Mother and children: an endogenous approach

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Most literature on intra-family allocation of time focuses on two-person households, especially on the substitution of the wife’s labour for that of the husband in home production. The husband and the wife allocate their time based on their comparative advantage in the production of market and home goods. Comparative advantage is a function of their relative wage rates and their efficiency in different activities. Since the husband’s wage rate is usually higher than that of the wife and, traditionally, the efficiency of the husband in home production is regarded as equal or less than that of the wife, the husband specialises in market work, allocating his time between market work and leisure. The wife either specialises in home production or is involved in market activities, home production and leisure. Given this division of labour, a rise in the wife’s wage rate should see the wife increase her market time (provided that the substitution effect is not offset by the income effect) but reduce her leisure and/or her home production time. The wife may consume fewer leisure goods especially if home production technology is not more readily available. The impact of a rise in the wife’s wage on the sum of her leisure and home production is negative. However, how her time is likely to be split between the two is of an unknown sign a priori.

Extending the model to include children may change the predicted family division of labour. In this case, an increase in the wage of the wife might also induce a substitution of children’s time for that of the mother in household production as the mother increases her work time, thus lowering children’s leisure time and possibly children’s school attendance (provided that such a substitution effect more than offsets the income effect at the household level).

The importance of the presence of children, their age and number, on the labour supply of married women has been addressed by many studies (Apps and Savage 1989; Beggs and Chapman 1988; Connelly 1992; Perry 1990). In these studies, the presence of children is regarded as exogenous and their presence adds to the opportunity cost of married women’s labour supply decisions. There are few econometric studies which examine how children contribute positively to a mother’s labour supply by doing housework. This paper attempts to model children’s time allocated to household chores and treats their time allocation as an endogenous variable, along with the time allocation of the mother.

Data collected in Vietnam is used to test the hypotheses generated from the theoretical model. In the case of Vietnam, given the large number of school-age children not in school (World Bank 1996:89) and their significant involvement in economic activities within the household, it is not unreasonable to expect that this kind of substitution between home production of the mother and children exists.
A theoretical model

Studies on time use have been based on the classic paper by Becker (1965). Becker’s time allocation model, together with the work of Mincer (1962) on the labour supplies of married women, gave rise to the ‘new economics of the family’ within which a wide range of socioeconomic phenomena are analysed. The examination of intra-family decision making on allocating household activities among household members, on family size and the labour supply decision of family members are some examples.

Aside from the literature on intra-family decisions on fertility (Rosenzweig 1981) most models are limited to a two-person household, focusing on the time allocation choices of the husband and the wife between market and non-market activities (Gramm 1974, 1975).

Maximising a household utility function subject to time and financial constraints forms the basis of the intra-family time allocation models. Utility maximisation requires time to be allocated in such a way that the marginal value in alternative uses (in home production, market work, and leisure) equals the market wage. A change in the market wage of the mother will change the cost of her time use in different activities. If the mother’s wage rate increases, her labour supply will rise if the substitution effect is dominant. The impact of a wage increase on her time-use pattern will, however, be more complicated if children are included. In most of the time use models (Gronau 1973, 1980) where children are not included, the higher the mother’s wages, the more time she spends working in the labour market (if the substitution effect is stronger than the income effect), and the less time she can allocate for her leisure and home production. Under this situation, she has to cut down the time spent on one or both of the activities. By including children in the model, however, a greater amount of time allocated by the mother to market work does not necessarily lead to her reducing the sum of leisure and home production (though her home production time does fall). Now, children can substitute for the mother in home production. Four cases could be derived. First, at one extreme, the child’s home production time is not substitutable for that of the mother. In this case, an increase in her market time will reduce her home production and/or leisure time. The latter depends on the elasticity of substitution between time and market-purchased goods.

Second, at the other extreme, if the child’s and the mother’s time are perfect substitutes in home production, then the decrease of the mother’s time in home production, due to an increase in her market time in response to the rise in her wage rate, could be completely compensated for by an equivalent increase in the child’s home time. Assuming the labour market always clears, the mother could completely substitute her market time for home production time. In this case, if the reduction in the mother’s home production time is totally absorbed by the increase in her market time, she could maintain the same amount of leisure that she consumed prior to the wage increase and at the same time increase her market time. Children have to spend more time in home production, forcing a choice on the children between their leisure time and the time that they spend in school.

Third, if the reduction of the mother’s home production time cannot be completely substituted for by the increase in the child’s home production time, then the mother could enjoy
less leisure than she consumed prior to the wage increase provided that the increase in her market time is greater than the fall in her home production time.

Finally, if the increase in the mother’s market time is less than the reduction in her home production time, then the mother could increase market time and consume more leisure than she consumed prior to the wage rise. The extent of the change in the mother’s leisure depends on the elasticity of substitution between the home production time of mother and child and the elasticity of the mother’s labour supply with respect to her wage rate. Nevertheless, in all cases, the child will have to reduce the sum of his or her leisure and school time.

The model

For the purpose of illustration we will adopt a very simple model which includes time allocation between market work, home production and schooling, to illustrate the substitution between a mother and children in home production. The following assumptions are made.

• There is no joint production. That is, home production does not generate leisure.
• There is perfect substitution between market goods and home production goods.
• There is additive separability of the household utility function, the marginal utility of any good is unaffected by the quantities consumed of all other goods.
• The sum of all individuals’ home production times is the sole input in the home production function.

Consider the household, which maximises the household utility function, Equation 1, subject to the full income constraint, Equation 3. In this case, the commodities that the household consumes consist of two goods: those purchased from the market, \( x_m \), and those that are produced at home

\[
z = g\left(\sum_{i} a_i t^h_i\right)
\]

The home production function, \( z \), is a function of the total time that individual members put into such activity. In addition, in order to reflect different marginal productivities of different members in home production, the time that each member spends in home production, \( t^h_i \), is weighted by a scalar, \( a_i \). Also, \( P_z \) is assumed to be equal to one. Together \( x_m \) and \( z \) comprise the composite goods that the household consumes.

Recall the Hicks’ composition good theorem, which states that if several goods have the same price (or, more specifically, the prices are proportional), then the goods may be aggregated together. However, if we can observe the aggregate quantity empirically, but not the individual components, then the theorem implies that we have to aggregate them in order to get an identifiable empirical model. In our case, since leisure and home production time have the same price, namely the wage rate, only the sum of these is identified. Under these circumstances, we have to assume that leisure time is exogenously given for all family members. Of course, an
alternative is to allow each individual’s home production time to enter the function of market purchased goods separately, that is $x_m = g(t_i^h)$.

Since the theoretical model is for illustrative purposes only, we opt for the first approach in order to make the mathematics more tractable. Thus, in what follows we shall assume leisure time is exogenously given at zero for all members of the household. For the sake of simplicity, only a three-person household is considered. In addition to the assumptions made, instead of modeling the child’s schooling as a function of the time a child spends in school and other inputs such as books, we only use the time spent in school as an argument in the utility function. We also assume each individual in the household has the same time endowment, $t_i = t_0 > 0$. The prices of the commodities consumed $p_x$, $p_z$ and $p_m$ are normalised to one. Now the model is

(1) Household utility function $u = u(x, t_3^s)$

(2) Composite goods 

$$x = x_m + g\left(\sum_{i=1}^{3} a_i t_i^h\right)$$

(3) Full budget constraint 

$$y = v + \sum_{i=1}^{3} t_0 w_i = \sum_{i=1}^{3} w_i t_i^h + w_3 t_3^s + x_m$$

Re-arranging Equation 3,

(5) 

$$y = v + \sum_{i=1}^{3} w_i t_i^m = x_m$$

(6) Time endowment 

$$t_0 > 0$$

(7) Time constraints 

$$t_0 = t_1^h + t_1^m, \quad t_0 = t_2^h + t_2^m$$

for the father and mother

(8) Child’s time constraint 

$$t_0 = t_3^h + t_3^m + t_3^s$$

(9) Non-negativity constraints 

$$t_i^m \geq 0; \quad t_i^h \geq 0; \quad t_3^s \geq 0$$

Setting up a Lagrangian problem, we have
Differentiating the Lagrangian function with respect to $t_i^m$, $t_i^h$, $t_3^s$, given the Kuhn-Tucker conditions

\begin{align}
\frac{\partial L}{\partial t_i^m} &= u w_i - \gamma_i + \mu_i = 0 \\
\frac{\partial L}{\partial t_i^h} &= u a_i g - \gamma_i + \xi_i = 0 \\
\frac{\partial L}{\partial t_3^s} &= u_s - \gamma_3 + \delta_3 = 0 \\
\frac{\partial L}{\partial \gamma_i} &= t_0 - t_i^m - t_i^h = 0 \quad \text{where } i=1, 2 \\
\frac{\partial L}{\partial \gamma_3} &= t_0 - t_3^m - t_3^h - t_3^s = 0 \\
\end{align}

The complementary slackness conditions are

\begin{align}
\mu_i t_i^m &= 0; \quad \mu_i \geq 0; \quad t_i^m \geq 0 \quad \text{where } i=1, 2, 3 \\
\xi_i t_i^h &= 0; \quad \xi_i \geq 0; \quad t_i^h \geq 0 \quad \text{where } i=1, 2, 3 \\
\delta_3 t_3^s &= 0; \quad \delta_3 \geq 0; \quad t_3^s \geq 0 \\
\end{align}

The fact that the total time, $t_0$, in the time constraints $t_i^m + t_i^h = t_0$ is positive rules out the case where $\xi_i$ and $\mu_i$ are both positive for $i=1, 2$. If they are both positive, this implies
\( t_i^m = t_i^h = 0 \) which violates the time constraints. However, it is possible to have \(^{12}\)

\[
\xi_i > 0 \text{ and } \mu_i = 0 \quad \Rightarrow t_i^h = 0, t_i^m > 0
\]

or

\[
\mu_i > 0 \text{ and } \xi_i = 0 \quad \Rightarrow t_i^h > 0, t_i^m = 0
\]

or

\[
\xi_i = \mu_i = 0 \quad \Rightarrow t_i^h > 0, t_i^m > 0
\]

Subtracting Equation 11 from Equation 10 yields

\[(18) \quad u_x(w_i - a_i g') = \xi_i - \mu_i \]

Assume \( w_1 > w_2 > w_3 \) and the child allocates some of his or her time for schooling, \( t_3^s > 0 \) (from Equation 17 and 12, this implies \( \delta = 0 \)). Using these assumptions and the equations above, we can derive different cases involving different individual patterns of individual time use.

Assume \( w_1 > w_2 > w_3 \) and \( w_2 \) is significantly higher than \( w_3 \). Furthermore, assume children go to school, \( t_3^s > 0 \) and therefore, \( \delta = 0 \). Here only commonly observed cases are examined.

**Case 1.** Both the father and mother engage in market work. Children are involved in home production and attend school.

The necessary condition for the case is

\[ w_1 > w_2 > a_i g' (0) > w_3 \]

Given the diminishing marginal productivity of home production, it is possible to find some positive \( t_i^h \), where \( i = 1, 2 \), to satisfy \( w_1 > a_i g' \), \( w_2 > a_2 g' \). Therefore, we can derive from Equation 18

Father

\[ w_1 - a_1 g' > 0 \Rightarrow \xi_1 - \mu_1 > 0 \]

\[ \Rightarrow \xi_1 > 0, \mu_1 = 0 \]

Mother

\[ w_2 - a_2 g' > 0 \Rightarrow \xi_2 - \mu_2 > 0 \]

\[ \Rightarrow \xi_2 > 0, \mu_2 = 0 \]
Therefore, the father and mother specialise in market work. That is,
\[ t^m_1 = t^m_2 = t^m_0 \geq 0 \]
and
\[ t^h_1 = t^h_2 = 0 \]

Child

The child allocates time to home production and schooling. That is,
\[ t^s_3 = t^s_0 - t^h_3 \]
\[ u_s(t^s_3) = u_x a^s_3 g^i(t^h_3) \]

Case 2. The husband engages in market work. The wife divides her time between home production and market work, and the children undertake home production and attend school.

The necessary condition for this case is as follows

\[ w_1 > a_i g'(0) > w_2 > w_3 \]

The diminishing marginal productivity of home production implies \( g'' < 0 \). Thus, \( g'(0) \) defines the upper bound of the marginal product.

Father

Since \( w_1 > a_i g'(0) \) at the outset, there is no \( t^h_1 \) which can induce the husband to participate in home production. Therefore, we have

\[ w_1 - a_i g'(0) > 0 \Rightarrow \xi_1 - \mu_1 > 0 \quad \text{from Equation 18} \]

Given such a condition, two possible scenarios can be derived

i) \( \xi_1 > 0, \mu_1 > 0 \)

This is ruled out since \( t^h_1 = 0 \) and \( t^m_1 = 0 \), which are implied by the complementary slackness conditions, and this violates the assumption of a positive time endowment for each individual member, that is \( t^0 > 0 \).

ii) \( \xi_1 > 0, \mu_1 = 0 \)
By the complementary slackness conditions this implies \( t_1^h = 0 \) and \( t_1^m > 0 \), i.e. the husband specialises in market work but does not engage in home production at all.

Mother

With \( a_2 g'(0) > w_2 \) and the diminishing marginal product of home production, it is possible to have some \( t_2^h, t_3^h > 0 \) with the marginal product of home production equated to the mother’s wage rate, that is \( w_2 = a_2 g'(a_2 t_2^h + a_3 t_3^h) > w_3 \). We can then rewrite \( w_2 = a_2 g'(a_2 t_2^h + a_3 t_3^h) \) as \( w_2 - a_2 g'(a_2 t_2^h + a_3 t_3^h) = 0 \). This implies from Equation 18

\[
\xi_2 = \mu_2 = 0
\]

Therefore, \( t_2^m > 0, t_2^h > 0 \). The wife engages in both market work and home production.

Child

From the above analysis, when equality of the marginal product of home production to the mother’s wage rate is achieved, children’s home production time can be positive or zero. In addition, since \( a_2 g'(a_2 t_2^h + a_3 t_3^h) > w_3 \), \( t_3^m = 0 \). Given all these, the following two cases are possible

i) \( t_2^h > 0, t_2^m > 0, t_3^h > 0, t_3^m = 0, t_3^s > 0 \)

ii) \( t_2^h > 0, t_2^m > 0, t_3^h = 0, t_3^m = 0, t_3^s > 0 \)

Participation by children in home production is fairly common in Vietnam. Hence, we only focus our analysis on case i) when examining children’s behaviour.

Assume that \( w_2 \) is significantly greater than \( w_3 \) and \( a_2 \) is not significantly lower than \( a_2 \), then when there are positive \( t_2^h \) and \( t_3^h \) that equate \( a_2 g' \) and \( w_2 \), so that \( a_3 g' > w_3 \) can be derived. The first assumption is in accordance with what is observed. The second assumption is not far from the reality. The productivity of children in home production, such as taking care of siblings, cooking and cleaning up, is not less than that of an adult. In some cases, children are the ones who run the house when all the adults are at work. The extent of involvement of children above 5 years of age in home production in developing countries is well documented (Cain 1980; Hill 1983). If \( a_3 g' > w_3 \), then from Equation 18, we have

\[
\xi_3 = \mu_3 < 0 \quad \text{or} \quad \xi_3 = 0, \mu_3 > 0
\]

From the complementary slackness conditions, we then have
That is, the child may engage in home production, but not market work.

To summarise Case 2; the father specialises in market work, the mother allocates her time between market work and home production, while the child allocates his or her time between home production and schooling. That is

\[ t_1^m = t_0, \quad t_1^h = 0 \]

(19) \[ t_2^m = t_0 - t_2^h > 0, \quad t_2^h > 0 \]

\[ t_3^s = t_0 - t_3^h > 0, \quad t_3^h > 0 \text{ and } t_3^m = 0 \]

Limiting ourselves to this plausible case, the rest of the section will derive comparative statics to examine the impact of the change in the mother’s wage rate, say, a rise of her wage rate, on the child’s time allocation.

Substituting the conditions expressed in Equation 19 into the first order conditions, we have

(20) \[ w_2 = a_2 g'(a_2 t_2^h + a_3 t_3^h) \quad \text{from Equations 10 and 11} \]

(21) \[ u_s(t_3^s) = u_x a_3 g'(a_2 t_2^h + a_3 t_3^h) = u_x w_2 \frac{a_3}{a_2} \quad \text{from Equation 12} \]

Differentiating Equations 20 and 21 with respect to \( w_2 \), we get

(22) \[ 1 = a_2 g'' (a_2 \frac{\partial t_2^h}{\partial w_2} + a_3 \frac{\partial t_3^h}{\partial w_2}) \iff \frac{\partial t_2^h}{\partial w_2} = \frac{1}{a_2 g''} - a_3 \frac{\partial t_3^h}{\partial w_2} \]

(23) \[-u_{ss} \left( \frac{\partial t_3^h}{\partial w_2} \right) = a_3 \left\{ g'' (a_2 \frac{\partial t_2^h}{\partial w_2} + a_3 \frac{\partial t_3^h}{\partial w_2}) u_x + g' \left[ u_{xx} \left[ t_2^m - w_2 \frac{\partial t_2^h}{\partial w_2} + g' (a_2 \frac{\partial t_2^h}{\partial w_2} + a_3 \frac{\partial t_3^h}{\partial w_2}) \right] \right] \right\} \]

Substituting Equation 22 into 23, we can simplify 23 to yield

(24) \[ \frac{\partial t_3^h}{\partial w_2} = -g' u_{xx} t_2^m + \left[ (-g' a_2^2 g'') \right] u_x u_{xx} \frac{u_x}{a_2^2} + \left[ (g')^2 u_{xx} / a_2 g'' \right] \]

\[ \frac{u_{ss}}{a_2^2} + w_2 u_{xx} g' a_3 a_2 \]

\[ \frac{u_{ss}}{a_2^2} + w_2 u_{xx} g' a_3^2 \]
The first term, which captures the income effect, is negative; whereas the second term, which represents the substitution effect, is positive. The reason why the income effect has a negative sign is straightforward; the higher is the $w_2$, the higher the household income and the lower the marginal utility of goods. Therefore, the marginal benefit of the child doing housework is lower and the child will spend less time in housework until the marginal cost equals the marginal benefit of doing housework. The substitution effect is positive; an increase in $w_2$ causes her time cost of home production to be higher relative to that of the child, implying that the wife will spend more time in market work and less time in housework, and the child will substitute for her and spend more time in home production.

As long as the second term is greater than the first term, then we have a positive sign. That is, the child spends more time in home production to substitute for the mother as she cuts down her home production time when her wage rate increases. However, which term is dominant is purely an empirical question.

Similarly, we can derive

$$
\frac{\partial t^h}{\partial w_2} = \frac{g' u_{xx} t^m_2}{u_{xx} a_3 + w_2 u_{xx} g'} + \frac{(u_{xx} / a_2 a_3 g'') + \left\{ \left(\frac{g'}{a_3} \right)^2 u_{xx} \right\} / a_2 g''} + u_x a_2
\]

The first term is positive, which captures the income effect. The second term is negative, which captures the substitution effect. The increase in $w_2$ increases the income level of the household and reduces the time that the wife will put into market work. Given that she only spends time on market work and housework, then the income effect increases the time that she will put into housework. Hence, the income effect is positive. The substitution effect works in the usual way as we mentioned earlier.

If the substitution effect is greater than the income effect, then an increase in the mother’s wages reduces he > me in home production. Given that the mother only allocates her time between market work and home production, this implies that an increase in the mother’s wages increases her time in market work. That is

$$
\frac{\partial t^m}{\partial w_2} < 0
$$

Provided that the income effect is not dominant, then from Equations 24 and 25, and Equations 24 and 26, we can derive

$$
\frac{\partial t^h}{\partial t^h} = \frac{\partial t^h}{\partial w_2} / \partial w_2 = \left( \frac{+}{(-)} \right) < 0
$$
Equation 27 highlights the possible cross substitution effect of time allocation between the child and the mother in home production. Equation 28 says that if the mother increases her market time, then the child’s time in home production will rise. This implies that if the income effect is not dominant, children with mothers earning a higher wage will spend more time in home production and, given the time constraint of the child, less time in school.

The main argument of this theoretical model is that children’s home production time could be substitutable for that of the mother. By being the enabling labour, children indirectly contribute to the household, allowing the mother to increase her labour supply.

Limitations

The theoretical model derived in the previous section is for purely illustrative purposes due to the inherent limitations. Very restrictive assumptions are made so as to make the mathematics more tractable. With the extension of the traditional 2-person household model to incorporate children, the theoretical framework becomes very complicated in order to capture the interactions among individual members. If we take a minimum of a 3-person household, in which adults participate in work, housework and leisure, and the child is involved in the three activities plus school time, a meaningful interpretation of the mathematics becomes difficult.

Conclusion

This paper has extended the traditional 2-person household time allocation model to a 3-person household model to capture the activities of children in households. By including children in the theoretical model, the paper has highlighted that children’s time in producing home goods is substitutable for the mother’s. This allows the mother to adjust her labour supply behaviour via her home production time in response to changes in her wage rates without necessarily reducing her leisure time as suggested by the traditional intra-family time allocation models. The model provides the first step of endogenising children’s time allocation and allows us to map out a preliminary time allocation relationship between the mother and children.

Notes

1 The author thanks Dr Neil Vousden for his help in developing the theoretical model. The author also gratefully acknowledges the useful comments by Professor Allan Woodland who acted as the discussant in the PhD Conference in 1997.

2 In developed countries where home production technology is readily available, the wife may not necessarily reduce her leisure. For instance, instead of washing the clothes by hand, a washing machine can be used. However, in developing countries, such as Vietnam, where technology is less accessible, the wife has to consume fewer leisure goods to maintain her home production time.
Most studies on the enabling role of children are descriptive (Evenson 1978; Khandker 1988; Mueller 1984; Rosenzweig and Evenson 1977).

4 Becker’s model highlighted the fact that time is an input in a household production function: ‘Households will be assumed to combine time and market goods to produce more basic commodities that directly enter their utility functions’ (Becker 1965: 495). In other words, the household maximises the household utility level subject to time and financial constraints where utility is a function of goods which are produced using market goods and time. By maximising the utility function subject to the constraints, the demand for market goods, the allocation of nonmarket time, and the household labour supply are determined simultaneously.

5 This school applies the home production theory in the area of fertility. It looks at the demand of children and children’s schooling with the mother facing the choice of allocating time between home production and childcare activities, and her market work (Banskota and Evenson 1978; Cabanero 1978; Evenson 1978).

6 Here we assume the husband’s wage rate is higher than the wife’s and also the husband’s efficiency in home production relative to the wife is lower. This leads to his specialisation in market work. Therefore, he will only allocate his time between market work and leisure. This leaves the wife to do all the home production.

7 The rise of the wife’s wage rate increases the income of the household. This may cause the members of the household to reduce their labour supply if the wealth effect is sufficiently strong. In this case, the reduction of children’s market time may offset the increase of their home production time. Nevertheless, as long as this income effect is not strong enough to induce a reduction of children’s market time which completely offsets the increase of their home production time, then children will face a trade-off between their leisure time and their school time.

8 Strictly speaking, it also depends on the relative weight of schooling and leisure in the utility function.

9 All these cases are based on the assumption that the two inputs for producing home goods - market purchased goods and time - are imperfect substitutes.

10 At this point, household enterprise activities have not been incorporated into the model. The term ‘home production’ and ‘housework’ are, therefore, used interchangeably throughout the paper.

11 For the sake of simplicity, time is assumed to be the sole input of home production. In Becker’s model the home production function is a function of market goods and time.

12 Strictly speaking, the complementary slackness conditions could imply that the time allocated to a particular activity can be equal to zero. Thus, \( \xi_i > 0 \) and \( \mu_i = 0 \Rightarrow t_i^h = 0 \), \( t_i^m \geq 0 \), or \( \mu_i > 0 \) and \( x_i = 0 \Rightarrow t_i^h \geq 0 \), \( t_i^m = 0 \), or \( \xi_i = \mu_i = 0 \Rightarrow t_i^h \geq 0 \), \( t_i^m \geq 0 \). For the sake of simplicity, we just focus on three more commonly observable possibilities here.

References


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