A Lower Bound on Network Layer Control Information

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ABSTRACT

Any asymptotic mean stationary mobility model generates, at the network layer, a route process that satisfies the Asymptotic Equipartition Partition Theorem of Information Theory. This permits an information theoretic lower bound for network layer control information.

By recasting the mobility problem as a Dynamical System, we provide a unique and rigorous examination of mobility models and routing protocols. In particular, new results on stationary and ergodic properties of common mobility models – via two useful generalizations – are provided. The important concept 'perfect simulation' for reliable, and repeatable, results is formalized. A fixed to variable length encoding lemma – of independent interest – for asymptotically mean stationary sources is developed. Finally, a lower bound on network layer control information is presented.

1. INTRODUCTION

It is well known that decode-and-forward communication strategies for wireless networks are, in general, not scalable. This result was originally provided by Gupta *et al.* in [1]. Recent works [2–6] generalize [1] by allowing more complex protocol models, however, each work assumes the network is physically stationary. Grossglauser *et al.* in [7, 8] showed that under certain circumstances the limits of [1] can be increased by exploiting node movement. Jafar [9] demonstrates that node movement is not always beneficial to network throughput.

We contend that mobile wireless network size – in addition to being constrained by throughput *e.g.* [1] – is also limited by network layer control information. The history of protocol information at the network layer is short [10–13]. We present a definition of network layer control information that generalizes these works to include a wide class of probabilistic mobility models and deterministic routing protocols. Protocol control information was originally defined by Gallager in [14] at the Medium Access Control (MAC) Layer.

Section 2 presents the dynamical system model for probabilistic node movement. This model – the subject of Ergodic Theory – is appropriate for any probabilistic, or deterministic, dynamical system. Asymptotic mean stationary (AMS), stationary and ergodic mobility models are defined. The AMS condition for a Generalized Random Waypoint Mobility Model is proven and the Random Waypoint Mobility Model is shown to be AMS and variable-length ergodic. In Section 3, the concept of perfect simulation is shown to produce a stationary system from an AMS mobility model. Section 4 develops a model for deterministic protocols and demonstrates how AMS mobility models induce AMS 'routing process'. Our main result – a lower bound on network layer control information – is then proven using the AMS properties of the routing process. In Section 5 we conclude the paper. All proofs are banished to the appendix.

The work of R. C. Timo and L. W. Hanlen was sponsored by National ICT Australia. National ICT Australia is funded through the Australian Government's *Backing Australia's Ability initiative*, in part through the Australian Research Council

2. NODE MOVEMENT: DYNAMICAL SYSTEM MODEL

Consider a finite collection of nodes $\mathcal{V} = \{v_1, \ldots, v_{|\mathcal{V}|}\}$, where each node is located within a discrete finite state-space $\mathcal{S}$. Assume $\mathcal{S}$ is a collection of points from $\mathbb{R}^2$, so the Euclidean distance between any two points is defined. Let $s_i(t) \triangleq \big(j_i(t), k_i(t)\big)$ denote the location of node v_i in $\mathcal{S}$ at time t and define the *node position vector* $\omega_t \triangleq \big(s_1(t), \ldots, s_{|\mathcal{V}|}(t)\big)$. For the node position vector, define the *node position space* $\Omega \triangleq \mathcal{S}^{|\mathcal{V}|} = \mathcal{S} \times \cdots \times \mathcal{S}$. As ω_t evolves in time, it drives physical and network layer change. Let nodes move 'instantaneously' at times $n = 0, 1, \ldots$ and define the *node position vector space* $\Omega^N \triangleq \big\{\omega_0^{N-1} \ : \ \omega_n \in \Omega, \ n = 0, \ldots, N-1\big\}$. Where $\omega_m^n = (\omega_m, \ldots, \omega_n)$, for any non-negative integers $m \leq n$, denotes a particular *sample path*[1].

The movement of nodes in $\mathcal{S}$ is a discrete random process $\mathbf{X}^\infty = X_0, X_1, \ldots$ with finite alphabet Ω. Write the joint distribution of the random $(n-m)$-vector $\mathbf{X}_m^n$, as $\mu_m^n(\omega_m^n) = \Pr\{\mathbf{X}_m^n = \omega_m^n\}$, where $\omega_m^n \in \Omega^{(n-m)}$. The *distribution* of $\mathbf{X}^\infty$, is the set of all joint distributions for all integers $n \geq m \geq 0$.

Mobility models are constructed by first specifying an *initial node position distribution*, $\mu_0(\omega_0) = \Pr\big\{X_0 = \omega_0\big\}$. Movement after the initial placement is defined by[2], *c.f.* [16]

$$\mu_0^{N-1}\big(\omega_0^{N-1}\big) = \mu_0(\omega_0)\mu_1(\omega_1|\omega_0)\cdots\mu_{N-1}\big(\omega_{N-1}|\omega_0^{N-2}\big) \qquad (1)$$

The mobility model description, *e.g.* (1), and the underlying space, *e.g.* Ω, are not important. What is important is the distribution of $\mathbf{X}^\infty$. An alternative description is given by the probability measure of a dynamical system.

Define the *node position sequence space* $\Omega^\infty \triangleq \big\{\omega^\infty \ : \ \omega_n \in \Omega, \ \forall n \in \mathbb{N}\big\}$ where $\omega^\infty = \omega_0, \omega_1, \ldots$ and $\mathbb{N} = 0, 1, \ldots$ is the non-negative integers. The *cylinder set* determined by ω_m^n, is the set $\big[\omega_m^n\big] = \big\{\widehat{\omega}^\infty \in \Omega^\infty \ : \ \widehat{\omega}_i = \omega_i, \ i = m, m+1, \ldots, n\big\}$.

The joint probability (1) on Ω^N generated directly by the mobility model uniquely extends, via the Kolmogorov Extension Theorem [17, 18], to a unique probability measure μ on the sequence space Ω^∞: μ is called the Kolmogorov measure. If $\mathcal{S}$ is 'closed' this offers no restriction to the mobility model – Appendix Remark 1. The Kolmogorov Extension Theorem ensures $\mu\big([\omega_m^n]\big) = \mu_m^n\big(\omega_m^n\big)$, $\forall n, m: \ n \geq m \geq 0$. Let $\mathcal{F}\left(\Omega^\infty\right)$ denote the σ-algebra generated by the set of all cylinder sets. The probability space $\left(\Omega^\infty, \mathcal{F}\left(\Omega^\infty\right), \mu\right)$ is then suitable for any mobility model on a 'closed' discrete space.

In (1) it is clear how the process evolves over time. The *left shift (time) transform* $T: \ \Omega^\infty \to \Omega^\infty$ represents the action of time in Ω^∞. $T\omega^\infty = T(\omega_0, \omega_1, \omega_2, \ldots) = \omega_1, \omega_2, \ldots$, $\forall \omega^\infty \in \Omega^\infty$ and $T^{-1}A = \big\{\omega^\infty \ : \ T\omega^\infty \in A\big\}, \forall A \in \mathcal{F}\left(\Omega^\infty\right)$.

Let $\Pi_0\big(\omega^\infty\big) = \Pi_0\big(\omega_0, \omega_1, \omega_2, \ldots\big) = \omega_0$. Then $\mathbf{X}^\infty$ defined by the mobility model (1) can be rewritten as

$$\mathbf{X}^\infty = \{X_n\}_{n=0}^\infty = \Big\{\Pi_0\big(T^n\omega^\infty\big)\Big\}_{n=0}^\infty \qquad (2)$$

[1]In networking literature, *c.f.* [15], this *sample path* is referred to as a *mobility trace* since it 'records' the location of nodes from time m to time n.

[2]Note (1) is without loss of generality.

The quadruple $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ defines a *Dynamical System* and leads us to view any 'network operation' that is entirely specified by ω, as a measurable function taking values from $\mathcal{F}(\omega^\infty)$ to some 'decision space' $\mathfrak{R}$. We have constructed the dynamical system model on a random process (1), however the Ergodic Theory model for a dynamical system is far more general *c.f.* [17–22].

The model (2) is suitable for the simulation-based analysis of mobility models. For example, suppose we simulate a mobility model and get ω_0^{N-1} as the output. The sample path ω_0^{N-1} has been randomly chosen with probability $\mu([\omega_0^{N-1}])$ from $|\Omega^N|$ possibilities. We make repeated time-based measurements, $f: \Omega^N \to \mathfrak{R} \subseteq \mathbb{R}$, *samples*, which we use to draw conclusions about the average behavior of either the sample path or the routing protocol. The *sample averages* are given by

$$\langle f \rangle_N = \frac{1}{N} \sum_{n=0}^{N-1} f\left(T^n \omega_0^{N-1}\right) \tag{3}$$

Example 1 (Protocol Failures). Let $A \subset \Omega$ be a set of node location vectors where the routing protocol fails. For example, A could be the set of position vectors where the source is disconnected from the sink. To estimate the probability of protocol failure due to A, choose f to be the *indicator function*,

$$f(\omega_0^{N-1}) = \mathbf{1}_A(\omega_0^{N-1}) = \begin{cases} 1, & \text{if } \omega_0 \in A \\ 0, & \text{Otherwise.} \end{cases} \tag{4}$$

Under (4), (3) becomes the *relative frequency* of protocol failures on ω_0^{N-1}. The function f is a *discrete random variable*.

More generally and analogous to the definition of $\mathbf{X}^\infty$ in (3), a random process of measurements $\{f_n\}_{n=0}^\infty$ is given by

$$\{f_n\}_{n=0}^\infty = \left\{ f\left(T^n \omega^\infty\right) \right\}_{n=0}^\infty \tag{5}$$

where f is *measurable*[3] with domain Ω^∞ and range $\mathfrak{R} \subseteq \mathbb{R}$.

The sequence $\{f_n\}$ is a random process, therefore $\{\langle f \rangle_N\}_{N=0}^\infty$ is a random process. The first concern of any asymptotic study of mobility models and routing, is to provide necessary conditions for (3) to converge[4] $N \to \infty$.

In $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$, the Kolmogorov measure μ induced by the mobility model, has *ergodic properties* with respect to the measurement f if

$$\langle f \rangle(\omega^\infty) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} f\left(T^n \omega^\infty\right), \quad f: \Omega^\infty \to \mathfrak{R} \subseteq \mathbb{R}$$

exists almost everywhere (a.e.)[5].

Example 2 (Stationary Node Locations). Consider the limiting relative frequency of each[6] $\omega \in \Omega$. Choose $|\Omega|$ indicator functions

$$\mathbf{1}_\omega(\widehat{\omega}^\infty) = \begin{cases} 1, & \text{if } \widehat{\omega}_0 = \omega \\ 0, & \text{Otherwise.} \end{cases} \quad \forall \omega \in \Omega$$

Let $\lim_{N\to\infty} \omega_0^{N-1} = \omega^\infty$ denote the limiting output of the simulator, then

$$\langle \mathbf{1}_\omega \rangle(\omega^\infty) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{1}_\omega\left(T^n \omega^\infty\right) \tag{6}$$

If the mobility model possess ergodic properties with respect $\{\mathbf{1}_\omega(\cdot)\}_{\omega\in\Omega}$, the simulator will a.e. produce a sequence ω^∞ for which the limit (6) exists. We are then justified to approximate the limit $\langle \mathbf{1}_\omega \rangle(\omega^\infty)$ with $\langle \mathbf{1}_\omega \rangle_N(\omega^\infty)$, for some sufficiently large N. Specifically, for any $\varepsilon > 0$, there exists some N_0, such that for all $N > N_0$, $\left|\langle \mathbf{1}_\omega \rangle - \langle \mathbf{1}_\omega \rangle_N\right| \leq \varepsilon$. A *necessary* requirement for a mobility model to possess a stationary location distribution is that it possess ergodic properties with respect to $\{\mathbf{1}_\omega(\cdot)\}_{\omega\in\Omega}$.

More generally, we should consider the limiting properties of any measurable event A. This leads us to consider the set of all indicator functions $\{\mathbf{1}_A(\cdot)\}_{A\in\mathcal{F}(\Omega^\infty)}$.

Observation 1. *If $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ has ergodic properties with respect to $\{\mathbf{1}_A(\cdot)\}_{A\in\mathcal{F}(\Omega^\infty)}$, then[7] it possess ergodic properties with respect to all bounded functions, and the limit*

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \mu\left(T^{-n} A\right) = E_\mu\left[\langle \mathbf{1}_A \rangle(\omega^\infty)\right] = \overline{\mu}(A)$$

exists a.e. for all $A \in \mathcal{F}(\Omega^\infty)$. Moreover, $\overline{\mu}(\cdot)$ is a unique stationary probability measure *with respect to T.*

The dynamical system $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ is *asymptotically mean stationary*[8] (AMS) if it possess ergodic properties with respect to $\{\mathbf{1}_A(\cdot)\}_{A\in\mathcal{F}(\Omega^\infty)}$. The measure μ induced by the mobility model is called *asymptotically mean stationary*, with *stationary mean* $\overline{\mu}$.

Observation 1 shifts our attention from mobility models with stationary node locations, *c.f.* [16, 24–31], to the general class of AMS mobility models. Since AMS mobility models possess ergodic properties with respect to all bounded functions, they are useful for testing asymptotic properties of protocols. The following lemma is necessary for the proof of Theorem 1 and is potentially of independent interest.

Lemma 1 (Variable Length Encoding). *A variable length encoding of an asymptotically mean stationary process produces an asymptotically mean stationary process as the output.*

Example 3 (GRWPMM). The Random Waypoint Mobility Model (RWPMM) is common in the literature, *c.f.* [30]. Consider a generalize discrete-space version (GRWPMM) where the *waypoint selection process* is any AMS random process[9]. Assume $\mathcal{S}$ is a $M_j \times M_k$ regular lattice.

A nodes movement is statistically independent of all other nodes, so it is sufficient to describe the movement of a single node. Let $S_n = (j_n, k_n)$ denote the location of the node at time n, where $j_n \in \{1, 2, \ldots, M_j\}$ and $k_n \in \{1, 2, \ldots, M_k\}$. Let $\mathbf{W}^\infty = W_0, W_1, \ldots$ denote the sequence of waypoints and let $W_w = (j_w, k_w)$ denote the current waypoint. At each time shift T, the node moves toward the current waypoint using the rules

$$S_{n+1} = \begin{cases} (j+1, k), & \text{if } s_n = (j,k),\ j < j_w \\ (j-1, k), & \text{if } s_n = (j,k),\ j > j_w \\ (j, k+1), & \text{if } s_n = (j,k),\ j = j_w,\ k < k_w \\ (j, k-1), & \text{if } s_n = (j,k),\ j = j_w,\ k > k_w \end{cases}$$

When the node reaches the waypoint (j_w, k_w), the process is repeated with the next waypoint W_{w+1}.

Theorem 1 (GRWPMM). *The Generalized Random Waypoint Mobility Model is an asymptotically mean stationary mobility model.*

Corollary 1.1 (RWPMM). *The Random Walk and Random Waypoint are asymptotically mean stationary mobility models.*

Sample averages (3) for the GRWPMM converge a.e., thus ensuring the existence of a unique *stationary*[10] mean $\overline{\mu}$. The RWPMM

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[3] Measurable function: Let $\mathcal{F}(\mathfrak{R}^\infty)$ be a σ-field of 'routing events.' Then $f^{-1}(B) \in \mathcal{F}(\Omega^\infty)$, $\forall B \in \mathcal{F}(\mathfrak{R}^\infty)$.

[4] Convergence: The limit of a sequence of measurements f_n exists iff $\limsup_{n\to\infty} f_n = \liminf_{n\to\infty} f_n$, *c.f.* [18, Page 70].

[5] Almost everywhere: let $F = \{\omega^\infty : \langle f \rangle(\omega^\infty) \text{ exists}\}$, then $\mu(F) = 1$. Ergodic Property: [18, Chapter 6].

[6] If the network consisted of only one node, then the *limiting relative frequency* of each $\omega \in \Omega$ is the relative frequency of the nodes location.

[7] Combine [18, Lemmas 6.2.1 and 6.2.2 & Corollaries 6.2.1 and 6.2.2].

[8] AMS processes: [23], and [18, Page 131].

[9] The waypoints in the RWPMM are selected i.i.d. (uniform) from $\mathcal{S}$.

[10] Sometimes called a *steady-state* distribution, *c.f.* [29].

is AMS with respect to T, but is not stationary with respect to T – it is variable length stationary. The GRWPMM forms a wide class random waypoint mobility models that includes the Random Walk Mobility Model (RWMM).

3. PERFECT SIMULATION

The existence of stationary means for node locations, and transients caused by convergence to these stationary means, has lead to the following works [24, 28–30]. Navidi *et. al.* [30] suggests that if $\overline{\mu}$ is not known, one should run the simulation for a *sufficiently long* time before taking protocol measurements. Intuitively, this 'warm up' allows the node location distribution, and any function thereof, to converge to something close to $\overline{\mu}$ and thus removes 'inconsistencies' caused by finite sample averages of transient distributions. Conversely if the stationary mean is known, Navidi *et. al.* [30], Yoon *et. al.* [29, Section 5], and Boudec *et. al.* [26], suggest one should 'create' a *stationary* mobility model by selecting initial node locations[11] values from their respective stationary means.

Consider an AMS mobility model where the stationary mean $\overline{\mu}(\cdot)$ is not know. Warm the mobility model up for N_w time steps and sample for N_s steps. Repeat for a large number of trials.

Lemma 2 (Warm Up). *Fix $c = N_W - N_S$ constant. Then if $N_w \to \infty$, the sample average of any event $A \in \mathcal{F}(\Omega^\infty)$ is equal to its stationary mean $\overline{\mu}(A)$.*

The warm up process creates a 'new' dynamical system $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \overline{\mu}, T)$ where the measure μ induced by the mobility model is replaced by $\overline{\mu}$. Conversely, suppose the stationary mean $\overline{\mu}(\cdot)$ is known. Select the initial distribution equal to the stationary mean $\overline{\mu}$ and obtain $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \overline{\mu}, T)$. A *perfect simulation*, or *sound mobility model*, is the dynamical system $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \overline{\mu}, T)$ obtained from $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ by either the warm up or randomized start process.

The mean stationary measure $\overline{\mu}$ is T-*invariant*, $\overline{\mu}(A) = \overline{\mu}(T^{-1}A)$, $\forall A \in \mathcal{F}(\Omega^\infty)$. This, together with the concept of a perfect simulation together, motivate a formal definition of a stationary mobility model. The Kolmogorov measure μ induced by the mobility model is T-*invariant* (or stationary) if $\mu(A) = \mu(T^{-1}A)$, $\forall A \in \mathcal{F}(\Omega^\infty)$.

The transient nature of AMS mobility models, *c.f.* [28–30], is considered 'harmful' to reliable, reproducible, results.

Observation 2. *The limiting sample average $\langle f \rangle(\omega^\infty)$ of an AMS mobility model $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ is equal to that of its associated stationary system $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \overline{\mu}, T)$.*

Potential errors resulting from the transient nature of the mobility model can be neutralized by choosing N sufficiently large. If N is limited in size by simulation complexity, convergence of sample averages cannot be guaranteed and one should consider $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \overline{\mu}, T)$.

Let $F = \{\omega^\infty : \langle f \rangle(\omega^\infty) \text{ exists}\}$, then $\omega^\infty \in F \Leftrightarrow \omega^\infty \in T^{-1}F$ and F is an *invariant set* under T. I.e. it does not change under T. More generally, $A \in \mathcal{F}(\Omega^\infty)$ is T-*invariant* if $T^{-1}A = A$. A mobility model is *ergodic*[12] if $\mu(A)$ is 0 or 1 for all invariant sets.

In ergodic systems, sample averages converge a.e. to a constant. Thus one sequence $\omega^\infty \in A$ where $A = T^{-1}$ and $\mu(A) = 1$, is sufficient to describe asymptotic behavior of protocols.

Lemma 3. *The Random Walk Mobility Model is ergodic. The Random Waypoint mobility model is variable length ergodic*[13]

[11] Including other parameters of interest, e.g. speed.

[12] Ergodicity: [18, Chapter 6.7].

[13] We conject that the Random Waypoint Mobility Model is ergodic. The final version of the paper will have this conjecture resolved. See [32].

4. ROUTING AS A RANDOM PROCESS

We now demonstrate that any routing protocol that imposes a deterministic mapping of the node position sequence space to some route sequence space, ensures the resulting route process inherits the AMS properties of the mobility model. It then follows that any AMS mobility model satisfies the Asymptotic Equipartition (AEP) Theorem at the network layer. Well known noiseless source coding techniques are then used to proved our main result of an information theoretic lower bound on network-layer control overhead.

Consider a network of $|\mathcal{V}|$ nodes with one source v_s and one sink v_t. Since v_s only communicates with v_t, ω^∞ is the only input driving the routing process. Let the movement of nodes be defined by some AMS mobility model and let $\{Routes\}$ be the set of 'route decisions' connecting v_s and v_t. For example, the set $\{Routes\}$ might consist of all possible paths connecting v_s to v_t. Label each element of $\{Routes\}$ with an element of $\mathfrak{R} = \{r_1, r_2, \ldots, r_{|\mathfrak{R}|}\} \subset \mathbb{R}$. Define $f: \Omega^\infty \to \mathfrak{R}$ such that for each $\omega^\infty \in \Omega^\infty$, $f(\omega^\infty) = r$ represents the 'routing decision' of the routing protocol. For example, the route $f(\omega^\infty) = r$ may be a 'least cost route' that was chosen via an optimization problem, *c.f.* [33]. Then f defines a random process of route decisions

$$\mathbf{R}^\infty = \{R_n\}_{n=0}^\infty = \{F(\omega^\infty)_n\}_{n=0}^\infty = \{f(T^n\omega^\infty)\}_{n=0}^\infty \qquad (7)$$

$F: \Omega^\infty \to \mathfrak{R}^\infty$ is called a *stationary coder* if $F(T_\Omega\omega^\infty) = T_\mathfrak{R} F(\mathbf{r}^\infty)$, where $T_\mathfrak{R}$ is the left shift on the route space $\mathfrak{R}$. $T_\mathfrak{R}(r_0, r_1, r_2, \ldots) = r_1, r_2, \ldots$. The theory of stationary coders can be found in any introductory text on Ergodic Theory, *c.f.* [17, 18].

We define a *stationary routing protocol* to be any routing protocol that generates a stationary coder $F: \Omega^\infty \to \mathfrak{R}^\infty$. If F is a stationary routing protocol, then $\mathbf{R}^\infty$ is a *stationary encoding* of μ. The Kolmogorov measure η of the routing process induced by the mobility model μ and transported by the stationary routing protocol F, is given by

$$\eta(A) = \mu(F^{-1}A), \ \forall A \in \mathcal{F}(\mathfrak{R}^\infty) \qquad (8)$$

The dynamical system $(\mathfrak{R}^\infty, \mathcal{F}(\mathfrak{R}^\infty), \eta, T_\mathfrak{R})$ describes the routing process in the same way as $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ describes the mobility model. We define AMS, stationary and ergodic routing processes in exactly the same way as we did for $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$.

Observation 3. *A stationary routing protocol operating on an AMS mobility model produces an AMS routing process. A stationary routing protocol operating on a stationary mobility model produces a stationary routing process. A stationary routing protocol operating on an AMS ergodic mobility model produces an AMS ergodic routing process process. In short, a stationary routing protocol inherits or preserves the statistical properties of the mobility model.*

In general, a routing protocol will only depend on a finite number of inputs, $f(\omega^\infty) = f(\widehat{\omega}^\infty)$, where $\omega_{-m}^d = \widehat{\omega}_{-m}^d$. In such cases, f is called a *sliding block code* with memory m and delay d. The theory of sliding block codes is developed in [34, Page 169] and [35, Page 644]. If $d > 0$, then the routing system is non-causal. If $m > 0$ then the routing protocol has memory, i.e. the current routing decision will depend on the current and past m node locations. A protocol that depends only on the current node locations is called *instantaneous*. The least cost routing protocol (via Dijkstra's Algorithm) is an instantaneous stationary coding of ω^∞.

Lemma 4. *If a mobility model possess a stationary node location distribution and the routing protocol is instantaneous, then the mobility model must be asymptotically mean stationary.*

Consider a dynamical system $(\Omega^\infty, \mathcal{F}(\Omega^\infty), \mu, T)$ and coding F that describe the mobility model and routing protocol respectively. The *N-th order route entropy* is

$$\begin{aligned} H_\eta^{(N)}(R) &= \frac{1}{N} H_\mu\left(\{fT^n(\omega^\infty)\}_{n=0}^{N-1}\right) \\ &= -\frac{1}{N} \sum_{\omega_0^{N-1} \in \Omega^N} \mu\left([\omega_0^{N-1}]\right) \log_2 \mu\left([\omega_0^{N-1}]\right) \end{aligned}$$

and the *entropy rate* is

$$\overline{H}_\eta(R) = \limsup_{N\to\infty} H_\eta^{(N)}(R)$$

Theorem 2 (Control Information Lower Bound). *Consider an AMS model together with a stationary routing protocol. Then $\overline{H}_\eta(R)$ is a lower bound on routing overhead.*

5. CONCLUSION

The entropy rate of the AMS route process, Theorem 2, provides an information theoretic lower bound on network layer control information for any AMS mobility model. This control information must be transmitted as overhead in some form to ensure reliable routing decisions at the network layer. The result rests on the application several results from Ergodic and Information Theory. We define a class of AMS mobility models, where the asymptotic stationarity of measurable events is guaranteed. AMS mobility models are most useful for the simulation based analysis of networks, since time-averaged results converge almost everywhere. We define and prove the AMS property for a Generalized Random Waypoint Mobility Model – the Random Walk and Random Waypoint Mobility Models being special cases. The proof rests on a Lemma that demonstrates the AMS properties of a variable-length encoding of an AMS process. Perfect simulation, or sound mobility models, are discussed and shown to be the stationary system associated with an AMS mobility model. We define Ergodic Mobility Models and prove ergodicity, and variable length ergodicity, for the Random Walk and Random Waypoint Mobility Models respectively.

A determinsitic routing protocol was shown to induce a stationary coding of the underlying mobility models. The resulting route decision process was defined and shown to inherit AMS, stationary and ergodic properties from the mobility model. Gray's [23] extension of the Asymptotic Equipartition Theorem (AEP) to one-sided AMS process shows the AEP holds at the network layer when the mobility models is AMS and the routing protocol is stationary. A well known source coding result is then used to lower bound control overhead.

This work concerns mobility models operating on discrete-finite state-spaces, however, each result may be extended to more general state-space models.

Appendix.

Remark 1 *(Kolmogorov Extension Theorem).* The alphabet of node movement is finite therefore, the Kolmogorov Extension Theorem holds if (9) is satisfied [17]

$$\mu_0^{N-1}\left(\omega_0^{N-1}\right) = \sum_{\omega_N\in\Omega} \mu_0^N\left(\omega_0^N\right), \quad \omega_0^{N-1}\in\Omega^N \tag{9}$$

Consider ω_0^{N-1} with probability $\mu_0^{N-1}\left(\omega_0^{N-1}\right)$ given by (1). Let $A = \left\{\widehat{\omega}_0^N \in \Omega^{N+1} \,:\, \widehat{\omega}_n = \omega_n,\ n = 0,1,2,\ldots,N-1\right\}$. If the state-space is closed[14] the sample path ω_0^N must 'move' into A and (9) must hold.

Proof: 1 *(Outline Theorem 1).* Let $\mathbf{W}^\infty = W_0, W_1, W_2, \ldots$ denote the waypoint process. The waypoint process in the RWPMM is i.i.d., therefore Corollary 1.1 follows immediately. Let $\mathbf{E}^\infty = E_0, E_1, \ldots$ be the (epoch) process recording the start and end points to each epoch. $\mathbf{E}^\infty$ is a memoryless coding of $\mathbf{W}^\infty$ of delay one with alphabet $\mathcal{S}^2$. $f(W_0,W_1,W_2,\ldots) = (W_0,W_1)$, and $\mathbf{E}^\infty = \left\{E_n\right\}_{n=0}^\infty = \left\{f(T^n\mathbf{w}^\infty)\right\}_{n=0}^\infty$. A stationary coding of an AMS process yields and AMS process [34, Lemma 4.2.3]. Therefore, $\mathbf{E}^\infty$ is AMS.

The node moves between waypoints in a deterministic fashion. Thus for each pair $(s,\hat{s}) \in \mathcal{S}\times\mathcal{S}$, there exists a vector of 'transition' positions describing the nodes position over the epoch. The length of this vector depends on the distance $L(s,\hat{s})$ between s and $\widehat{s}$. The Node Location Process $\mathbf{X}^\infty$ is therefore a fixed to variable length coding of $\mathbf{E}^\infty$. By Lemma 1, $\mathbf{X}^\infty$ is AMS. ■

[14] I.e. A node may not move out of $\mathcal{S}$.

Proof: 2 *(Outline Lemma 1).* Suppose μ is AMS with respect to T^* with stationary mean $\overline{\mu}^*$. Then $\overline{\mu}^*$ asymptotically dominates μ, $\overline{\mu}^* \gg \mu$. By definition $\overline{\mu}^*$ is T^*-invariant and therefore AMS with respect to T [23, Example 6]. Where $T^* = T^{L(s,\hat{s})}$ and $L(s,\hat{s}) < \infty$, $\forall \hat{s}, s \in \mathcal{S}$. Then $\overline{\mu} \gg \overline{\mu}^*$ for some stationary mean $\overline{\mu}$. Since $\overline{\mu} \gg \overline{\mu}^*$, then $\overline{\mu} \gg \mu$ and by [23, Theorems 3 & 4] μ is AMS with respect to T – its stationary mean is $\overline{\mu}$. ■

Proof: 3 *(Warm Up Lemma 4).* The sample average of any event $A \in \mathcal{F}\left(\Omega^\infty\right)$ with warm up is given by

$$\left\langle \mathbf{1}_A \right\rangle_{N_w}^{N_w+N_s}(\omega^\infty) = \frac{1}{N_s}\sum_{n=N_w}^{N_w+N_s-1} \mathbf{1}_A(T^n\omega^\infty) \tag{10}$$

Let $N_w \to \infty$, $n = j + N_w$ and hold $\overline{N} = N_w - N_s$ constant, then (10) becomes

$$\begin{aligned}
\left\langle \mathbf{1}_A \right\rangle_{N_w}^{N_w+N_s}(\omega^\infty) &= \lim_{N_w\to\infty} \frac{1}{N_w-\overline{N}} \sum_{j=0}^{N_w-\overline{N}} \mathbf{1}_A(T^{j+N_w}\omega^\infty) \\
&= \lim_{N_w\to\infty} \frac{1}{N_w} \sum_{j=0}^{N_w} \mathbf{1}_A T^j\left(T^{N_w}\omega^\infty\right) \\
&= \left\langle \mathbf{1}_A \right\rangle\left(T^{N_w}\omega^\infty\right) \\
&= \left\langle \mathbf{1}_A \right\rangle\left(\omega^\infty\right)
\end{aligned}$$

which gives the expectation

$$E_\mu\left[\left\langle \mathbf{1}_A \right\rangle_{N_w}^{N_w+N_s}(\omega^\infty)\right] = \overline{\mu}(A)$$

■

Proof: 4 *(Outline Lemma 4).* The σ-field $\mathcal{F}$ induced by an instantaneous routing protocol is generated by cylinder sets that extend only over one time instance. That is, $[\omega_m^m]$ for $m \geq 0$. Since $\mathcal{F} \subseteq \mathcal{F}\left(\Omega\right)$, where $\mathcal{F}\left(\Omega\right)$ is the σ-field generated by $\{\omega\}_{\omega\in\Omega}$, the mobility model is AMS if it has a stationary node location distribution. ■

Proof: 5 *(Outline Lemma 3).* Ergodicity of the Random Walk Mobility Model follows directly from the irreducibility of its transition matrix. I.e. the Random Walk Mobility Model is ergodic iff its transition matrix is irreducible. Variable length ergodicity of the Random Waypoint Mobility Model follows directly from the fact that it is a variable-length encoding of a AMS ergodic source (the epoch process). The epoch process is AMS and ergodic since it is a stationary coding of a stationary and ergodic process (the waypoint process is i.i.d.). ■

Proof: 6 *(Outline Theorem 2).* The entropy rate of a one-sided finite alphabet AMS process is well known to be a lower bound for compressibility, *c.f.* [34]. The theorem then follows if the route process generated by a stationary protocol on a AMS mobility model is a one-sided finite alphabet AMS process. This is true by Observation 3. ■

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