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Anderson delocalization in one dimensional μ or ε-near-zero metamaterial stacks and other dispersion effects on localization

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Abstract: We have carried out a comprehensive study of dispersion and absorption effects on Anderson localization in one-dimensional metamaterial stacks and have shown that the field is delocalized in μ or ε-near-zero media at normal incidence.

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1. Introduction

Anderson localization is a fundamental and general phenomenon of wave propagation in random media [1]. Despite considerable efforts, the theoretical framework of Anderson localization in higher dimensions ($D > 1$) is far from complete, especially in the case of classical waves where the effects of absorption, gain, and polarization are of significance. In contrast, the one-dimensional case ($D = 1$) has been studied extensively for both quantum mechanical and classical waves, and it has been proved that scalar waves are localized for every wavelength and for any disorder [2].

Recently, we have studied localization in the presence of metamaterials and have shown that in stacks of alternating layers of normal materials and metamaterials, localization is suppressed at long wavelengths for disorder in the dielectric permittivity of the layers. That paper [3], however, did not taken into accounts the effects of dispersion. This contribution extends our previous work and presents a comprehensive theoretical and computational study of the effects of dispersion and absorption on localization properties in the the presence of metamaterials.

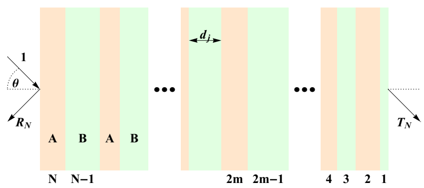

Fig. 1. The geometry of the problem

2. Model and the Theoretical Consideration

We consider one-dimensional stacks composed of homogeneous metamaterial layers only or normal material layers only, as well as stacks of alternating layers of metamaterials A and normal material B (mixed stacks) (see Fig.1). In all cases, the dielectric permittivity and the magnetic permeability are [4]:

$$\varepsilon(\omega) = 1 - \frac{\omega_{ep}^2 - \omega_{eo}^2}{\omega^2 - \omega_{eo}^2 + i\gamma\omega}, \qquad \mu(\omega) = 1 - \frac{\omega_{mp}^2 - \omega_{mo}^2}{\omega^2 - \omega_{mo}^2 + i\gamma\omega} \tag{1}$$

where we take $f_{mp} = 10.95$GHz, $f_{mo} = 10.05$GHz, $f_{ep} = 12.8$GHz, $f_{eo} = 10.3$GHz and $\gamma = 10$MHz ($f = \omega/2\pi$), and where $\omega = 2\pi f$. For the given model, the refractive index is negative in the range of frequencies $10.40GHz <$

$f < 10.95GHz$ (see the inset of Fig.2). At frequency $f = 10.95GHz$, the magnetic permittivity changes sign (from negative to positive) when $\gamma = 0$, and so the metamaterial changes its character from double negative to single negative. Below, we show that such changes have a profound effect on Anderson localization. The thicknesses of the layers are given by $d = 0.003m$, which is comparable to the experimental set up in Ref.[4]. The disorder is introduced by randomizing the resonance frequencies ω_{eo} or ω_{fo}. Arising from this is an exponential decay of the field amplitudes, induced by disorder, with a decay length known as the localization length l which is calculated from

$$l_T = -\frac{2N}{\langle \ln |T_N|^2 \rangle}, \tag{2}$$

where T_N is the stack amplitude transmission coefficient that is computed by recurrence relations [3]. Extending our asymptotic analysis [5] to accommodate dispersion and absorption, we may express the localization length according to

$$\frac{1}{l} = -\mathrm{Re}\,\langle \ln t \rangle - \mathrm{Re}\frac{\langle r \rangle^2}{1 - \langle t^2 \rangle}, \quad \frac{1}{l} = -\mathrm{Re}\,\langle \ln t \rangle - \frac{|\langle r \rangle^2| + \mathrm{Re}\left(\langle r \rangle^2 \langle t^2 \rangle^*\right)}{1 - |\langle t^2 \rangle|^2}, \tag{3}$$

where the left and right expressions respectively correspond to metamaterial and mixed stacks. Here, $\langle \ln t \rangle$, $\langle r \rangle$ and $\langle t^2 \rangle$ are ensemble averages of the logarithm of the transmission, the reflection and the square of the transmission for a single layer, where the single layer is embedded in a homogeneous medium with average parameters $\overline{\varepsilon}$ and $\overline{\mu}$.

3. Results

As a representative example of our results, we plot in Fig. 2 the localization length as a function of frequency for normal incidence and for a metamaterial stack with disordered dielectric permittivity, where the resonance frequency ω_{eo} is disordered according to a uniform distribution within 0.5% of its average value. The upper curve corresponds to the case of zero absorption. The localization length increases as the frequency approaches the value f_{mp} at which μ vanishes. We show that at the frequency f_{mp}, the Furstenberg theorem is not applicable and that the field is delocalized

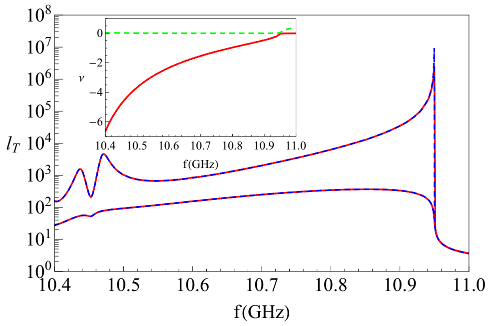

Fig. 2. The localization length l versus frequency f at normal incidence $\theta_a = 0$ for a metamaterial stack without absorbtion (top curve) and at the presence of absorbtion (bottom curve). The red curves represent the numerical calculations and the blue curves are the theoretical predictions (3) and they are in excellent agreement. Inset: the real part of the metamaterial refractive index layer (red curve) and the imaginary part (green line).

even in one dimension. We have calculated the transfer matrix $\mathcal{T}_n$ of a layer and have shown that $\mathcal{T}_n$ has a single eigenvector $\mathbf{v} = [1 \quad 1]^T$, which is independent of the disorder of ε at $f = f_{mp}$. These findings violate the applicability of the Furstenberg theorem, which requires that the transfer matrices should contain at least two elements with no common eigenvectors [2]. The absorption has a strong effect on localization (bottom curve) and must be reduced in order to observe the delocalization.

In summary, we have carried out a comprehensive study of the dispersion and absorption effects on the Anderson localization. The presentation will show a number of examples that illustrate the effects of polarization on localization at the presence of dispersive metamaterials and the suppression of the localization at the presence of the dispersion.

References

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