfy their risk preferences, and firms can do this with a full set of conventional securities; in a frictionless common information setting where firms know the risk preferences of consumers, and face incentives to satisfy them in a tax effective manner when the market is competitive. These insights will be useful when personal taxes are introduced in the next section.

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\(^{19}\) The constraints could bound these trades from below where this allows the equilibrium to move down along RR beyond point B. This would make consumers better off by the additional wealth they obtain from tax arbitrage.

\(^{20}\) Consumers cannot satisfy their risk preferences with a full set of conventional bonds if the selling constraints on them are binding in equilibrium. Recall that the bond market is complete for consumers when they have access to a full set of linearly independent conventional bonds, and face no constraints on their security trades.
4.2 Equilibrium with Corporate and Personal Taxes

Once security returns are subject to personal tax, the consumer budget constraint in \( (1b) \) becomes:

\[
p_s x^i_s = \alpha^i_p p_b \{1 + r_{bs}(1 - t^i_b)\} + \alpha^i_e p_e \{1 + r_{es}(1 - t^i_e)\} \quad \forall s = 1, \ldots, S.
\]

The only returns to equity are capital gains; no dividends are paid because the personal taxes on them are higher for all consumers. The first order conditions for the representative consumer and firm are summarised in the following table.

<table>
<thead>
<tr>
<th>Security</th>
<th>Consumer i</th>
<th>Firm j</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_s )</td>
<td>( q_s {1 + r_{bs}(1 - t^i_b)} \leq 1 )</td>
<td>( q_s {1 + r_{bs}(1 - t^c)} \leq 1 )</td>
</tr>
</tbody>
</table>

Firms supply both securities when \( r_{bs}(1 - t^i_b) = r_{es}(1 - t^i_e) \) in each state \( s \), while consumers buy both securities when \( r_{bs}(1 - t^i_b) = r_{es}(1 - t^c) \) in each state \( s \). These conditions are met simultaneously when:

\[
(1 - t^i_e)(1 - t^i_b) = (1 - t^i_c).
\]

Any consumers with personal taxes that satisfy this relationship are marginal investors. Notice how payouts to equity are double taxed under the corporate tax system.

When personal tax rates differ across consumers, we can have:

(i) \( (1 - t^i_e)/(1 - t^i_b) > (1 - t^i_c) \) - for **equity specialists**;

(ii) \( (1 - t^i_e)/(1 - t^i_b) < (1 - t^i_c) \) - for **debt specialists**; and,

(iii) \( (1 - t^i_e)/(1 - t^i_b) = (1 - t^i_c) \) - for **marginal investors**.

To simplify the analysis let us assume all equity specialists have the same tax rates as do all debt specialists. This gives consumers in each tax clientele illustrated in Figure 5 the same sloped indifference schedules. Once again, they are linear because debt and equity are equally risky at the margin. The different slopes reflect the different personal tax rates, where those with schedules \( U^i_e \) have a tax preference for equity, those with schedules \( U^i_b \) have a tax preference for debt, and those with schedules \( U^i_m \) are marginal investors.

\[
21 \text{ All equity income is paid as a capital gain in this model because dividends are subject to a higher rate of personal tax than are capital gains for all consumers.}
\]

\[
22 \text{ Interest dominates dividends, despite being subject to the same personal tax rate, because dividends are also subject to corporate tax. Even though capital gains are double taxed, they can dominate interest for consumers with personal cash tax rates that exceed the corporate tax rate.}
\]
It is clear from figure 5 that the debt and equity specialists have infinite demands for their tax preferred securities unless they are constrained from selling debt and equity; debt specialists increase their wealth by going short in equity and long in debt, while equity specialists increase their wealth by taking the opposite positions. Unless this tax arbitrage is constrained no equilibrium exists.\textsuperscript{23} If consumers are completely constrained from selling securities, equity specialists locate at point A, debt specialist at point B, and marginal investors anywhere along their budget constraints between the points A and B.

Miller’s equilibrium is illustrated in Figure 6 where consumers divide into strict tax clienteles. Point A is the minimum amount of debt that must be supplied to satisfy debt specialists, while point B is the minimum amount of equity that must be supplied to satisfy equity specialists. The allocations between points A and B are those which marginal investors are willing to hold. The aggregate supplies of debt and equity in equilibrium must therefore lie along the line segment AB on the asset production frontier RR.

It is easy to see how consumers specialise in one security when there is no risk to spread. DeAngelo and Masulis (1980b) obtain the same result in an uncertainty setting when there is a complete set of primitive bonds and a complete set of primitive shares; consumers can spread risk by just holding their tax-preferred securities. Notice how the aggregate debt-equity ratio is indeterminate along the line segment AB in Figure 6. There are two ways of demonstrating Miller’s leverage irrelevance proposition for individual firms. The first relies on the arbitrage activity by investors, while the second relies on the arbitrage activity by firms. If one firm changes its capital structure and moves the aggregate debt-equity ratio along the line segment AB, it clearly has no real effects on consumers when marginal investors absorb the change.

\textsuperscript{23} In this model the marginal personal tax rates are endowed exogenously on consumers so tax preferences are unaffected by changes in wealth from tax arbitrage. Dammon and Green (1987) provide a diagrammatic analysis of the equilibrium outcomes when marginal personal tax rates are increasing functions of income, while Jones and Milne (1992) include a government budget constraint to bound the equilibrium.
But marginal investors are not required for this irrelevance result.

To see this, consider Miller’s equilibrium in Figure 7 without them, where there is an optimal aggregate debt-equity ratio at point A. When one firm raises its leverage and moves the equilibrium below A along RR, there is an excess supply of debt and excess demand for equity. This puts downward pressure on the price of debt and upward pressure on the price of equity which induces other firms to shift out of debt and into equity (with the same risk). Thus, the aggregate debt-equity ratio stays constant, and there is no change in the real consumption opportunities for consumers. Geometrically, it is the linearity in the indifference schedules and/or linearity in the aggregate production frontier that drives Miller’s irrelevance proposition.

Finally, if the primitive securities in DeAngelo and Masulis are replaced with conventional securities (that have payouts in more than one state), consumers will have to rely on firms to supply the risky securities that satisfy their tax preferences. If firms cannot supply these securities (because, for example, the capital market is incomplete) consumers may bundle debt and equity together to spread risk, even though they have a tax preference for one of the securities. This generates the “investor leverage clienteles” identified by Kim et al., Taggart, Kim and Auerbach and King. Once investors break out of strict tax clienteles, firm debt-equity choices can change their real consumption opportunities by altering the riskiness of their tax-preferred securities; Miller’s leverage irrelevance result fails.

Sarig and Scott argue these investor leverage clienteles disappear in a competitive capital market. The previous analysis demonstrates the role played by firms (or their agents) in the spanning arguments used by Sarig and Scott. In a frictionless common information setting, firms will costlessly create the tax-preferred securities that satisfy the risk preferences of every investor. This separates investors into strict tax clienteles, and Miller’s leverage irrelevance holds because firms preserve the aggregate supplies of debt and equity instruments that investors prefer.

5. Conclusion

This paper highlights the arbitrage activity by firms in Miller’s equilibrium. Arbitrage is crucial to all the MM irrelevance propositions, and can be undertaken by consumers and/or firms. Once consumers face (short) selling constraints to restrict tax arbitrage, competition is

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24 This is demonstrated by Auerbach and King when consumers have mean variance preferences.
preserved in Miller’s equilibrium if firms can arbitrage away any profits. And this is the invisible hand that drives firms to supply investors with the securities that satisfy their risk and tax preferences. If firms are also constrained, investor leverage clienteles arise, and Miller’s leverage irrelevance result fails to apply.

There are good practical reasons to expect that firms do not costlessly obtain information about their investor’s risk preferences, nor is it costless for them to create these securities. It is quite likely that, once these costs are made explicit in the formal analysis, investor leverage clienteles will arise. But clearly, this takes the analysis outside the “classical finance” model that underpins the MM irrelevance propositions.
References:


