Persistent inequality with endogenous rate of time preference

Persistent
inequality

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Abstract

Purpose – The purpose of this paper is to provide a theory that can explain the persistence of inequality in an economy where household agents are identical in terms of their preferences and have access to the same production technology.

Design/methodology/approach – An overlapping generations model is developed where agents are imperfectly altruistic and face uncertain lifetimes. The rate of time preference of an agent depends on her probability of survival which is increasing in her level of consumption. An agent's initial endowment of human capital jointly determines her patience and willingness to accumulate human capital and other productive assets.

Findings – It was found that inequality may persist in the economy as a result of endogenous rate of time preference. Agents with low initial endowment of human capital are impatient and choose not to invest in human capital. Agents with initial endowment of human capital above a certain threshold choose to invest in human capital as they expect to survive long enough to reap the benefits of their investment.

Originality/value – This paper adds to the present literature by providing an alternative mechanism to explain the persistence of inequality. In future research this model framework can be used to evaluate the impact of alternative government and tax policies on the long run distributions of income and wealth.

Keywords Human capital, Statistical analysis, Wealth and income

Paper type Research paper

1. Introduction

The persistence of inequality in the distribution of income and wealth is of interest to both economists and policy makers. Understanding the reasons behind the existence of inequality is crucial to understand the impact of macroeconomic and tax policy on the economy. Inequality can also have important socioeconomic implications for an economy. Higher inequality can lead to social and distributive conflicts which may be harmful for growth[1].

Traditionally, the phenomenon of inequality was thought to be an issue which was particularly pervasive among developing countries. For instance, Deaton and Dreze (2002) find that even in 1999-2000 a substantial proportion of the population of India was below the poverty line. They also find that in 1990s there has been an increase in regional disparities across the Indian states. However, recent empirical evidence on inequality in developed countries is more surprising. In recent papers, Alderson and Nielson (2002) and Atkinson (2002) find that over the last three decades there has been an increase in income inequality in many OECD countries. The increase in inequality has been particularly significant in the USA and the UK and to a lesser extent in Australia, Belgium and New-Zealand. In USA, the Gini coefficient of income

JEL classification codes – D91, O15, D31

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has increased by more than five points over the period 1967-2002 (Atkinson, 2002). In Australia, the income ratio of the richest 10 per cent to the poorest 10 per cent of the population has increased from 3.77 in 1994-1995 to 4.00 in 2002-2003 (Australian Bureau of Statistics, 2004). The distribution of wealth is far more concentrated than the distribution of income. In the USA, the Gini coefficient for income in 1992 was 0.57, whereas for wealth it was 0.78. Also the distribution of wealth has become more unequal over the previous two decades. Tables I and II give us an idea regarding the magnitude of wealth concentration in the USA and Australia. In the USA, among the households surveyed in the Panel Study of Income Dynamics in 1989-1990, the top ten percentile held more than 61 per cent of total wealth. In the Survey of Consumer Finances, the same number was more than 69 per cent. In Australia, top ten percentile of the households surveyed in 2002 accounted for 45 per cent of the total wealth.

Why does income inequality persist over time? Why is the distribution of wealth more concentrated? What kind of model framework should we use to understand these facts? The current theoretical literature uses the overlapping generations (OLG) framework to explain the existence of income and wealth inequality. The basic idea in this literature is that when credit markets are imperfect, only sufficiently wealthy individuals are able to engage in productive investment. The poorer sections of the population are unable to meet collateral requirements to borrow from banks. They are forced to engage in less productive subsistence occupations[2]. In an OLG model, individual agents live for finite periods and do not explicitly take into account the welfare of their future generations when they make their economic decisions. In order to link different generations, it is usually assumed that parents leave a proportion of their income as bequests. As a result, bequests received by every new generation of agents is purely by accident and not due to any optimization exercise. Poorer agents leave low bequests. In the presence of credit market imperfections, it means that the

	1	1-5	5-10	10-20	20-40	40-60	60-100	Gini
PSID	25.3	21.8	14	16.3	15.8	6	0.9	0.76
SCF	34.7	23.1	11.3	12.7	12.2	5	1	0.80

Table I.
The concentration of
household wealth in the
USA

Notes: The calculations are based on Panel Study of Income Dynamics (PSID) survey in 1989-1990 and Survey of Consumer Finances (SCF) in 1998. The rows show the per cent of wealth held by percentile groups. Top 1 per cent of the household surveyed in the SCF own 34.7 per cent of the total wealth

Source: Reproduced from Hendricks (2007)

	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	90-100	Gini
HILDA	45	18	13	9	7	5	3	1	0	0	0.61

Table II.
The concentration of
household wealth in
Australia, 2002

Notes: The calculations are based on the Household Income and Labor Dynamics in Australia (HILDA) survey. The rows show the per cent of wealth held by percentile groups. Top 10 per cent of the household surveyed own 45 per cent of the total net worth

Source: Reproduced from Headey *et al.* (2005)

next generation is also unable to meet collateral requirements to engage in productive economic activities. Poverty is passed on from one generation to the next.

While the presence of credit market imperfections is a plausible reason for inequality it also leads to a number of puzzles. Credit markets are far more efficient today. According to these models, as we observe improvements in the efficiency of credit markets we should see a gradual decline in inequality, something contradicting the behavior of inequality in developed countries such as USA and Australia. Why then is there an increase in inequality? Perhaps credit market imperfections alone cannot explain the existence of inequality.

The infinite horizon (IH) model is the other commonly used framework in macroeconomics. An IH model is a special case of the OLG model where individual agents are perfectly altruistic i.e. they care about the welfare of future generations as much as themselves. Hence, they maximize the welfare of their entire dynasty[3]. Not surprisingly in an IH model, the non-convexity created by credit market imperfections is not sufficient to perpetuate inequality. As the dynasty lives forever, poorer agents forego consumption and save for future generations. Eventually, some generation is able to meet the collateral requirements. In the steady-state, there is no income inequality. We may observe wealth inequality in an IH model if there is some sort of uncertainty in the economy. Aiyagari (1994) simulated the wealth distribution in an IH model where agents faced an idiosyncratic labor productivity shock every period. The distribution of the productivity shocks were estimated to match the US data. The model predicted a Gini coefficient of 0.41 for wealth while the top five percentile of the population held 15.6 per cent of total wealth. We know that wealth distribution is much more skewed. There is a parallel literature in macroeconomics that attempts to construct models which would be able to generate wealth distributions similar to the US data[4]. In this literature inequality is typically generated by introducing some sort of exogenous preference heterogeneity[5].

In this paper, we take a different approach. As the OLG and the IH models are commonly used frameworks in macroeconomics, it would be instructive to develop a general framework which would nest these two models as special cases. We study the dynamics of inequality when economic agents are imperfectly altruistic and face uncertain lifetimes. Upon death an agent is replaced by an identical offspring with her parent's endowment of human capital. Agents give a relative weight $\theta \in [0, 1]$, to the welfare of their offsprings. This parameter captures the degree of altruism. When the parameter θ , approaches 1 we are in the realm of perfect altruism and the model becomes an IH model. On the other hand, when θ is zero we have Blanchard's (1985) OLG model. Our framework allows us to combine these two well-known models and study the dynamics of inequality in a general context.

In Blanchard (1985), agents face a probability of survival which is taken to be an exogenously given parameter. We endogenize the survival probability by assuming it to be increasing in the level of consumption. In our model, individual agents differ only in terms of their initial endowment of human capital. In each period, agents decide the proportion of income to consume or invest in human capital. Consumption changes their probability of surviving to the next period while human capital determines the wage income in that period. If an agent dies, she is replaced by an offspring with her parent's human capital endowment. We solve the dynamic optimization problem facing an agent and characterize the global dynamics of the model. We find that with an endogenous probability of survival there is a possibility of multiple steady-state equilibria. Agent's below a certain threshold level of human capital have a low degree of patience. As a result, they behave in myopic fashion and

choose not to invest in human capital. They reach a bad equilibrium in the steady state with low levels of income and human capital. Agents above the threshold have sufficiently high degree of patience to induce them to invest in human capital. They attain the good equilibrium with higher levels of human capital and income. We provide an alternate explanation for the existence of inequality. Although our model was motivated by the observed empirical pattern of inequality in developed countries it is equally applicable to developing countries.

The paper is organized as follows. Next, we review the literature that is related to our paper. In section 2, we describe our model and derive our basic results in section 3. Results concerning the steady states are presented in section 4. In section 5, we characterize the global dynamics of the model with its stability properties. In section 6, we study some extensions of our model and conclude in section 7.

1.1 Related literature

The main innovation of our paper is that we allow an agent's probability of survival to be determined endogenously. We assume that as an agent's consumption increases she is more likely to survive to the next period. Is the link between survival probability and consumption likely to be empirically important? There is a lot of evidence linking economic prosperity and life expectancy. In the cross country data, a significant amount of variation in adult life-expectancy can be explained simply by the level of per capita GDP (Deaton, 2003; Kalemli-Ozcan, 2002). More surprisingly, even in developed countries, there is a lot variation in adult life expectancy across different income groups. Duleep (1986), Smith (1999) and Warren and Hernandez (2007) find a strong relationship between economic factors and different measures of health including mortality. In the USA, since the 1960s, there has been a deterioration in the health status of the lower income groups (Pappas *et al.*, 1993). Our model provides a theory linking health status and economic factors.

In a recent paper, Chakraborty and Das (2004) show that if there is a positive relationship between probability of survival and private health spending inequality may emerge endogenously. Although they use an OLG framework the reason for the existence of inequality in their paper is similar to ours. In their model every agent lives for two periods (young and old age) and has one offspring. Agents derive utility from their consumption in two periods and bequests that they leave for their children. However, survival to old age is random and is dependent on the level of consumption of a "health good" in young age. If an agent lives till the old age her bequest to her offspring is higher than if she does not survive. In their model, inequality exists because of two reasons. First, an agent whose parent is wealthier is more likely to survive to old age as she is able to consume more of the "health good". Secondly, agents whose parents survive till their old age receive larger bequests. There is a deadweight loss in bequests when an agent's parent dies prematurely. One weakness of their model especially in the context of the US economy is that in the USA, the majority of bequests are accidental[6]. In our model, when $\theta = 0$ all bequests are accidental. Also there is no loss in bequests when a parent dies. The entire stock of the parent's asset is passed on to the offspring. Hence, the results derived in Chakraborty and Das (2004) are not crucially dependent on the assumption that agents whose parents die prematurely face a loss in terms of inheritance of assets from their parents.

In our model, the discount factor or the level of patience of the agents is endogenous. Agents with higher income save more. This is consistent with recent empirical evidence on savings behavior. Lawrance (1991) finds that in the USA the subjective

rate of time preference is about 3-5 per cent points higher for households with lower incomes than those with higher incomes. Controlling for race and education widens this difference even more. Dynan *et al.* (2004) find that households with higher levels of permanent income have higher savings rates. Ventura (2003) uses survey data collected by Bank of Italy to directly estimate the rate of time preference. He finds that the degree of patience is increasing in income as well as wealth. We provide a plausible micro foundation for these empirical findings.

2. The model

2.1 Production technology

Consider a small open economy producing a single homogeneous commodity. Time is discrete and is indexed by $t = 1, 2, \dots, \infty$. Output (Y_t) is produced using a standard neoclassical technology

$$Y_t = F(K_t, H_t),$$

where K_t and H_t are capital and human capital, respectively. $F(\cdot)$ is constant returns to scale. Assuming the market for capital and human capital are competitive the rental rates (r_t) and wage rates (w_t) are given by

$$r_t = f'(k_t), \quad (1)$$

and

$$w_t = f(k_t) - f'(k_t)k_t \quad (2)$$

respectively. k_t is the capital to human capital ratio i.e. (K_t/H_t), and $f(k_t) = F(K_t/H_t, 1)$. Let the world interest rate be r . If capital is perfectly mobile k_t will be pinned down by the world interest rate. Capital will flow into and out of the economy depending on the level of domestic human capital. Note that the wage rate (equation (2)) just depends on k_t . From equation (1), it follows that the domestic wage rate is simply a decreasing function of the world interest rate i.e.

$$w_t = w(r). \quad (3)$$

2.2 Preferences

The economy consists of a continuum of agents who are born into dynastic households. The main difference of our paper from the rest of the literature is the nature of preferences of the individual agents. It will be helpful to explain the objective function of an agent through a series of steps. First, we have the period utility function which describes the utility of an agent at each time period. Suppose, the period utility function of an agent is denoted by $u(c_t)$ where c_t is the consumption at time period t . We assume that $u(\cdot)$ is increasing and concave in c_t .

(Assumption 1). $u(0) = 0, u'(\cdot) > 0, u''(\cdot) < 0$.

In an OLG model the welfare of an agent who lives for T periods would be described by

$$u(c_0) + \beta u(c_1) + \beta^2 u(c_2) + \dots + \beta^T u(c_t),$$

where $\beta \in (0, 1)$ is the subjective discount factor of the agent. The IH model is a special case where an agent upon her death is replaced by an offspring with her parents endowment of productive factors. If the agent cares about the welfare of her offspring as much as herself, her preference would be represented by

$$\sum_{t=0}^{\infty} \beta^t u(c_t).$$

It is as if the agent really wants to maximize the discounted sum of the utility of the entire dynasty which lives forever. In our model, the life of an agent is uncertain. In every period an agent faces a probability of not surviving to the next period. If an agent living in period t consumes c_t units of the consumption good, her probability of surviving to period $t + 1$ is given by a function $\alpha(c_t)$ where $\alpha(\cdot)$ satisfies the following properties.

(Assumption 2). $0 < \alpha(c_t) < 1, \alpha'(\cdot) > 0, \alpha''(\cdot) < 0$.

Figure 1 shows the behavior of $\alpha(\cdot)$ as consumption increases. We assume that the probability of survival function is increasing in the level of consumption. The additional assumption regarding concavity of $\alpha(\cdot)$ ensures that the first order conditions for maximum are also sufficient.

In case, an agent does not survive she is replaced by an identical offspring with her parents endowment of human capital. Agents are imperfectly altruistic. They take into account the utility of their offsprings while making their decisions, however they give a lower weight to the utility of their offsprings. Let θ denote the degree of altruism where $\theta \in [0, 1]$. The role of the altruism parameter and its significance in the agent's decision-making problem is best explained thinking of just two periods. Suppose an agent is making a decision over for two periods say time period $t = 0$ and $t = 1$. The objective function of an agent is given by

$$u(c_0) + \beta\{\alpha(c_0)u(c_1) + [1 - \alpha(c_0)]\theta u(c_1)\},$$

since $\alpha(c_0)$ is the probability that the agent will survive to period 1 and in case of her death i.e. with probability $[1 - \alpha(c_0)]$ she will be replaced by her offspring whose

Figure 1.

utility is valued slightly lower, $\theta u(c_1)$. The welfare of an agent can be re-written as

$$\begin{aligned} & u(c_0) + \beta\{\alpha(c_0)u(c_1) + [1 - \alpha(c_0)]\theta u(c_1)\}, \\ & = u(c_0) + \beta\{\alpha(c_0) + [1 - \alpha(c_0)]\theta\}u(c_1), \\ & = u(c_0) + \phi(c_0, \theta)u(c_1), \end{aligned}$$

where, $\phi(c_0, \theta) = \beta\{\alpha(c_0) + [1 - \alpha(c_0)]\theta\}$. The term, $\phi(\cdot)$, is the effective discount factor of the agent. Notice as long as $\theta < 1$, the discount factor is not an exogenous parameter but determined by the decision of the agent in period 0.

Now we can define the objective function of an agent. Individual agents seek to maximize expected lifetime welfare. Since the probability of survival is endogenous in our model the weight given to each period's utility is determined by the probability of surviving to that period. Let ρ_t denote the weight given to the period t utility. The objective of an individual agent is to maximize

$$\sum_{t=0}^{\infty} \rho_t u(c_t), \quad (4)$$

where, $\rho_0 = 1$ and

$$\rho_{t+1} = \phi(c_t, \theta)\rho_t. \quad (5)$$

Equation (5) describes the evolution of the discount factor. The agent is alive for sure in period $t = 0$, so $\rho_0 = 1$. As illustrated in the two period case earlier, the weight given to a period's utility will depend on the probability of surviving to that period. The expression, $\phi(c_t, \theta) = \beta\{\alpha(c_t) + [1 - \alpha(c_t)]\theta\}$, captures this effect. Figure 2 plots the function $\phi(c_t, \theta)$ as c_t increases. As we can see $\phi(\cdot, \theta)$ is increasing and concave in c_t as long as $\theta < 1$. As θ increases $\phi(c_t, \theta)$ shifts upwards and becomes flatter. Formally, the

Figure 2.

partial derivatives of $\phi(\cdot)$ with respect to c_t and θ are given by

$$\phi_c(c_t, \theta) = \alpha'(c_t)\beta(1 - \theta), \quad \phi_{cc}(c_t, \theta) = \alpha''(c_t)\beta(1 - \theta),$$

and

$$\phi_\theta(c_t, \theta) = \beta[1 - \alpha(c_t)].$$

When $\theta = 1$, $\phi(c_t, \theta) = \beta$ for any value of c_t , and the discount factors for each period $\rho_t = \beta^t$. The preference of the agent becomes time separable as is the case in the standard IH model. When $\theta = 0$, $\phi(c_t, \theta) = \beta\alpha(c_t)$, the discount factors have the same interpretation as Blanchard's (1985) perpetual youth model except in our case the probability of survival is endogenously determined.

2.3 Human capital accumulation

To begin with we are going to assume that agents can accumulate only one kind of asset called human capital[7]. Agents differ in terms of their initial endowments of human capital i.e. h_0 . Every agent is born with one unit of labor time which she combines with her human capital as effective units of labor. Given a competitive wage rate w_t the wage income of an agent in period t is, $w_t h_t$, where h_t is the human capital of the agent. In every period, the agent has to decide the proportion of wage income to allocate between consumption and investment in human capital. The human capital of an agent evolves according to the following equation:

$$h_{t+1} = g(w_t h_t - c_t) + (1 - \delta)h_t, \quad (6)$$

where $\delta \in [0, 1]$ is the depreciation rate of human capital.

(Assumption 3). $g(0) = 0, g'(\cdot) > 0, g''(\cdot) < 0$. In addition, $\lim_{h \rightarrow 0} g'(\cdot) = \infty$.

The Inada condition ensures that every agent will choose to invest in human capital in the steady state. Let the initial distribution of human capital be given by $\Phi_0(h_0)$. Depending on the human capital accumulation of every agent this distribution function will change over time. We should point out that although we refer to h_t as human capital it could be more generally thought of as any accumulable factor whose return is specific to the agent.

3. Solving the model

The maximization problem faced by an agent is

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \rho_t u(c_t),$$

subject to

$$h_{t+1} = g(w_t h_t - c_t) + (1 - \delta)h_t,$$

and

$$\rho_{t+1} = \phi(c_t, \theta)\rho_t.$$

From equation (3), we know that the wage rate is constant as long as the world interest rate is constant. The Lagrangian for the agent's problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \rho_t u(c_t) + \tilde{\lambda}_t [g(wh_t - c_t) + (1 - \delta)h_t - h_{t+1}] + \mu_t [\phi(c_t, \theta)\rho_t - \rho_{t+1}] \right\}.$$

The first-order conditions for maximum are

$$\begin{aligned} \frac{\partial}{\partial c_t} &= \rho_t u'(c_t) - \tilde{\lambda}_t g'(wh_t - c_t) + \rho_t \mu_t \phi_c(c_t, \theta) = 0, \\ \frac{\partial \mathcal{L}}{\partial h_{t+1}} &= -\tilde{\lambda}_t + \tilde{\lambda}_{t+1} [wg'(wh_{t+1} - c_{t+1}) + (1 - \delta)] = 0, \end{aligned}$$

and

$$\frac{\partial \mathcal{L}}{\partial \rho_{t+1}} = u(c_{t+1}) - \mu_t + \mu_{t+1} \phi(c_{t+1}, \theta) = 0. \quad (7)$$

The first order conditions can be re-written as

$$u'(c_t) + \mu_t \phi_c(c_t, \theta) = \lambda_t g'(wh_t - c_t), \quad (8)$$

$$\lambda_t = \lambda_{t+1} \phi(c_t, \theta) [wg'(wh_{t+1} - c_{t+1}) + (1 - \delta)], \quad (9)$$

where $\lambda_t = \tilde{\lambda}_t / \rho_t$. Note that $u''(c_t) + \mu_t \phi_{cc}(c_t, \theta) < 0$ since the functions $u(\cdot)$ and $\phi(\cdot)$ are concave. Thus, the first-order conditions for a maximum are also sufficient. Substituting equations (8) in (9) we get,

$$\frac{g'(wh_{t+1} - c_{t+1}) [u'(c_t) + \mu_t \phi_c(c_t, \theta)]}{g'(wh_t - c_t) [u'(c_{t+1}) + \mu_{t+1} \phi_c(c_{t+1}, \theta)]} = \phi(c_t, \theta) [wg'(wh_{t+1} - c_{t+1}) + (1 - \delta)]. \quad (10)$$

Equation (10) tells us that at an optimum, the loss in welfare due to foregoing consumption in period t has to equal the discounted value of gain in welfare from period $t+1$ onwards.

Let us now consider equation (7). Re-arranging we have

$$\mu_t = u(c_{t+1}) + \mu_{t+1} \phi(c_{t+1}, \theta).$$

Substituting one period forward for μ_{t+1} gives us,

$$\mu_t = u(c_{t+1}) + \phi(c_{t+1}, \theta) [u(c_{t+2}) + \mu_{t+2} \phi(c_{t+2}, \theta)].$$

If we carry out this substitution repeatedly, we find that μ_t is the present discounted value of utility from period $t+1$ onwards i.e.

$$\mu_t = (\rho_{t+1})^{-1} \sum_{s=t+1}^{\infty} \rho_s u(c_s). \quad (11)$$

In our model when an agent is making her consumption decision her probability of survival is affected as well. Hence her valuation of future utilities μ_t enters in equation (10).

Definition 1. For a given $(w, \Phi_0(h_0))$ a perfect foresight equilibrium (PFE) of the economy is given by sequences $\{c_t\}_{t=0}^{\infty}$, $\{h_{t+1}\}_{t=0}^{\infty}$, $\{\mu_t\}_{t=1}^{\infty}$ such that equations (6), (10) and (11) hold for all agents.

Equation (6) is the inter-temporal budget constraint of the agent while equation (10) is the inter-temporal optimality condition. We should point out that we have ruled out any borrowing or lending between agents[8].

4. Steady-state solution

We will first characterize the stationary solution to an agent's problem. In a steady state $c_t = c_{t+1} = c$, $h_t = h_{t+1} = h$ and $\mu_t = u(c)/(1 - \phi(c, \theta))$ for all t . Equations (6) and (10) reduce to

$$\delta h = g(wh - c), \quad (\text{BC})$$

and

$$\phi(c, \theta)[wg'(wh - c) + (1 - \delta)] = 1. \quad (\text{EE})$$

Equation (EE) is the steady-state counterpart of the Euler equation of the agent. Equation (BC) implicitly defines the locus of points along which the agent's consumption and human capital are constant and satisfy the budget constraint. It is easy to see from (BC) that when $h = 0$, $c = 0$. Let $c = \eta(h, w, \delta)$ be the function defined by the steady-state condition (BC) where $\eta(h, w, \delta) = wh - (g^{-1})(\delta h)$.

Lemma 1. Let $h_{\max} = \arg \max \eta(h, w, \delta)$. At a steady-state solution to the agent's maximization problem $h < h_{\max}^h$.

Proof. Notice the steady-state condition (BC) implies $\eta(0, w, \delta) = 0$ and $\eta_h = (\partial/\partial h)\eta(\cdot) = w - \delta/g'(\cdot)$. From the properties of the function $g(\cdot)$ it follows that for small values of h , η_h is positive. As h increases η_h falls and is equal to zero when $wg'(\delta h) = \delta$. Let $h_{\max}$ be the value of h that satisfies this condition. Once $h > h_{\max}$, c is decreasing in h according to the steady-state condition (BC). However, $h > h_{\max}$ can never be a steady-state equilibrium. $h > h_{\max} \Leftrightarrow wg'(\cdot) - \delta < 0$. From (EE) if $wg'(\cdot) - \delta < 0$ it follows that $\phi(c, \theta) > 1$ which is a contradiction. ■

Figure 3 shows the relationship between consumption and human capital in the steady state. We are going to restrict our attention to the positively sloped part of the (BC) curve. The steady-state condition (EE), commonly called the fisher equation, is also positively sloped. Taking total derivatives of the equation (EE) we get

$$\phi_c(c, \theta)[wg'(\cdot) + (1 - \delta)]dc - \phi(c, \theta)wg''(\cdot)dc = -\phi(c, \theta)w^2g''(\cdot)dh,$$

or

$$\frac{dc}{dh} = \frac{-\phi(\cdot)w^2g''(\cdot)}{\phi_c(\cdot)[wg'(\cdot) + (1 - \delta)] - \phi(\cdot)wg''(\cdot)} > 0,$$

Figure 3.

due to the concavity of the function $g(\cdot)$. We will soon derive the conditions for the existence of a steady state solution. However, just plotting the (BC) and (EE) curves we see that there could be multiple steady state solutions to an agent's problem. In Figure 3, we illustrate such a possibility.

In order to prove the existence of a steady state solution it will be helpful to write the (EE) condition in terms of human capital alone. We substitute $c = \eta(h, \cdot)$ derived from equation (BC) into equation (EE). Define $\Psi(h)$ as

$$\Psi(h) = \phi(\eta(h, \cdot), \theta)[wg'(wh - \eta(h, \cdot)) + (1 - \delta)] - 1. \quad (\text{SS})$$

Notice we have reduced the steady-state condition of an agent to just one equation. When $\Psi(h) = 0$ both the steady-state conditions (BC) and (EE) are satisfied.

- P1. There exists at least one steady-state solution to an agent's maximization problem. The steady-state level of human capital is bounded i.e. $h \in (0, h_{\max})$. The number of steady-state solutions is odd.

Proof. $\Psi(0) = \beta\theta[wg'(0) + (1 - \delta)] - 1$ is positive because $g(\cdot)$ satisfies the Inada condition. As $h \rightarrow h_{\max}$, we know that $wg'(\cdot) + (1 - \delta)$ will fall and from the definition of $h_{\max}$ it follows that $\Psi(h_{\max}) = -1$. Hence, there must exist some intermediate value of $h \in (0, h_{\max})$ where $\Psi(h) = 0$. This proves the existence of at least one steady-state solution. Suppose, there exist two steady-state solutions. From the continuity of the function $\Psi(h)$, it follows that there exists an $\bar{h} < h_{\max}$ such that $\Psi(\bar{h}) > 0$ and $\Psi(h) = 0$ has two solutions in the interval $(0, \bar{h})$. However, we know that $\Psi(h_{\max}) = -1$. Hence, there must exist another steady-state solution in the interval $(\bar{h}, h_{\max})$. This argument can be generalized to prove that the number of steady-state solutions is odd. ■

We are implicitly ruling out cases where the (EE) curve is tangential to the (BC) curve. The local stability properties of such solutions are not well defined. It will become clear

in the next section that this assumption is not particularly important for our analysis. Let us understand the intuition behind the existence of more than one steady-state equilibria. In an infinite horizon model when $\theta = 1$, $\phi(\eta(h, \cdot), \theta) = \beta$ for any value of h . The return from investing in human capital $[wg'(wh - \eta(h, \cdot)) + (1 - \delta)] = wg'(g^{-1}(\delta h)) + (1 - \delta)$ is strictly decreasing in h . Therefore, $\Psi(h)$ is strictly decreasing in h and $\Psi(h) = 0$ will have a unique solution in the interval $(0, h_{\max})$. When $\theta < 1$, the discount factor $\phi(\eta(h, \cdot), \theta)$ is increasing with h . Depending on which effect dominates $\Psi(h)$ may be increasing or decreasing in h and hence the possibility of more than one steady state solution. In the subsequent analysis, we are going to assume that there are three steady-state solutions as is shown in Figure 4. All the interesting qualitative properties of our model can be studied in such a scenario.

5. Global dynamics

Now we will study the global dynamics of the model. There is a possibility of more than one steady-state solution to an agent's problem. We need to understand which solution will be preferred by an agent. The following proposition will help us characterize the dynamics of human capital accumulation.

- P2.* Let h_0 be the initial endowment of human capital of an agent. The agent will accumulate human capital if the marginal return from human capital exceeds the rate of time preference, i.e.

$$wg'(wh_0 - \eta(h_0)) + (1 - \delta) > \frac{1}{\phi(\eta(h_0), \theta)},$$

and vice-versa[9].

Proof. The inverse of the discount factor, $1/\phi(\eta(h_0), \theta)$, is called the rate of time preference. We need to show that if the return from human capital accumulation at the

Figure 4.

initial endowment of human capital exceeds the rate of time preference then $\{h_t = h_0\}_{t=0}^{\infty}$ and $\{c_t = \eta(h_0, w, \delta) = wh_0 - (g^{-1})(\delta h_0)\}_{t=0}^{\infty}$ cannot be an optimum for an agent. Furthermore $h_1 > h_0$. Suppose

$$wg'(wh_0 - \eta(h_0)) + (1 - \delta) > \frac{1}{\phi(\eta(h_0), \theta)}. \quad (12)$$

If we reduce consumption for only one period by ϵ , where ϵ is close to zero the resultant loss in utility will be $[u'(c_0) + \mu_0 \phi_c(c_0, \theta)]$, where $\mu_0 = u(c_0)/(1 - \phi(c_0, \theta))$. The saved amount can be used to accumulate human capital which will provide higher wage income in the next period. If that increased income were to be consumed the discounted value of its utility will be $\phi(c_0, \theta)[wg'(wh_0 - c_0) + (1 - \delta)][u'(c_0) + \mu_0 \phi_c(c_0, \theta)]$. Since $\phi(\eta(h_0), \theta)[wg'(wh_0 - \eta(h_0)) + (1 - \delta)] > 1$, from the condition above, it follows that the gain in utility will outweigh the loss. Hence, $\{h_t = h_0\}_{t=0}^{\infty}$ and $\{c_t = \eta(h_0, w, \delta) = wh_0 - (g^{-1})(\delta h_0)\}_{t=0}^{\infty}$ cannot be an optimal sequence for an agent and the agent will accumulate human capital i.e. $h_1 > h_0$. We can use a similar argument to show that if

$$wg'(wh_0 - \eta(h_0)) + (1 - \delta) < \frac{1}{\phi(\eta(h_0), \theta)} \quad (13)$$

then human capital must be strictly decreasing.

- P2. Characterizes the path of human capital preferred by an agent depending on her initial endowment of human capital. When

$$wg'(wh_0 - \eta(h_0)) + (1 - \delta) > \frac{1}{\phi(\eta(h_0), \theta)},$$

where $\Psi(h)$ is positive and an agent will choose to accumulate human capital. When the opposite inequality holds $\Psi(h)$ is negative and the agent will reduce her stock of human capital. We can see from Figure 5 the steady state $\hat{h}$ is unstable[10]. If the initial human capital endowment of an agent is less than $\hat{h}$ the agent will end up with a stock of human capital, h_B . If the initial human capital endowment of an agent exceeds $\hat{h}$ then the agent will accumulate human capital and end up with a stock of human capital h_G in steady state. We are going to refer to agent's converging to h_B and h_G as low- and high-income agents, respectively. Given the initial distribution of human capital, $\Phi_0(h_0)$, $\Phi_0(\hat{h})$ proportion of the agents will end up with a lower level of human capital and lifetime income. $(1 - \Phi_0(\hat{h}))$ proportion of the agents with sufficiently high endowment of human capital will end up with high levels of human capital and lifetime income. In the long run, we will see persistence of income inequality. The economy gets polarized into two groups. Notice agents with high levels of lifetime income have higher savings rates compared to the other group[11]. This is consistent with the empirical evidence found by Dynan *et al.* (2004).

In our model, income inequality exists because a certain proportion of agents end up with low levels of human capital in the steady state. The reason for the persistence of inequality in our paper is different from the other papers in the literature. Agents with low endowment of human capital have low income. They have very little chance of surviving to the next period. Such agents have low discount factors and do not

Figure 5.

accumulate human capital. The following proposition helps us in understanding why inequality disappears in an infinite horizon model.

P3. The proportion of agents converging to the low level of steady-state human capital h_B is decreasing in θ .

Proof. By definition $\Psi(\hat{h}) = 0$. Consider, $\Psi(h) = \phi(\eta(h, \cdot), \theta) [wg'(wh - \eta(h, \cdot)) + (1 - \delta)] - 1$. $(\partial/\partial\theta) \Psi(h, \cdot) = \phi_\theta(\cdot, \theta) [wg'(wh - \eta(h, \cdot)) + (1 - \delta)] > 0$. Hence, the entire term, $\Psi(h)$, shifts upward as θ increases. This means that there exists an $h_0 < \hat{h}$ such that $\Psi(h_0) > 0$. Hence, the proportion of agents converging to h_B will fall as θ increases. ■

In every period, there are two motives for an agent to invest in human capital. The first one is a purely selfish motive. Investing in human capital allows an agent to earn a wage income in the future if she survives to the next period. There is also a strategic motive. Even if an agent does not survive to the next period her future generations inherit her human capital. As agents care about the welfare of their dynasty they choose to accumulate human capital. As θ increases agents start to behave more altruistically and start giving more importance to the welfare of their future generations. We can see from Figure 6, an increase in θ shifts the entire $\Psi(h)$ curve upwards. Even when there is a possibility of multiple steady-state solutions the proportion of agents converging to the high steady-state human capital is now higher. This leads to a larger proportion of the population to attain higher lifetime income. In fact, when $\theta = 1$, agents care as much about the welfare of their dynasty as themselves. There is no inequality in the long run.

P4. An increase in the world interest rate will result in a lower proportion of agents attaining high lifetime income.

Proof. As the world interest rate r increases the wage rate will decrease. We want to know the effect of this on the function, $\Psi(h) = \phi(\eta(h, w, \delta), \theta) \{wg'[(g^{-1})(\delta h)] + (1 - \delta)\} - 1$. We know that $\eta(h, w, \delta)$ is increasing in w . Hence, the entire $\Psi(h)$ curve will shift downwards as a result of an increase in r . In case, there are

Figure 6.

two stable steady-state solutions to an agent's problem, a downward shift of $\Psi(h)$ implies the threshold that is needed for an agent to prefer to accumulate human capital is now higher. It is easy to see from Figure 6 that a larger proportion of agents will choose not to accumulate human capital and reach h_B in long run. ■

Of course, *P4* needs to be qualified. In our model, an increase in the world interest rate leads to an outflow of capital and leads to a decrease in the wage rate. The reduction in wage rate means that a larger proportion of agents choose not to invest in human capital and end up with lower lifetime income. However, the close link between the world interest rate and the wage rate is a stylized feature of our model. It may not be a generally observed phenomenon.

6. Extensions

In this section, we study a couple of interesting extensions of our model. First, we allow the agents in the economy to accumulate another asset which can be thought of as physical capital. With perfect capital mobility, it is reasonable to think that the agents would be able to invest in another asset which would give them a risk free rate of return equal to the world interest rate r . Secondly, in the previous section, we have assumed that human capital accumulation only requires investment in the form of the consumption good. We assume that human capital evolves according to

$$h_{t+1} = B(v_t h_t)^\gamma x_t^\sigma + (1 - \delta)h_t, \quad (14)$$

where $B > 0$ is a shift parameter and $\gamma, \sigma \in (0, 1)$. v_t is the proportion of labor time spent for human capital accumulation. x_t is the expenditure in terms of the consumption good. The notion that accumulation of human capital becomes easier with higher levels of human capital is implicitly captured in equation (14) through the interaction of v_t with h_t . Empirical estimates of γ and σ suggest that $\gamma + \sigma < 1$, which is an assumption we will also make [12]. The dynamic optimization problem facing an agent is similar to the previous section. The budget constraint facing an agent is now given by

$$a_{t+1} = ra_t + w(1 - v_t)h_t - c_t - x_t, \quad (15)$$

where a_{t+1} denotes the investment in the risk free asset in period t and $(1-v_t)$ is the proportion of time spent working. The choice of v_t is essentially a static one. Using equation (15) to substitute for x_t in equation (14) we have

$$h_{t+1} = B(v_t h_t)^\gamma [ra_t + w(1 - v_t)h_t - c_t - a_{t+1}]^\sigma + (1 - \delta)h_t, \quad (16)$$

At an optimum the amount of time spent in human capital accumulation is given by

$$v_t = \frac{\gamma(ra_t + wh_t - c_t - a_{t+1})}{(\gamma + \sigma)wh_t}. \quad (17)$$

Regarding the decision to invest in the risk free asset, let us focus again on equation (16). An agent will invest in the risk free asset only if its marginal return, the world interest rate r , exceeds the return from human capital. Let the marginal return from human capital be denoted by R_{t+1} [13]. Using equation (17) to substitute for v_t we have

$$R_{t+1} = \tilde{B}w_t^{1-\gamma}(ra_{t+1} + wh_{t+1} - c_{t+1} - a_{t+2})^{\gamma+\sigma-1} + (1 - \delta),$$

where $\tilde{B} = B(\gamma/(\gamma + \sigma))^\gamma(\sigma/(\gamma + \sigma))^\sigma$. Since $\gamma + \sigma < 1$, the rate of return from investment in human capital is high for low values of $(ra_{t+1} + wh_{t+1})$. An agent will invest in the risk free asset only after achieving a sufficiently high level of income.

P5. There exists a $\underline{h}$ such that $a_{t+1} = 0$ if $h_{t+1} \leq \underline{h}$.

Proof. Let $\underline{h}$ solve the following equation:

$$\tilde{B}w^{1-\gamma}(ra_{t+1} + wh_{t+1} - c_{t+1})^{\gamma+\sigma-1} + (1 - \delta) = r.$$

The left-hand side is monotonically decreasing in h_{t+1} . Hence, if $h_{t+1} \leq \underline{h}$, the rate of return from investing in human capital R_{t+1} exceeds the return from investing in the risk free asset. Hence, $a_{t+1} = 0$. ■

The proposition above means that poorer agents will choose to invest entirely in human capital. We still have the possibility of multiple solutions to an agents' problem and inequality persisting in the long run. The introduction of the risk free asset creates a wedge between income and wealth distribution. Wolf (1998) reports that the proportion of households with zero or negative financial wealth in USA was 26 per cent in 1983. This number increased to 29 per cent by 1995.

7. Conclusion

In this paper, we have provided an alternative explanation for the existence of inequality in an economy. We endogenize the expected lifetime of individual agents in an economy by introducing a probability of survival function. Endogenous probability of survival means that the rate of time preference or the level of patience of agents depends on their income. This has an important effect on their savings behavior and their decision to accumulate human capital. We find that there is a possibility of multiple steady-state equilibria. If an agent's endowment of human capital is below a threshold level she chooses not to invest in human capital. Such agents converge to a

bad equilibrium with low levels of human capital and income. Agents above the threshold choose to invest in human capital and converge to the good equilibrium. In the long run, we can have persistence of inequality even though agents have identical preferences and there are no non-convexities in the production process. Introduction of another asset with a risk free rate of return creates a wedge between the distribution of income and wealth. Poorer agents i.e. with low endowment of human capital choose to use their savings to accumulate human capital as they get a higher marginal return from human capital than from physical capital. As such they may still end up at the bad equilibrium if their probability of survival is too low.

Is our model likely to able to explain the recent trends in income and wealth inequality that we observe in many developed countries? In many developed countries, the pattern of increase in income and wealth inequality started since mid 1980s. Interestingly, we also see a significant reduction in the marginal tax rates in almost every developed country over this period[14]. Although we do not model taxes explicitly, our model can be easily extended to show that any change in policy that provides tax breaks to the rich is going to raise inequality. In Australia, a consumption tax of 10 per cent was introduced in 2001. In our model, the poor have lower savings rates so a consumption tax will also lead to an increase in inequality. A careful quantitative evaluation of our model is left for future research.

Notes

1. Fajnzylber *et al.* (2002) find inequality to be associated with higher incidence of violent crime while You and Khagram (2004) find higher inequality to be associated with higher levels of corruption.
2. See for instance, Aghion and Bolton (1997), Banerjee and Newman (1993), and Galor and Zeira (1993). In these types of models, income and wealth distributions are identical.
3. This is the case in Becker *et al.* (1990).
4. Cagetti and De Nardi (2005) provide an excellent review of this literature.
5. Hendricks (2007) for instance introduces heterogeneity in discount factors across households.
6. See for instance Hendricks (2002).
7. In section 6, we show that the our results are not qualitatively affected if we allow the agents to accumulate another asset, say, physical capital.
8. Since agents are accumulating only human capital it is not unreasonable to assume that credit markets do not exist. However, our results will hold as long as the agents face some sort of credit rationing.
9. For notational convenience, we suppress the terms δ and w in the function $\eta(\cdot)$.
10. In general, if we rank the steady-state equilibria in increasing order, all the odd numbered equilibria will be stable while the even numbered equilibria will be unstable.
11. We get a two point distribution in steady state. However, that can be changed quite easily by introducing some uncertainty in the production function.
12. Refer to Mincer (1993) for more details.
13. We are simply taking the derivative of the right-hand side of equation (16) with respect to h_{t+1} . In the previous sections, this marginal return was denoted by, $wg'(wh_{t+1} - c_{t+1}) + (1 - \delta)$.
14. Mendoza *et al.* (1994) provide estimates of the trends in income and corporate tax rates.

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Further reading

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