THE AUSTRALIAN NATIONAL UNIVERSITY

Working Paper Series in Finance 00-11

ZERO-NON-ZERO PATTERNED VECTOR ERROR CORRECTION MODELLING FOR I(2) COINTEGRATED TIME-SERIES WITH APPLICATIONS IN TESTING PPP AND STOCK MARKET RELATIONSHIPS

Jack H.W. Penm, T.J. Brailsford and R.D. Terrell

Faculty of Economics and Commerce
The Australian National University

JEL Classification: C10, C63, F30, G10

Keywords: Vector error correction modelling; Computational finance; Cointegration; Purchasing power parity; Stock market

Last revision date: August 2000

Contact author: Professor Tim Brailsford
Faculty of Economics and Commerce
The Australian National University
Canberra ACT 0200 AUSTRALIA
Telephone: 02 6249 3596 (International +61 2 6249 3596)
Facsimile: 02 6249 0414 (International +61 2 6249 0414)
Email: dean.ecocomm@anu.edu.au

Zero-Non-Zero Patterned Vector Error Correction Modelling for I(2) Cointegrated Time-Series with Applications in Testing PPP and Stock Market Relationships

Jack H.W. Penm, T.J. Brailsford and R.D. Terrell

The Australian National University
Canberra ACT 0200, Australia

Abstract

Vector error-correction models (VECMs) have become increasingly popular in their applications to financial markets. Standard VECM models assume that the cointegrating vectors are of full rank such that they contain no zero elements. However, applications of VECM models to financial market data have revealed that zero entries are indeed possible. The existence of zero entries has not been fully discussed in cointegration theory. In such cases, the use of standard VECM models may lead to incorrect inferences. Specifically, if the underlying true VECM and the associated cointegrating and loading vectors contain zero entries, the resultant specifications can produce different conclusions concerning the cointegrating relationships among the variables. In this paper, we provide a new efficient and effective algorithm to select cointegrating and loading vectors that can contain zero entries in the context of a VECM framework for time-series of integrated order I(2). We employ two case studies to demonstrate the usefulness of the algorithm in tests of purchasing power parity and a three-variable system concerning the stock market.

1. INTRODUCTION

The use of vector autoregressive (VAR) models with unit roots and the associated vector error correction models (VECMs) for analysing possible cointegrating relations among economic variables has become common in the literature [eg. Granger (1981), Granger and Lee (1989)]. The advantage utilization of these models provides a device which has proved to be a more efficient tool than conventional time-series approaches.

The concept of cointegrated variables was first introduced by Granger (1981) and Granger and Weiss (1983). Engle and Granger (1987) show that, if cointegrating relations exist between the variables, then an I(1) system may be more usefully specified as a VECM. Johansen (1988), (1991) derives the maximum likelihood estimator of the space of cointegrating vectors. Stock and Watson (1993) propose a simple computational procedure for the estimation of cointegrating vectors which is asymptotically efficient. Granger and Lee (1989) suggest multi-cointegration to improve short- and long-term forecasts. Further, Engle and Yoo (1991) propose an I(2) cointegration system which coincides with Granger's multi-cointegration.

The VECM supports hypotheses which imply the presence of zero entries in the optimal specification. An optimal VECM specification with zero entries suggests that the cointegrating vectors and the loading vectors may also contain zero entries. However, the estimation of model specification with zero entries has not been fully discussed in cointegration theory.[1] This issue becomes problematic if the underlying true VECM and the associated cointegrating and loading vectors have zero entries, as different model specifications can produce different cointegrating relationships, thus leading to different and incorrect inferences.

---

[1] Johansen (1991) undertakes hypothesis testing for specific zero-non-zero patterned a and ß. However, that approach depends on *a priori* hypotheses on the zero-non-zero patterns of a and ß. It is difficult to use Johansen's method to obtain the correct zero-non-zero patterns for the *cointegrating* and loading vectors when the number of possible hypotheses is large. This is especially the case if the number of the cointegrating vectors in the system and the number of variables involved in the cointegrating relations are high.

Penm *et al* (1997) have developed an effective and efficient search algorithm to select zero-non-zero (ZNZ) patterned cointegrating and loading vectors in a subset VECM with zero entries for an I(1) system. However that paper did not deal with the application to a higher order integrated system. Several financial variables typically display properties consistent with an I(2) process. Hence, there is a lack of guidance on how to deal with zero entries in an I(2) system. In this paper we provide an appropriate algorithm to select ZNZ patterned cointegrating and loading vectors in a subset VECM with zero entries for an I(2) system. Thus, the contribution of the paper is to specify procedures for analyzing financial variables of order I(2).

The paper is constructed as follows. First, to begin the algorithm we identify the optimal specification for a subset VECM with zero entries using appropriate model selection criteria. After the optimal subset VECM with zero entries is identified, the rank of the long-term impact matrix is then computed using the singular value decomposition (SVD) method such that the number of cointegrating vectors in the system is known, with allowance for possible zero entries in the impact matrix. Once the ZNZ pattern of the impact matrix has been determined, along with the number of cointegrating vectors in the system, a tree-pruning algorithm is then proposed for the search of all acceptable ZNZ patterns of the cointegrating and loading vectors. The acceptable ZNZ patterns of these vectors are discussed in detail in Section 3. The estimation of the associated candidates for the ZNZ patterned loading vectors in the VECM framework can be carried out by the regression method with linear restrictions as proposed in Penm *et al* (1997). Section 4 presents two applications of the procedure to financial markets. The first application deals with a three-variable system concerning the stock market while the second application examines purchasing power parity (PPP). Concluding remarks are presented in Section 5.

2. VECM MODELLING FOR AN I(2) SYSTEM

We begin by considering the general VAR(q) model of the form:

$$y(t) + \sum_{\tau=1}^{q} B_\tau y(t-\tau) = \varepsilon(t), \tag{1}$$

where $\varepsilon(t)$ is a sx1 I(0) vector process with $E\{\varepsilon(t)\} = 0$ and:

$$\begin{aligned} E\{\varepsilon(t)\varepsilon'(t-\tau)\} &= G, \quad \tau = 0, \\ &= 0, \quad \tau > 0. \end{aligned}$$

$B_\tau, \tau = 1,2,\ldots,q$ are sxs parameter matrices,

$$B^q(L) = I + \sum_{\tau=1}^{q} B_\tau L^\tau,$$

where it is assumed that the roots of $|B^q(L)| = 0$ lie outside or on the unit circle, and L denotes the lag operator.

Further, y(t) is integrated of order d, I(d), if it contains at least one element which must be differenced d times before it becomes I(0). We also call y(t) cointegrated with the cointegrating vector, β, of order g, if β'y(t) is integrated of order (d-g), where y(t) has to contain at least two I(d) variables.[2]

Under this I(2) assumption, we have:

$$\begin{aligned} B^q(L) &= B^q(1)L + (I-L)B^{q-1}(L) \\ &= B^q(1)L + B^{q-1}(1)L - B^{q-1}(1)L^2 + (I-L)^2 B^{q-2}(L) \end{aligned}$$

[2] In this paper, we consider only the case d=2, although the procedure can be generally applied to models with d>2.

Following Engle and Yoo (1991), the associated VECM for (1) can be expressed as:

$$\left[B^{q}(1) \quad , \quad B^{q-1}(1)\right]\begin{bmatrix} y(t-1) \\ \Delta y(t-1) \end{bmatrix} + B^{q-2}(L)\Delta^{2} y(t) = \varepsilon(t), \tag{2}$$

where y(t) contains variables of three types, namely I(0), I(1) and I(2) and $\Delta = (I - L)$. We can rewrite (2) as:

$$B^{*}\begin{bmatrix} y(t-1) \\ \Delta y(t-1) \end{bmatrix} + B^{q-2}(L)\Delta^{2} y(t) = \varepsilon(t), \tag{3}$$

where $B^{*} = \left[B^{q}(1) \quad , \quad B^{q-1}(1)\right]$ and $B^{*}\begin{bmatrix} y(t-1) \\ \Delta y(t-1) \end{bmatrix}$ is stationary and the first term in (3) is the error correction term. We call the term $B^{q-2}(L)\Delta^{2} y(t)$ the vector autoregressive part of the VECM.

Because y(t) is cointegrated of order 2, the long-term impact matrix, B^{*}, must be singular. As a result, $B^{*} = \alpha\beta'$ and $\beta'\begin{bmatrix} y(t-1) \\ \Delta y(t-1) \end{bmatrix}$ is stationary, where the rank of B^{*} is r and α is a matrix of (s x r) and β' is (r x 2s). The columns of β are the cointegrating vectors and the rows of α are the loading vectors.

Model development is more convenient using VECMs, rather than the equivalent VARs, if the systems under study include cointegrated time series. Engle and Granger (1987) note that, for I(1) systems, the VARs in first difference will be mis-specified and the VARs in levels will ignore important constraints on the coefficient matrices. Although these constraints may be satisfied asymptotically, efficiency gains and improvements in forecasts are likely to result by imposing them. The analogous conclusion applies to I(2) systems.

Comparisons of forecasting performance of the VECMs versus VARs for cointegrated systems have been reported in studies such as Engle and Yoo (1987) and LeSage (1990). The results of these studies indicate that, while in the short-run there may be gains in using unrestricted VAR models, the VECMs produce long-run forecasts with smaller errors when the variables used in the models satisfy the test for cointegration.

3. SEARCH ALGORITHM

In the proposed algorithm for an I(2) system, the identification of ZNZ patterned B^* and the determination of ZNZ patterned α and β are carried out in the following way. First, model selection criteria are used to select the optimal subset VECM with zero entries to determine the ZNZ patterned B^*. Penm and Terrell (1984) have proposed a search method in conjunction with model selection criteria to select the optimal subset VAR with zero entries. This method is now extended to select the optimal subset VECM with zero entries for an I(2) system.

Second, after the ZNZ patterned B^* is determined, the rank of the matrix B^* is then computed using the singular value decomposition (SVD) method, and the number of cointegrating vectors in the system will be known.

Third, given the ZNZ patterned B^* has been determined and the rank of B^* has been computed, we then proceed with the tree-pruning algorithm as adapted for an I(2) system to obtain all acceptable ZNZ patterned αs and βs which are consistent with the ZNZ patterned B^*. Let a_p and $ß_p$ denote a ZNZ pattern of a and ß respectively and B_p the ZNZ pattern of B^*. If the (i,j)-th entry of the product, $a_p ß_p'$ is zero, and the corresponding (i,j)-th entry of B_p is also zero, then both a_p and $ß_p$ are acceptable. This tree-pruning algorithm, which avoids the need to evaluate all possible ZNZ patterned αs and βs, is discussed in the Appendix.

The ZNZ patterns of acceptable αs and βs depend on the pattern of B^* determined earlier by model selection criteria. Of note, the imposition of zero entries on β does not preclude a similar restriction on α. One example is that if the determined B^* contains a zero row, such as:

$$B^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix},$$

where 1 denotes a non-zero entry.

In this case zero restrictions will have to be imposed on the first row of α. This is because the pattern of B^* implies that the cointegrating relations in the system have no influence on the first variable in the system. Noting that the number of zeros in α and β are not fixed even with a given ZNZ patterned B^*, many differently patterned αs and βs can be obtained using the tree-pruning algorithm. A simple example can be used for demonstration.

Let $B^* = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$, where the rank of B^* is 2.

At least three candidate sets of α and β can be obtained, which are:

$$\alpha = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } \quad \beta = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

$$\alpha = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } \quad \beta = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

$$\alpha = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } \quad \beta = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (6)$$

The cointegrating relationships implied by (4), (5) and (6) are different. While (4) and (5) imply that y_1, y_2 and y_3 are cointegrated, (6) indicates that y_1 and y_2 are cointegrated and y_3 is an I(0) series. It is obvious that we cannot take the zero-maximising approach of choosing the β with the maximum number of zero entries to determine the ZNZ patterns of α and β. If we do, then we will select (6), not (4) nor (5), while the true model could be either (4) or (5). As a result we again utilise model selection criteria to determine the optimal ZNZ patterns for α and β. Although (4) and (5), in theory, both indicate that y_1, y_2 and y_3 are cointegrated, in practice different forecasting performance will result from (4) and (5). Using model selection criteria in this situation will aid in the selection between (4) and (5) in terms of forecasting performance.

To obtain the correct specification for α and β, we next check to see whether α and β can be uniquely obtained by factorising B^*. If this is possible, the factorisation can be carried out. If it is not possible, we employ the efficient estimation of I(2) cointegrated systems based on a triangular ECM representation [see Stock and Watson (1993)] to estimate β. Since any non-zero entry in β could be normalised as unity, we repeat the estimation procedure with all possible normalisations. Again different normalisations in practice may result in different forecasting performances for the model. The normalisation, which produces the smallest value for model selection, is then selected as the candidate β. After the optimal normalisation is determined for every candidate β, we then estimate the associated acceptable ZNZ patterned αs in the VECM framework and employ model selection criteria again to determine the optimal α and β.

There are two reasons for employing model selection criteria again to determine the optimal α and β. In the example given above, model selection criteria will help to select between (4), (5) and (6), since the approach of zero-maximisation cannot be used to determine β. In addition, Engle and Granger (1987) have demonstrated that efficiency gains could be obtained in such estimation.

4. APPLICATIONS

In this section, two applications to financial market data are presented to demonstrate the usefulness of the proposed procedure.

4.1 The Three-Variable Stock Market System

The first application examines the relationships among the stock market, money supply and inflation. Prior research has shown that these three variables are linked. First, despite the Fisher effect, inflation has generally been shown to exhibit a negative relationship with the stock market [see Fama and Schwert (1981) and DeFina (1991)]. The reasons that have been advanced to explain the relationship include inflationary expectations, fixed price nominal contracts and the tax shield effects associated with depreciable fixed assets.[3] However as Stulz (1986) argues this relationship is dependent also on money growth.

Second, announcements of the money supply have been shown to convey a valuable information signal to the stock market. While there is some conjecture as to the sign of the relationship, it is generally accepted that a negative relationship exists between the money supply and stock returns. The general theory advanced is that the linkage between the money supply and interest rates affects economic activity and corporate profits. However, there are questions over whether the real rate of interest is affected. Two main hypotheses have emerged. First, changes in the money supply may alter expectations about monetary policy. An increase in the money supply may signal a future tightening of monetary policy from the Central Bank resulting in expectations of higher interest rates, which in turn act to depress stock prices through both a rise in the real rate and a reduction in economic activity. Second, an increase in the money supply may raise expectations of higher inflation which in turn leads to higher interest rates through the inflation premium in nominal interest rates. As discussed above, higher expected inflation decreases stock prices. Both of these hypotheses suggest a negative sign on the relationship between money supply and the stock market which is generally supported by the evidence. Hardouvelis (1988) shows that increases in the money supply induce rises in interest rates. Moreover, Pearce and Roley (1985) and Jain

---

[3] DeFina (1991) provides a good overview of the various arguments.

(1988) find evidence of a significant negative relationship between unexpected money supply signals and stock market movements.

Finally, the third interaction in the system is the linkage between inflation and the money supply. This relationship is well-known and rooted in monetary theory [eg. Mishkin (1992)]. Despite arguments over the influence of lags and the money multiplier, the economic relationship is well established. Of note, the purpose here is not to test in detail hypotheses surrounding these variables, but rather to illustrate how relationships in the financial markets can be tested.

The following data are used in the test. We focus on the Australian market due to the ease of data availability and the lack of previous research in this area in the Australian market.[4] The All Ordinaries Index (AOI) is used as the stock market indicator. The AOI is a broad market indicator with coverage of around 320 stocks representing about 90-95% of total market capitalisation. The index is value-weighted and calculated on the basis of market capitalisation of the constituent stocks traded on the Australian Stock Exchange. Money supply is measured by the standard stock of money (M3).[5] Inflation is measured as the seasonally adjusted consumer price indices for Australia (CPI_{AUS}). The CPI measures the aggregate price behaviour of all consumer goods and services and is commonly used by government and industry in Australia to adjust for the cost-of-living allowances in wage and benefit contracts. Data are collected from DataStream™ over the period June 1981 through December 1999. While money supply and the stock market index are available over shorter frequencies, the CPI figures are produced on a quarterly basis, and hence this forms the sampling frequency.

The Dickey and Pantula (1987) procedure is used to test for the presence of more than one unit root. The procedure rejects the hypothesis of three unit roots for both

[4] The three variable system examined here could be tested in any other market.
[5] M3 is a common measure of the money supply and is used in Reserve Bank targeting. While an alternative measure of M2 comprises money that can be spent immediately and assets invested for the short term, M3 consists of the sum of M2 plus large deposits. These deposits include institutional money-market funds and agreements among banks. Since M3 comprises M2, we employ M3 in the test.

log(CPI_{AUS}) and log(M3) at the 5 percent level, and the hypothesis of two unit roots for log(AOI). Subsequently, the procedure accepts the hypothesis that both log(CPI_{AUS}) and log(M3) have two unit roots and log(AOI) has one unit root.[6]

We then make use of the procedure described in Section 3 to identify the specification for the VECM formed by these variables. For brevity only the results using Hannan-Quinn Information Criterion (HQC) [see Hannan and Quinn (1979)] are presented in Table 1.[7] In addition, to make certain of this selected specification, we also apply HQC to the residual vector. The results support the conclusion that the residual vector is a white noise process.

Given the determined specification of this VECM, the SVD method is then applied to the estimated matrix B^*. The estimated singular values indicate that the rank of the matrix B^* is 2. We then utilise the tree-pruning algorithm to select all acceptable ZNZ patterns of α and β which are consistent with the ZNZ pattern of the matrix B^*. The proposed estimated procedures for α and β in conjunction with HQC are conducted to select the optimal α and β. To select all acceptable ZNZ patterns of a and ß which are consistent with the ZNZ pattern of the matrix B^*, the tree-pruning algorithm described in the Appendix is utilised. The results indicate that only three sets of a and ß are acceptable. Subsequently we utilise the factorisation method which is now available to estimate each acceptable ß. After this, we then estimate each a in the VECM framework. HQC is again utilised to finally select the optimal zero-non-zero patterns of a and ß as presented in Table 1.

Despite the relatively small sample size, the results are generally consistent with economic intuition and prior evidence. The causality identified in the selected ZNZ patterned VECM confirms that M3 is an independent source of financial and economic disturbance, and an indirect causality exists from M3 through CPI_{AUS} to AOI. This result supports the impact

[6] For simplicity, and to keep the paper to a reasonable size, the relevant test results are not presented, but can be supplied to readers upon request.

[7] The results are obtained using GLS estimation on a SUN 7800 running Unix.

that money supply has on stock prices through inflationary pressures. Among CPI_{AUS}, M3 and AOI, two cointegrating vectors are found. The first selected cointegrating vector supports that Δlog(AOI) is stationary. The second selected cointegrating vector confirms that log(CPI_{AUS}), log(AOI) and Δlog(AOI) are cointegrated with log(M3). This indicates that both log(CPI_{AUS}) and log(M3) are CI(2,1) processes, not CI(2,2) processes as described in Engle and Granger (1987). The positive sign between log(M3) and log(CPI_{AUS}) and the negative sign between log(M3) and log(AOI) is consistent with the hypothesis discussed above that increases in the money supply leading to an increase in inflation thereby exhibiting a negative effect on the stock market.

4.2 Purchasing Power Parity

The second application concerns testing of purchasing power parity (PPP) using the bilateral exchange rate between the Australian and the US dollar. Formally the PPP condition can be expressed as $E_t = P_t / P_t^*$, where E_t denotes units of domestic currency per unit of foreign currency, P_t domestic price level and P_t^* foreign price level.

Recently the theory of cointegration has been utilised to test for PPP in an I(1) system. Following Engle and Granger (1987), $\ln(E_t)$ and $\ln(P_t / P_t^*)$ are characterised as integrated of order 1. If there is a long-term cointegrating relationship between them, where $\beta' X_t = (\beta_1, \beta_2)(\ln(E_t), \ln(P_t / P_t^*))' = \varepsilon_t$ with ε_t as stationary, then it can be concluded that in the I(1) system the necessary condition of the PPP hypothesis is acceptable in the long-term. If $\beta' = (1, -1)$, then both sufficient and necessary conditions of PPP are acceptable [see Corbae and Ouliaris (1990) and Oh (1996)]. Dutt and Ghosh (1995) adopt the Phillips-Hansen Fully Modified Ordinary Least Squares procedure to regress $\ln(E_t)$ against $\ln(P_t/P_t^*)$. The Phillips-Hansen procedure corrects for both endogeneity in the data and asymptotic bias in the coefficient estimates. The Phillips and Ouliaris (1990) test is then applied to determine the order of integration of the residuals for the necessary condition.

Following early work on PPP relationships such as Gailliott (1971), cointegration theory has been used in recent years to test for PPP. Fisher and Park (1991) have tested bilateral exchange rates for the currencies of 10 major industrial economies and found support for the necessary condition of PPP in the exchange-rate behaviour of many of them. Conejo and Shields (1993) test for PPP in Latin American exchange rates and find that the necessary condition of PPP holds for Brazil and Mexico.

Typically, the ratio ln(P/ P*) is treated as one variable, which does not necessarily have the same integration order as lnP and lnP* [see Corbae and Ouliaris (1990) and Oh (1996)]. However, the three variables, (lnE, lnP and lnP*), in one model do not necessarily result in the same order of integration of each variable. For instance, lnE is well-known to be I(1), but both lnP and lnP* are often found to be I(2). These results imply a need for an I(2) model, rather than I(1).

For illustration, the quarterly seasonally-adjusted consumer price indices for Australia (CPI_{AUS}) and the United States (CPI_{US}), and the exchange rates (EXCH) per US dollar from March 1972 through December 1998 are used. The data are obtained from DataStream™.[8] The y vector comprises log(CPI_{AUS}), log(CPI_{US}), as well as log(EXCH), measured as the values of Australian dollars in US dollars. The unit root tests indicate that both log(CPI_{AUS}) and log(CPI_{US}) are I(2) while log(EXCH) is I(1). The results identified by HQC are presented in Table 2.

The selected pattern of the cointegrating vector demonstrates some interesting findings. The first selected cointegrating vector indicates that Δlog(EXCH) is stationary. The second selected cointegrating vector confirms that both log(CPI_{AUS}) and log(CPI_{US}) are cointegrated with log(EXCH). The same sign occurring in log(EXCH) and log(CPI_{AUS}), as shown in Table 2, indicates that, *ceteris paribus*, an increase in CPI_{AUS} leads to an increase in an appreciation in the US dollar, and the opposite sign occurring in log(EXCH) and log(CPI_{US}) indicates that, *ceteris paribus*, an increase in CPI_{US} leads to a

[8] The calculation of the CPI in the USA changed after 1 January 1999, hence the fourth quarter of 1998 is chosen as the end period.

depreciation in the US dollar. The presence of the long-term cointegrating relationships is consistent with PPP holding within the I(2) system and across the Australian and US exchange market.

Since a sophisticated lag structure is determined for the VAR part of the selected VECM, this VECM is useful in explaining the short-term dynamics between the nominal exchange rate, domestic and foreign price levels. The causal relations detected by the optimal VECM show an indirect causality from CPI_{US} through CPI_{AUS} to the exchange rate. This in turn offers some insight into the short-term deviations from PPP which have been observed despite PPP holding in the long-run [see Abuaf and Jorion (1990)].

5. CONCLUSIONS

The use of cointegration to examine relationships among financial variables is common. However, current techniques do not explicitly allow for restrictions of zero entries whereas applications of VECM models to financial market data have revealed that zero entries are indeed possible. In such cases, the use of standard VECM models may lead to incorrect inferences. In this paper we have developed an effective and efficient algorithm to select the optimal ZNZ patterned cointegrating and loading vectors in a subset VECM with zero entries for an I(2) system. Many financial series are of order I(2) and hence the procedure developed in the paper has substantial applicability.

Two case studies are analysed to demonstrate the usefulness of this algorithm. The first case study deals with the inter-relationships between the stock market, money supply and inflation and the results are generally consistent with both theory and prior evidence. In the second case study, we examine PPP and confirm support for the necessary condition of the PPP hypothesis for the bilateral exchange rate between the Australian and US dollar. These case studies are not complete in any sense and as detailed studies they represent major pieces of research in their own right. Even so, they demonstrate that the proposed algorithm is effective and leads to an efficient analysis of the cointegrating relationships for an I(2) system.

APPENDIX: TREE-PRUNING ALGORITHM

The tree-pruning algorithm presented here provides us with a means of finding all acceptable patterns for a and ß without evaluating all possible patterns that arise from the relation $B^* = aß'$. This algorithm is an extension of the tree-pruning algorithm proposed in Penm *et al* (1997) for the I(1) system.

To begin this tree-pruning algorithm we first construct an inverse tree for ß, where each node of the tree represents a pattern for ß. The ß tree is then traversed in binary order. Furthermore there is an a inverse tree embedded in each node of the ß tree, with the nodes of this inverse tree representing all possible patterns of a. The a tree is also traversed in binary order. This tree traversal method is simple to implement and efficient in terms of computing time and storage requirements.

Suitable tree-pruning rules are then set up in the algorithm for restricting the search to the acceptable patterns of a and ß only. Since these rules avoid searching along unfavourable branches, a complete search through all possible patterns of a and ß is not required. Thus a considerable saving of computation time and storage can be achieved. After this tree-pruning algorithm is conducted, all acceptable possible patterns of a and ß will be found.

The procedure for constructing inverse trees consists of two stages as follows.

A. A t-entry inverse tree for ß

The first step is to decide the size of an inverse tree for ß. As noted in Section 3, a_p, $ß_p$ and B_p denote a ZNZ pattern of a, ß and B^* respectively. For instance, when a = $\begin{bmatrix} 0 & 0 & 0.1 \\ 0.2 & 0 & 0 \end{bmatrix}$, then a_p can be expressed as $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, where 1 represents a non-zero entry and 0 a zero entry. Analogously both $ß_p$ and B_p can be constructed.

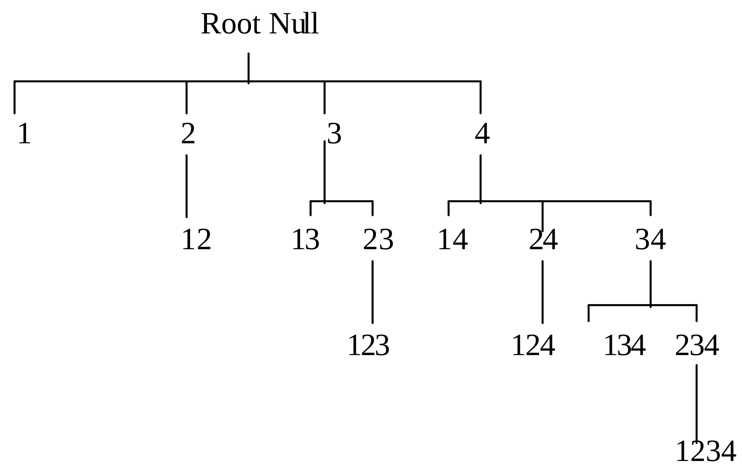
Assuming that the v-th entry of $ß_p$ is zero and the other entries are non-zero, we test the matrix $a_p ß_p'(v)$. If for every a_p there exists a zero entry of $a_p ß_p'(v)$, but the corresponding entry of B_p is non-zero, then this represents a contradiction. This means that the v-th entry of ß$_p$ must be set to non-zero. On the other hand if the corresponding entry of B_p is zero, the v-th entry of ß$_p$ is undetermined.

If there are t undetermined entries in ß$_p$ after testing all k entries of ß$_p$, then a t-entry inverse tree for ß needs to be constructed.

The root of the tree represents a pattern with all t undetermined entries. The n-th generation, n=1, 2, ..., t-1, is taken by interior nodes, of which there are C^t_n nodes in the n-th generation. Those nodes represent the possible ß$_p$ patterns in which the t entries have n zero entries.

To move from one generation to the next we make use of the rule that the a-th offspring in generation n has a-1 offspring in generation n+1 (the next generation down the tree). In setting up the second and later generations, the ordering of the nodes from left to right is controlled by natural ordering. For instance in the 4-entry case we would have in the second generation the 2 zero entry subsets, i.e. 12, 13, 14, 23, 24, 34. Therefore, a node describes a pattern in terms of the zero entries, as indicated in Figure A.1.

Figure A.1 A four variable inverse tree



It is noted that the amount of both computation time and storage increases exponentially as t becomes larger. We therefore propose the following pruning principles to avoid travelling along unfavourable branches during the search.

Pruning principles

After the inverse tree is constructed we start with the pruning. This is undertaken using the following criteria:

Let S be a set of zero entries of $ß_p$ and U a superset of S.

1) If $ß_p$'(S) has one or more zero rows, the node representing S or U can be pruned because both the ranks of ß'(S) and ß'(U) are not full, and they need to be.

2) If the nonzero entries of a row of $ß_p$' correspond to either of the following conditions, then the node associated with $ß_p$ can be ignored.

a) only one I(2) variable
b) no I(2) variable and only one I(1) variable.

3) If there are r cointegrating vectors in the system, but only N components of y(t) are involved in these cointegrating relationships ($N \leq r$), then the node associated with $ß_p$ can be ignored. For instance, if

$$\beta' = \begin{bmatrix} \beta_1 & \beta_2 & 0 & 0 \\ \beta_3 & \beta_4 & 0 & 0 \end{bmatrix},$$

then this means that the first two components of y(t) are cointegrated by two cointegrating vectors. This contradicts cointegration theory.

4) If a $ß_p$ is examined, then any node represented by $Pß_p$, where P is a rxr row permutation matrix, can be ignored. This is because both $Pß_p$ and $ß_p$ represent the same cointegrating

relation. For instance, consider $\beta'_p = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$. In this example there are 3! $ß_p$ patterns representing the same cointegrating relation.

5) For a given $ß_p(S)$ if there exists a non-zero entry of B_p, but the corresponding entry of $a_p ß_p'(S)$ is zero for all possible a_p, then this $ß_p(S)$ is not acceptable. The node representing $ß_p(S)$ can be pruned and so can the node representing $ß_p(U)$.

(B) An m-entry inverse tree for a

In the second step we decide the size of the inverse tree for a using the algorithm similar to that for ß. For a given $ß_p$ we test the matrix $a_p(k_1)ß_p'$, where the k_1-th entry of a_p is zero and the other entries are non-zero. If there exists a zero entry in the matrix $a_p(k_1)ß_p'$, but the corresponding entry of B_p is non-zero, then this represents a contradiction. This means that the k_1-th entry of a_p must be a non-zero entry. Thus this k_1-th entry of a_p is determined and must be set to non-zero. On the other hand if the corresponding entry of B_p is also zero, the k_1-th entry of a_p remains undetermined. This can be demonstrated by the following example. Consider

$$B_p = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \ \alpha_p = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \text{ and } \beta'_p = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix},$$

where k_1=(1,1). The (1,1)-th entry of $a_p ß_p'$ in this example is zero, but the (1,1)-th entry of B_p is in fact non-zero. Therefore the (1,1)-th entry of a_p must be set to non-zero.

Assuming that there are m undetermined entries of a_p after testing all k entries of a_p, an m-entry inverse tree for a_p needs to be constructed. The procedure for constructing the inverse tree for a is similar to that for ß.

Pruning principles

The pruning is performed using the following criteria[9]:

Let E be a set of zero entries of a_p and denote the $a_p(E)$ node as the node representing the $a_p(E)$ pattern. Also let R be a superset of E and the $a_p(R)$ node represent the $a_p(R)$ pattern.

1) If there exists a zero entry of $a_p(E)ß_p'$ but the corresponding entry of B_p is non-zero, then the node representing $a_p(E)$ can be pruned and so can the node representing $a_p(R)$. For instance, consider

$$\alpha_p(E) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \ \beta_p' = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \text{ and } B_p = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

In this example, the (1,2)-th entry of $a_p(E)ß_p'$ is zero, but the (1,2)-th entry of B_p is non-zero. This represents a contradiction and the $\alpha_p(E)$ node can be pruned. Any $a_p(R)$ whose zero entry set is a superset of E will also fail the test, and, therefore these $a_p(R)$ nodes can also be pruned.

2) If $a_p(E)$ has one or more zero columns then these $\alpha_p(E)$ and $a_p(R)$ nodes can be pruned. This is because the rank of the loading vectors a(E) is not full, and neither is a(R).

3) If an entry of $a_p(E)ß_p'$ is non-zero but the corresponding entry of B_p is zero, then this entry of $a_p(E)ß_p'$ has to be restricted to zero. If either of the following two conditions is met, the $\alpha_p(E)$ node can be ignored:

9 The amount of computation time and storage increases exponentially with m. The tree-pruning principles are required to reduce these amounts by avoiding travelling along unfavourable branches.

(a) If the number of non-zero entries of $a_p(E)$ involved is less than the number of restrictions then there will be no acceptable solution for a(E). For instance, consider

$$\alpha(E) = \begin{bmatrix} \alpha_{11} & 0 \\ \alpha_{21} & \alpha_{22} \end{bmatrix}, \ \beta' = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & 0 \end{bmatrix}, \text{ and } B_p = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In this example we have the following three restrictions:

$\alpha_{21}\beta_{1k} + \alpha_{22}\beta_{2k} = 0$, k=1,2,3.

Although β' can be estimated by using the estimation method proposed in Section 3, there will be no solution for α_{21} and α_{22} because we have only tow unknowns, α_{21} and α_{22}.

(b) If any non-zero entry of $a_p(E)$ has to be zero to satisfy restrictions then the given $a_p(E)$ is unacceptable. For instance, consider

$$\alpha(E) = \begin{bmatrix} \alpha_{11} & 0 \\ \alpha_{21} & \alpha_{22} \end{bmatrix}, \ \beta' = \begin{bmatrix} 0 & \beta_{12} & 0 & 0 \\ \beta_{21} & 0 & \beta_{23} & 0 \end{bmatrix}, \text{ and } B_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

Now we have the restriction:

$\alpha_{22}\beta_{21} = 0$.

This indicates that α_{22}=0. Thus $a_p(E)$ is unacceptable.

4) If the (i,j)-th entry of $a_p(E)$ is the only non-zero entry of the ith row, then the zero-non-zero pattern of the jth row of $ß_p$' should be identical to that of the ith row of B_p. If this is not true then the $\alpha_p(E)$ node can be ignored.

5) If the (i,j)-th entry of $ß_p$' is the only non-zero entry of the jth column, then the zero-non-zero pattern of the ith column of $a_p(E)$ should be identical to that of the jth column of B_p. If this is not the case then the $a_p(E)$ node can be ignored.

REFERENCES

Abuaf, N. and Jorion, P. (1990), 'Purchasing Power Parity in the Long Run', *Journal of Finance*, 45, 157-174.

Conejo, C. and Shield, M.P. (1993), 'Relative PPP and the Long-term Terms of Trade for Five Latin American Countries, A Cointegration Approach', *Applied Economics*, 25(12), 1511-1515.

Corbae, D. and Ouliaris, S. (1990), 'A Test of Long-Run Purchasing Power Parity Allowing for Structural Breaks', *The Economic Record*, 167(196), 26-33.

DeFina, R. (1991), 'Does Inflation Depress the Stock Market?', *Business Review, Federal Reserve Bank of Philadelphia*, Nov-Dec, 3-12.

Dickey, D.A. and Pantula, S.G. (1987), 'Determining the Order of Differencing in Autoregressive Processes', *Journal of Business and Economic Studies*, 5(4), 455-461.

Dutt, S.D. and Ghosh, D. (1995), 'Purchasing Power Parity Doctrine, Weak and Strong Form Tests', *Applied Economics Letters*, 2, 316-20.

Engle, R.F. and Granger, C.W.J. (1987), 'Cointegration and Error Correction, Representation, Estimation and Testing', *Econometrica*, 55, 69-104.

Engle, R.F. and Yoo, B.S. (1987), 'Forecasting and Testing in Co-integrated System', *Journal of Econometrics*, 35, 143-159.

Engle, R.F. and Yoo, B.S. (1991), 'Cointegrated Economic Time-series, An Overview with New Results', in *Long-Run Economic Relationships,* R.F. Engle and C.W.J. Granger (eds), New York, Oxford University Press, 237-266.

Fama, E.F. and Schwert, G.W. (1977), 'Asset Returns and Inflation', *Journal of Financial Economics*, 7, 115-146.

Fisher, E.O. and Park, J.Y. (1991), 'Testing Purchasing Power Parity under the Null Hypothesis of Co-integration', *The Economic Journal,* 101(409), 1476-1484.

Gailliott, H.J. (1971), 'Purchasing Power Parity as an Explanation of Long-Term Changes in Exchange Rates', *Journal of Money, Credit and Banking*, 2, 348-357.

Granger, C.W.J. (1981), 'Some Properties of Time Series Data and their Use in Econometric Model Specification', *Journal of Econometrics*, 16, 121-130.

Granger, C.W.J. and Lee, T.H. (1989), 'Multicointegration', *Advances in Econometrics*, 8, 71-84.

Granger, C.W.J. and Weiss, A.A. (1983), ‘Time Series Analysis of Error Correcting Models’, in *Studies in Econometrics, Time Series and Multivariate Statistics*, S. Kerlin (ed), New York, Academic Press, 255-278.

Hannan, E.J. and Quinn, B.G. (1979), ‘The Determination of the Order of an Autoregression’, *Journal of the Royal Statistical Society B* 41, 190-195.

Hardouvelis, G.A. (1988), 'Economic News, Exchange Rates and Interest Rates', *Journal of International Money and Finance*, 7, 23-35.

Jain, P.C. (1988), 'Response of Hourly Stock Prices and Trading Volume to Economic News', *Journal of Business*, 61, 219-231.

Johansen, S. (1988), ‘Statistical Analysis of Cointegration Vectors’, *Journal of Economic Dynamics and Control*, 12, 231-255.

Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegrating Vector in Gaussian Vector Autoregression Models", *Econometrica*, 59, 1551-1580.

LeSage, J.P. (1990), ‘A Comparison of the Forecasting Ability of ECM and VAR Models’, *Review of Economics and Statistics*, 72, 664-71.

Mishkin, F.S. (1992), *The Economics of Money, Banking and Financial Markets*, 3rd edition, New York, Harper Collins.

Oh, K.Y. (1996), 'Purchasing Power Parity and Unit Root Tests using Panel Data', *Journal of International Money and Finance*, 15(3), 405-18

Pearce, D.K. and Roley, V.V. (1985), 'Stock Prices and Economic News', *Journal of Business*, 58, 49-67.

Penm, J.H.W. and Terrell, R.D. (1984), ‘Multivariate Subset Autoregressive Modelling with Zero Constraints for Detecting Overall Causality’, *Journal of Econometrics*, 24, 311-330.

Penm, J.H., Penm, J.H.W. and Terrell, R.D. (1997), ‘The Selection of ZNZ Patterned Cointegrating Vectors in Error-Correction Modelling’, *Econometric Reviews* 16, No. 3, pp. 281-304.

Penm, J.H.W., Penm, J.H.C. and Terrell, R.D. (1999), ‘Testing Purchasing Power Parity in the Framework of Error Correction Modelling,’ *Asian-Pacific Economic Review*, in press.

Phillips, P.C.B. and Ouliaris, S. (1990), ‘Asymptotic Properties of Residual Based Tests of Cointegration’, *Econometrica*, 58, 165-93.

Stock, J.H. and Watson, M.W. (1993), ‘A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems’, *Econometrica*, 61, 783-820.

Stulz, R.M. (1986), 'Asset Pricing and Expected Inflation', *Journal of Finance*, 41, 209-223.

Table 1. The VECM for the Relationship, Linking Money Supply, Inflation and Stock Market Indicator for Australia Selected by HQC Using the GLS Procedure

Variables: $y_t^1 = \log(M3)$, $y_t^2 = \log(CPI_{AUS})$, $y_t^3 = \log(AOI)$.
Sample Period: 1981.II to 1999.IV

The VECM:

$$\begin{bmatrix} \Delta^2 y_t^1 \\ \Delta^2 y_t^2 \\ \Delta^2 y_t^3 \end{bmatrix} + \begin{bmatrix} \underset{(3.07)}{0.340} & 0.0 & 0.0 \\ \underset{(1.32)}{0.148} & \underset{(5.09)}{0.468} & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-1}^1 \\ \Delta^2 y_{t-1}^2 \\ \Delta^2 y_{t-1}^3 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & \underset{(1.68)}{0.155} & 0.0 \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-3}^1 \\ \Delta^2 y_{t-3}^2 \\ \Delta^2 y_{t-3}^3 \end{bmatrix} +$$

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 \\ \underset{(-2.21)}{-0.045} & \underset{(3.70)}{0.388} & \underset{(-1.45)}{-0.105} \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} y_{t-1}^1 \\ y_{t-1}^2 \\ y_{t-1}^3 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & \underset{(1.81)}{0.220} \\ 0.0 & 0.0 & \underset{(9.41)}{1.081} \end{bmatrix} \begin{bmatrix} \Delta y_{t-1}^1 \\ \Delta y_{t-1}^2 \\ \Delta y_{t-1}^3 \end{bmatrix} = \varepsilon(t)$$

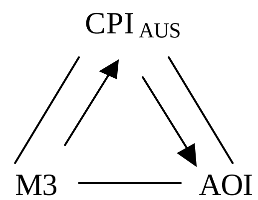
$$\hat{\alpha} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & \underset{(-4.38)}{-0.045} \\ \underset{(9.35)}{1.081} & 0.0 \end{bmatrix} \quad \hat{\beta}' = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & \underset{(9.41)}{1.0} \\ \underset{(2.21)}{1.0} & \underset{(-3.69)}{-8.433} & \underset{(1.45)}{2.278} & 0.0 & 0.0 & \underset{(-1.80)}{-4.789} \end{bmatrix}$$

Long-term Cointegrating Relationship Identified:

1) $\Delta \log(AOI)$ is stationary

2) $\log(M3) = 8.433\log(CPI_{AUS}) - 2.278\log(AOI) + 4.789\Delta\log(AOI)$

Short-term Causality[a] and Long-term Cointegration Pattern Recognised:



The short-term causality pattern is detected from the vector autoregressive part of the VECM
T-values in parentheses. Δ denotes first difference

a) x Granger-causes y: (Notation : x ⟶ y)
x and y are cointegrated in the long-term: (Notation : x —— y).

Table 2. The VECM for the Relationship, Linking Exchange Rates and Consumer Price Indices, between Australia and the USA Selected by HQC Using the GLS Procedure

---

Variables: $y_t^1 = \log(CPI_{AUS})$, $y_t^2 = \log(CPI_{US})$, $y_t^3 = \log(EXCH_{AUS/US})$

Sample Period: 1972.I to 1998.IV

The VECM:

$$\begin{bmatrix} \Delta^2 y_t^1 \\ \Delta^2 y_t^2 \\ \Delta^2 y_t^3 \end{bmatrix} + \begin{bmatrix} 0.767 & -0.299 & 0.0 \\ (7.85) & (-2.23) & \\ 0.0 & 0.0 & 0.0 \\ & & \\ 0.0 & 0.0 & 0.185 \\ & & (1.35) \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-1}^1 \\ \Delta^2 y_{t-1}^2 \\ \Delta^2 y_{t-1}^3 \end{bmatrix} + \begin{bmatrix} 0.695 & 0.0 & 0.0 \\ (6.65) & & \\ 0.055 & 0.0 & 0.0 \\ (1.97) & & \\ 0.088 & 0.0 & 0.168 \\ (1.05) & & (1.75) \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-2}^1 \\ \Delta^2 y_{t-2}^2 \\ \Delta^2 y_{t-2}^3 \end{bmatrix} +$$

$$\begin{bmatrix} 0.641 & 0.232 & 0.0 \\ (6.37) & (1.53) & \\ 0.072 & 0.0 & 0.0 \\ (2.38) & & \\ 0.100 & 0.0 & 0.0 \\ (1.15) & & \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-3}^1 \\ \Delta^2 y_{t-3}^2 \\ \Delta^2 y_{t-3}^3 \end{bmatrix} + \begin{bmatrix} -0.191 & -0.415 & 0.0 \\ (-2.01) & (-2.59) & \\ -0.115 & 0.0 & 0.0 \\ (-4.27) & & \\ 0.125 & .0.0 & 0.0 \\ (1.45) & & \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-4}^1 \\ \Delta^2 y_{t-4}^2 \\ \Delta^2 y_{t-4}^3 \end{bmatrix} +$$

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 \\ & & \\ 0.0 & 0.343 & 0.0 \\ & (3.72) & \\ 0.101 & 0.0 & 0.0 \\ (1.21) & & \end{bmatrix} \begin{bmatrix} \Delta^2 y_{t-5}^1 \\ \Delta^2 y_{t-5}^2 \\ \Delta^2 y_{t-5}^3 \end{bmatrix} +$$

$$\begin{bmatrix} 0.018 & -0.043 & -0.083 \\ (2.88) & (-1.62) & (-1.73) \\ 0.0 & 0.0 & 0.0 \\ & & \\ 0.0 & 0.0 & 0.0 \\ & & \end{bmatrix} \begin{bmatrix} y_{t-1}^1 \\ y_{t-1}^2 \\ y_{t-1}^3 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ & & \\ 0.0 & 0.0 & 0.0 \\ & & \\ 0.0 & 0.0 & 0.855 \\ & & (5.05) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1}^1 \\ \Delta y_{t-1}^2 \\ \Delta y_{t-1}^3 \end{bmatrix} = \varepsilon(t)$$

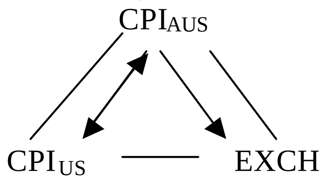
$$\hat{\alpha} = \begin{bmatrix} 0.0 & -0.083 \\ & (-3.05) \\ 0.0 & 0.0 \\ & \\ 0.855 & 0.0 \\ (5.06) & \end{bmatrix} \quad \hat{\beta}' = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ & & & & & (5.05) \\ -0.210 & 0.533 & 1.0 & 0.0 & 0.0 & 0.0 \\ (2.88) & (-1.62) & (-1.73) & & & \end{bmatrix}$$

Long-term Cointegrating Relationship Identified:

1) A stationary $\Delta \log(EXCH_{AUS/US})$

2) $\log(EXCH_{AUS/US}) = 0.210\log(CPI_{AUS}) - 0.533\log(CPI_{US})$

Short-term Causality[a] and Long-term Cointegration Pattern Recognised:



b) The short-term causality pattern is detected from the vector autoregressive part of the VECM
c) T-values in parentheses. Δ denotes first difference
d) x Granger-causes y: (Notation : x ⟶ y)
x and y are cointegrated in the long-term: (Notation : x —— y).