PARITY OF NEUTRAL PION

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The paper discusses the parity of the neutral pion. It is shown that the recent determination of the parity of the neutral pion using the double Dalitz decay $\pi^0 \rightarrow e^+e^-e^+e^-$ is compatible with the mixed-parity nature of the neutral pion predicted by a recent composite model.

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1. Introduction

Direct determination of the parity of the neutral pion presents a difficult experimental challenge. Since 1950 when Yang¹ indicated that the parity could in principle be determined by measuring the polarizations of the two photons in the decay $\pi^0 \rightarrow \gamma\gamma$, no method has yet been developed to measure the polarization of the high-energy (≈ 70 MeV) photons involved in the decay. Yang showed that the polarizations of the two photons are perpendicular or parallel depending whether the π^0 is pseudoscalar or scalar, respectively. However, it was soon pointed out² that the double Dalitz decay $\pi^0 \rightarrow e^+e^-e^+e^-$, which proceeds through an intermediate state of two virtual photons, is sensitive to the parity of the neutral pion, since the plane of a Dalitz pair is correlated with the polarization of the associated virtual photon.

Recently, a new determination of the parity of the neutral pion employing the double Dalitz decay technique has been carried out.³ Analysis of this experiment concluded that the neutral pion had parity $P_0 = -1$ and placed a limit on scalar contributions to the $\pi^0 \rightarrow e^+e^-e^+e^-$ decay amplitude of less than 3.3% assuming CPT conservation.

In Sec. 2 we shall examine the parity of the neutral pion within the framework of the Standard Model^{4,5} (SM) of particle physics. It will be seen that the experiment of Abouzaid *et al.*³ is compatible with the parity of the neutral pion predicted by the

SM. In Sec. 3 we shall discuss the parity of the neutral pion within the framework of the Composite Generation Model^{6,7} (CGM) which is an alternative model to the SM. It will be shown that the experiment of Abouzaid *et al.*³ is also compatible with the mixed-parity nature of the π^0 arising in the CGM. Section 4 states the conclusions.

2. Parity of Neutral Pion in the Standard Model

In the SM,^{4,5} the parity of the neutral pion is assumed to be $P_0 = -1$. In 1954 Chinowsky and Steinberger established,⁸ within the theoretical framework of the time, that the negative pion was pseudoscalar and in 1955 they determined⁹ that the branching ratio between the capture reactions in deuterium: $\pi^- + D \rightarrow 2n + \pi^0$ and $\pi^- + D \rightarrow 2n + \gamma$, was $< 0.1\%$ and showed that this provided strong evidence that the neutral pion was also pseudoscalar. This was prior to the quark model^{10–12} of mesons and baryons so that in the analyses of the experiments, the pions, the proton and the neutron were each assumed to be fundamental particles with no substructure.

Following the adoption of the quark model as part of the SM, the parity of the neutral pion remained accepted as $P_0 = -1$: in the quark model, the π^0 was proposed to be the linear combination

$$\pi^0 = ([u\bar{u}] - [d\bar{d}])/\sqrt{2}, \quad (1)$$

where u and d represent the up quark and down quark, respectively, and $\bar{u}$ and $\bar{d}$ represent the corresponding antiparticles. Assuming that the fundamental quarks have parity $P_q = +1$ while their fundamental antiquarks have parity $P_{\bar{q}} = -1$, the neutral pion has parity $P_0 = -1$.

Early attempts^{13,14} to determine directly the parity of the neutral pion via its double Dalitz decay $\pi^0 \rightarrow e^+e^-e^+e^-$ were only sufficiently accurate to confirm that the neutral pion was a pseudoscalar ($P_0 = -1$) rather than a scalar ($P_0 = +1$) particle. An underlying assumption in the analyses of these experiments was that the neutral pion had a definite intrinsic parity $P_0 = +1$ or $P_0 = -1$.

Recently, a new precise measurement of the properties of the double Dalitz decay of the neutral pion has been reported.³ This experiment used 30511 candidate decays, collected from the decay of the long-lived neutral kaon into three neutral pions, $K_L^0 \rightarrow \pi^0\pi^0\pi^0$. Analysis of the data confirmed the negative parity of the neutral pion with much higher statistical significance than previous results and also placed stringent limits on possible scalar contributions to the decay. Assuming CPT conservation, the analysis found that the scalar contributions to the decay amplitude were less than 3.3%, which is consistent with the prediction of the SM (0%).

3. Parity of Neutral Pion in Composite Generation Model

The Composite Generation Model^{6,7} (CGM) was developed from the classification scheme for leptons and quarks associated with the Generation Model¹⁵⁻¹⁷ (GM). The GM differs from the SM in that it treats quark mixing in hadronic weak interaction processes differently from the method introduced by Cabibbo¹⁸ and employed in the SM in order to preserve the universality of the weak interaction. Essentially, in the GM, the quark mixing is placed in the quark states (wavefunctions) rather than in the weak interactions. This allows a unified and simpler classification of both the leptons and quarks in terms of only three additive quantum numbers, each of which is conserved in all interactions.

The three conserved additive quantum numbers of the GM: electric charge (Q), particle number (p) and generation quantum number (g), are sufficient to describe all the observed transition amplitudes for both hadronic and leptonic processes, provided each “force” particle, mediating the various interactions, has $p = g = 0$. This unified classification scheme for leptons and quarks indicated that leptons and quarks are intimately related and led to the development of a composite version of the GM: the CGM.

In the GM, physical quarks are considered to be mixed quark states, rather than pure quark states. As is conventional, the mixing is restricted to the down-type quarks (down (d), strange (s) and bottom (b)), although in principle one could equally well describe the weak interaction processes in terms of mixing of the corresponding up-type quarks (up (u), charmed (c) and top (t)).

For simplicity, the following discussion will be restricted to the first two generations of quarks, involving only the original Cabibbo mixing angle θ_c , which has a value $\approx 13^\circ$. The extension to three generations is straightforward¹⁹ but for our purposes inclusion of the third generation b -quark in the mixing would have only a negligible impact on the results.

Thus in the GM, hadrons are considered to be composed of mixed-quark states, i.e. a proton essentially consists of two u -quarks and a d' -quark, rather than two u -quarks and a d -quark as in the SM. Such mixed-quark states in hadrons readily account for the reduced transition probabilities relative to leptonic transition probabilities observed in weak interaction processes involving quarks, as described by the alternative quark-mixing mechanism introduced by Cabibbo.

The existence of mixed-quark states in hadrons has several consequences.¹⁷ One important consequence is that mixed-quark states may have mixed parity.

In the CGM the constituent particles of quarks are fermions, which we have called “rishons,” following the introduction of such particles into an early composite model of leptons and quarks by Harari.²⁰ If one adopts the simple convention that all rishons have positive parity, and all their antiparticles have negative parity, one finds that the d -quark and the s -quark have opposite intrinsic parity, according to the proposed structures^{6,7} of these quarks in the CGM: the d -quark consists of two rishons and one antirishon ($P_d = -1$), while the s -quark consists of three

rishons and two antirishons ($P_s = +1$). The u -quark consists of two rishons and one antirishon so that $P_u = -1$, and the antiparticles of these three quarks have the corresponding opposite parities: $P_{\bar{d}} = +1$, $P_{\bar{s}} = -1$ and $P_{\bar{u}} = +1$.

In the CGM the π^0 meson has, in the two generation approximation, the structure

$$\pi^0 = ([u\bar{u}] - [d'\bar{d}'])/\sqrt{2}, \quad (2)$$

where

$$d' = d \cos \theta_c + s \sin \theta_c \quad (3)$$

and

$$\bar{d}' = \bar{d} \cos \theta_c + \bar{s} \sin \theta_c. \quad (4)$$

Thus

$$\pi^0 = ([u\bar{u}] - [d\bar{d}] \cos^2 \theta_c - [s\bar{s}] \sin^2 \theta_c - [d\bar{s}] \sin \theta_c \cos \theta_c - [s\bar{d}] \sin \theta_c \cos \theta_c)/\sqrt{2}. \quad (5)$$

For the proposed rishon parities the u - and d -quarks have negative parity, the s -quark has positive parity and their antiparticles have the corresponding opposite parities. Furthermore, defining the phase of the charge-conjugation operator (C) by the relation: $C|q\rangle = +|\bar{q}\rangle$, it is seen that the first three components of Eq. (5) are eigenstates of CP with eigenvalue $CP = -1$, while the remaining two components, $[d\bar{s}]$ and $[s\bar{d}]$, with amplitude $\sin \theta_c \cos \theta_c/\sqrt{2}$ are not individually eigenstates of CP. However, taken together, the state $([d\bar{s}] + [s\bar{d}])$ is an eigenstate of CP with eigenvalue $CP = +1$.

It should be noted that each of the components with eigenvalue $CP = -1$ also has eigenvalues $C = +1$ and $P = -1$, while the state with eigenvalue $CP = +1$ also has eigenvalues $C = +1$ and $P = +1$, so that the pseudoscalar or scalar nature of the neutral pion is determined by its existence in either $CP = -1$ or $CP = +1$ eigenstates, respectively.

Taking the square of the product of the amplitudes comprising the $CP = +1$ eigenstates to be the joint probability²¹ of these two states existing together simultaneously, one can calculate that this probability is $\frac{1}{4} \sin^4 \theta_c \cos^4 \theta_c = 5.84 \times 10^{-4}$, using $\cos \theta_c = 0.9742$.²²

Let us now consider the recent experiment of Abouzaid *et al.*³ Analysis of the data found that the scalar contributions to the decay *amplitude* were less than 3.3%. In the CGM, since CP is conserved in the electromagnetic decay, the occurrence of scalar contributions to the decay amplitude arises entirely from the joint probability *amplitude* of the $CP = +1$ eigenstate component of the neutral pion. This is given by $\frac{1}{2} \sin^2 \theta_c \cos^2 \theta_c = 2.42 \times 10^{-2}$. Normalizing to the total decay amplitude, one finds that the scalar contribution to the decay amplitude is $\approx 2.5\%$, which is compatible with the experimental result of less than 3.3%.

4. Conclusion

It has been shown that the recent determination³ of the parity of the neutral pion using the double Dalitz decay $\pi^0 \rightarrow e^+e^-e^+e^-$ is compatible with the mixed-parity nature of the neutral pion predicted by the CGM. The CGM predicts a scalar contribution to the neutral pion decay amplitude of $\approx 2.5\%$. The experimental result is a scalar contribution of less than 3.3% , which is of the same order of magnitude as the CGM prediction, suggesting that further experimentation may determine a non-zero scalar contribution to the decay amplitude.

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