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Available information in 2D motional Stark effect imaging^{a)}

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Recent advances in imaging techniques have allowed the extension of the standard polarimetric 1D motional Stark effect (MSE) diagnostic to 2D imaging of the internal magnetic field of fusion devices [J. Howard, *Plasma Phys. Controlled Fusion* **50**, 125003 (2008)]. This development is met with the challenge of identifying and extracting the new information, which can then be used to increase the accuracy of plasma equilibrium and current density profile determinations. This paper develops a 2D analysis of the projected MSE polarization orientation and Doppler phase shift. It is found that, for a standard viewing position, the 2D MSE imaging system captures sufficient information to allow imaging of the internal vertical magnetic field component $B_z(r, z)$ in a tokamak. © 2010 American Institute of Physics. [doi:10.1063/1.3492410]

I. INTRODUCTION

Motional Stark effect (MSE) polarimetry has become a routine diagnostic for high power fusion devices that employ diagnostic or heating neutral beams. In tokamaks, measurement of the polarization state of the Stark split multiplet is used to infer the internal toroidal current density profile.^{1,2} Until now, because of various technical limitations, especially in low field compact machines where the Stark multiplet is difficult to resolve, MSE systems have been limited to a maximum of ~ 30 discrete viewing positions across the injected beam. However, recent developments in spectropolarimetric instrumentation have opened the possibility of 2D imaging of the internal magnetic field of tokamak fusion devices.^{3,4}

The MSE technique relies on the splitting of the Doppler-shifted neutral beam Balmer α light into orthogonally polarized σ and π components, a result of the motion-induced strong electric field $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ experienced in the rest frame of the neutral atoms. Currently conventional 1D MSE systems view only in the midplane, allowing internal magnetic field information to be extracted since the beam velocity is also in the midplane. However, in the case of 2D MSE imaging,⁵ or when the neutral heating beam is directed off-axis,⁵ the velocity profile of the neutral beam must be carefully considered to enable extraction of accurate magnetic field information.

The 2D MSE imaging spectropolarimeter is sensitive to both the projected polarization orientation of the Stark multiplet and to the interferometric phase shift produced by the Doppler shift of the beam emission. Both the polarization orientation and the Doppler phase are captured as phase modulations of the 2D spatial heterodyne fringe pattern produced by the instrument. In this paper we establish the link between these phase modulations and the properties of the

neutral beam, the magnetic field, and the viewing geometry. It is demonstrated that the ability of the new instrument to simultaneously capture both neutral beam and magnetic information allows the structure of the vertical component of the internal magnetic field of a tokamak to be recovered from a single image of the neutral beam.

II. RELATION BETWEEN POLARIZATION ANGLE AND INTERNAL MAGNETIC FIELD

This section establishes the link between the polarization image and the properties of the neutral beam, the magnetic field, and viewing geometry. The analysis includes the unknown divergent behavior of the neutral heating beam.

Given the origin and the orientation of the detecting array, a local basis $(\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$ for each individual sightline (pixel) is established such that the unit vector $\hat{\mathbf{i}}$ points from the pixel to its image point P in the vertical neutral beam sheet. It is convenient to orient the unit vector $\hat{\mathbf{j}}$ so as to have no component parallel to the Z axis. The basis can be expressed in terms of the natural cylindrical coordinate system of a tokamak as

$$\hat{\mathbf{i}} = \sin \Omega \cos \psi \hat{\mathbf{R}} + \sin \psi \hat{\mathbf{Z}} + \cos \Omega \cos \psi \hat{\phi}, \quad (1)$$

$$\hat{\mathbf{j}} = -\cos \Omega \hat{\mathbf{R}} + \sin \Omega \hat{\phi}, \quad (2)$$

$$\hat{\mathbf{k}} = -\sin \Omega \sin \psi \hat{\mathbf{R}} + \cos \psi \hat{\mathbf{Z}} - \cos \Omega \sin \psi \hat{\phi}. \quad (3)$$

where ψ is the vertical angle between the line-of-sight and the equatorial midplane. Ω is the angle between the projection of $\hat{\mathbf{i}}$ onto the equatorial plane and the toroidal unit vector $\hat{\phi}$ while α is the angle between the beam and $\hat{\phi}$. The coordinate system and viewing geometry are best visualized with reference to Fig. 1.

Ignoring line-of-sight integration effects, we consider the emission only from a central vertical beam sheet which is aligned parallel to the X -axis (see Fig. 1). The beam velocity unit vector $\hat{\mathbf{v}} = \mathbf{v}/v_0$ where $v_0 \equiv |\mathbf{v}|$ is assumed to be contained

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FIG. 1. (Color online) Diagram of the MSE viewing geometry showing the angle α between the beam velocity vector $\mathbf{v}$ and the toroidal unit vector $\hat{\phi}$ and the angle Ω between $\mathbf{v}$ and $\hat{\phi}$. Both angles are taken positive in the direction $\hat{\mathbf{Z}} \times \hat{\phi}$. The inset shows the viewing coordinate basis and vertical angle ψ , as well as the inclination angle ξ of the beam velocity vector at observation point P .

within this sheet and is described in terms of the horizontal angle α with respect to the toroidal unit vector at P and the vertical inclination angle ξ to the horizontal plane (see Fig. 1)

$$\hat{\mathbf{v}} = \sin \alpha \cos \xi \hat{\mathbf{R}} + \sin \xi \hat{\mathbf{Z}} + \cos \alpha \cos \xi \hat{\phi}. \quad (4)$$

Expressing the magnetic field in terms of cylindrical coordinates, the induced electric field $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ for an atom at P is readily calculated as

$$\begin{aligned} \mathbf{E} = & v_0 (B_Z \cos \alpha \cos \xi - B_\phi \sin \xi) \hat{\mathbf{R}} \\ & + v_0 (B_\phi \sin \alpha \cos \xi - B_R \cos \alpha \cos \xi) \hat{\mathbf{Z}} \\ & + v_0 (B_R \sin \xi - B_Z \sin \alpha \cos \xi) \hat{\phi}. \end{aligned} \quad (5)$$

The orientation θ of the polarized emission at the detector is obtained from the ratio of the projected transverse components of the induced electric field

$$R \equiv \tan \theta = \frac{-\mathbf{E} \cdot \hat{\mathbf{j}}}{\mathbf{E} \cdot \hat{\mathbf{k}}}, \quad (6)$$

leading to

$$R = \frac{\sin \xi (B_R \sin \Omega + B_\phi \cos \Omega) - B_Z \cos \gamma \cos \xi}{a_1 B_R + a_2 B_Z - a_3 B_\phi},$$

$$a_1 = \cos \Omega \sin \psi \sin \xi + \cos \alpha \cos \psi \cos \xi, \quad (7)$$

$$a_2 = -\sin \gamma \sin \psi \cos \xi,$$

$$a_3 = \sin \Omega \sin \psi \sin \xi + \sin \alpha \cos \psi \cos \xi,$$

where we have introduced the difference angle $\gamma = \alpha - \Omega$ between the line-of-sight and the beam velocity vector.

This is the basic result that describes the MSE polarization angle observed at each detector pixel. In general, the net polarization angle must be obtained from the sum of Stokes vectors over the horizontal extent of the neutral beam. However, this expression is sufficient to demonstrate the key dependencies without regard to integration effects. As it is the ratio of magnitudes of the two orthogonal induced electric fields, there is no dependence on the magnitude of the velocity in the final form, only the velocity direction. We show later that the beam velocity magnitude is encoded by the interferometric phase. In reality there will be an angular spread of beam velocities about some mean direction ξ which will affect both the projected polarization angle and Doppler phase shift. These effects are not considered here.

Physical constraints on possible viewing geometries limit the typical vertical viewing angles such that $|\psi| < 5^\circ$, while the neutral beam divergence is also usually small $|\xi| < 1^\circ$ to 2° . Together with the fact that for tokamaks the toroidal magnetic field component is generally an order of magnitude larger than the poloidal magnetic field, Eq. (8) can be well approximated by

$$R \approx \frac{-B_\phi \xi \cos \Omega + B_Z \cos \gamma}{B_\phi \sin \alpha}. \quad (8)$$

The validity of this approximate form has also been confirmed numerically for the view used by the 2D system employed on the TEXTOR tokamak.⁶

As seen from Eq. (8), a 2D MSE system in the standard viewing geometry produces a polarization image that is primarily sensitive to B_Z and B_ϕ . As the geometrical factors and the toroidal magnetic field are well known, it is clear that knowledge of the beam angular divergence ξ is required for extraction of B_Z . In passing, however, we note that for the important case of synchronous imaging of fluctuating internal magnetic fields (e.g., magnetohydrodynamic activity and sawteeth) a perturbation analysis shows that structural information can be recovered without the need to know the beam velocity profile.

III. NEUTRAL BEAM VELOCITY PROFILE FROM THE PHASE OF THE MSE IMAGE

The 2D MSE imaging instrument uses a polarization interferometer to provide the sinusoidal spectral discrimination required to reveal the polarization orientation of the multiplet components. One or more Savart shearing plates⁷ are used to produce the spatial heterodyne carrier fringes for encoding the polarization angle θ and optical phase ϕ . An in-depth

description of the principles of the 2D MSE instrument can be found elsewhere.^{3,4} The basic equation which describes the interferometric image is of the form

$$S = I_0[1 + \zeta \cos(\phi \pm 2\theta)], \quad (9)$$

where I_0 is the brightness of the MSE multiplet, ζ is the fringe visibility at phase offset ϕ , and θ is the polarization angle orientation with respect to the final polarizer of the system. A variety of modulation strategies are available to separate the polarization and Doppler phase shift angles.

The phase ϕ depends on the center wavelength of the Doppler-shifted Stark emission $\lambda = \lambda_0(1 + \eta v_0/c)$, where

$$\eta(\gamma, \psi) \equiv \hat{\mathbf{v}} \cdot \hat{\mathbf{i}} = \cos \xi \cos \psi \cos \gamma + \sin \xi \sin \psi. \quad (10)$$

It is clear that the phase shift image can deliver information on the magnitude of the velocity v_0 and the divergence ξ of the neutral beam.

The total phase shift ϕ is the sum of the shifts introduced by the bias wave-plate [thickness L and birefringence $B(\lambda)$] and the Savart shearing plate [angle-dependent optical path delay $\Delta(\lambda)$] which is used to generate the spatial heterodyne carrier. To first order in the Doppler shift, the total phase offset is given by

$$\phi = \phi_0(1 + \gamma \eta v_0/c) + \phi_S(1 + \gamma_S \eta v_0/c), \quad (11)$$

where $\phi_0 = 2\pi L B_0/\lambda_0$ is the wave-plate phase offset at the unshifted wavelength λ_0 and $\phi_S = 2\pi \Delta_0/\lambda_0$ is the Savart plate delay with $B_0 \equiv B(\lambda_0)$ and $\Delta_0 \equiv \Delta(\lambda_0)$. The factors γ and γ_S account for the dispersive properties of the delay wave-plate and Savart plates.³ The large constant phase offsets ϕ_0 and ϕ_S can be obtained from an image of a Balmer-alpha (or equivalent) calibration light source $\phi_C = \phi_0 + \phi_S$ and the residual phase image is given by

$$\delta\phi \equiv \phi - \phi_C = (\phi_0\gamma + \phi_S\gamma_S) \eta v_0/c. \quad (12)$$

When using a compensated Savart plate to generate the carrier pattern, the Savart delay is negligible $\phi_S \ll \phi_0$. Using Eqs. (12) and (10) it is possible to extract the velocity profile ξ from the residual phase image and thereby estimate the internal magnetic field B_Z .

Figure 2 shows contours of η versus image angles (γ, ψ) for the TEXTOR experimental viewing geometry.⁴ Figure 2(a) shows calculated contours for the simple case in which ξ increases linearly with elevation angle ψ , spanning the angle range $\sim \pm 2.5^\circ$. (When the beam has zero divergence, the contours are approximately straight and parallel). Figure 2(b) shows experimental contours of η_B inferred from the measured Doppler interferometric phase image $\delta\phi$. The images have been masked according to the intersection of the regions of non-negligible brightness in the observed beam emission and calibration images. The right edge of the experimental image exhibits some distortion due to demodulation artifacts generated at the edge of the circular calibration

FIG. 2. (Color online) Calculated and measured contours of projected normalized velocity $\eta(\psi, \gamma)$. (a) Beam angle ξ increases linearly with ψ spanning the range $\sim \pm 2.5^\circ$ and (b) measured contours of η .

image, while poorer fringe contrast degrades the phase shift signal to noise ratio at the left of the image (plasma center). The experimental image clearly exhibits beam divergence and could, in principle, be unfolded for the beam velocity profile to allow recovery of B_Z from the image of the polarimetric tilt angle.

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